Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.7-d-trig-^m-a+b-c-sec-^n-^p

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July 17, 2021 Compiled on July 17, 2021 at 8:52pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [470]. This is test number [126].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.57 (468)	% 0.43 (2)
Mathematica	% 90.21 (424)	% 9.79 (46)
Maple	% 91.70 (431)	% 8.30 (39)
Maxima	% 60.85 (286)	% 39.15 (184)
Fricas	% 85.32 (401)	% 14.68 (69)
Sympy	% 4.47 (21)	% 95.53 (449)
Giac	% 34.68 (163)	% 65.32 (307)
Mupad	% 51.70 (243)	% 48.30 (227)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

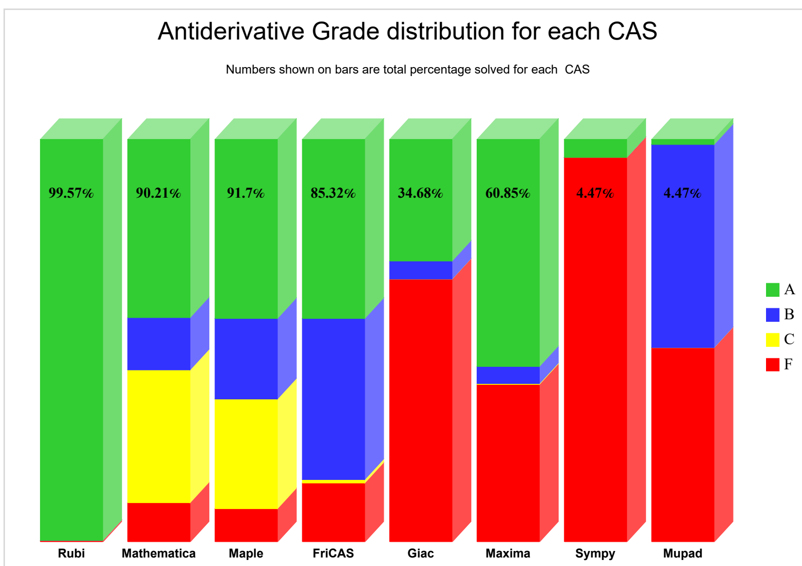
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

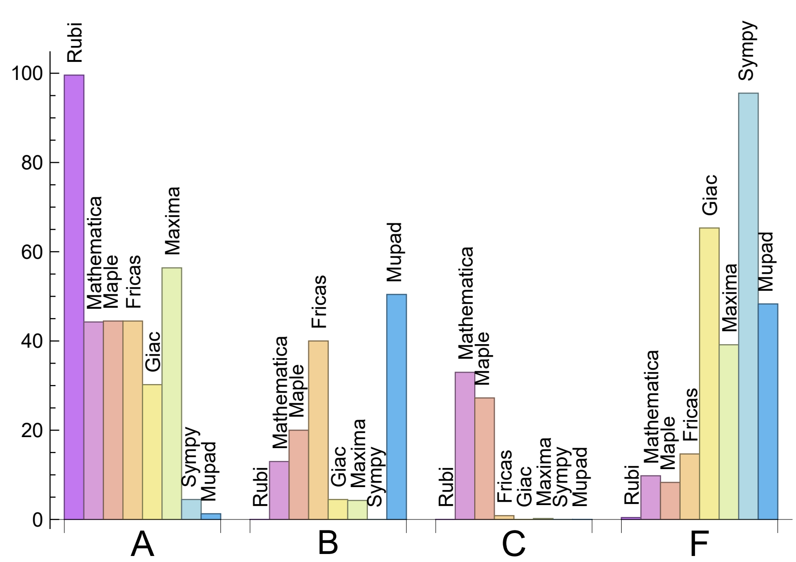
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.57	0.00	0.00	0.43
Mathematica	44.26	12.98	32.98	9.79
Maple	44.47	20.00	27.23	8.30
Maxima	56.38	4.26	0.21	39.15
Fricas	44.47	40.00	0.85	14.68
Sympy	4.47	0.00	0.00	95.53
Giac	30.21	4.47	0.00	65.32
Mupad	1.28	50.43	0.00	48.30

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	46	100.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Maxima	184	83.70 %	14.13 %	2.17 %
Fricas	69	100.00 %	0.00 %	0.00 %
Sympy	449	63.92 %	36.08 %	0.00 %
Giac	307	59.61 %	0.00 %	40.39 %
Mupad	227	96.04 %	3.96 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

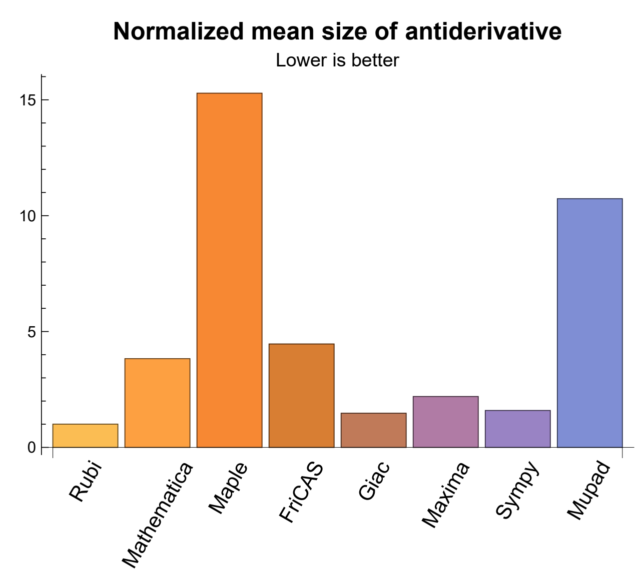
1.3 Performance

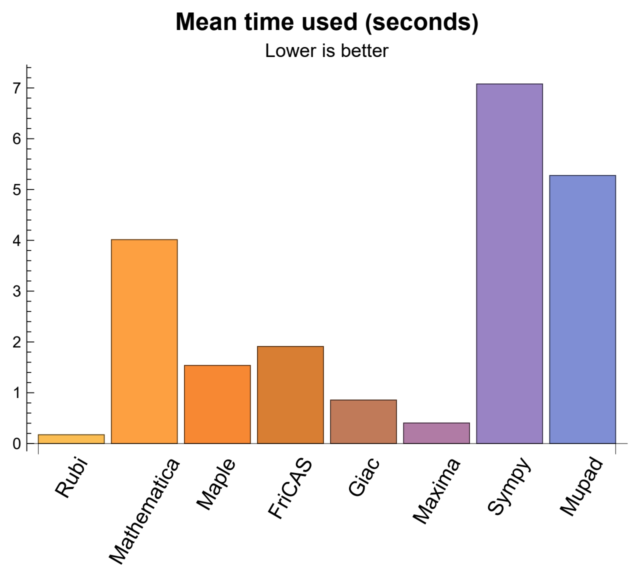
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	127.59	1.00	105.00	1.00
Mathematica	4.01	447.74	3.83	171.00	1.62
Maple	1.54	2747.28	15.29	260.00	1.89
Maxima	0.40	187.64	2.20	98.00	1.17
Fricas	1.91	600.44	4.46	390.00	3.63
Sympy	7.08	71.52	1.59	61.00	1.31
Giac	0.86	146.61	1.47	119.00	1.31
Mupad	5.28	1563.04	10.73	105.00	1.16

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{462, 466, 467, 468, 469, 470}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {34, 47, 48, 49, 53, 60, 61, 62, 63, 64, 65, 66, 78, 91, 125, 126, 128, 132, 133, 136, 137, 138, 139, 140, 180, 181, 193, 196, 199, 205, 208, 209, 211, 215, 217, 218, 219, 240, 254, 264, 268, 269, 276, 277, 280, 281, 282, 284, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 358, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 392, 393, 394, 416, 428, 437, 441, 447, 448, 449, 450, 451, 463, 464}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

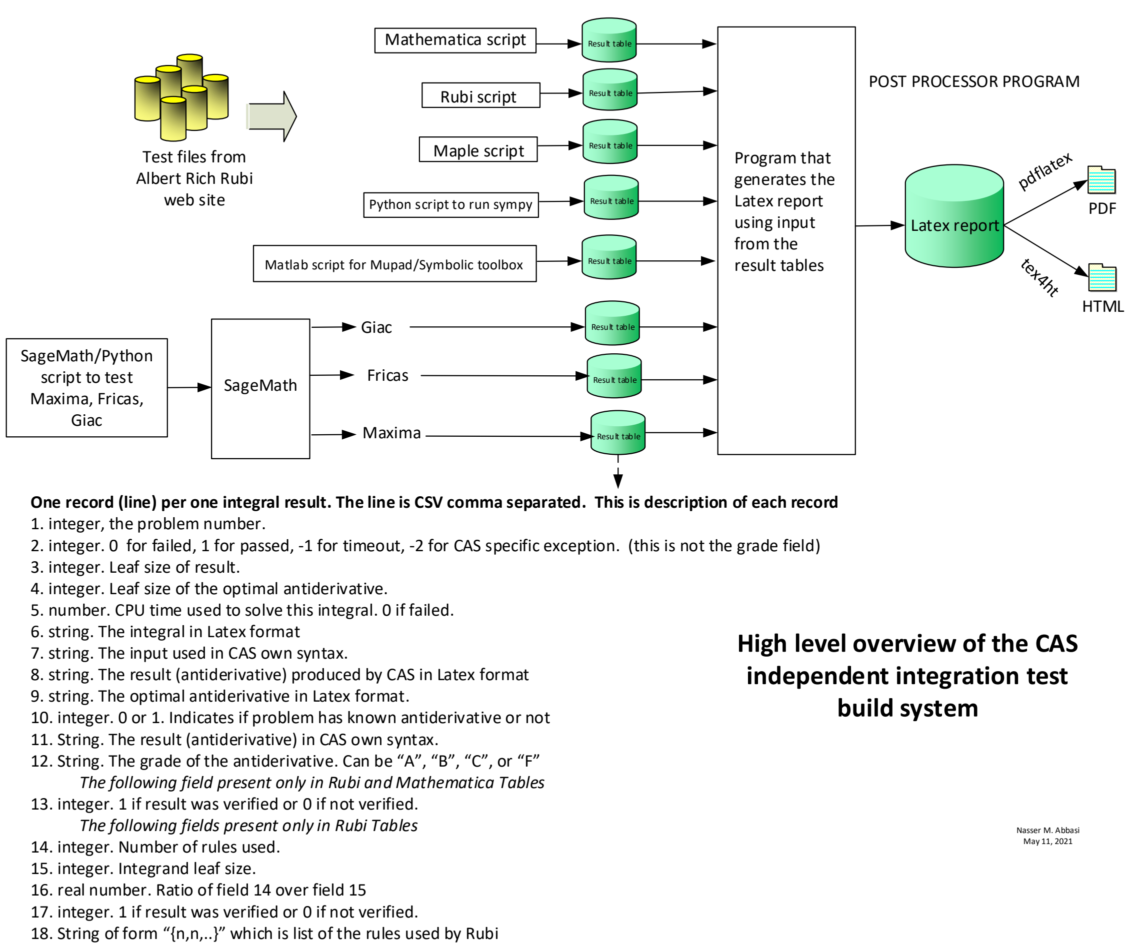
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470 }

B grade: { }

C grade: { }

F grade: { 132, 298 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 26, 67, 68, 69, 70, 71, 72, 80, 81, 83, 84, 85, 87, 93, 94, 95, 96, 97, 99, 100, 101, 103, 104, 105, 107, 108, 109, 112, 113, 114, 116, 117, 118, 119, 120, 121, 129, 130, 131, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 182, 183, 184, 185, 188, 190, 191, 192, 194, 195, 197, 198, 200, 203, 204, 206, 207, 210, 212, 216, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 245, 252, 253, 259, 267, 271, 278, 283, 290, 304, 305, 306, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 382, 383, 386, 387, 388, 395, 396, 408, 409, 412, 413, 414, 421, 422, 425, 426, 427, 438, 439, 440, 442, 443, 444, 445, 446, 452, 453, 454, 455, 456, 459, 462, 463, 464, 465, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 7, 18, 19, 21, 24, 25, 27, 102, 106, 115, 125, 126, 127, 132, 136, 137, 138, 139, 140, 174, 178, 179, 236, 256, 265, 266, 279, 291, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 330, 331, 332, 333, 334, 335, 336, 378, 411, 423, 424, 434, 435, 436, 441, 447, 448, 449, 450, 451 }

C grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 77, 78, 79, 82, 88, 89, 90, 91, 92, 98, 110, 111, 122, 123, 124, 128, 148, 149, 150, 151, 180, 181, 186, 187, 189, 193, 196, 199, 201, 202, 205, 208, 209, 211, 213, 214, 215, 217, 218, 219, 232, 234, 235, 240, 246, 247, 248, 249, 250, 251, 254, 255, 260, 263, 264, 268, 269, 272, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 321, 322, 323, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 391, 392, 393, 394, 397, 398, 399, 400, 401, 416, 417, 428, 429, 430, 437, 457, 458, 460, 461 }

F grade: { 73, 74, 76, 86, 228, 229, 230, 233, 237, 241, 242, 243, 244, 257, 258, 261, 262, 270, 273, 274, 275, 285, 286, 287, 288, 376, 377, 379, 381, 385, 389, 390, 402, 403, 404, 405, 406, 407, 410, 415, 418, 419, 420, 431, 432, 433 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 64, 66, 69, 82, 93, 94, 95, 103, 104, 105, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 216, 217, 218, 220, 221, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 358, 359, 360, 361, 362, 363, 364, 365, 374, 375, 378, 391, 404, 417, 430, 454, 455, 456, 457, 458, 459, 460, 462, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 18, 19, 20, 28, 32, 33, 34, 35, 46, 47, 57, 58, 59, 60, 63, 65, 67, 68, 70, 71, 72, 80, 81, 83, 84, 85, 96, 97, 98, 106, 107, 109, 110, 111, 122, 123, 124, 192, 212, 213, 215, 219, 222, 223, 224, 225, 226, 227, 341, 342, 343, 344, 354, 355, 356, 357, 366, 367, 368, 369, 370, 371, 372, 373, 376, 377, 379, 380, 381, 389, 390, 392, 393, 394, 402, 403, 405, 406, 407, 415, 416, 418, 419, 420, 428, 429, 431, 432, 433, 452, 453, 461 }

C grade: { 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 99, 100, 101, 102, 112, 113, 114, 115, 125, 126, 127, 128, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 382, 383, 384, 385, 386, 387, 388, 395, 396, 397, 398, 399, 400, 401, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 426, 427, 434, 435, 436, 437, 438, 439, 440 }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 247, 248, 249, 263, 264, 265, 276, 277, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 466, 467, 468, 469, 470 }

B grade: { 33, 46, 58, 59, 64, 65, 102, 115, 219, 266, 279, 289, 297, 355, 367, 368, 411, 416, 423, 424 }

C grade: { 379 }

F grade: { 70, 71, 72, 73, 74, 75, 76, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 109, 110, 111, 112, 113, 114, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 41, 42, 43, 47, 48, 49, 54, 55, 56, 60, 61, 67, 68, 69, 70, 71, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 118, 119, 120, 121, 125, 126, 129, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 198, 203, 204, 205, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 239, 240, 247, 248, 249, 253, 263, 268, 269, 276, 281, 282, 290, 295, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 363, 364, 365, 382, 395, 396, 452, 453, 454, 455, 456, 459, 462, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 7, 11, 18, 19, 20, 33, 39, 40, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 62, 63, 64, 65, 66, 72, 75, 76, 77, 78, 79, 88, 89, 101, 102, 109, 110, 111, 114, 115, 122, 123, 124, 127, 128, 130, 131, 149, 150, 151, 161, 186, 187, 188, 197, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 236, 237, 238, 250, 251, 252, 254, 255, 256, 264, 265, 266, 267, 277, 278, 279, 280, 289, 291, 292, 293, 294, 296, 297, 320, 322, 323, 335, 336, 342, 343, 344, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade: { 457, 458, 460, 461 }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.6 Sympy

A grade: { 162, 311, 312, 313, 317, 318, 319, 324, 325, 326, 339, 417, 430, 452, 453, 454, 459, 466, 468, 469, 470 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 316, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 467 }

2.1.7 Giac

A grade: { 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 34, 35, 36, 37, 38, 39, 40, 42, 43, 47, 48, 49, 50, 51, 52, 53, 55, 56, 60, 61, 62, 63, 64, 65, 66, 144, 145, 146, 147, 156, 157, 158, 159, 160, 161, 162, 163, 164, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 317, 318, 319, 320, 330, 331, 332, 333, 334, 343, 344, 345, 346, 347, 348, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 462, 466, 467, 468, 469, 470 }

B grade: { 1, 2, 15, 28, 41, 54, 148, 149, 150, 151, 222, 223, 225, 226, 227, 321, 322, 323, 335, 336, 349 }

C grade: { }

F grade: { 5, 6, 7, 18, 19, 20, 31, 32, 33, 44, 45, 46, 57, 58, 59, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 152, 153, 154, 155, 165, 166, 167, 168, 169, 180, 181, 193, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 324, 325, 326, 327, 328, 329, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465 }

2.1.8 Mupad

A grade: { 462, 466, 467, 468, 469, 470 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 103, 104, 105, 108, 116, 117, 121, 129, 135, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

C grade: { }

F grade: { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 107, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	120	102	73	75	0	288	70
normalized size	1	1.00	1.45	1.23	0.88	0.90	0.00	3.47	0.84
time (sec)	N/A	0.061	0.084	1.004	0.337	1.085	0.000	1.371	0.096

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	88	82	58	60	0	213	55
normalized size	1	1.00	1.33	1.24	0.88	0.91	0.00	3.23	0.83
time (sec)	N/A	0.051	0.040	1.099	0.333	0.816	0.000	0.496	4.212

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	62	40	42	0	61	39
normalized size	1	1.00	1.20	1.41	0.91	0.95	0.00	1.39	0.89
time (sec)	N/A	0.039	0.030	1.050	0.339	1.174	0.000	0.228	0.071

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	25	25	27	0	28	25
normalized size	1	1.00	1.46	1.04	1.04	1.12	0.00	1.17	1.04
time (sec)	N/A	0.020	0.016	0.357	0.331	1.304	0.000	1.792	0.044

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	84	57	44	60	0	0	29
normalized size	1	1.00	3.11	2.11	1.63	2.22	0.00	0.00	1.07
time (sec)	N/A	0.031	0.047	0.492	0.334	0.740	0.000	0.000	0.094

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	236	100	76	124	0	0	62
normalized size	1	1.00	4.45	1.89	1.43	2.34	0.00	0.00	1.17
time (sec)	N/A	0.052	0.380	0.967	0.393	0.815	0.000	0.000	4.203

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	198	142	101	178	0	0	86
normalized size	1	1.00	2.44	1.75	1.25	2.20	0.00	0.00	1.06
time (sec)	N/A	0.075	1.827	0.899	0.331	0.570	0.000	0.000	4.287

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	112	111	86	0	113	105
normalized size	1	1.00	0.80	1.14	1.13	0.88	0.00	1.15	1.07
time (sec)	N/A	0.104	0.296	0.903	0.447	1.105	0.000	1.342	4.887

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	92	82	68	0	89	79
normalized size	1	1.00	0.77	1.31	1.17	0.97	0.00	1.27	1.13
time (sec)	N/A	0.064	0.297	0.942	0.446	0.685	0.000	0.233	4.400

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	54	46	47	50	0	51	35
normalized size	1	1.00	1.29	1.10	1.12	1.19	0.00	1.21	0.83
time (sec)	N/A	0.044	0.096	0.716	0.454	0.680	0.000	0.259	4.244

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	16	17
normalized size	1	1.00	1.00	1.07	1.00	2.07	0.00	1.07	1.13
time (sec)	N/A	0.012	0.002	0.872	0.346	0.761	0.000	1.436	4.305

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	36	43	26	39	0	28	28
normalized size	1	1.00	1.38	1.65	1.00	1.50	0.00	1.08	1.08
time (sec)	N/A	0.034	0.061	0.919	0.344	0.933	0.000	0.242	4.208

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	73	43	66	0	54	46
normalized size	1	1.00	1.83	1.59	0.93	1.43	0.00	1.17	1.00
time (sec)	N/A	0.046	0.044	1.122	0.331	0.724	0.000	0.304	4.307

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	128	101	64	91	0	82	60
normalized size	1	1.00	1.88	1.49	0.94	1.34	0.00	1.21	0.88
time (sec)	N/A	0.056	0.042	1.175	0.342	0.553	0.000	0.945	4.523

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	118	155	89	90	0	446	87
normalized size	1	1.00	1.22	1.60	0.92	0.93	0.00	4.60	0.90
time (sec)	N/A	0.090	0.614	0.905	0.324	0.504	0.000	0.492	4.302

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	125	67	67	0	97	66
normalized size	1	1.00	1.15	1.74	0.93	0.93	0.00	1.35	0.92
time (sec)	N/A	0.072	0.442	0.894	0.324	0.462	0.000	0.401	4.137

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	42	44	0	47	45
normalized size	1	1.00	1.63	0.91	0.91	0.96	0.00	1.02	0.98
time (sec)	N/A	0.035	0.113	0.275	0.325	0.577	0.000	1.360	0.062

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	108	117	82	101	0	0	53
normalized size	1	1.00	2.08	2.25	1.58	1.94	0.00	0.00	1.02
time (sec)	N/A	0.066	0.513	0.775	0.359	0.662	0.000	0.000	0.118

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	1021	195	126	193	0	0	96
normalized size	1	1.00	9.82	1.88	1.21	1.86	0.00	0.00	0.92
time (sec)	N/A	0.110	6.589	1.381	0.329	0.614	0.000	0.000	4.291

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	218	264	165	286	0	0	135
normalized size	1	1.00	1.55	1.87	1.17	2.03	0.00	0.00	0.96
time (sec)	N/A	0.138	1.850	1.207	0.350	0.635	0.000	0.000	4.419

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	499	199	164	131	0	197	163
normalized size	1	1.00	3.37	1.34	1.11	0.89	0.00	1.33	1.10
time (sec)	N/A	0.177	1.351	0.918	0.445	0.564	0.000	1.273	4.791

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	153	123	120	107	0	132	116
normalized size	1	1.00	1.34	1.08	1.05	0.94	0.00	1.16	1.02
time (sec)	N/A	0.121	1.619	0.904	0.451	0.647	0.000	0.404	4.325

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	126	71	67	81	0	72	94
normalized size	1	1.00	1.73	0.97	0.92	1.11	0.00	0.99	1.29
time (sec)	N/A	0.099	1.004	0.516	0.435	0.576	0.000	1.367	4.460

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	44	58	0	53	42
normalized size	1	1.00	2.65	1.20	1.10	1.45	0.00	1.32	1.05
time (sec)	N/A	0.029	0.364	0.980	0.342	1.656	0.000	0.217	4.333

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	109	96	54	71	0	64	56
normalized size	1	1.00	2.18	1.92	1.08	1.42	0.00	1.28	1.12
time (sec)	N/A	0.058	1.140	0.898	0.356	0.620	0.000	0.312	4.402

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	151	144	80	101	0	105	85
normalized size	1	1.00	1.99	1.89	1.05	1.33	0.00	1.38	1.12
time (sec)	N/A	0.076	1.355	1.377	0.342	0.707	0.000	0.797	4.449

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	353	190	107	139	0	151	108
normalized size	1	1.00	3.43	1.84	1.04	1.35	0.00	1.47	1.05
time (sec)	N/A	0.097	1.568	1.692	0.337	0.619	0.000	0.385	4.803

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	425	183	102	229	0	373	123
normalized size	1	1.00	4.34	1.87	1.04	2.34	0.00	3.81	1.26
time (sec)	N/A	0.105	3.258	0.898	0.430	0.717	0.000	0.269	4.307

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	376	103	63	154	0	89	76
normalized size	1	1.00	5.30	1.45	0.89	2.17	0.00	1.25	1.07
time (sec)	N/A	0.084	1.392	0.930	0.438	0.874	0.000	0.251	0.121

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	329	46	40	118	0	44	39
normalized size	1	1.00	7.00	0.98	0.85	2.51	0.00	0.94	0.83
time (sec)	N/A	0.041	0.546	0.402	0.422	0.561	0.000	0.542	0.067

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	239	76	64	156	0	0	123
normalized size	1	1.00	4.35	1.38	1.16	2.84	0.00	0.00	2.24
time (sec)	N/A	0.071	0.764	0.784	0.443	0.763	0.000	0.000	0.206

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	371	158	128	327	0	0	392
normalized size	1	1.00	4.31	1.84	1.49	3.80	0.00	0.00	4.56
time (sec)	N/A	0.100	1.550	0.938	0.448	1.191	0.000	0.000	4.912

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	549	296	231	693	0	0	870
normalized size	1	1.00	4.26	2.29	1.79	5.37	0.00	0.00	6.74
time (sec)	N/A	0.154	4.702	0.963	0.458	0.845	0.000	0.000	7.895

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	357	460	209	428	0	250	1448
normalized size	1	1.00	2.15	2.77	1.26	2.58	0.00	1.51	8.72
time (sec)	N/A	0.335	4.086	0.854	0.426	0.749	0.000	0.264	5.706

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	303	260	137	332	0	160	494
normalized size	1	1.00	2.59	2.22	1.17	2.84	0.00	1.37	4.22
time (sec)	N/A	0.169	1.935	1.015	0.444	0.567	0.000	0.263	4.583

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	245	124	77	257	0	97	111
normalized size	1	1.00	3.22	1.63	1.01	3.38	0.00	1.28	1.46
time (sec)	N/A	0.098	0.843	0.947	0.448	0.532	0.000	1.448	4.448

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	48	44	231	0	68	460
normalized size	1	1.00	4.04	1.07	0.98	5.13	0.00	1.51	10.22
time (sec)	N/A	0.044	0.286	1.000	0.448	0.504	0.000	0.761	4.562

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	189	54	50	271	0	74	46
normalized size	1	1.00	3.50	1.00	0.93	5.02	0.00	1.37	0.85
time (sec)	N/A	0.073	0.642	0.827	0.437	0.594	0.000	0.755	4.277

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	226	74	82	397	0	107	80
normalized size	1	1.00	2.97	0.97	1.08	5.22	0.00	1.41	1.05
time (sec)	N/A	0.094	2.100	0.960	0.425	0.614	0.000	0.957	4.311

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	318	116	137	587	0	180	112
normalized size	1	1.00	3.03	1.10	1.30	5.59	0.00	1.71	1.07
time (sec)	N/A	0.139	1.718	1.014	0.439	0.532	0.000	0.304	5.051

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	454	276	148	405	0	545	195
normalized size	1	1.00	2.82	1.71	0.92	2.52	0.00	3.39	1.21
time (sec)	N/A	0.178	6.305	0.949	0.531	0.696	0.000	1.405	0.161

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	403	165	104	297	0	143	130
normalized size	1	1.00	3.54	1.45	0.91	2.61	0.00	1.25	1.14
time (sec)	N/A	0.112	3.471	0.906	0.428	0.577	0.000	0.639	0.145

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	393	75	70	201	0	76	72
normalized size	1	1.00	4.68	0.89	0.83	2.39	0.00	0.90	0.86
time (sec)	N/A	0.051	3.139	0.619	0.445	0.475	0.000	0.492	4.561

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	384	172	138	390	0	0	2188
normalized size	1	1.00	3.88	1.74	1.39	3.94	0.00	0.00	22.10
time (sec)	N/A	0.107	1.274	1.187	0.429	1.041	0.000	0.000	5.945

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	468	250	231	698	0	0	1845
normalized size	1	1.00	3.18	1.70	1.57	4.75	0.00	0.00	12.55
time (sec)	N/A	0.173	1.940	1.309	0.446	0.592	0.000	0.000	5.646

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	450	390	369	1202	0	0	4338
normalized size	1	1.00	2.28	1.98	1.87	6.10	0.00	0.00	22.02
time (sec)	N/A	0.246	2.436	0.969	0.434	0.662	0.000	0.000	9.326

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	2738	555	302	674	0	311	1461
normalized size	1	1.00	10.25	2.08	1.13	2.52	0.00	1.16	5.47
time (sec)	N/A	0.426	23.512	0.952	0.461	0.676	0.000	0.379	6.658

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	1105	323	205	522	0	204	435
normalized size	1	1.00	5.79	1.69	1.07	2.73	0.00	1.07	2.28
time (sec)	N/A	0.255	13.242	0.893	0.483	0.623	0.000	0.345	5.767

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	825	155	126	441	0	158	816
normalized size	1	1.00	6.35	1.19	0.97	3.39	0.00	1.22	6.28
time (sec)	N/A	0.169	11.307	0.820	0.600	0.657	0.000	0.313	5.209

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	240	127	106	435	0	119	2056
normalized size	1	1.00	2.61	1.38	1.15	4.73	0.00	1.29	22.35
time (sec)	N/A	0.086	1.984	0.896	0.478	0.790	0.000	0.229	6.350

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	242	86	117	407	0	133	91
normalized size	1	1.00	2.66	0.95	1.29	4.47	0.00	1.46	1.00
time (sec)	N/A	0.085	2.216	1.050	0.467	0.642	0.000	0.468	4.403

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	303	160	193	663	0	192	141
normalized size	1	1.00	2.46	1.30	1.57	5.39	0.00	1.56	1.15
time (sec)	N/A	0.175	6.180	1.424	0.546	0.628	0.000	0.457	5.494

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	777	189	268	987	0	263	198
normalized size	1	1.00	4.13	1.01	1.43	5.25	0.00	1.40	1.05
time (sec)	N/A	0.264	3.218	1.252	0.443	0.746	0.000	0.510	6.305

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	1641	374	204	579	0	837	255
normalized size	1	1.00	7.67	1.75	0.95	2.71	0.00	3.91	1.19
time (sec)	N/A	0.255	10.557	1.023	0.433	0.677	0.000	0.583	4.509

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1153	231	149	439	0	183	172
normalized size	1	1.00	7.49	1.50	0.97	2.85	0.00	1.19	1.12
time (sec)	N/A	0.188	9.775	0.942	0.445	0.684	0.000	0.499	0.168

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	656	108	103	299	0	97	105
normalized size	1	1.00	5.66	0.93	0.89	2.58	0.00	0.84	0.91
time (sec)	N/A	0.068	5.745	0.581	0.451	0.650	0.000	0.402	0.147

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	447	352	261	779	0	0	3557
normalized size	1	1.00	2.90	2.29	1.69	5.06	0.00	0.00	23.10
time (sec)	N/A	0.196	2.403	0.933	0.433	0.805	0.000	0.000	8.601

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	532	430	399	1332	0	0	2728
normalized size	1	1.00	2.50	2.02	1.87	6.25	0.00	0.00	12.81
time (sec)	N/A	0.312	3.570	1.164	0.437	0.879	0.000	0.000	6.921

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	549	567	528	1833	0	0	5613
normalized size	1	1.00	2.14	2.21	2.05	7.13	0.00	0.00	21.84
time (sec)	N/A	0.366	5.105	1.240	0.443	1.000	0.000	0.000	9.604

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	1639	689	418	930	0	372	2117
normalized size	1	1.00	5.22	2.19	1.33	2.96	0.00	1.18	6.74
time (sec)	N/A	0.505	18.871	1.277	0.446	0.634	0.000	0.608	7.717

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	2469	423	299	803	0	325	1317
normalized size	1	1.00	10.37	1.78	1.26	3.37	0.00	1.37	5.53
time (sec)	N/A	0.340	25.011	1.176	0.437	0.596	0.000	0.509	7.193

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	1915	314	272	815	0	219	2628
normalized size	1	1.00	10.41	1.71	1.48	4.43	0.00	1.19	14.28
time (sec)	N/A	0.278	17.644	0.960	0.439	0.703	0.000	0.862	7.834

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	332	321	231	819	0	205	3271
normalized size	1	1.00	2.31	2.23	1.60	5.69	0.00	1.42	22.72
time (sec)	N/A	0.188	5.625	0.968	0.430	0.593	0.000	0.316	8.606

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	987	157	219	615	0	184	146
normalized size	1	1.00	7.96	1.27	1.77	4.96	0.00	1.48	1.18
time (sec)	N/A	0.111	6.820	1.071	0.437	0.610	0.000	0.501	5.107

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	994	306	323	1009	0	275	207
normalized size	1	1.00	6.06	1.87	1.97	6.15	0.00	1.68	1.26
time (sec)	N/A	0.250	4.054	1.231	0.444	0.650	0.000	0.672	6.885

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	479	411	434	1423	0	382	267
normalized size	1	1.00	1.98	1.70	1.79	5.88	0.00	1.58	1.10
time (sec)	N/A	0.370	5.769	1.178	0.450	0.668	0.000	1.092	7.490

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	152	1840	171	295	0	0	-1
normalized size	1	1.00	1.09	13.24	1.23	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.847	4.584	0.434	0.877	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	120	1525	116	232	0	0	-1
normalized size	1	1.00	1.20	15.25	1.16	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.377	1.841	0.421	0.846	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	98	93	88	182	0	0	87
normalized size	1	1.00	1.48	1.41	1.33	2.76	0.00	0.00	1.32
time (sec)	N/A	0.053	0.135	0.528	0.422	0.836	0.000	0.000	6.768

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	119	688	0	496	0	0	-1
normalized size	1	1.00	1.45	8.39	0.00	6.05	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.129	2.013	0.000	0.785	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	163	3015	0	867	0	0	-1
normalized size	1	1.00	1.31	24.31	0.00	6.99	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.429	1.719	0.000	0.720	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	198	9758	0	1476	0	0	-1
normalized size	1	1.00	1.08	53.32	0.00	8.07	0.00	0.00	-0.01
time (sec)	N/A	0.220	1.382	1.897	0.000	1.297	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	2665	0	1715	0	0	-1
normalized size	1	1.00	0.00	11.10	0.00	7.15	0.00	0.00	-0.00
time (sec)	N/A	0.384	8.911	3.356	0.000	4.909	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	0	1939	0	1565	0	0	-1
normalized size	1	1.00	0.00	10.71	0.00	8.65	0.00	0.00	-0.01
time (sec)	N/A	0.221	5.349	1.490	0.000	1.834	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	432	1290	0	1417	0	0	-1
normalized size	1	1.00	3.51	10.49	0.00	11.52	0.00	0.00	-0.01
time (sec)	N/A	0.131	5.796	1.182	0.000	1.087	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	588	0	1227	0	0	-1
normalized size	1	1.00	0.00	7.44	0.00	15.53	0.00	0.00	-0.01
time (sec)	N/A	0.051	1.637	4.737	0.000	0.907	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	1003	50	306	0	0	-1
normalized size	1	1.00	0.90	14.75	0.74	4.50	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.202	2.062	0.331	0.669	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	285	3847	82	436	0	0	-1
normalized size	1	1.00	2.71	36.64	0.78	4.15	0.00	0.00	-0.01
time (sec)	N/A	0.097	7.217	1.857	0.346	1.040	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	422	8587	143	656	0	0	-1
normalized size	1	1.00	2.83	57.63	0.96	4.40	0.00	0.00	-0.01
time (sec)	N/A	0.140	7.983	1.835	0.336	2.470	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	188	2537	277	351	0	0	-1
normalized size	1	1.00	0.96	12.94	1.41	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.307	1.773	0.431	1.283	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	164	1913	250	278	0	0	-1
normalized size	1	1.00	1.01	11.81	1.54	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.756	1.430	0.436	0.850	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	121	142	231	0	0	61
normalized size	1	1.00	0.73	1.21	1.42	2.31	0.00	0.00	0.61
time (sec)	N/A	0.070	0.599	0.355	0.432	0.874	0.000	0.000	6.111

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	171	2563	0	715	0	0	-1
normalized size	1	1.00	1.40	21.01	0.00	5.86	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.548	1.581	0.000	0.860	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	202	5178	0	984	0	0	-1
normalized size	1	1.00	1.25	32.16	0.00	6.11	0.00	0.00	-0.01
time (sec)	N/A	0.203	1.421	1.825	0.000	1.003	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	262	10199	0	1511	0	0	-1
normalized size	1	1.00	1.20	46.78	0.00	6.93	0.00	0.00	-0.00
time (sec)	N/A	0.336	3.353	2.132	0.000	0.914	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	0	3067	0	1855	0	0	-1
normalized size	1	1.00	0.00	10.29	0.00	6.22	0.00	0.00	-0.00
time (sec)	N/A	0.474	9.991	2.275	0.000	17.852	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	211	2309	0	1667	0	0	-1
normalized size	1	1.00	0.97	10.64	0.00	7.68	0.00	0.00	-0.00
time (sec)	N/A	0.341	5.306	1.796	0.000	6.161	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	493	1583	0	1535	0	0	-1
normalized size	1	1.00	3.06	9.83	0.00	9.53	0.00	0.00	-0.01
time (sec)	N/A	0.198	5.762	1.260	0.000	2.069	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	1457	0	0	-1
normalized size	1	1.00	4.47	13.19	0.00	12.35	0.00	0.00	-0.01
time (sec)	N/A	0.096	4.986	1.471	0.000	1.397	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	2032	98	370	0	0	-1
normalized size	1	1.00	0.61	19.35	0.93	3.52	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.182	1.401	0.337	1.287	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	135	3925	243	472	0	0	-1
normalized size	1	1.00	0.78	22.82	1.41	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.153	8.095	1.570	0.342	2.526	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	512	8726	273	682	0	0	-1
normalized size	1	1.00	2.45	41.75	1.31	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.205	10.327	2.443	0.349	10.592	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	93	105	162	87	0	0	-1
normalized size	1	1.00	0.76	0.85	1.32	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.933	1.960	0.350	0.544	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	69	83	57	0	0	-1
normalized size	1	1.00	0.86	0.93	1.12	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.277	1.494	0.344	0.502	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	48	31	28	37	0	0	46
normalized size	1	1.00	1.60	1.03	0.93	1.23	0.00	0.00	1.53
time (sec)	N/A	0.042	0.113	0.360	0.330	0.514	0.000	0.000	5.384

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	86	280	0	140	0	0	-1
normalized size	1	1.00	2.00	6.51	0.00	3.26	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.100	1.741	0.000	0.628	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	140	2199	0	305	0	0	-1
normalized size	1	1.00	1.61	25.28	0.00	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.990	1.860	0.000	0.619	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	78	4983	0	491	0	0	-1
normalized size	1	1.00	0.57	36.11	0.00	3.56	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.190	1.919	0.000	0.705	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	163	2425	0	639	0	0	-1
normalized size	1	1.00	0.84	12.56	0.00	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.278	1.388	1.759	0.000	2.244	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	145	1701	0	565	0	0	-1
normalized size	1	1.00	1.07	12.60	0.00	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.454	1.439	0.000	0.945	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	125	1055	0	497	0	0	-1
normalized size	1	1.00	1.47	12.41	0.00	5.85	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.235	1.562	0.000	0.713	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	992	408	0	0	-1
normalized size	1	1.00	2.23	9.74	25.44	10.46	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.066	2.470	0.617	0.598	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	48	33	47	0	0	74
normalized size	1	1.00	1.67	1.45	1.00	1.42	0.00	0.00	2.24
time (sec)	N/A	0.071	0.108	1.699	0.362	0.666	0.000	0.000	4.646

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	66	96	96	0	0	123
normalized size	1	1.00	0.95	0.85	1.23	1.23	0.00	0.00	1.58
time (sec)	N/A	0.097	0.194	1.635	0.363	0.551	0.000	0.000	9.922

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	100	101	191	171	0	0	723
normalized size	1	1.00	0.76	0.77	1.45	1.30	0.00	0.00	5.48
time (sec)	N/A	0.140	0.325	1.681	0.351	1.152	0.000	0.000	15.145

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	432	35190	250	136	0	0	-1
normalized size	1	1.00	2.53	205.79	1.46	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.187	7.506	3.236	0.377	0.750	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	93	12782	141	98	0	0	-1
normalized size	1	1.00	0.82	112.12	1.24	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.117	3.399	1.903	0.362	0.561	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	64	59	57	67	0	0	155
normalized size	1	1.00	1.03	0.95	0.92	1.08	0.00	0.00	2.50
time (sec)	N/A	0.056	1.271	0.290	0.346	0.507	0.000	0.000	10.869

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	113	1094	0	344	0	0	-1
normalized size	1	1.00	1.41	13.68	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.736	2.126	0.000	0.667	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	97	3289	0	547	0	0	-1
normalized size	1	1.00	0.77	26.10	0.00	4.34	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.310	2.292	0.000	0.732	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	100	8268	0	873	0	0	-1
normalized size	1	1.00	0.56	46.71	0.00	4.93	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.468	2.068	0.000	1.040	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	256	2437	0	813	0	0	-1
normalized size	1	1.00	1.06	10.07	0.00	3.36	0.00	0.00	-0.00
time (sec)	N/A	0.374	8.168	1.933	0.000	10.051	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	229	1714	0	703	0	0	-1
normalized size	1	1.00	1.31	9.79	0.00	4.02	0.00	0.00	-0.01
time (sec)	N/A	0.217	3.433	1.296	0.000	3.211	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	190	1069	0	607	0	0	-1
normalized size	1	1.00	1.57	8.83	0.00	5.02	0.00	0.00	-0.01
time (sec)	N/A	0.150	1.167	1.103	0.000	1.089	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	2055	601	0	0	-1
normalized size	1	1.00	2.18	13.08	26.69	7.81	0.00	0.00	-0.01
time (sec)	N/A	0.050	1.307	1.839	0.921	0.907	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	89	64	102	0	0	2151
normalized size	1	1.00	1.12	1.31	0.94	1.50	0.00	0.00	31.63
time (sec)	N/A	0.091	1.617	1.384	0.365	0.663	0.000	0.000	12.122

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	102	137	156	189	0	0	124682
normalized size	1	1.00	0.83	1.11	1.27	1.54	0.00	0.00	1013.67
time (sec)	N/A	0.128	0.618	1.697	0.429	1.464	0.000	0.000	27.528

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	126	204	282	314	0	0	-1
normalized size	1	1.00	0.69	1.11	1.54	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.919	2.111	0.458	4.673	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	182	229	334	189	0	0	-1
normalized size	1	1.00	0.83	1.05	1.53	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.215	3.214	3.411	0.424	0.964	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	129	159	195	138	0	0	-1
normalized size	1	1.00	0.88	1.09	1.34	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.138	2.315	2.105	0.408	0.710	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	90	86	101	0	0	26927
normalized size	1	1.00	0.91	0.93	0.89	1.04	0.00	0.00	277.60
time (sec)	N/A	0.068	1.322	0.303	0.391	0.616	0.000	0.000	17.230

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	108	5056	0	592	0	0	-1
normalized size	1	1.00	0.85	39.81	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.142	5.111	2.136	0.000	0.700	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	151	11110	0	941	0	0	-1
normalized size	1	1.00	0.88	64.97	0.00	5.50	0.00	0.00	-0.01
time (sec)	N/A	0.206	1.400	2.605	0.000	0.774	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	129	15551	0	1307	0	0	-1
normalized size	1	1.00	0.55	66.46	0.00	5.59	0.00	0.00	-0.00
time (sec)	N/A	0.324	1.799	3.230	0.000	1.004	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	1705	4477	0	1003	0	0	-1
normalized size	1	1.00	5.92	15.55	0.00	3.48	0.00	0.00	-0.00
time (sec)	N/A	0.442	19.164	2.849	0.000	39.594	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	1315	3223	0	873	0	0	-1
normalized size	1	1.00	5.79	14.20	0.00	3.85	0.00	0.00	-0.00
time (sec)	N/A	0.302	13.950	2.125	0.000	12.092	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	983	3158	0	879	0	0	-1
normalized size	1	1.00	5.89	18.91	0.00	5.26	0.00	0.00	-0.01
time (sec)	N/A	0.202	10.528	1.713	0.000	3.660	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	881	0	0	-1
normalized size	1	1.00	15.42	24.19	0.00	7.05	0.00	0.00	-0.01
time (sec)	N/A	0.099	16.946	2.205	0.000	1.231	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	146	94	192	0	0	336
normalized size	1	1.00	1.02	1.38	0.89	1.81	0.00	0.00	3.17
time (sec)	N/A	0.108	1.974	1.610	0.360	1.461	0.000	0.000	16.300

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	138	225	216	320	0	0	-1
normalized size	1	1.00	0.87	1.42	1.37	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.161	4.591	1.673	0.358	4.854	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	173	324	373	460	0	0	-1
normalized size	1	1.00	0.77	1.43	1.65	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.240	7.412	1.745	0.378	15.771	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	0	286	0	0	0	0	0	-1
normalized size	1	0.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	4.005	3.480	0.000	0.898	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	253	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	7.770	4.768	0.000	0.562	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	178	0	0	0	0	0	-1
normalized size	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	3.885	4.520	0.000	0.522	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0	79
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.16
time (sec)	N/A	0.049	1.650	1.367	0.000	0.512	0.000	0.000	4.919

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	1532	0	0	0	0	0	-1
normalized size	1	1.00	19.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	16.878	1.540	0.000	0.574	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	266	0	0	0	0	0	-1
normalized size	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	4.407	1.561	0.000	0.587	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	5878	0	0	0	0	0	-1
normalized size	1	1.00	66.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	25.723	6.090	0.000	0.609	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	3781	0	0	0	0	0	-1
normalized size	1	1.00	42.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	21.133	4.441	0.000	0.719	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	-1
normalized size	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	15.168	1.346	0.000	0.624	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.088	1.834	0.000	0.628	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	132	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	2.246	1.679	0.000	0.765	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	149	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	2.001	1.749	0.000	0.593	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	125	129	82	0	66	61
normalized size	1	1.00	0.97	1.69	1.74	1.11	0.00	0.89	0.82
time (sec)	N/A	0.047	0.041	1.488	0.329	0.454	0.000	0.357	4.619

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	81	81	69	0	53	49
normalized size	1	1.00	1.04	1.45	1.45	1.23	0.00	0.95	0.88
time (sec)	N/A	0.039	0.030	1.262	0.326	0.442	0.000	0.422	4.468

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	49	45	56	0	39	33
normalized size	1	1.00	1.11	1.29	1.18	1.47	0.00	1.03	0.87
time (sec)	N/A	0.030	0.019	1.018	0.325	0.538	0.000	0.215	4.396

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	16	32	0	16	16
normalized size	1	1.00	1.62	1.06	1.00	2.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.006	0.779	0.336	0.499	0.000	0.243	4.442

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	31	31	26	31	0	45	19
normalized size	1	1.00	1.63	1.63	1.37	1.63	0.00	2.37	1.00
time (sec)	N/A	0.021	0.030	0.517	0.416	0.414	0.000	0.238	4.704

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	47	40	75	0	80	31
normalized size	1	1.00	0.97	1.27	1.08	2.03	0.00	2.16	0.84
time (sec)	N/A	0.030	0.026	0.681	0.422	0.451	0.000	0.641	4.410

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	63	50	109	0	111	41
normalized size	1	1.00	0.65	1.15	0.91	1.98	0.00	2.02	0.75
time (sec)	N/A	0.039	0.050	0.895	0.422	0.462	0.000	0.246	4.538

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	36	79	60	147	0	139	51
normalized size	1	1.00	0.49	1.08	0.82	2.01	0.00	1.90	0.70
time (sec)	N/A	0.045	0.017	0.786	0.428	0.456	0.000	0.378	4.926

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	75	138	126	114	0	0	102
normalized size	1	1.00	0.77	1.41	1.29	1.16	0.00	0.00	1.04
time (sec)	N/A	0.059	0.339	0.801	0.340	0.488	0.000	0.000	4.655

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	98	97	95	0	0	78
normalized size	1	1.00	0.77	1.40	1.39	1.36	0.00	0.00	1.11
time (sec)	N/A	0.046	0.131	1.027	0.341	0.515	0.000	0.000	0.143

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	59	58	72	0	0	41
normalized size	1	1.00	1.20	1.48	1.45	1.80	0.00	0.00	1.02
time (sec)	N/A	0.025	0.015	0.794	0.339	0.469	0.000	0.000	4.410

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	32	38	40	0	0	22
normalized size	1	1.00	1.46	1.33	1.58	1.67	0.00	0.00	0.92
time (sec)	N/A	0.028	0.016	0.671	0.336	0.505	0.000	0.000	0.060

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	33	27	28	0	37	28
normalized size	1	1.00	1.67	1.10	0.90	0.93	0.00	1.23	0.93
time (sec)	N/A	0.045	0.021	1.485	0.337	0.494	0.000	0.170	0.047

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	71	54	43	45	0	62	43
normalized size	1	1.00	1.42	1.08	0.86	0.90	0.00	1.24	0.86
time (sec)	N/A	0.066	0.021	1.880	0.324	0.445	0.000	0.268	4.318

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	60	74	0	86	56
normalized size	1	1.00	0.93	0.90	0.69	0.85	0.00	0.99	0.64
time (sec)	N/A	0.049	0.291	1.044	0.346	0.445	0.000	0.227	4.308

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	43	56	0	62	42
normalized size	1	1.00	0.94	0.89	0.66	0.86	0.00	0.95	0.65
time (sec)	N/A	0.043	0.203	1.055	0.340	0.444	0.000	0.818	4.503

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	34	37	0	37	28
normalized size	1	1.00	0.84	0.81	0.79	0.86	0.00	0.86	0.65
time (sec)	N/A	0.038	0.088	0.964	0.338	0.515	0.000	0.358	4.459

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	16	17
normalized size	1	1.00	1.00	1.07	1.00	2.07	0.00	1.07	1.13
time (sec)	N/A	0.013	0.003	0.752	0.337	0.481	0.000	0.203	4.416

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	37	28	51	40	25
normalized size	1	1.00	1.06	1.19	1.19	0.90	1.65	1.29	0.81
time (sec)	N/A	0.027	0.029	0.687	0.439	0.496	5.790	0.184	4.313

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	73	49	0	79	67
normalized size	1	1.00	0.74	1.07	1.20	0.80	0.00	1.30	1.10
time (sec)	N/A	0.041	0.089	1.293	0.438	0.721	0.000	1.149	4.478

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	86	103	68	0	104	91
normalized size	1	1.00	0.76	0.97	1.16	0.76	0.00	1.17	1.02
time (sec)	N/A	0.053	0.106	1.680	0.439	0.572	0.000	0.232	4.937

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	119	256	200	168	0	0	170
normalized size	1	1.00	0.72	1.55	1.21	1.02	0.00	0.00	1.03
time (sec)	N/A	0.141	0.533	1.287	0.342	0.475	0.000	0.000	4.684

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	94	191	166	143	0	0	134
normalized size	1	1.00	0.73	1.48	1.29	1.11	0.00	0.00	1.04
time (sec)	N/A	0.134	0.390	1.313	0.334	0.489	0.000	0.000	4.555

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	63	125	119	116	0	0	86
normalized size	1	1.00	0.69	1.37	1.31	1.27	0.00	0.00	0.95
time (sec)	N/A	0.074	0.136	1.154	0.326	0.620	0.000	0.000	4.465

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	78	87	94	0	0	55
normalized size	1	1.00	1.43	1.39	1.55	1.68	0.00	0.00	0.98
time (sec)	N/A	0.067	0.031	1.061	0.325	0.666	0.000	0.000	0.109

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	72	63	66	0	0	48
normalized size	1	1.00	1.47	1.47	1.29	1.35	0.00	0.00	0.98
time (sec)	N/A	0.061	0.023	1.039	0.321	0.821	0.000	0.000	4.529

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	106	67	55	59	0	82	44
normalized size	1	1.00	2.00	1.26	1.04	1.11	0.00	1.55	0.83
time (sec)	N/A	0.066	0.025	1.598	0.323	0.533	0.000	0.214	4.440

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	134	103	120	0	164	94
normalized size	1	1.00	0.91	1.26	0.97	1.13	0.00	1.55	0.89
time (sec)	N/A	0.089	0.405	1.260	0.341	0.763	0.000	0.291	4.544

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	104	81	94	0	123	70
normalized size	1	1.00	0.94	1.30	1.01	1.18	0.00	1.54	0.88
time (sec)	N/A	0.076	0.370	1.253	0.341	0.594	0.000	0.276	4.673

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	71	71	69	0	82	44
normalized size	1	1.00	0.91	1.34	1.34	1.30	0.00	1.55	0.83
time (sec)	N/A	0.063	0.262	1.521	0.345	0.903	0.000	0.902	4.543

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	44	58	0	53	42
normalized size	1	1.00	2.65	1.20	1.10	1.45	0.00	1.32	1.05
time (sec)	N/A	0.029	0.384	1.280	0.349	1.114	0.000	0.229	4.550

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	53	56	0	57	66
normalized size	1	1.00	1.11	1.09	1.13	1.19	0.00	1.21	1.40
time (sec)	N/A	0.072	0.150	1.217	0.431	1.209	0.000	0.742	4.521

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	58	78	87	62	0	93	76
normalized size	1	1.00	0.72	0.96	1.07	0.77	0.00	1.15	0.94
time (sec)	N/A	0.086	0.113	1.226	0.460	1.390	0.000	0.592	4.566

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	116	135	89	0	161	123
normalized size	1	1.00	0.83	0.97	1.13	0.75	0.00	1.35	1.03
time (sec)	N/A	0.146	0.191	1.615	0.430	1.734	0.000	0.254	5.323

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	268	84	83	90	0	91	73
normalized size	1	1.00	3.67	1.15	1.14	1.23	0.00	1.25	1.00
time (sec)	N/A	0.044	1.019	1.249	0.352	0.749	0.000	0.329	4.507

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	455	130	134	130	0	148	119
normalized size	1	1.00	4.10	1.17	1.21	1.17	0.00	1.33	1.07
time (sec)	N/A	0.065	1.574	1.479	0.363	0.628	0.000	0.223	4.579

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	1195	141	124	272	0	0	591
normalized size	1	1.00	13.90	1.64	1.44	3.16	0.00	0.00	6.87
time (sec)	N/A	0.121	6.119	0.640	0.453	1.991	0.000	0.000	4.949

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	1022	68	83	157	0	0	456
normalized size	1	1.00	18.58	1.24	1.51	2.85	0.00	0.00	8.29
time (sec)	N/A	0.072	1.349	0.619	0.440	1.963	0.000	0.000	4.583

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	50	117	0	39	28
normalized size	1	1.00	1.00	0.78	1.39	3.25	0.00	1.08	0.78
time (sec)	N/A	0.042	0.076	0.888	0.448	1.156	0.000	0.262	0.111

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	67	164	0	55	44
normalized size	1	1.00	1.00	0.87	1.29	3.15	0.00	1.06	0.85
time (sec)	N/A	0.063	0.097	1.397	0.444	0.460	0.000	0.231	4.403

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	105	70	88	230	0	89	72
normalized size	1	1.00	1.38	0.92	1.16	3.03	0.00	1.17	0.95
time (sec)	N/A	0.089	0.283	1.479	0.436	0.698	0.000	0.227	4.452

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	136	110	117	305	0	136	111
normalized size	1	1.00	1.26	1.02	1.08	2.82	0.00	1.26	1.03
time (sec)	N/A	0.102	0.685	1.394	0.456	0.474	0.000	0.245	0.144

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	224	79	65	354	0	101	72
normalized size	1	1.00	2.91	1.03	0.84	4.60	0.00	1.31	0.94
time (sec)	N/A	0.088	2.259	0.580	0.454	0.865	0.000	0.244	4.368

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	192	47	45	286	0	69	44
normalized size	1	1.00	3.69	0.90	0.87	5.50	0.00	1.33	0.85
time (sec)	N/A	0.068	0.627	0.431	0.451	0.539	0.000	0.354	4.416

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	27	209	0	50	31
normalized size	1	1.00	1.00	0.78	0.75	5.81	0.00	1.39	0.86
time (sec)	N/A	0.057	0.075	0.551	0.433	0.565	0.000	0.222	4.508

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	48	44	231	0	68	460
normalized size	1	1.00	4.04	1.07	0.98	5.13	0.00	1.51	10.22
time (sec)	N/A	0.042	0.286	0.776	0.431	0.625	0.000	0.213	4.708

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	92	72	272	0	99	373
normalized size	1	1.00	0.89	1.23	0.96	3.63	0.00	1.32	4.97
time (sec)	N/A	0.103	0.232	1.564	0.439	0.621	0.000	0.232	5.238

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	194	126	343	0	149	1114
normalized size	1	1.00	0.81	1.66	1.08	2.93	0.00	1.27	9.52
time (sec)	N/A	0.157	0.399	1.753	0.435	0.870	0.000	1.231	5.260

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	359	189	424	0	229	1979
normalized size	1	1.00	0.82	2.20	1.16	2.60	0.00	1.40	12.14
time (sec)	N/A	0.245	0.853	1.654	0.450	0.783	0.000	0.240	6.153

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	980	151	146	392	0	0	2039
normalized size	1	1.00	9.61	1.48	1.43	3.84	0.00	0.00	19.99
time (sec)	N/A	0.140	3.890	0.559	0.452	2.071	0.000	0.000	5.869

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	88	68	98	262	0	79	62
normalized size	1	1.00	1.19	0.92	1.32	3.54	0.00	1.07	0.84
time (sec)	N/A	0.070	0.272	0.638	0.438	1.343	0.000	0.301	0.135

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	80	111	301	0	94	71
normalized size	1	1.00	0.99	0.96	1.34	3.63	0.00	1.13	0.86
time (sec)	N/A	0.068	0.378	0.839	0.431	1.080	0.000	0.662	4.434

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	945	92	133	391	0	117	94
normalized size	1	1.00	9.36	0.91	1.32	3.87	0.00	1.16	0.93
time (sec)	N/A	0.132	3.005	1.155	0.428	0.690	0.000	0.328	0.182

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	139	120	154	490	0	152	124
normalized size	1	1.00	1.10	0.95	1.22	3.89	0.00	1.21	0.98
time (sec)	N/A	0.155	1.114	1.400	0.440	0.742	0.000	0.286	4.623

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	171	158	183	583	0	197	173
normalized size	1	1.00	1.09	1.01	1.17	3.71	0.00	1.25	1.10
time (sec)	N/A	0.168	2.031	1.850	0.448	1.968	0.000	0.269	0.183

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	248	128	110	516	0	125	113
normalized size	1	1.00	2.48	1.28	1.10	5.16	0.00	1.25	1.13
time (sec)	N/A	0.135	2.372	0.701	0.449	0.700	0.000	1.421	4.788

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	84	106	88	406	0	93	70
normalized size	1	1.00	1.02	1.29	1.07	4.95	0.00	1.13	0.85
time (sec)	N/A	0.082	0.281	0.694	0.457	0.768	0.000	0.274	4.409

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	211	66	71	368	0	87	69
normalized size	1	1.00	2.89	0.90	0.97	5.04	0.00	1.19	0.95
time (sec)	N/A	0.074	0.889	0.566	0.433	0.573	0.000	0.670	4.471

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	240	127	106	435	0	119	2056
normalized size	1	1.00	2.61	1.38	1.15	4.73	0.00	1.29	22.35
time (sec)	N/A	0.082	1.914	0.868	0.437	0.773	0.000	0.967	6.668

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	103	174	175	544	0	203	2401
normalized size	1	1.00	0.73	1.23	1.23	3.83	0.00	1.43	16.91
time (sec)	N/A	0.194	1.329	1.244	0.453	0.795	0.000	0.253	7.692

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	138	276	268	656	0	205	2880
normalized size	1	1.00	0.68	1.36	1.32	3.23	0.00	1.01	14.19
time (sec)	N/A	0.290	1.620	1.322	0.457	0.771	0.000	0.775	11.907

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	499	442	369	789	0	285	3310
normalized size	1	1.00	1.79	1.59	1.33	2.84	0.00	1.03	11.91
time (sec)	N/A	0.347	4.419	1.531	0.462	0.952	0.000	0.525	8.558

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	128	108	179	472	0	122	113
normalized size	1	1.00	1.19	1.00	1.66	4.37	0.00	1.13	1.05
time (sec)	N/A	0.099	0.476	0.901	0.434	1.676	0.000	0.359	0.223

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	163	124	212	544	0	162	129
normalized size	1	1.00	1.30	0.99	1.70	4.35	0.00	1.30	1.03
time (sec)	N/A	0.106	0.608	0.930	0.446	1.284	0.000	0.744	4.681

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	927	142	233	613	0	185	149
normalized size	1	1.00	6.44	0.99	1.62	4.26	0.00	1.28	1.03
time (sec)	N/A	0.131	6.824	0.962	0.451	1.198	0.000	1.104	0.254

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	2382	149	253	727	0	205	175
normalized size	1	1.00	15.27	0.96	1.62	4.66	0.00	1.31	1.12
time (sec)	N/A	0.194	7.511	1.405	0.439	1.641	0.000	0.479	4.742

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	194	177	272	856	0	239	256
normalized size	1	1.00	1.07	0.98	1.50	4.73	0.00	1.32	1.41
time (sec)	N/A	0.243	4.386	1.236	0.444	0.903	0.000	0.443	0.390

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	2670	214	303	995	0	284	257
normalized size	1	1.00	12.48	1.00	1.42	4.65	0.00	1.33	1.20
time (sec)	N/A	0.258	7.597	1.320	0.456	0.925	0.000	0.319	0.327

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	125	294	210	722	0	193	149
normalized size	1	1.00	0.88	2.07	1.48	5.08	0.00	1.36	1.05
time (sec)	N/A	0.157	0.899	0.638	0.449	1.447	0.000	0.358	5.211

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	283	238	187	654	0	171	125
normalized size	1	1.00	2.30	1.93	1.52	5.32	0.00	1.39	1.02
time (sec)	N/A	0.102	3.502	0.618	0.458	0.885	0.000	1.362	5.102

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	265	97	156	580	0	130	112
normalized size	1	1.00	2.50	0.92	1.47	5.47	0.00	1.23	1.06
time (sec)	N/A	0.082	2.541	0.791	0.457	0.957	0.000	0.416	5.001

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	332	321	231	819	0	205	3271
normalized size	1	1.00	2.31	2.23	1.60	5.69	0.00	1.42	22.72
time (sec)	N/A	0.175	5.567	1.115	0.464	0.780	0.000	1.420	9.331

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	366	338	970	0	239	3708
normalized size	1	1.00	0.78	1.82	1.68	4.83	0.00	1.19	18.45
time (sec)	N/A	0.336	3.469	1.606	0.461	0.600	0.000	0.337	9.973

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	1430	470	464	1129	0	491	4158
normalized size	1	1.00	5.32	1.75	1.72	4.20	0.00	1.83	15.46
time (sec)	N/A	0.377	6.531	1.668	0.473	0.841	0.000	0.656	9.586

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	1770	636	611	1296	0	369	4594
normalized size	1	1.00	5.03	1.81	1.74	3.68	0.00	1.05	13.05
time (sec)	N/A	0.475	6.613	1.595	0.477	0.705	0.000	0.362	10.107

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1411	649	401	1323	0	324	4506
normalized size	1	1.00	6.92	3.18	1.97	6.49	0.00	1.59	22.09
time (sec)	N/A	0.335	6.864	0.819	0.446	0.608	0.000	1.050	9.462

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	70	168	81	100	0	217	-1
normalized size	1	1.00	0.52	1.25	0.60	0.75	0.00	1.62	-0.01
time (sec)	N/A	0.062	2.161	2.327	0.450	0.451	0.000	0.712	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	60	158	62	87	0	182	-1
normalized size	1	1.00	0.59	1.56	0.61	0.86	0.00	1.80	-0.01
time (sec)	N/A	0.051	0.543	1.990	0.454	0.890	0.000	0.493	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	148	40	68	0	137	-1
normalized size	1	1.00	0.75	2.31	0.62	1.06	0.00	2.14	-0.02
time (sec)	N/A	0.041	0.106	2.340	0.430	1.334	0.000	1.150	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	109	21	53	0	141	-1
normalized size	1	1.00	1.00	3.30	0.64	1.61	0.00	4.27	-0.03
time (sec)	N/A	0.030	0.048	3.072	0.439	0.626	0.000	0.347	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	76	37	56	0	0	-1
normalized size	1	1.00	1.25	2.38	1.16	1.75	0.00	0.00	-0.03
time (sec)	N/A	0.035	0.059	1.959	0.435	2.868	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	141	60	94	0	212	-1
normalized size	1	1.00	0.85	2.10	0.90	1.40	0.00	3.16	-0.01
time (sec)	N/A	0.042	0.144	1.664	0.434	0.837	0.000	0.854	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	203	79	125	0	273	-1
normalized size	1	1.00	0.69	2.03	0.79	1.25	0.00	2.73	-0.01
time (sec)	N/A	0.051	0.282	1.778	0.450	1.655	0.000	1.110	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	79	265	94	162	0	275	-1
normalized size	1	1.00	0.59	1.99	0.71	1.22	0.00	2.07	-0.01
time (sec)	N/A	0.061	0.336	1.834	0.454	0.644	0.000	1.699	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	471	0	6562	0	0	0	0	-1
normalized size	1	1.27	0.00	17.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	27.032	2.288	0.000	0.635	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	364	0	4739	0	0	0	0	-1
normalized size	1	1.26	0.00	16.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	11.547	1.503	0.000	1.198	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	271	0	3454	0	0	0	0	-1
normalized size	1	1.24	0.00	15.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	11.659	1.483	0.000	0.831	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	103	69	3408	0	0	0	0	-1
normalized size	1	1.29	0.86	42.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.255	2.017	0.000	0.698	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	299	539	4623	0	0	0	0	-1
normalized size	1	1.22	2.19	18.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	8.500	2.331	0.000	1.432	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	400	0	6394	0	0	0	0	-1
normalized size	1	1.18	0.00	18.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.571	11.509	2.535	0.000	0.807	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	968	2518	317	468	0	0	-1
normalized size	1	1.00	5.20	13.54	1.70	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.170	11.399	2.119	0.357	1.980	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	390	1769	173	390	0	0	-1
normalized size	1	1.00	3.20	14.50	1.42	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.106	8.133	1.716	0.358	0.846	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	210	1098	69	320	0	0	-1
normalized size	1	1.00	2.76	14.45	0.91	4.21	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.607	1.362	0.339	0.740	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	588	0	1227	0	0	-1
normalized size	1	1.00	0.00	7.44	0.00	15.53	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.091	2.385	0.000	1.080	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	136	1064	0	499	0	0	-1
normalized size	1	1.00	1.66	12.98	0.00	6.09	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.666	1.762	0.000	1.354	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	152	1713	0	567	0	0	-1
normalized size	1	1.00	1.09	12.24	0.00	4.05	0.00	0.00	-0.01
time (sec)	N/A	0.123	1.233	2.309	0.000	0.838	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	1902	2436	0	641	0	0	-1
normalized size	1	1.00	9.70	12.43	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.206	16.368	2.463	0.000	2.012	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	572	0	7996	0	0	0	0	-1
normalized size	1	1.27	0.00	17.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	9.683	2.831	0.000	1.041	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	470	0	6562	0	0	0	0	-1
normalized size	1	1.27	0.00	17.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.700	16.240	1.816	0.000	0.736	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	366	0	5185	0	0	0	0	-1
normalized size	1	1.26	0.00	17.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	11.482	1.468	0.000	0.579	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	277	0	3632	0	0	0	0	-1
normalized size	1	1.24	0.00	16.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	12.581	1.546	0.000	0.578	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	294	179	5069	0	0	0	0	-1
normalized size	1	1.22	0.74	21.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	1.879	1.707	0.000	0.485	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	395	350	6396	0	0	0	0	-1
normalized size	1	1.24	1.10	20.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.641	8.980	2.095	0.000	0.684	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	512	3343	415	566	0	0	-1
normalized size	1	1.00	2.11	13.76	1.71	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.223	10.854	2.801	0.355	8.545	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	400	2519	243	470	0	0	-1
normalized size	1	1.00	2.42	15.27	1.47	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.131	9.172	2.318	0.364	1.990	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	1768	104	390	0	0	-1
normalized size	1	1.00	0.76	15.93	0.94	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.245	1.502	0.341	0.954	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	1457	0	0	-1
normalized size	1	1.00	4.47	13.19	0.00	12.35	0.00	0.00	-0.01
time (sec)	N/A	0.092	1.875	1.346	0.000	1.143	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	466	1512	0	1403	0	0	-1
normalized size	1	1.00	3.76	12.19	0.00	11.31	0.00	0.00	-0.01
time (sec)	N/A	0.136	7.556	1.448	0.000	1.366	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	191	1713	0	563	0	0	-1
normalized size	1	1.00	1.53	13.70	0.00	4.50	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.025	1.723	0.000	0.888	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	165	2439	0	647	0	0	-1
normalized size	1	1.00	0.85	12.64	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.162	2.006	2.075	0.000	2.242	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	706	2231	0	1611	0	0	-1
normalized size	1	1.00	4.25	13.44	0.00	9.70	0.00	0.00	-0.01
time (sec)	N/A	0.172	10.514	1.804	0.000	2.250	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	109	429	0	160	0	0	-1
normalized size	1	1.00	2.60	10.21	0.00	3.81	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.175	1.510	0.000	0.527	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	57	190	0	131	0	0	-1
normalized size	1	1.00	2.38	7.92	0.00	5.46	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.049	2.704	0.000	0.574	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	380	0	4753	0	0	0	0	-1
normalized size	1	1.15	0.00	14.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	11.483	2.163	0.000	0.549	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	202	0	3029	0	0	0	0	-1
normalized size	1	1.19	0.00	17.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.371	10.627	1.719	0.000	0.480	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	103	69	269	0	0	0	0	-1
normalized size	1	1.29	0.86	3.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.179	1.553	0.000	0.486	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	128	279	2985	0	0	0	0	-1
normalized size	1	1.22	2.66	28.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	5.187	1.729	0.000	0.495	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	296	0	4640	0	0	0	0	-1
normalized size	1	1.16	0.00	18.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	7.343	2.051	0.000	0.595	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	345	395	0	6382	0	0	0	0	-1
normalized size	1	1.14	0.00	18.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	14.745	2.485	0.000	0.562	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	326	1756	160	396	0	0	-1
normalized size	1	1.00	2.38	12.82	1.17	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.135	9.527	2.199	0.360	0.865	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	326	1086	74	324	0	0	-1
normalized size	1	1.00	4.02	13.41	0.91	4.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	10.278	2.040	0.330	0.743	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	379	23	215	0	0	-1
normalized size	1	1.00	2.23	9.72	0.59	5.51	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.124	1.597	0.330	0.596	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	992	408	0	0	-1
normalized size	1	1.00	2.23	9.74	25.44	10.46	0.00	0.00	-0.03
time (sec)	N/A	0.030	0.079	1.630	0.626	0.641	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	126	1056	0	502	0	0	-1
normalized size	1	1.00	1.45	12.14	0.00	5.77	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.239	1.983	0.000	0.650	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	1840	1701	0	567	0	0	-1
normalized size	1	1.00	12.87	11.90	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.142	16.228	2.366	0.000	1.188	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1739	2425	0	643	0	0	-1
normalized size	1	1.00	8.52	11.89	0.00	3.15	0.00	0.00	-0.00
time (sec)	N/A	0.207	16.809	2.511	0.000	1.954	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	367	0	12510	0	0	0	0	-1
normalized size	1	1.27	0.00	43.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	20.817	2.218	0.000	0.494	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	182	113	6601	0	0	0	0	-1
normalized size	1	1.21	0.75	44.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.384	2.350	1.950	0.000	0.496	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	284	822	6593	0	0	0	0	-1
normalized size	1	1.24	3.59	28.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	9.601	2.250	0.000	0.520	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	295	0	8684	0	0	0	0	-1
normalized size	1	1.23	0.00	36.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	14.371	2.504	0.000	0.559	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	399	0	11939	0	0	0	0	-1
normalized size	1	1.19	0.00	35.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	16.291	2.506	0.000	0.570	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	509	0	15199	0	0	0	0	-1
normalized size	1	1.17	0.00	34.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	15.466	2.756	0.000	0.673	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	260	4338	161	524	0	0	-1
normalized size	1	1.00	1.88	31.43	1.17	3.80	0.00	0.00	-0.01
time (sec)	N/A	0.146	7.698	1.881	0.358	0.917	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	405	3068	78	410	0	0	-1
normalized size	1	1.00	5.26	39.84	1.01	5.32	0.00	0.00	-0.01
time (sec)	N/A	0.097	7.922	1.825	0.351	0.641	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	57	59	30	65	0	0	199
normalized size	1	1.00	1.78	1.84	0.94	2.03	0.00	0.00	6.22
time (sec)	N/A	0.075	0.627	1.449	0.330	0.550	0.000	0.000	6.291

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	2055	601	0	0	-1
normalized size	1	1.00	2.18	13.08	26.69	7.81	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.382	1.910	0.841	0.743	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	2059	1645	0	699	0	0	-1
normalized size	1	1.00	15.72	12.56	0.00	5.34	0.00	0.00	-0.01
time (sec)	N/A	0.155	15.325	2.392	0.000	1.178	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	2046	2372	0	811	0	0	-1
normalized size	1	1.00	10.55	12.23	0.00	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.226	16.668	2.341	0.000	2.522	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	2068	3171	0	941	0	0	-1
normalized size	1	1.00	7.63	11.70	0.00	3.47	0.00	0.00	-0.00
time (sec)	N/A	0.315	19.725	2.756	0.000	7.326	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	383	167	14353	0	0	0	0	-1
normalized size	1	1.19	0.52	44.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	2.763	2.393	0.000	0.525	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	381	1156	10271	0	0	0	0	-1
normalized size	1	1.19	3.62	32.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	10.165	2.103	0.000	0.546	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	389	0	14353	0	0	0	0	-1
normalized size	1	1.19	0.00	43.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	12.276	2.085	0.000	0.596	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	411	0	17502	0	0	0	0	-1
normalized size	1	1.18	0.00	50.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	16.095	2.613	0.000	0.641	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	441	512	0	20922	0	0	0	0	-1
normalized size	1	1.16	0.00	47.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	12.192	3.525	0.000	1.066	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	559	639	0	26983	0	0	0	0	-1
normalized size	1	1.14	0.00	48.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.906	22.283	5.251	0.000	0.938	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	357	3018	275	688	0	0	-1
normalized size	1	1.00	2.68	22.69	2.07	5.17	0.00	0.00	-0.01
time (sec)	N/A	0.139	11.701	2.097	0.386	0.926	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	74	76	117	134	0	0	153
normalized size	1	1.00	0.94	0.96	1.48	1.70	0.00	0.00	1.94
time (sec)	N/A	0.093	4.298	1.964	0.363	0.862	0.000	0.000	12.666

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	215	85	61	134	0	0	172
normalized size	1	1.00	3.03	1.20	0.86	1.89	0.00	0.00	2.42
time (sec)	N/A	0.090	6.116	1.607	0.370	0.703	0.000	0.000	13.902

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	881	0	0	-1
normalized size	1	1.00	15.42	24.19	0.00	7.05	0.00	0.00	-0.01
time (sec)	N/A	0.102	6.495	1.788	0.000	1.441	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1775	4270	0	1023	0	0	-1
normalized size	1	1.00	9.49	22.83	0.00	5.47	0.00	0.00	-0.01
time (sec)	N/A	0.243	17.331	2.407	0.000	3.357	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	1777	5600	0	1187	0	0	-1
normalized size	1	1.00	6.81	21.46	0.00	4.55	0.00	0.00	-0.00
time (sec)	N/A	0.338	20.476	2.936	0.000	9.318	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	1776	6934	0	1337	0	0	-1
normalized size	1	1.00	5.35	20.89	0.00	4.03	0.00	0.00	-0.00
time (sec)	N/A	0.429	27.569	3.751	0.000	26.445	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	1777	6116	0	1241	0	0	-1
normalized size	1	1.00	9.93	34.17	0.00	6.93	0.00	0.00	-0.01
time (sec)	N/A	0.194	18.539	2.643	0.000	3.722	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	37	142	388	53	0	0	-1
normalized size	1	1.00	2.64	10.14	27.71	3.79	0.00	0.00	-0.07
time (sec)	N/A	0.020	0.027	1.096	0.577	0.568	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	2195	0	0	0	0	0	-1
normalized size	1	0.00	19.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	18.900	4.431	0.000	0.566	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	124	1989	0	0	0	0	0	-1
normalized size	1	1.20	19.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	17.154	1.556	0.000	0.505	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	124	1995	0	0	0	0	0	-1
normalized size	1	1.20	19.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	16.583	1.449	0.000	0.519	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	122	1983	0	0	0	0	0	-1
normalized size	1	1.21	19.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	16.500	1.907	0.000	0.556	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	124	1987	0	0	0	0	0	-1
normalized size	1	1.20	19.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	17.036	5.567	0.000	0.509	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	124	1997	0	0	0	0	0	-1
normalized size	1	1.20	19.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	17.407	6.060	0.000	0.516	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	2.106	2.477	0.000	0.558	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	126	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	2.260	1.736	0.000	0.487	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.011	1.308	0.000	0.555	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	-1
normalized size	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	6.256	0.046	0.000	0.472	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	-1
normalized size	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	15.765	2.594	0.000	0.498	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1912	0	0	0	0	0	-1
normalized size	1	1.00	23.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	16.282	3.015	0.000	0.503	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	-1
normalized size	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	17.120	3.746	0.000	0.579	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	65	95	69	116	0	52
normalized size	1	1.00	0.76	0.90	1.32	0.96	1.61	0.00	0.72
time (sec)	N/A	0.062	0.155	0.625	0.326	0.489	5.175	0.000	4.699

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	50	64	50	80	0	46
normalized size	1	1.00	0.88	1.02	1.31	1.02	1.63	0.00	0.94
time (sec)	N/A	0.049	0.078	0.533	0.327	0.540	1.671	0.000	4.785

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	28	33	37	42	0	32
normalized size	1	1.00	1.00	0.93	1.10	1.23	1.40	0.00	1.07
time (sec)	N/A	0.024	0.015	0.226	0.320	0.487	0.465	0.000	4.927

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	26	33	35	0	0	32
normalized size	1	1.00	1.57	0.93	1.18	1.25	0.00	0.00	1.14
time (sec)	N/A	0.047	0.029	0.597	0.338	0.507	0.000	0.000	4.868

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	43	29	50	0	0	51
normalized size	1	1.00	1.62	1.34	0.91	1.56	0.00	0.00	1.59
time (sec)	N/A	0.051	0.158	0.738	0.322	0.452	0.000	0.000	6.143

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	64	64	49	83	0	0	61
normalized size	1	1.00	1.25	1.25	0.96	1.63	0.00	0.00	1.20
time (sec)	N/A	0.071	0.174	0.668	0.324	0.458	0.000	0.000	6.099

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	73	61	56	89	66	61	51
normalized size	1	1.00	1.14	0.95	0.88	1.39	1.03	0.95	0.80
time (sec)	N/A	0.062	0.026	0.582	0.415	0.501	3.496	7.146	4.986

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	50	45	72	54	49	40
normalized size	1	1.00	1.19	1.04	0.94	1.50	1.12	1.02	0.83
time (sec)	N/A	0.057	0.020	0.580	0.411	0.461	2.212	1.674	4.681

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	41	33	53	42	36	29
normalized size	1	1.00	1.28	1.28	1.03	1.66	1.31	1.12	0.91
time (sec)	N/A	0.052	0.015	0.633	0.419	0.490	1.595	0.748	4.520

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	16	17
normalized size	1	1.00	1.00	1.07	1.00	2.07	0.00	1.07	1.13
time (sec)	N/A	0.013	0.002	0.874	0.331	0.485	0.000	0.872	4.515

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	33	25	34	0	57	19
normalized size	1	1.00	2.26	1.74	1.32	1.79	0.00	3.00	1.00
time (sec)	N/A	0.054	0.029	0.886	0.431	0.494	0.000	0.221	4.498

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	51	48	41	76	0	119	35
normalized size	1	1.00	1.55	1.45	1.24	2.30	0.00	3.61	1.06
time (sec)	N/A	0.059	0.021	0.741	0.419	0.487	0.000	0.258	4.671

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	63	52	110	0	182	46
normalized size	1	1.00	1.00	1.24	1.02	2.16	0.00	3.57	0.90
time (sec)	N/A	0.063	0.028	0.835	0.420	0.494	0.000	0.399	4.886

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	126	120	147	99	190	0	124
normalized size	1	1.00	1.26	1.20	1.47	0.99	1.90	0.00	1.24
time (sec)	N/A	0.101	0.431	0.692	0.344	0.563	14.180	0.000	4.509

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	107	103	114	79	128	0	92
normalized size	1	1.00	1.39	1.34	1.48	1.03	1.66	0.00	1.19
time (sec)	N/A	0.084	0.247	0.625	0.325	0.476	5.081	0.000	4.544

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	46	67	53	61	0	61
normalized size	1	1.00	1.71	0.96	1.40	1.10	1.27	0.00	1.27
time (sec)	N/A	0.042	0.110	0.337	0.330	0.458	1.667	0.000	4.501

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	84	60	64	79	0	0	58
normalized size	1	1.00	1.58	1.13	1.21	1.49	0.00	0.00	1.09
time (sec)	N/A	0.074	0.241	0.935	0.342	0.518	0.000	0.000	4.483

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	81	78	60	100	0	0	68
normalized size	1	1.00	1.42	1.37	1.05	1.75	0.00	0.00	1.19
time (sec)	N/A	0.081	0.176	1.182	0.333	0.461	0.000	0.000	4.582

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	77	87	61	97	0	0	83
normalized size	1	1.00	1.51	1.71	1.20	1.90	0.00	0.00	1.63
time (sec)	N/A	0.087	0.247	0.845	0.338	0.496	0.000	0.000	4.613

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	275	105	84	137	0	98	126
normalized size	1	1.00	2.89	1.11	0.88	1.44	0.00	1.03	1.33
time (sec)	N/A	0.108	2.074	0.770	0.437	0.485	0.000	4.736	4.541

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	395	94	71	113	0	84	97
normalized size	1	1.00	5.13	1.22	0.92	1.47	0.00	1.09	1.26
time (sec)	N/A	0.097	1.101	0.596	0.434	0.504	0.000	5.145	4.575

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	281	85	58	86	0	70	69
normalized size	1	1.00	4.76	1.44	0.98	1.46	0.00	1.19	1.17
time (sec)	N/A	0.093	0.803	0.586	0.436	0.512	0.000	1.084	4.658

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	44	58	0	53	42
normalized size	1	1.00	2.65	1.20	1.10	1.45	0.00	1.32	1.05
time (sec)	N/A	0.029	0.370	1.033	0.340	0.535	0.000	0.221	4.581

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	66	46	67	0	49	44
normalized size	1	1.00	2.28	1.83	1.28	1.86	0.00	1.36	1.22
time (sec)	N/A	0.080	0.778	0.928	0.449	0.579	0.000	0.394	4.642

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	160	73	59	98	0	187	53
normalized size	1	1.00	3.56	1.62	1.31	2.18	0.00	4.16	1.18
time (sec)	N/A	0.087	0.938	1.284	0.455	0.488	0.000	0.573	4.608

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	256	107	72	136	0	290	68
normalized size	1	1.00	3.94	1.65	1.11	2.09	0.00	4.46	1.05
time (sec)	N/A	0.093	1.031	1.165	0.434	0.628	0.000	1.012	4.837

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	99	112	81	84	0	0	103
normalized size	1	1.00	1.43	1.62	1.17	1.22	0.00	0.00	1.49
time (sec)	N/A	0.100	0.270	0.731	0.341	0.712	0.000	0.000	4.612

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	59	50	41	0	0	64
normalized size	1	1.00	0.91	1.31	1.11	0.91	0.00	0.00	1.42
time (sec)	N/A	0.077	0.099	0.640	0.357	0.704	0.000	0.000	4.486

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	37	26	21	128	0	63
normalized size	1	1.00	1.13	1.61	1.13	0.91	5.57	0.00	2.74
time (sec)	N/A	0.031	0.189	0.227	0.342	0.529	12.698	0.000	4.558

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	73	50	42	0	0	65
normalized size	1	1.00	0.93	1.59	1.09	0.91	0.00	0.00	1.41
time (sec)	N/A	0.080	0.096	0.748	0.345	0.537	0.000	0.000	4.695

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	100	158	87	126	0	0	98
normalized size	1	1.00	1.35	2.14	1.18	1.70	0.00	0.00	1.32
time (sec)	N/A	0.110	0.228	1.028	0.343	0.940	0.000	0.000	4.935

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	138	293	145	265	0	0	160
normalized size	1	1.00	1.28	2.71	1.34	2.45	0.00	0.00	1.48
time (sec)	N/A	0.148	0.617	1.042	0.349	0.918	0.000	0.000	4.905

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	229	186	95	373	0	132	1109
normalized size	1	1.00	2.76	2.24	1.14	4.49	0.00	1.59	13.36
time (sec)	N/A	0.273	2.790	0.839	0.442	0.538	0.000	6.308	4.923

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	206	121	66	297	0	91	410
normalized size	1	1.00	3.49	2.05	1.12	5.03	0.00	1.54	6.95
time (sec)	N/A	0.167	1.097	0.679	0.441	0.581	0.000	3.025	4.684

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	184	75	45	226	0	69	126
normalized size	1	1.00	4.00	1.63	0.98	4.91	0.00	1.50	2.74
time (sec)	N/A	0.126	0.293	0.717	0.437	0.539	0.000	0.722	4.717

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	48	44	231	0	68	460
normalized size	1	1.00	4.04	1.07	0.98	5.13	0.00	1.51	10.22
time (sec)	N/A	0.045	0.233	0.581	0.454	0.592	0.000	0.295	4.856

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	204	73	66	310	0	91	637
normalized size	1	1.00	3.29	1.18	1.06	5.00	0.00	1.47	10.27
time (sec)	N/A	0.174	1.354	0.739	0.454	0.496	0.000	1.063	6.252

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	390	110	106	533	0	140	2644
normalized size	1	1.00	4.53	1.28	1.23	6.20	0.00	1.63	30.74
time (sec)	N/A	0.248	3.379	1.059	0.459	0.533	0.000	0.326	8.973

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	671	173	161	833	0	222	4324
normalized size	1	1.00	5.59	1.44	1.34	6.94	0.00	1.85	36.03
time (sec)	N/A	0.344	2.843	1.252	0.437	0.623	0.000	0.836	10.170

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	109	126	98	118	0	0	170
normalized size	1	1.00	1.42	1.64	1.27	1.53	0.00	0.00	2.21
time (sec)	N/A	0.106	0.426	1.025	0.346	0.664	0.000	0.000	4.655

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	68	59	53	0	0	97
normalized size	1	1.00	1.59	1.33	1.16	1.04	0.00	0.00	1.90
time (sec)	N/A	0.082	0.718	0.999	0.357	0.537	0.000	0.000	4.591

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	79	59	57	52	0	0	90
normalized size	1	1.00	1.61	1.20	1.16	1.06	0.00	0.00	1.84
time (sec)	N/A	0.056	0.482	0.327	0.357	0.531	0.000	0.000	4.481

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	112	155	117	138	0	0	106
normalized size	1	1.00	1.35	1.87	1.41	1.66	0.00	0.00	1.28
time (sec)	N/A	0.114	0.331	1.092	0.349	0.720	0.000	0.000	4.851

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	130	240	192	312	0	0	160
normalized size	1	1.00	1.17	2.16	1.73	2.81	0.00	0.00	1.44
time (sec)	N/A	0.157	1.311	1.273	0.357	0.945	0.000	0.000	5.051

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	162	374	279	557	0	0	206
normalized size	1	1.00	1.16	2.67	1.99	3.98	0.00	0.00	1.47
time (sec)	N/A	0.197	1.837	1.224	0.343	1.557	0.000	0.000	6.094

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	286	242	127	514	0	163	765
normalized size	1	1.00	2.40	2.03	1.07	4.32	0.00	1.37	6.43
time (sec)	N/A	0.267	4.626	0.768	0.433	0.580	0.000	8.754	4.989

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	249	168	96	393	0	126	285
normalized size	1	1.00	2.77	1.87	1.07	4.37	0.00	1.40	3.17
time (sec)	N/A	0.176	2.560	0.897	0.427	0.533	0.000	1.762	4.780

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	346	108	75	458	0	99	711
normalized size	1	1.00	4.07	1.27	0.88	5.39	0.00	1.16	8.36
time (sec)	N/A	0.156	7.681	0.953	0.424	0.678	0.000	1.474	4.999

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	240	127	106	435	0	119	2056
normalized size	1	1.00	2.61	1.38	1.15	4.73	0.00	1.29	22.35
time (sec)	N/A	0.085	1.921	0.917	0.424	0.559	0.000	0.235	6.393

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	288	149	163	604	0	183	3146
normalized size	1	1.00	2.38	1.23	1.35	4.99	0.00	1.51	26.00
time (sec)	N/A	0.254	3.938	1.063	0.443	0.600	0.000	1.307	9.296

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	1588	186	235	979	0	220	4987
normalized size	1	1.00	9.92	1.16	1.47	6.12	0.00	1.38	31.17
time (sec)	N/A	0.354	6.800	1.344	0.447	0.608	0.000	0.504	10.462

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	3028	248	319	1505	0	310	6017
normalized size	1	1.00	14.63	1.20	1.54	7.27	0.00	1.50	29.07
time (sec)	N/A	0.437	7.360	1.351	0.472	0.608	0.000	1.162	10.865

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	136	138	112	116	0	0	166
normalized size	1	1.00	1.74	1.77	1.44	1.49	0.00	0.00	2.13
time (sec)	N/A	0.108	2.155	0.864	0.350	0.541	0.000	0.000	4.602

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	131	115	113	111	0	0	153
normalized size	1	1.00	1.62	1.42	1.40	1.37	0.00	0.00	1.89
time (sec)	N/A	0.112	0.930	0.813	0.351	0.556	0.000	0.000	4.449

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	129	81	102	102	0	0	142
normalized size	1	1.00	1.74	1.09	1.38	1.38	0.00	0.00	1.92
time (sec)	N/A	0.074	1.331	0.391	0.367	0.527	0.000	0.000	4.404

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	158	304	243	307	0	0	190
normalized size	1	1.00	1.22	2.34	1.87	2.36	0.00	0.00	1.46
time (sec)	N/A	0.165	1.001	1.067	0.371	1.074	0.000	0.000	4.863

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	176	389	344	584	0	0	272
normalized size	1	1.00	1.14	2.53	2.23	3.79	0.00	0.00	1.77
time (sec)	N/A	0.209	1.735	1.514	0.354	1.949	0.000	0.000	6.033

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	208	522	454	859	0	0	327
normalized size	1	1.00	1.08	2.72	2.36	4.47	0.00	0.00	1.70
time (sec)	N/A	0.269	5.265	1.459	0.355	3.170	0.000	0.000	6.420

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	523	356	193	664	0	211	615
normalized size	1	1.00	3.56	2.42	1.31	4.52	0.00	1.44	4.18
time (sec)	N/A	0.292	6.410	1.087	0.438	0.637	0.000	6.254	4.899

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	1473	264	163	763	0	168	1117
normalized size	1	1.00	10.75	1.93	1.19	5.57	0.00	1.23	8.15
time (sec)	N/A	0.257	14.156	0.863	0.446	0.577	0.000	2.373	5.403

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	1334	263	191	860	0	181	2405
normalized size	1	1.00	9.67	1.91	1.38	6.23	0.00	1.31	17.43
time (sec)	N/A	0.227	10.601	0.721	0.441	0.551	0.000	1.309	7.442

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	332	321	231	819	0	205	3271
normalized size	1	1.00	2.31	2.23	1.60	5.69	0.00	1.42	22.72
time (sec)	N/A	0.178	5.562	0.844	0.465	0.625	0.000	0.230	8.427

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	2089	337	311	1060	0	257	4890
normalized size	1	1.00	11.54	1.86	1.72	5.86	0.00	1.42	27.02
time (sec)	N/A	0.380	7.092	1.154	0.464	0.684	0.000	2.405	10.706

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	3340	374	409	1649	0	314	7057
normalized size	1	1.00	14.52	1.63	1.78	7.17	0.00	1.37	30.68
time (sec)	N/A	0.461	7.756	1.480	0.460	0.877	0.000	4.141	11.983

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	976	437	520	2229	0	404	7460
normalized size	1	1.00	3.42	1.53	1.82	7.82	0.00	1.42	26.18
time (sec)	N/A	0.607	8.342	1.634	0.467	0.974	0.000	1.274	15.210

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	924	0	456	0	0	-1
normalized size	1	1.00	0.00	8.32	0.00	4.11	0.00	0.00	-0.01
time (sec)	N/A	0.141	2.217	2.052	0.000	3.278	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	648	0	386	0	0	-1
normalized size	1	1.00	0.00	8.10	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.105	1.010	1.557	0.000	1.242	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	119	61	0	312	0	0	46
normalized size	1	1.00	2.20	1.13	0.00	5.78	0.00	0.00	0.85
time (sec)	N/A	0.065	0.438	0.251	0.000	0.691	0.000	0.000	5.487

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	C	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	570	3317	963	0	0	-1
normalized size	1	1.00	0.00	8.14	47.39	13.76	0.00	0.00	-0.01
time (sec)	N/A	0.110	1.612	1.851	0.932	0.925	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	527	2528	0	1342	0	0	-1
normalized size	1	1.00	4.83	23.19	0.00	12.31	0.00	0.00	-0.01
time (sec)	N/A	0.149	5.681	1.710	0.000	1.275	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	0	7346	0	1953	0	0	-1
normalized size	1	1.00	0.00	45.63	0.00	12.13	0.00	0.00	-0.01
time (sec)	N/A	0.229	5.081	1.811	0.000	2.712	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	263	2756	0	1775	0	0	-1
normalized size	1	1.00	1.20	12.58	0.00	8.11	0.00	0.00	-0.00
time (sec)	N/A	0.436	3.571	2.283	0.000	7.637	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	208	2005	0	1621	0	0	-1
normalized size	1	1.00	1.26	12.15	0.00	9.82	0.00	0.00	-0.01
time (sec)	N/A	0.314	2.633	1.952	0.000	2.196	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	526	1331	0	1471	0	0	-1
normalized size	1	1.00	4.46	11.28	0.00	12.47	0.00	0.00	-0.01
time (sec)	N/A	0.223	4.193	1.576	0.000	1.154	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	589	0	1227	0	0	-1
normalized size	1	1.00	0.00	7.46	0.00	15.53	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.084	1.763	0.000	0.806	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	130	1004	0	499	0	0	-1
normalized size	1	1.00	1.88	14.55	0.00	7.23	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.572	1.701	0.000	0.687	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	176	3855	0	629	0	0	-1
normalized size	1	1.00	1.54	33.82	0.00	5.52	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.735	1.573	0.000	1.243	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	178	8605	0	849	0	0	-1
normalized size	1	1.00	1.07	51.53	0.00	5.08	0.00	0.00	-0.01
time (sec)	N/A	0.333	1.651	1.898	0.000	3.951	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	2606	0	527	0	0	-1
normalized size	1	1.00	0.00	19.30	0.00	3.90	0.00	0.00	-0.01
time (sec)	N/A	0.164	2.959	1.692	0.000	12.892	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	0	2150	0	443	0	0	-1
normalized size	1	1.00	0.00	20.67	0.00	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.943	1.587	0.000	3.587	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	84	81	0	373	0	0	66
normalized size	1	1.00	1.08	1.04	0.00	4.78	0.00	0.00	0.85
time (sec)	N/A	0.082	0.232	0.304	0.000	1.058	0.000	0.000	7.158

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	506	2424	0	1075	0	0	-1
normalized size	1	1.00	5.56	26.64	0.00	11.81	0.00	0.00	-0.01
time (sec)	N/A	0.136	5.245	1.839	0.000	1.049	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	622	1609	0	1300	0	0	-1
normalized size	1	1.00	5.46	14.11	0.00	11.40	0.00	0.00	-0.01
time (sec)	N/A	0.170	5.440	1.858	0.000	1.032	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	684	4955	0	1801	0	0	-1
normalized size	1	1.00	4.30	31.16	0.00	11.33	0.00	0.00	-0.01
time (sec)	N/A	0.243	5.586	1.658	0.000	3.157	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	353	3583	0	1973	0	0	-1
normalized size	1	1.00	1.22	12.36	0.00	6.80	0.00	0.00	-0.00
time (sec)	N/A	0.571	6.188	2.436	0.000	25.228	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	258	2757	0	1777	0	0	-1
normalized size	1	1.00	1.21	12.88	0.00	8.30	0.00	0.00	-0.00
time (sec)	N/A	0.478	4.103	1.913	0.000	7.523	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	703	2002	0	1627	0	0	-1
normalized size	1	1.00	4.23	12.06	0.00	9.80	0.00	0.00	-0.01
time (sec)	N/A	0.365	6.181	1.532	0.000	2.138	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	1457	0	0	-1
normalized size	1	1.00	4.47	13.19	0.00	12.35	0.00	0.00	-0.01
time (sec)	N/A	0.096	1.793	1.435	0.000	1.120	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	410	1952	0	1446	0	0	-1
normalized size	1	1.00	3.69	17.59	0.00	13.03	0.00	0.00	-0.01
time (sec)	N/A	0.228	6.124	2.032	0.000	1.210	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	100	2014	0	597	0	0	-1
normalized size	1	1.00	0.89	17.98	0.00	5.33	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.311	1.873	0.000	1.540	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	139	5850	0	767	0	0	-1
normalized size	1	1.00	0.84	35.45	0.00	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.355	1.353	2.015	0.000	5.496	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	358	0	410	0	0	-1
normalized size	1	1.00	0.00	4.02	0.00	4.61	0.00	0.00	-0.01
time (sec)	N/A	0.133	1.919	1.780	0.000	1.148	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	0	303	0	328	0	0	-1
normalized size	1	1.00	0.00	5.41	0.00	5.86	0.00	0.00	-0.02
time (sec)	N/A	0.095	1.503	1.503	0.000	0.693	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	B	F	F(-2)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	0	42	0	261	0	0	27
normalized size	1	1.00	0.00	1.27	0.00	7.91	0.00	0.00	0.82
time (sec)	N/A	0.055	0.121	0.253	0.000	0.581	0.000	0.000	5.090

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	376	0	1015	0	0	-1
normalized size	1	1.00	0.00	5.37	0.00	14.50	0.00	0.00	-0.01
time (sec)	N/A	0.106	2.203	1.875	0.000	0.770	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	4245	0	1550	0	0	-1
normalized size	1	1.00	0.00	36.59	0.00	13.36	0.00	0.00	-0.01
time (sec)	N/A	0.164	4.760	2.012	0.000	1.019	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	0	10441	0	2257	0	0	-1
normalized size	1	1.00	0.00	62.90	0.00	13.60	0.00	0.00	-0.01
time (sec)	N/A	0.244	7.301	2.674	0.000	2.373	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	230	1993	0	1673	0	0	-1
normalized size	1	1.00	1.33	11.52	0.00	9.67	0.00	0.00	-0.01
time (sec)	N/A	0.319	4.348	2.342	0.000	2.848	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	196	1322	0	1507	0	0	-1
normalized size	1	1.00	1.63	11.02	0.00	12.56	0.00	0.00	-0.01
time (sec)	N/A	0.230	2.808	1.731	0.000	1.133	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	404	0	1259	0	0	-1
normalized size	1	1.00	0.00	5.05	0.00	15.74	0.00	0.00	-0.01
time (sec)	N/A	0.197	2.708	1.515	0.000	0.907	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	992	408	0	0	-1
normalized size	1	1.00	2.23	9.74	25.44	10.46	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.106	1.640	0.626	0.726	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	127	1865	0	525	0	0	-1
normalized size	1	1.00	1.72	25.20	0.00	7.09	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.204	1.862	0.000	0.716	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	168	5619	0	723	0	0	-1
normalized size	1	1.00	1.41	47.22	0.00	6.08	0.00	0.00	-0.01
time (sec)	N/A	0.249	1.830	1.923	0.000	1.189	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	199	11267	0	987	0	0	-1
normalized size	1	1.00	1.16	65.51	0.00	5.74	0.00	0.00	-0.01
time (sec)	N/A	0.345	4.171	2.269	0.000	3.438	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	6593	0	458	0	0	-1
normalized size	1	1.00	0.00	74.92	0.00	5.20	0.00	0.00	-0.01
time (sec)	N/A	0.151	4.921	1.979	0.000	1.422	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	187	2368	2532	417	0	0	-1
normalized size	1	1.00	2.97	37.59	40.19	6.62	0.00	0.00	-0.02
time (sec)	N/A	0.116	3.909	1.697	0.954	0.814	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	382	64	0	392	53	0	49
normalized size	1	1.00	6.70	1.12	0.00	6.88	0.93	0.00	0.86
time (sec)	N/A	0.072	6.361	0.210	0.000	0.624	14.815	0.000	5.558

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	9693	0	1569	0	0	-1
normalized size	1	1.00	0.00	96.93	0.00	15.69	0.00	0.00	-0.01
time (sec)	N/A	0.147	5.939	2.056	0.000	1.218	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	0	19968	0	2347	0	0	-1
normalized size	1	1.00	0.00	130.51	0.00	15.34	0.00	0.00	-0.01
time (sec)	N/A	0.242	9.971	2.253	0.000	2.445	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	0	32565	0	3501	0	0	-1
normalized size	1	1.00	0.00	152.89	0.00	16.44	0.00	0.00	-0.00
time (sec)	N/A	0.334	13.430	2.939	0.000	8.709	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	247	3860	0	1895	0	0	-1
normalized size	1	1.00	1.44	22.44	0.00	11.02	0.00	0.00	-0.01
time (sec)	N/A	0.349	9.693	1.789	0.000	3.100	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	201	2662	0	1655	0	0	-1
normalized size	1	1.00	1.73	22.95	0.00	14.27	0.00	0.00	-0.01
time (sec)	N/A	0.249	4.257	1.722	0.000	2.218	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	169	569	2005	548	0	0	-1
normalized size	1	1.00	2.38	8.01	28.24	7.72	0.00	0.00	-0.01
time (sec)	N/A	0.209	2.436	1.643	0.801	1.088	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	2055	601	0	0	-1
normalized size	1	1.00	2.18	13.08	26.69	7.81	0.00	0.00	-0.01
time (sec)	N/A	0.046	1.362	1.986	0.870	0.909	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	182	2841	0	741	0	0	-1
normalized size	1	1.00	1.53	23.87	0.00	6.23	0.00	0.00	-0.01
time (sec)	N/A	0.272	4.259	1.964	0.000	1.252	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	224	7541	0	1061	0	0	-1
normalized size	1	1.00	1.29	43.34	0.00	6.10	0.00	0.00	-0.01
time (sec)	N/A	0.362	5.397	2.115	0.000	3.818	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	237	14137	0	1517	0	0	-1
normalized size	1	1.00	0.98	58.66	0.00	6.29	0.00	0.00	-0.00
time (sec)	N/A	0.471	10.756	2.262	0.000	11.853	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	187	10947	0	564	0	0	-1
normalized size	1	1.00	1.93	112.86	0.00	5.81	0.00	0.00	-0.01
time (sec)	N/A	0.166	7.815	3.577	0.000	1.762	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	613	10839	0	522	0	0	-1
normalized size	1	1.00	6.89	121.79	0.00	5.87	0.00	0.00	-0.01
time (sec)	N/A	0.128	10.165	3.203	0.000	1.497	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	613	86	0	494	78	0	68
normalized size	1	1.00	7.39	1.04	0.00	5.95	0.94	0.00	0.82
time (sec)	N/A	0.088	7.434	0.230	0.000	1.374	24.143	0.000	8.168

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	68989	0	2279	0	0	-1
normalized size	1	1.00	0.00	503.57	0.00	16.64	0.00	0.00	-0.01
time (sec)	N/A	0.207	8.761	5.236	0.000	2.614	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	0	105237	0	3507	0	0	-1
normalized size	1	1.00	0.00	526.18	0.00	17.54	0.00	0.00	-0.00
time (sec)	N/A	0.320	19.508	7.392	0.000	8.706	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	145925	0	4751	0	0	-1
normalized size	1	1.00	0.00	544.50	0.00	17.73	0.00	0.00	-0.00
time (sec)	N/A	0.449	30.303	18.803	0.000	33.456	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	316	2256	0	2035	0	0	-1
normalized size	1	1.00	2.01	14.37	0.00	12.96	0.00	0.00	-0.01
time (sec)	N/A	0.345	11.264	1.937	0.000	3.884	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	409	1142	0	661	0	0	-1
normalized size	1	1.00	3.41	9.52	0.00	5.51	0.00	0.00	-0.01
time (sec)	N/A	0.267	6.020	1.651	0.000	1.615	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	410	2112	0	773	0	0	-1
normalized size	1	1.00	3.45	17.75	0.00	6.50	0.00	0.00	-0.01
time (sec)	N/A	0.257	4.555	1.523	0.000	1.539	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	881	0	0	-1
normalized size	1	1.00	15.42	24.19	0.00	7.05	0.00	0.00	-0.01
time (sec)	N/A	0.099	6.498	2.196	0.000	1.359	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	247	7586	0	1097	0	0	-1
normalized size	1	1.00	1.42	43.60	0.00	6.30	0.00	0.00	-0.01
time (sec)	N/A	0.378	7.212	2.271	0.000	4.096	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	234	15128	0	1579	0	0	-1
normalized size	1	1.00	0.99	64.10	0.00	6.69	0.00	0.00	-0.00
time (sec)	N/A	0.482	13.954	2.489	0.000	13.454	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	272	22712	0	2059	0	0	-1
normalized size	1	1.00	0.86	72.10	0.00	6.54	0.00	0.00	-0.00
time (sec)	N/A	0.604	24.433	2.516	0.000	43.486	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	259	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	3.488	2.892	0.000	0.976	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	94	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.527	1.482	0.000	0.529	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	61	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.129	1.153	0.000	0.507	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.046	1.312	0.000	0.510	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	115	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	2.125	2.031	0.000	0.525	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	139	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	3.510	1.463	0.000	0.532	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	2777	0	0	0	0	0	-1
normalized size	1	1.00	31.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	18.152	1.602	0.000	0.590	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	2465	0	0	0	0	0	-1
normalized size	1	1.00	28.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	16.539	1.159	0.000	0.528	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	-1
normalized size	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	6.264	0.046	0.000	0.489	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	2469	0	0	0	0	0	-1
normalized size	1	1.00	29.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	16.902	1.273	0.000	0.588	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	3033	0	0	0	0	0	-1
normalized size	1	1.00	34.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	18.347	1.542	0.000	0.548	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	87	183	73	81	119	0	227
normalized size	1	1.00	0.95	1.99	0.79	0.88	1.29	0.00	2.47
time (sec)	N/A	0.069	0.264	1.095	0.341	0.732	8.408	0.000	8.788

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	126	51	59	82	0	167
normalized size	1	1.00	0.97	2.07	0.84	0.97	1.34	0.00	2.74
time (sec)	N/A	0.054	0.121	0.979	0.345	0.598	2.982	0.000	8.392

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	28	28	37	42	0	83
normalized size	1	1.00	1.00	0.93	0.93	1.23	1.40	0.00	2.77
time (sec)	N/A	0.023	0.014	0.363	0.344	0.522	0.877	0.000	5.257

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	65	48	45	61	0	0	72
normalized size	1	1.00	1.20	0.89	0.83	1.13	0.00	0.00	1.33
time (sec)	N/A	0.070	0.057	0.688	0.346	0.462	0.000	0.000	4.638

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	114	69	62	99	0	0	86
normalized size	1	1.00	1.58	0.96	0.86	1.38	0.00	0.00	1.19
time (sec)	N/A	0.063	1.012	1.104	0.339	0.540	0.000	0.000	4.653

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	251	274	218	4427	0	0	7402
normalized size	1	1.00	1.15	1.25	1.00	20.21	0.00	0.00	33.80
time (sec)	N/A	0.321	0.360	0.661	0.441	1.672	0.000	0.000	7.258

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	242	141	159	2278	0	0	1620
normalized size	1	1.00	1.46	0.85	0.96	13.72	0.00	0.00	9.76
time (sec)	N/A	0.147	0.254	0.774	0.443	36.120	0.000	0.000	8.540

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	21	21	170	0	114
normalized size	1	1.00	1.00	1.61	0.91	0.91	7.39	0.00	4.96
time (sec)	N/A	0.031	0.018	0.166	0.342	0.500	43.386	0.000	5.189

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	290	393	306	6482	0	0	11182
normalized size	1	1.00	0.98	1.33	1.04	21.97	0.00	0.00	37.91
time (sec)	N/A	0.517	0.399	1.041	0.443	1.661	0.000	0.000	7.898

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	336	676	488	10746	0	0	58699
normalized size	1	1.00	0.85	1.72	1.24	27.34	0.00	0.00	149.36
time (sec)	N/A	0.632	1.971	1.334	0.432	3.845	0.000	0.000	19.524

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	3.193	6.115	0.000	0.580	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	245	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	10.123	1.925	0.000	0.504	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	162	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	4.589	1.636	0.000	0.571	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.081	1.272	0.000	0.595	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	3.768	1.822	0.000	0.509	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	36.197	2.128	0.000	0.515	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	2.439	1.645	0.000	0.514	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	1.232	1.699	0.000	0.537	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	1.853	1.755	0.000	0.594	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [255] had the largest ratio of [.6000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	21	0.095
2	A	3	2	1.00	21	0.095
3	A	3	2	1.00	21	0.095
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	4	4	1.00	21	0.190
7	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	6	6	1.00	21	0.286
9	A	5	5	1.00	21	0.238
10	A	4	4	1.00	21	0.190
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	21	0.095
13	A	3	2	1.00	21	0.095
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	23	0.087
16	A	3	2	1.00	23	0.087
17	A	3	2	1.00	21	0.095
18	A	4	3	1.00	21	0.143
19	A	5	5	1.00	23	0.217
20	A	6	5	1.00	23	0.217
21	A	7	6	1.00	23	0.261
22	A	6	5	1.00	23	0.217
23	A	5	5	1.00	23	0.217
24	A	4	3	1.00	14	0.214
25	A	3	2	1.00	23	0.087
26	A	3	2	1.00	23	0.087
27	A	3	2	1.00	23	0.087
28	A	4	3	1.00	23	0.130
29	A	4	4	1.00	23	0.174
30	A	3	3	1.00	21	0.143
31	A	4	4	1.00	21	0.190
32	A	5	5	1.00	23	0.217
33	A	6	6	1.00	23	0.261
34	A	7	7	1.00	23	0.304
35	A	6	6	1.00	23	0.261
36	A	5	5	1.00	23	0.217
37	A	3	3	1.00	14	0.214
38	A	3	3	1.00	23	0.130
39	A	4	4	1.00	23	0.174
40	A	4	3	1.00	23	0.130
41	A	6	5	1.00	23	0.217
42	A	5	4	1.00	23	0.174
43	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	5	5	1.00	21	0.238
45	A	6	6	1.00	23	0.261
46	A	7	6	1.00	23	0.261
47	A	8	7	1.00	23	0.304
48	A	7	6	1.00	23	0.261
49	A	6	6	1.00	23	0.261
50	A	5	5	1.00	14	0.357
51	A	4	4	1.00	23	0.174
52	A	5	4	1.00	23	0.174
53	A	6	5	1.00	23	0.217
54	A	6	5	1.00	23	0.217
55	A	6	5	1.00	23	0.217
56	A	5	4	1.00	21	0.190
57	A	6	6	1.00	21	0.286
58	A	7	7	1.00	23	0.304
59	A	8	7	1.00	23	0.304
60	A	9	7	1.00	23	0.304
61	A	8	6	1.00	23	0.261
62	A	7	6	1.00	23	0.261
63	A	6	6	1.00	14	0.429
64	A	5	4	1.00	23	0.174
65	A	6	5	1.00	23	0.217
66	A	7	6	1.00	23	0.261
67	A	6	6	1.00	25	0.240
68	A	5	5	1.00	25	0.200
69	A	4	4	1.00	23	0.174
70	A	6	6	1.00	23	0.261
71	A	7	7	1.00	25	0.280
72	A	8	8	1.00	25	0.320
73	A	9	8	1.00	25	0.320
74	A	8	8	1.00	25	0.320
75	A	7	7	1.00	25	0.280
76	A	6	6	1.00	16	0.375
77	A	4	4	1.00	25	0.160
78	A	5	5	1.00	25	0.200
79	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	7	7	1.00	25	0.280
81	A	6	6	1.00	25	0.240
82	A	5	5	1.00	23	0.217
83	A	7	7	1.00	23	0.304
84	A	8	8	1.00	25	0.320
85	A	9	9	1.00	25	0.360
86	A	10	10	1.00	25	0.400
87	A	9	9	1.00	25	0.360
88	A	8	8	1.00	25	0.320
89	A	7	7	1.00	16	0.438
90	A	5	5	1.00	25	0.200
91	A	6	6	1.00	25	0.240
92	A	7	7	1.00	25	0.280
93	A	4	4	1.00	25	0.160
94	A	3	3	1.00	25	0.120
95	A	2	2	1.00	23	0.087
96	A	3	3	1.00	23	0.130
97	A	5	5	1.00	25	0.200
98	A	6	6	1.00	25	0.240
99	A	7	7	1.00	25	0.280
100	A	6	6	1.00	25	0.240
101	A	5	5	1.00	25	0.200
102	A	3	3	1.00	16	0.188
103	A	2	2	1.00	25	0.080
104	A	3	3	1.00	25	0.120
105	A	4	4	1.00	25	0.160
106	A	5	5	1.00	25	0.200
107	A	4	4	1.00	25	0.160
108	A	3	3	1.00	23	0.130
109	A	4	4	1.00	23	0.174
110	A	6	6	1.00	25	0.240
111	A	7	6	1.00	25	0.240
112	A	8	7	1.00	25	0.280
113	A	7	6	1.00	25	0.240
114	A	6	6	1.00	25	0.240
115	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
116	A	3	3	1.00	25	0.120
117	A	4	4	1.00	25	0.160
118	A	5	5	1.00	25	0.200
119	A	6	6	1.00	25	0.240
120	A	5	5	1.00	25	0.200
121	A	4	4	1.00	23	0.174
122	A	6	6	1.00	23	0.261
123	A	7	6	1.00	25	0.240
124	A	8	6	1.00	25	0.240
125	A	9	7	1.00	25	0.280
126	A	8	6	1.00	25	0.240
127	A	7	6	1.00	25	0.240
128	A	6	6	1.00	16	0.375
129	A	4	4	1.00	25	0.160
130	A	5	5	1.00	25	0.200
131	A	6	6	1.00	25	0.240
132	F	0	0	N/A	0	N/A
133	A	5	5	1.00	23	0.217
134	A	4	4	1.00	23	0.174
135	A	3	3	1.00	21	0.143
136	A	3	3	1.00	21	0.143
137	A	3	3	1.00	23	0.130
138	A	3	3	1.00	23	0.130
139	A	3	3	1.00	23	0.130
140	A	3	3	1.00	14	0.214
141	A	3	3	1.00	23	0.130
142	A	4	4	1.00	23	0.174
143	A	5	5	1.00	23	0.217
144	A	6	3	1.00	15	0.200
145	A	5	3	1.00	15	0.200
146	A	4	3	1.00	15	0.200
147	A	3	2	1.00	13	0.154
148	A	3	3	1.00	15	0.200
149	A	4	3	1.00	15	0.200
150	A	5	3	1.00	15	0.200
151	A	6	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
152	A	4	3	1.00	21	0.143
153	A	3	3	1.00	21	0.143
154	A	2	2	1.00	19	0.105
155	A	2	2	1.00	19	0.105
156	A	3	2	1.00	21	0.095
157	A	4	3	1.00	21	0.143
158	A	3	2	1.00	21	0.095
159	A	3	2	1.00	21	0.095
160	A	3	3	1.00	21	0.143
161	A	3	2	1.00	12	0.167
162	A	2	2	1.00	21	0.095
163	A	3	3	1.00	21	0.143
164	A	4	3	1.00	21	0.143
165	A	6	5	1.00	23	0.217
166	A	5	5	1.00	23	0.217
167	A	4	4	1.00	21	0.190
168	A	5	4	1.00	21	0.190
169	A	4	3	1.00	23	0.130
170	A	3	2	1.00	23	0.087
171	A	3	2	1.00	23	0.087
172	A	3	2	1.00	23	0.087
173	A	3	2	1.00	23	0.087
174	A	4	3	1.00	14	0.214
175	A	5	4	1.00	23	0.174
176	A	4	4	1.00	23	0.174
177	A	5	5	1.00	23	0.217
178	A	4	3	1.00	14	0.214
179	A	4	3	1.00	14	0.214
180	A	5	5	1.00	23	0.217
181	A	4	4	1.00	23	0.174
182	A	2	2	1.00	21	0.095
183	A	3	3	1.00	21	0.143
184	A	4	3	1.00	23	0.130
185	A	4	3	1.00	23	0.130
186	A	4	3	1.00	23	0.130
187	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
188	A	2	2	1.00	23	0.087
189	A	3	3	1.00	14	0.214
190	A	5	5	1.00	23	0.217
191	A	6	6	1.00	23	0.261
192	A	7	6	1.00	23	0.261
193	A	5	5	1.00	23	0.217
194	A	3	3	1.00	23	0.130
195	A	3	3	1.00	21	0.143
196	A	5	4	1.00	21	0.190
197	A	5	4	1.00	23	0.174
198	A	5	4	1.00	23	0.174
199	A	5	4	1.00	23	0.174
200	A	3	3	1.00	23	0.130
201	A	3	3	1.00	23	0.130
202	A	5	5	1.00	14	0.357
203	A	6	6	1.00	23	0.261
204	A	7	6	1.00	23	0.261
205	A	8	6	1.00	23	0.261
206	A	4	3	1.00	23	0.130
207	A	4	4	1.00	23	0.174
208	A	4	4	1.00	21	0.190
209	A	6	5	1.00	21	0.238
210	A	6	5	1.00	23	0.217
211	A	6	5	1.00	23	0.217
212	A	4	4	1.00	23	0.174
213	A	4	4	1.00	23	0.174
214	A	4	3	1.00	23	0.130
215	A	6	6	1.00	14	0.429
216	A	7	6	1.00	23	0.261
217	A	8	6	1.00	23	0.261
218	A	9	6	1.00	23	0.261
219	A	7	6	1.00	14	0.429
220	A	6	4	1.00	17	0.235
221	A	5	4	1.00	17	0.235
222	A	4	4	1.00	17	0.235
223	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
224	A	3	3	1.00	17	0.176
225	A	4	4	1.00	17	0.235
226	A	5	4	1.00	17	0.235
227	A	6	4	1.00	17	0.235
228	A	11	10	1.27	25	0.400
229	A	10	10	1.26	25	0.400
230	A	10	10	1.24	23	0.435
231	A	5	5	1.29	23	0.217
232	A	9	9	1.22	25	0.360
233	A	10	10	1.18	25	0.400
234	A	6	6	1.00	25	0.240
235	A	5	5	1.00	25	0.200
236	A	4	4	1.00	25	0.160
237	A	6	6	1.00	16	0.375
238	A	4	4	1.00	25	0.160
239	A	5	5	1.00	25	0.200
240	A	7	6	1.00	25	0.240
241	A	12	10	1.27	25	0.400
242	A	11	10	1.27	25	0.400
243	A	10	10	1.26	23	0.435
244	A	9	9	1.24	23	0.391
245	A	9	9	1.22	25	0.360
246	A	10	10	1.24	25	0.400
247	A	7	6	1.00	25	0.240
248	A	6	5	1.00	25	0.200
249	A	5	4	1.00	25	0.160
250	A	7	7	1.00	16	0.438
251	A	7	7	1.00	25	0.280
252	A	5	4	1.00	25	0.160
253	A	6	5	1.00	25	0.200
254	A	8	8	1.00	16	0.500
255	A	6	6	1.00	10	0.600
256	A	5	5	1.00	10	0.500
257	A	10	10	1.15	25	0.400
258	A	7	7	1.19	25	0.280
259	A	5	5	1.29	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	5	5	1.22	23	0.217
261	A	9	9	1.16	25	0.360
262	A	10	10	1.14	25	0.400
263	A	5	5	1.00	25	0.200
264	A	4	4	1.00	25	0.160
265	A	3	3	1.00	25	0.120
266	A	3	3	1.00	16	0.188
267	A	4	4	1.00	25	0.160
268	A	6	6	1.00	25	0.240
269	A	7	6	1.00	25	0.240
270	A	10	10	1.27	25	0.400
271	A	7	7	1.21	25	0.280
272	A	9	9	1.24	23	0.391
273	A	9	9	1.23	23	0.391
274	A	10	10	1.19	25	0.400
275	A	11	10	1.17	25	0.400
276	A	5	5	1.00	25	0.200
277	A	4	4	1.00	25	0.160
278	A	2	2	1.00	25	0.080
279	A	4	4	1.00	16	0.250
280	A	6	6	1.00	25	0.240
281	A	7	6	1.00	25	0.240
282	A	8	6	1.00	25	0.240
283	A	10	10	1.19	25	0.400
284	A	10	10	1.19	25	0.400
285	A	10	10	1.19	23	0.435
286	A	10	10	1.18	23	0.435
287	A	11	11	1.16	25	0.440
288	A	12	11	1.14	25	0.440
289	A	5	5	1.00	25	0.200
290	A	3	3	1.00	25	0.120
291	A	3	3	1.00	25	0.120
292	A	6	6	1.00	16	0.375
293	A	7	6	1.00	25	0.240
294	A	8	6	1.00	25	0.240
295	A	9	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
296	A	7	6	1.00	16	0.375
297	A	3	3	1.00	10	0.300
298	F	0	0	N/A	0	N/A
299	A	5	5	1.20	23	0.217
300	A	5	5	1.20	21	0.238
301	A	5	5	1.21	21	0.238
302	A	5	5	1.20	23	0.217
303	A	5	5	1.20	23	0.217
304	A	5	5	1.00	23	0.217
305	A	4	4	1.00	23	0.174
306	A	3	3	1.00	23	0.130
307	A	3	3	1.00	14	0.214
308	A	3	3	1.00	23	0.130
309	A	3	3	1.00	23	0.130
310	A	3	3	1.00	23	0.130
311	A	4	3	1.00	21	0.143
312	A	4	3	1.00	21	0.143
313	A	3	2	1.00	19	0.105
314	A	4	3	1.00	19	0.158
315	A	4	3	1.00	21	0.143
316	A	4	3	1.00	21	0.143
317	A	4	3	1.00	21	0.143
318	A	4	3	1.00	21	0.143
319	A	4	3	1.00	21	0.143
320	A	3	2	1.00	12	0.167
321	A	4	3	1.00	21	0.143
322	A	4	3	1.00	21	0.143
323	A	4	3	1.00	21	0.143
324	A	4	3	1.00	23	0.130
325	A	4	3	1.00	23	0.130
326	A	4	3	1.00	21	0.143
327	A	4	3	1.00	21	0.143
328	A	4	3	1.00	23	0.130
329	A	4	3	1.00	23	0.130
330	A	4	3	1.00	23	0.130
331	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
332	A	4	3	1.00	23	0.130
333	A	4	3	1.00	14	0.214
334	A	4	3	1.00	23	0.130
335	A	4	3	1.00	23	0.130
336	A	4	3	1.00	23	0.130
337	A	4	3	1.00	23	0.130
338	A	4	3	1.00	23	0.130
339	A	2	2	1.00	21	0.095
340	A	4	3	1.00	21	0.143
341	A	4	3	1.00	23	0.130
342	A	4	3	1.00	23	0.130
343	A	7	7	1.00	23	0.304
344	A	6	6	1.00	23	0.261
345	A	5	5	1.00	23	0.217
346	A	3	3	1.00	14	0.214
347	A	6	6	1.00	23	0.261
348	A	7	7	1.00	23	0.304
349	A	8	7	1.00	23	0.304
350	A	4	3	1.00	23	0.130
351	A	4	3	1.00	23	0.130
352	A	4	3	1.00	21	0.143
353	A	4	3	1.00	21	0.143
354	A	4	3	1.00	23	0.130
355	A	4	3	1.00	23	0.130
356	A	7	7	1.00	23	0.304
357	A	6	6	1.00	23	0.261
358	A	6	6	1.00	23	0.261
359	A	5	5	1.00	14	0.357
360	A	7	7	1.00	23	0.304
361	A	8	7	1.00	23	0.304
362	A	9	7	1.00	23	0.304
363	A	4	3	1.00	23	0.130
364	A	4	3	1.00	23	0.130
365	A	4	3	1.00	21	0.143
366	A	4	3	1.00	21	0.143
367	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
368	A	4	3	1.00	23	0.130
369	A	7	7	1.00	23	0.304
370	A	7	7	1.00	23	0.304
371	A	7	7	1.00	23	0.304
372	A	6	6	1.00	14	0.429
373	A	8	8	1.00	23	0.348
374	A	9	8	1.00	23	0.348
375	A	10	8	1.00	23	0.348
376	A	7	6	1.00	25	0.240
377	A	6	6	1.00	25	0.240
378	A	5	5	1.00	23	0.217
379	A	7	5	1.00	23	0.217
380	A	8	6	1.00	25	0.240
381	A	9	7	1.00	25	0.280
382	A	10	9	1.00	25	0.360
383	A	9	9	1.00	25	0.360
384	A	8	8	1.00	25	0.320
385	A	6	6	1.00	16	0.375
386	A	6	6	1.00	25	0.240
387	A	7	7	1.00	25	0.280
388	A	8	7	1.00	25	0.280
389	A	8	6	1.00	25	0.240
390	A	7	6	1.00	25	0.240
391	A	6	5	1.00	23	0.217
392	A	8	6	1.00	23	0.261
393	A	8	6	1.00	25	0.240
394	A	9	7	1.00	25	0.280
395	A	11	9	1.00	25	0.360
396	A	10	9	1.00	25	0.360
397	A	9	9	1.00	25	0.360
398	A	7	7	1.00	16	0.438
399	A	8	8	1.00	25	0.320
400	A	7	7	1.00	25	0.280
401	A	8	7	1.00	25	0.280
402	A	6	5	1.00	25	0.200
403	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	A	4	4	1.00	23	0.174
405	A	7	5	1.00	23	0.217
406	A	8	6	1.00	25	0.240
407	A	9	7	1.00	25	0.280
408	A	9	9	1.00	25	0.360
409	A	8	8	1.00	25	0.320
410	A	7	7	1.00	25	0.280
411	A	3	3	1.00	16	0.188
412	A	6	6	1.00	25	0.240
413	A	7	7	1.00	25	0.280
414	A	8	7	1.00	25	0.280
415	A	6	5	1.00	25	0.200
416	A	5	5	1.00	25	0.200
417	A	5	5	1.00	23	0.217
418	A	8	6	1.00	23	0.261
419	A	9	7	1.00	25	0.280
420	A	10	8	1.00	25	0.320
421	A	9	9	1.00	25	0.360
422	A	8	8	1.00	25	0.320
423	A	5	5	1.00	25	0.200
424	A	4	4	1.00	16	0.250
425	A	7	7	1.00	25	0.280
426	A	8	7	1.00	25	0.280
427	A	9	7	1.00	25	0.280
428	A	6	5	1.00	25	0.200
429	A	6	6	1.00	25	0.240
430	A	6	5	1.00	23	0.217
431	A	9	7	1.00	23	0.304
432	A	10	7	1.00	25	0.280
433	A	11	8	1.00	25	0.320
434	A	9	9	1.00	25	0.360
435	A	7	7	1.00	25	0.280
436	A	7	7	1.00	25	0.280
437	A	6	6	1.00	16	0.375
438	A	8	8	1.00	25	0.320
439	A	9	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	10	8	1.00	25	0.320
441	A	4	4	1.00	25	0.160
442	A	5	4	1.00	23	0.174
443	A	4	4	1.00	23	0.174
444	A	3	3	1.00	21	0.143
445	A	5	5	1.00	21	0.238
446	A	6	6	1.00	23	0.261
447	A	4	4	1.00	23	0.174
448	A	4	4	1.00	23	0.174
449	A	3	3	1.00	14	0.214
450	A	4	4	1.00	23	0.174
451	A	4	4	1.00	23	0.174
452	A	3	2	1.00	21	0.095
453	A	3	2	1.00	21	0.095
454	A	3	2	1.00	19	0.105
455	A	3	2	1.00	19	0.105
456	A	5	4	1.00	21	0.190
457	A	11	10	1.00	23	0.435
458	A	9	9	1.00	23	0.391
459	A	2	2	1.00	21	0.095
460	A	11	10	1.00	21	0.476
461	A	11	10	1.00	23	0.435
462	A	0	0	0.00	0	0.000
463	A	15	8	1.00	25	0.320
464	A	11	8	1.00	25	0.320
465	A	5	5	1.00	23	0.217
466	A	0	0	0.00	0	0.000
467	A	0	0	0.00	0	0.000
468	A	0	0	0.00	0	0.000
469	A	0	0	0.00	0	0.000
470	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx$

Optimal. Leaf size=83

$$-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-3*b)*cos(f*x+e)/f+(a-b)*cos(f*x+e)^3/f-1/5*(3*a-b)*cos(f*x+e)^5/f+1/7*a
*cos(f*x+e)^7/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/f - ((3*a - b)*Cos
[e + f*x]^5)/(5*f) + (a*Cos[e + f*x]^7)/(7*f) + (b*Sec[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3(b+ax^2)}{x^2}\,dx,x,\cos(e+fx)\right)}{f}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(a\left(1-\frac{3b}{a}\right)+\frac{b}{x^2}-3(a-b)x^2+(3a-b)x^4-ax^6\right)dx,x,\cos(\right.}{f}$$
$$= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(3a-b)\cos^5(e+fx)}{5f}+$$

Mathematica [A] time = 0.08, size = 120, normalized size = 1.45

$$-\frac{35a\cos(e+fx)}{64f}+\frac{7a\cos(3(e+fx))}{64f}-\frac{7a\cos(5(e+fx))}{320f}+\frac{a\cos(7(e+fx))}{448f}+\frac{19b\cos(e+fx)}{8f}-\frac{3b\cos(3(e+fx))}{16f}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] (-35*a*Cos[e + f*x])/(64*f) + (19*b*Cos[e + f*x])/(8*f) + (7*a*Cos[3*(e + f
*x)])/(64*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (7*a*Cos[5*(e + f*x)])/(320*
f) + (b*Cos[5*(e + f*x)])/(80*f) + (a*Cos[7*(e + f*x)])/(448*f) + (b*Sec[e
+ f*x])/f
```

fricas [A] time = 1.08, size = 75, normalized size = 0.90

$$\frac{5\,a\cos\left(fx+e\right)^8-7\,(3\,a-b)\cos\left(fx+e\right)^6+35\,(a-b)\cos\left(fx+e\right)^4-35\,(a-3\,b)\cos\left(fx+e\right)^2+35\,b}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^8 - 7*(3*a - b)*cos(f*x + e)^6 + 35*(a - b)*cos(f*x
+ e)^4 - 35*(a - 3*b)*cos(f*x + e)^2 + 35*b)/(f*cos(f*x + e))
```

giac [B] time = 1.37, size = 288, normalized size = 3.47

$$\frac{2\left(\frac{35\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}+\frac{16\,a-77\,b-\frac{112\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{504\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{336\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{1337\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{560\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{1680\,b(\cos(fx+e)-1}{(\cos(fx+e)+1)^3}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^7}\right.}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] 2/35*(35*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (16*a - 77*b - 112
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 504*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 336*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1337*b*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 560*a*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3 + 1680*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 1015*b*(c
os(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 280*b*(cos(f*x + e) - 1)^5/(cos(f
*x + e) + 1)^5 - 35*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 1)^7)/f
```

maple [A] time = 1.00, size = 102, normalized size = 1.23

$$\frac{-\frac{a\left(\frac{16}{5}+\sin^{6}\left(fx+e\right)+\frac{6\left(\sin^{4}\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^{2}\left(fx+e\right)\right)}{5}\right)\cos\left(fx+e\right)}{7}+b\left(\frac{\sin^{8}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\frac{16}{5}+\sin^{6}\left(fx+e\right)+\frac{6\left(\sin^{4}\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^{2}\left(fx+e\right.\right.}{5}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x)
```

```
[Out] 1/f*(-1/7*a*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e
)+b*(sin(f*x+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*
x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.34, size = 73, normalized size = 0.88

$$\frac{5\,a\cos\left(fx+e\right)^{7}-7\,(3\,a-b)\cos\left(fx+e\right)^{5}+35\,(a-b)\cos\left(fx+e\right)^{3}-35\,(a-3\,b)\cos\left(fx+e\right)+\frac{35\,b}{\cos\left(fx+e\right)}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^7 - 7*(3*a - b)*cos(f*x + e)^5 + 35*(a - b)*cos(f*x
+ e)^3 - 35*(a - 3*b)*cos(f*x + e) + 35*b/cos(f*x + e))/f
```

mupad [B] time = 0.10, size = 70, normalized size = 0.84

$$\frac{\frac{a\cos\left(e+f\,x\right)^7}{7}-\cos\left(e+f\,x\right)\,\left(a-3\,b\right)-{\cos\left(e+f\,x\right)}^5\,\left(\frac{3\,a}{5}-\frac{b}{5}\right)+\frac{b}{\cos\left(e+f\,x\right)}+{\cos\left(e+f\,x\right)}^3\,\left(a-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*cos(e + f*x)^7)/7 - cos(e + f*x)*(a - 3*b) - cos(e + f*x)^5*((3*a)/5 -
b/5) + b/cos(e + f*x) + cos(e + f*x)^3*(a - b))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**7,x)
```

```
[Out] Timed out
```

3.2 $\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-2*b)*cos(f*x+e)/f+1/3*(2*a-b)*cos(f*x+e)^3/f-1/5*a*cos(f*x+e)^5/f+b*sec
(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4133, 448}

$$\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((2*a - b)*Cos[e + f*x]^3)/(3*f) - (a*Cos[e
 + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)}{x^2}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{2b}{a}\right)+\frac{b}{x^2}-(2a-b)x^2+ax^4\right)dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.33

$$-\frac{5a\cos(e+fx)}{8f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{a\cos(5(e+fx))}{80f}+\frac{7b\cos(e+fx)}{4f}-\frac{b\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (7*b*Cos[e + f*x])/(4*f) + (5*a*Cos[3*(e + f*x)
])/(48*f) - (b*Cos[3*(e + f*x)])/(12*f) - (a*Cos[5*(e + f*x)])/(80*f) + (b*
Sec[e + f*x])/f
```

fricas [A] time = 0.82, size = 60, normalized size = 0.91

$$-\frac{3\,a\cos\left(fx+e\right)^{6}-5\,(2\,a-b)\cos\left(fx+e\right)^{4}+15\,(a-2\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^6 - 5*(2*a - b)*cos(f*x + e)^4 + 15*(a - 2*b)*cos(f
*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

giac [B] time = 0.50, size = 213, normalized size = 3.23

$$\frac{2\left(\frac{15\,b}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1}+\frac{8\,a-25\,b-\frac{40\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{110\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{80\,a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{160\,b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{90\,b\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{15\,b\left(\cos\left(fx+e\right)-1\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-1\right)^{5}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/15*(15*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (8*a - 25*b - 40*a
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 110*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 80*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 160*b*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 - 15*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) -
1)/(cos(f*x + e) + 1) - 1)^5)/f
```

maple [A] time = 1.10, size = 82, normalized size = 1.24

$$\frac{-\frac{a\left(\frac{8}{3}+\sin^{4}\left(fx+e\right)+\frac{4\left(\sin^{2}\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+b\left(\frac{\sin^{6}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\frac{8}{3}+\sin^{4}\left(fx+e\right)+\frac{4\left(\sin^{2}\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x)
```

```
[Out] 1/f*(-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^6/
cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.33, size = 58, normalized size = 0.88

$$-\frac{3\,a\cos\left(fx+e\right)^{5}-5\,(2\,a-b)\cos\left(fx+e\right)^{3}+15\,(a-2\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos\left(fx+e\right)}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^5 - 5*(2*a - b)*cos(f*x + e)^3 + 15*(a - 2*b)*cos(f
*x + e) - 15*b/cos(f*x + e))/f
```

mupad [B] time = 4.21, size = 55, normalized size = 0.83

$$\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{2\,a}{3}-\frac{b}{3}\right)-\cos\left(e+f\,x\right)\,\left(a-2\,b\right)-\frac{a\,\cos\left(e+f\,x\right)^5}{5}+\frac{b}{\cos\left(e+f\,x\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (cos(e + f*x)^3*((2*a)/3 - b/3) - cos(e + f*x)*(a - 2*b) - (a*cos(e + f*x)^
5)/5 + b/cos(e + f*x))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.3 $\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-b)*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (a*Cos[e + f*x]^3)/(3*f) + (b*Sec[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.20

$$-\frac{3a\cos(e+fx)}{4f}+\frac{a\cos(3(e+fx))}{12f}+\frac{b\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

```
[Out] (-3*a*Cos[e + f*x])/(4*f) + (b*Cos[e + f*x])/f + (a*Cos[3*(e + f*x)])/(12*f
) + (b*Sec[e + f*x])/f
```

fricas [A] time = 1.17, size = 42, normalized size = 0.95

$$\frac{a\cos\left(fx+e\right)^4-3\,(a-b)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^4 - 3*(a - b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x + e))
```

giac [A] time = 0.23, size = 61, normalized size = 1.39

$$\frac{b}{f\cos\left(fx+e\right)}+\frac{af^5\cos\left(fx+e\right)^3-3\,af^5\cos\left(fx+e\right)+3\,bf^5\cos\left(fx+e\right)}{3\,f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - 3*a*f^5*cos(f*x + e) + 3*b
*f^5*cos(f*x + e))/f^6
```

maple [A] time = 1.05, size = 62, normalized size = 1.41

$$\frac{-\frac{a\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+b\left(\frac{\sin^4\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x)
```

```
[Out] 1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f
*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.34, size = 40, normalized size = 0.91

$$\frac{a\cos\left(fx+e\right)^3-3\,(a-b)\cos\left(fx+e\right)+\frac{3\,b}{\cos\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a*cos(f*x + e)^3 - 3*(a - b)*cos(f*x + e) + 3*b/cos(f*x + e))/f
```

mupad [B] time = 0.07, size = 39, normalized size = 0.89

$$\frac{\frac{a\cos\left(e+f\,x\right)^3}{3}-\cos\left(e+f\,x\right)\,\left(a-b\right)+\frac{b}{\cos\left(e+f\,x\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*cos(e + f*x)^3)/3 - cos(e + f*x)*(a - b) + b/cos(e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\sin^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**3, x)
```

3.4 $\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx$

Optimal. Leaf size=24

$$\frac{b\sec(e+fx)}{f}-\frac{a\cos(e+fx)}{f}$$

```
[Out] -a*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4133, 14}

$$\frac{b\sec(e+fx)}{f}-\frac{a\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

```
[Out] -((a*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a+\frac{b}{x^2}\right)\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$\frac{a\sin(e)\sin(fx)}{f}-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

```
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Sec[e + f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

fricas [A] time = 1.30, size = 27, normalized size = 1.12

$$-\frac{a\cos\left(fx+e\right)^{2}-b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="fricas")
[Out] -(a*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

giac [A] time = 1.79, size = 28, normalized size = 1.17

$$-\frac{a\cos\left(fx+e\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="giac")
[Out] -a*cos(f*x + e)/f + b/(f*cos(f*x + e))
```

maple [A] time = 0.36, size = 25, normalized size = 1.04

$$\frac{b\sec\left(fx+e\right)-\frac{a}{\sec\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e),x)
[Out] 1/f*(b*sec(f*x+e)-a/sec(f*x+e))
```

maxima [A] time = 0.33, size = 25, normalized size = 1.04

$$-\frac{a\cos\left(fx+e\right)-\frac{b}{\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="maxima")
[Out] -(a*cos(f*x + e) - b/cos(f*x + e))/f
```

mupad [B] time = 0.04, size = 25, normalized size = 1.04

$$-\frac{a\,\cos\left(e+f\,x\right)-\frac{b}{\cos\left(e+f\,x\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2),x)
[Out] -(a*cos(e + f*x) - b/cos(e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

3.5 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=27

$$\frac{b\sec(e+fx)}{f}-\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] -(a+b)*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4133, 453, 206}

$$\frac{b\sec(e+fx)}{f}-\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2\left(1-x^2\right)}\,dx,x,\cos(e+fx)\right)}{f}\\ &= \frac{b\sec(e+fx)}{f}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [B] time = 0.05, size = 84, normalized size = 3.11

$$\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{b\sec(e+fx)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) - (b*Log[Cos[(e + f*x)/2]])/f + (a*Log[Sin
[e/2 + (f*x)/2]])/f + (b*Log[Sin[(e + f*x)/2]])/f + (b*Sec[e + f*x])/f
```

fricas [B] time = 0.74, size = 60, normalized size = 2.22

$$-\frac{(a+b)\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-(a+b)\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2\,b}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - (a + b)*cos(f*x +
e)*log(-1/2*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)+(a+b)/4*ln(abs(
1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.49, size = 57, normalized size = 2.11

$$\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*a*ln(csc(f*x+e)-cot(f*x+e))+1/f*b/cos(f*x+e)+1/f*b*ln(csc(f*x+e)-cot(f*
x+e))
```

maxima [A] time = 0.33, size = 44, normalized size = 1.63

$$-\frac{(a+b)\log\left(\cos\left(fx+e\right)+1\right)-(a+b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,b}{\cos\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)*log(cos(f*x + e) + 1) - (a + b)*log(cos(f*x + e) - 1) - 2*b/c
os(f*x + e))/f
```

mupad [B] time = 0.09, size = 29, normalized size = 1.07

$$\frac{b}{f\,\cos\left(e+f\,x\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x),x)
[Out] b/(f*cos(e + f*x)) - (atanh(cos(e + f*x))*(a + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

3.6 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=53

$$-\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -1/2*(a+3*b)*arctanh(cos(f*x+e))/f-1/2*(a+b)*cot(f*x+e)*csc(f*x+e)/f+b*sec(
f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 456, 453, 206}

$$-\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 3*b)*ArcTanh[Cos[e + f*x]])/(2*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*
x])/(2*f) + (b*Sec[e + f*x])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{\text{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)}dx, x, \cos(e+f\right.}{2f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= -\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\,\text{se}}{}$$

Mathematica [B] time = 0.38, size = 236, normalized size = 4.45

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(e + f*x)/2]^2)/f - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(
e + f*x)/2]])/(2*f) - (3*b*Log[Cos[(e + f*x)/2]])/(2*f) + (a*Log[Sin[(e + f
*x)/2]])/(2*f) + (3*b*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)
/(8*f) + (b*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2]))
```

fricas [B] time = 0.81, size = 124, normalized size = 2.34

$$\frac{2\,(a+3\,b)\cos\left(fx+e\right)^2 - \left((a+3\,b)\cos\left(fx+e\right)^3 - (a+3\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right) + \left((a+3\,b)\cdots\right.}{4\left(f\cos\left(fx+e\right)^3 - f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 3*b)*cos(f*x + e)^2 - ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 3*b)*cos(f*x + e)^3 - (a + 3
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+(1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1)))*a)/16+(-3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*
```

```
b-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-14*(1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1)))*b+b+a)*1/16/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2-(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))-(-3*b-a)/8*ln(abs(1-cos(f*x+exp(1)))
/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.97, size = 100, normalized size = 1.89

$$-\frac{a\cot\left(fx+e\right)\csc\left(fx+e\right)}{2f}+\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}-\frac{b}{2f\sin\left(fx+e\right)^{2}\cos\left(fx+e\right)}+\frac{3b}{2f\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2*a*cot(f*x+e)*csc(f*x+e)/f+1/2/f*a*ln(csc(f*x+e)-cot(f*x+e))-1/2/f*b/si
n(f*x+e)^2/cos(f*x+e)+3/2/f*b/cos(f*x+e)+3/2/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.39, size = 76, normalized size = 1.43

$$-\frac{(a+3b)\log\left(\cos\left(fx+e\right)+1\right)-(a+3b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+3b)\cos\left(fx+e\right)^{2}-2b\right)}{\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 3*b)*log(cos(f*x + e) + 1) - (a + 3*b)*log(cos(f*x + e) - 1) - 2
*((a + 3*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

mupad [B] time = 4.20, size = 62, normalized size = 1.17

$$\frac{b-\cos\left(e+f\,x\right)^{2}\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f\,\left(\cos\left(e+f\,x\right)-\cos\left(e+f\,x\right)^{3}\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^3,x)
```

```
[Out] (b - cos(e + f*x)^2*(a/2 + (3*b)/2))/(f*(cos(e + f*x) - cos(e + f*x)^3)) -
(atanh(cos(e + f*x))*(a/2 + (3*b)/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\csc^{3}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.7 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=81

$$-\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b\sec(e+f}{f}$$

```
[Out] -3/8*(a+5*b)*arctanh(cos(f*x+e))/f-1/8*(3*a+7*b)*cot(f*x+e)*csc(f*x+e)/f-1/
4*(a+b)*cot(f*x+e)*csc(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 456, 453, 206}

$$-\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b\sec(e+f}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-3*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f) + (b*Sec[e +
f*x])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^3}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{-4b-3(a+b)x^2}{x^2(1-x^2)^2}dx, x, \cos(\right.}{4f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx}{4f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx}{4f}$$

$$= -\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}$$

Mathematica [B] time = 1.83, size = 198, normalized size = 2.44

$$\frac{-\left((a+b)\csc^4\left(\frac{1}{2}(e+fx)\right)\right) - 2(3a+7b)\csc^2\left(\frac{1}{2}(e+fx)\right) + \frac{-(a+b)\sec^4\left(\frac{1}{2}(e+fx)\right)+\tan^2\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)((3a+7b)\cos}{}}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*(3*a + 7*b)*Csc[(e + f*x)/2]^2 - (a + b)*Csc[(e + f*x)/2]^4 + (2*(-3*(a
 + 13*b) + 4*Cos[e + f*x]*(8*b + 3*(a + 5*b)*Log[Cos[(e + f*x)/2]] - 3*(a +
 5*b)*Log[Sin[(e + f*x)/2]]))*Sec[(e + f*x)/2]^2 - (a + b)*Sec[(e + f*x)/2]
^4 + (4*(a + 2*b) + (3*a + 7*b)*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Tan[(e + f
*x)/2]^2)/(-1 + Tan[(e + f*x)/2]^2))/(64*f)
```

fricas [B] time = 0.57, size = 178, normalized size = 2.20

$$\frac{6(a+5b)\cos\left(fx+e\right)^4 - 10(a+5b)\cos\left(fx+e\right)^2 - 3\left((a+5b)\cos\left(fx+e\right)^5 - 2(a+5b)\cos\left(fx+e\right)^3 + (\right.}{16(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 5*b)*cos(f*x + e)^4 - 10*(a + 5*b)*cos(f*x + e)^2 - 3*((a + 5*
b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 5*b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f
*x + e)^3 + (a + 5*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)+(-90*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*a-16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-8*(1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1)))*a-b-a)*1/128/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^2+(32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+32*((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^2*a+512*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+2
56*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)/4096+(15*b+3*a)/32*ln(abs(1-c
os(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.90, size = 142, normalized size = 1.75

$$-\frac{a\cot\left(fx+e\right)\left(\csc^{3}\left(fx+e\right)\right)}{4f}-\frac{3a\cot\left(fx+e\right)\csc\left(fx+e\right)}{8f}+\frac{3a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{}{4f\sin\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/4/f*a*cot(f*x+e)*csc(f*x+e)^3-3/8*a*cot(f*x+e)*csc(f*x+e)/f+3/8/f*a*ln(c
sc(f*x+e)-cot(f*x+e))-1/4/f*b/sin(f*x+e)^4/cos(f*x+e)-5/8/f*b/sin(f*x+e)^2/
cos(f*x+e)+15/8/f*b/cos(f*x+e)+15/8/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.33, size = 101, normalized size = 1.25

$$-\frac{3\,(a+5\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+5\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+5\,b)\cos\left(fx+e\right)^{4}-5\,(a+5\,b)\cos\left(fx+e\right)^{2}+8\,b\right)}{\cos\left(fx+e\right)^{5}-2\,\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a + 5*b)*log(cos(f*x + e) + 1) - 3*(a + 5*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 5*b)*cos(f*x + e)^4 - 5*(a + 5*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

mupad [B] time = 4.29, size = 86, normalized size = 1.06

$$\frac{\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)\,\cos\left(e+f\,x\right)^4+\left(-\frac{5\,a}{8}-\frac{25\,b}{8}\right)\,\cos\left(e+f\,x\right)^2+b}{f\,\left({\cos\left(e+f\,x\right)}^5-2\,{\cos\left(e+f\,x\right)}^3+\cos\left(e+f\,x\right)\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^5,x)
```

```
[Out] (b + cos(e + f*x)^4*((3*a)/8 + (15*b)/8) - cos(e + f*x)^2*((5*a)/8 + (25*b)
/8))/(f*(cos(e + f*x) - 2*cos(e + f*x)^3 + cos(e + f*x)^5)) - (atanh(cos(e
+ f*x))*((3*a)/8 + (15*b)/8))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.8 $\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$

Optimal. Leaf size=98

$$\frac{(13a-6b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-18b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-6b)-\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

```
[Out] 5/16*(a-6*b)*x-1/16*(11*a-18*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-6*b)*cos
(f*x+e)^3*sin(f*x+e)/f-1/6*a*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4132, 455, 1814, 1157, 388, 203}

$$\frac{(13a-6b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-18b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-6b)-\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (5*(a - 6*b)*x)/16 - ((11*a - 18*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a*Cos[e + f*x]^5*Sin[e +
f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)}{\left(1+x^2\right)^4}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}-\frac{\text{Subst}\left(\int \frac{-a+6ax^2-6ax^4-6bx^6}{\left(1+x^2\right)^3}\,dx,x,\tan(e+\right.}{6f}$$

$$=\frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{\text{Sub}}{}$$

$$=-\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-6b)\cos^3(e+fx)\sin(e+}{24f}$$

$$=-\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-6b)\cos^3(e+fx)\sin(e+}{24f}$$

$$=\frac{5}{16}(a-6b)x-\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-6b)\cos^3(e}{24}$$

Mathematica [A] time = 0.30, size = 78, normalized size = 0.80

$$\frac{(96b-45a)\sin(2(e+fx))+(9a-6b)\sin(4(e+fx))-a\sin(6(e+fx))+60ae+60afx+192b\tan(e+fx)-36}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (60*a*e - 360*b*e + 60*a*f*x - 360*b*f*x + (-45*a + 96*b)*Sin[2*(e + f*x)]
+ (9*a - 6*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + 192*b*Tan[e + f*x])/(
192*f)
```

fricas [A] time = 1.11, size = 86, normalized size = 0.88

$$\frac{15\,(a-6\,b)fx\cos\left(fx+e\right)-\left(8\,a\cos\left(fx+e\right)^6-2\,(13\,a-6\,b)\cos\left(fx+e\right)^4+3\,(11\,a-18\,b)\cos\left(fx+e\right)^2-48\,b}{48\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 6*b)*f*x*cos(f*x + e) - (8*a*cos(f*x + e)^6 - 2*(13*a - 6*b)*
cos(f*x + e)^4 + 3*(11*a - 18*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e))/(f*co
s(f*x + e))
```

giac [A] time = 1.34, size = 113, normalized size = 1.15

$$\frac{15\left(fx+e\right)(a-6b)+48b\tan\left(fx+e\right)-\frac{33a\tan\left(fx+e\right)^{5}-54b\tan\left(fx+e\right)^{5}+40a\tan\left(fx+e\right)^{3}-96b\tan\left(fx+e\right)^{3}+15a\tan\left(fx+e\right)-42b}{\left(\tan\left(fx+e\right)^{2}+1\right)^{3}}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (33*a*tan(f*x + e)^5 - 5
4*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 96*b*tan(f*x + e)^3 + 15*a*tan(f
*x + e) - 42*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

maple [A] time = 0.90, size = 112, normalized size = 1.14

$$\frac{a\left(-\frac{\left(\sin^{5}\left(fx+e\right)+\frac{5\left(\sin^{3}\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin^{7}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^{5}\left(fx+e\right)+\frac{5\left(\sin^{3}\left(fx+e\right)\right)}{4}+\frac{15\sin\left(\right.}{8}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x)
```

```
[Out] 1/f*(a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/
8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e))
```

maxima [A] time = 0.45, size = 111, normalized size = 1.13

$$\frac{15\left(fx+e\right)(a-6b)+48b\tan\left(fx+e\right)-\frac{3(11a-18b)\tan\left(fx+e\right)^{5}+8(5a-12b)\tan\left(fx+e\right)^{3}+3(5a-14b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{6}+3\tan\left(fx+e\right)^{4}+3\tan\left(fx+e\right)^{2}+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (3*(11*a - 18*b)*tan(f*x
 + e)^5 + 8*(5*a - 12*b)*tan(f*x + e)^3 + 3*(5*a - 14*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.89, size = 105, normalized size = 1.07

$$x\left(\frac{5a}{16}-\frac{15b}{8}\right)-\frac{\left(\frac{11a}{16}-\frac{9b}{8}\right)\tan\left(e+fx\right)^{5}+\left(\frac{5a}{6}-2b\right)\tan\left(e+fx\right)^{3}+\left(\frac{5a}{16}-\frac{7b}{8}\right)\tan\left(e+fx\right)}{f\left(\tan\left(e+fx\right)^{6}+3\tan\left(e+fx\right)^{4}+3\tan\left(e+fx\right)^{2}+1\right)}+\frac{b\tan\left(e+f\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/6 - 2*b) + tan(e + f*x)^5*
((11*a)/16 - (9*b)/8) + tan(e + f*x)*((5*a)/16 - (7*b)/8))/(f*(3*tan(e + f*
x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.9 $\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx$

Optimal. Leaf size=70

$$-\frac{(5a-4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 3/8*(a-4*b)*x-1/8*(5*a-4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*sin(
f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4132, 455, 1157, 388, 203}

$$-\frac{(5a-4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-4ax^2-4bx^4}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{4f}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{\text{Sub}}{}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\tan}{}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+}{4f}$$

Mathematica [A] time = 0.30, size = 54, normalized size = 0.77

$$\frac{12(a-4b)(e+fx)-8(a-b)\sin(2(e+fx))+a\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(e + f*x) - 8*(a - b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)] +
 32*b*Tan[e + f*x])/(32*f)
```

fricas [A] time = 0.69, size = 68, normalized size = 0.97

$$\frac{3\,(a-4\,b)fx\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^4-(5\,a-4\,b)\cos\left(fx+e\right)^2+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 4*b)*f*x*cos(f*x + e) + (2*a*cos(f*x + e)^4 - (5*a - 4*b)*cos(f
*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

giac [A] time = 0.23, size = 89, normalized size = 1.27

$$\frac{3\left(fx+e\right)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{5\,a\tan\left(fx+e\right)^3-4\,b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)-4\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - (5*a*tan(f*x + e)^3 - 4*b*t
an(f*x + e)^3 + 3*a*tan(f*x + e) - 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2
)/f
```

maple [A] time = 0.94, size = 92, normalized size = 1.31

$$\frac{a\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin^{5}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)-\frac{3fx}{2}-\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x)
```

```
[Out] 1/f*(a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+b*(sin
(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e
))
```

maxima [A] time = 0.45, size = 82, normalized size = 1.17

$$\frac{3\left(fx+e\right)(a-4b)+8b\tan\left(fx+e\right)-\frac{(5a-4b)\tan\left(fx+e\right)^{3}+(3a-4b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - ((5*a - 4*b)*tan(f*x + e)^3
 + (3*a - 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.40, size = 79, normalized size = 1.13

$$x\left(\frac{3a}{8}-\frac{3b}{2}\right)-\frac{\left(\frac{5a}{8}-\frac{b}{2}\right)\tan\left(e+fx\right)^{3}+\left(\frac{3a}{8}-\frac{b}{2}\right)\tan\left(e+fx\right)}{f\left(\tan\left(e+fx\right)^{4}+2\tan\left(e+fx\right)^{2}+1\right)}+\frac{b\tan\left(e+fx\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 - (3*b)/2) - (tan(e + f*x)^3*((5*a)/8 - b/2) + tan(e + f*x)*((3*
a)/8 - b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(e + f*x))
/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sin^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

3.10 $\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx$

Optimal. Leaf size=42

$$\frac{1}{2}x(a-2b)-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 1/2*(a-2*b)*x-1/2*a*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4132, 455, 388, 203}

$$\frac{1}{2}x(a-2b)-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

```
[Out] ((a - 2*b)*x)/2 - (a*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$
$$= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{2f}$$
$$= \frac{1}{2}(a-2b)x - \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 1.29

$$\frac{a(e+fx)}{2f} - \frac{a\sin(2(e+fx))}{4f} - \frac{b\tan^{-1}(\tan(e+fx))}{f} + \frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

```
[Out] (a*(e + f*x))/(2*f) - (b*ArcTan[Tan[e + f*x]])/f - (a*Sin[2*(e + f*x)])/(4*
f) + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.68, size = 50, normalized size = 1.19

$$\frac{(a-2\,b)fx\cos\left(fx+e\right) - \left(a\cos\left(fx+e\right)^2 - 2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a - 2*b)*f*x*cos(f*x + e) - (a*cos(f*x + e)^2 - 2*b)*sin(f*x + e))/(f
*cos(f*x + e))
```

giac [A] time = 0.26, size = 51, normalized size = 1.21

$$\frac{\left(fx+e\right)(a-2\,b) + 2\,b\tan\left(fx+e\right) - \frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

maple [A] time = 0.72, size = 46, normalized size = 1.10

$$\frac{a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + b\left(\tan\left(fx+e\right) - fx - e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x)
```

```
[Out] 1/f*(a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(tan(f*x+e)-f*x-e))
```

maxima [A] time = 0.45, size = 47, normalized size = 1.12

$$\frac{\left(fx+e\right)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

mupad [B] time = 4.24, size = 35, normalized size = 0.83

$$\frac{b\tan\left(e+f\,x\right)-\frac{a\,\sin\left(2\,e+2\,f\,x\right)}{4}+f\,x\,\left(\frac{a}{2}-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x) - (a*sin(2*e + 2*f*x))/4 + f*x*(a/2 - b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.11 $\int \left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\,\text{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

fricas [B] time = 0.76, size = 31, normalized size = 2.07

$$\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

giac [A] time = 1.44, size = 16, normalized size = 1.07

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

maple [A] time = 0.87, size = 16, normalized size = 1.07

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

maxima [A] time = 0.35, size = 15, normalized size = 1.00

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

mupad [B] time = 4.30, size = 17, normalized size = 1.13

$$\frac{b\tan\left(e + f\,x\right) + a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + a*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

3.12 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=26

$$\frac{b\tan(e+fx)}{f}-\frac{(a+b)\cot(e+fx)}{f}$$

```
[Out] -(a+b)*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 14}

$$\frac{b\tan(e+fx)}{f}-\frac{(a+b)\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 1.38

$$-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) - (b*Cot[e + f*x])/f + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.93, size = 39, normalized size = 1.50

$$-\frac{(a+2b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -((a + 2*b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

giac [A] time = 0.24, size = 28, normalized size = 1.08

$$\frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

maple [A] time = 0.92, size = 43, normalized size = 1.65

$$\frac{-a\cot\left(fx+e\right)+b\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)}-2\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
[Out] 1/f*(-a*cot(f*x+e)+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e)))
```

maxima [A] time = 0.34, size = 26, normalized size = 1.00

$$\frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

mupad [B] time = 4.21, size = 28, normalized size = 1.08

$$\frac{b\tan\left(e+f\,x\right)}{f}-\frac{a+b}{f\tan\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^2,x)
[Out] (b*tan(e + f*x))/f - (a + b)/(f*tan(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.13 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=46

$$-\frac{(a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+2*b)*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 448}

$$-\frac{(a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((a + b)*Cot[e + f*x]^3)/(3*f) + (b*Tan[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{a+2b}{x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a+2b)\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 1.83

$$-\frac{2a\cot(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}-\frac{5b\cot(e+fx)}{3f}-\frac{b\cot(e+fx)\csc^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (5*b*Cot[e + f*x])/(3*f) - (a*Cot[e + f*x]*Csc[
e + f*x]^2)/(3*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (b*Tan[e + f*x]
)/f
```

fricas [A] time = 0.72, size = 66, normalized size = 1.43

$$-\frac{2\,(a+4\,b)\cos\left(fx+e\right)^{4}-3\,(a+4\,b)\cos\left(fx+e\right)^{2}+3\,b}{3\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 4*b)*cos(f*x + e)^4 - 3*(a + 4*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

giac [A] time = 0.30, size = 54, normalized size = 1.17

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan\left(fx+e\right)^{2}+6\,b\tan\left(fx+e\right)^{2}+a+b}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 + a + b)/t
an(f*x + e)^3)/f
```

maple [A] time = 1.12, size = 73, normalized size = 1.59

$$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}\left(fx+e\right)\right)}{3}\right)\cot\left(fx+e\right)+b\left(-\frac{1}{3\sin\left(fx+e\right)^{3}\cos\left(fx+e\right)}+\frac{4}{3\sin\left(fx+e\right)\cos\left(fx+e\right)}-\frac{8\cot\left(fx+e\right)}{3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4
/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

maxima [A] time = 0.33, size = 43, normalized size = 0.93

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+2\,b)\tan\left(fx+e\right)^{2}+a+b}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + 2*b)*tan(f*x + e)^2 + a + b)/tan(f*x + e)^3
)/f
```

mupad [B] time = 4.31, size = 46, normalized size = 1.00

$$\frac{b\tan\left(e+f\,x\right)}{f}-\frac{(a+2\,b)\,\tan\left(e+f\,x\right)^{2}+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^4,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/3 + b/3 + tan(e + f*x)^2*(a + 2*b))/(f*tan(e + f*x)
^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)\csc^{4}\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.14 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=68

$$-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+3b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+3*b)*cot(f*x+e)/f-1/3*(2*a+3*b)*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
+b*tan(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4132, 448}

$$-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+3b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 3*b)*Cot[e + f*x])/f) - ((2*a + 3*b)*Cot[e + f*x]^3)/(3*f) - ((a +
b)*Cot[e + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+b+bx^2\right)}{x^6}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^6}+\frac{2a+3b}{x^4}+\frac{a+3b}{x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{(a+3b)\cot(e+fx)}{f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 128, normalized size = 1.88

$$-\frac{8a\cot(e+fx)}{15f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}+\frac{b\tan(e+fx)}{f}-\frac{11b\cot(e+fx)}{5f}-\frac{b\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-8*a*Cot[e + f*x])/(15*f) - (11*b*Cot[e + f*x])/(5*f) - (4*a*Cot[e + f*x]*
Csc[e + f*x]^2)/(15*f) - (3*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*f) - (a*Cot[e
 + f*x]*Csc[e + f*x]^4)/(5*f) - (b*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) + (b*
Tan[e + f*x])/f
```

fricas [A] time = 0.55, size = 91, normalized size = 1.34

$$-\frac{8\,(a+6\,b)\cos\left(fx+e\right)^{6}-20\,(a+6\,b)\cos\left(fx+e\right)^{4}+15\,(a+6\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\left(f\cos\left(fx+e\right)^{5}-2\,f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 6*b)*cos(f*x + e)^6 - 20*(a + 6*b)*cos(f*x + e)^4 + 15*(a + 6
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

giac [A] time = 0.94, size = 82, normalized size = 1.21

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,a\tan\left(fx+e\right)^{4}+45\,b\tan\left(fx+e\right)^{4}+10\,a\tan\left(fx+e\right)^{2}+15\,b\tan\left(fx+e\right)^{2}+3\,a+3\,b}{\tan\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 45*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 15*b*tan(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

maple [A] time = 1.18, size = 101, normalized size = 1.49

$$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^{4}\left(fx+e\right)\right)}{5}-\frac{4\left(\csc^{2}\left(fx+e\right)\right)}{15}\right)\cot\left(fx+e\right)+b\left(-\frac{1}{5\sin\left(fx+e\right)^{5}\cos\left(fx+e\right)}-\frac{2}{5\sin\left(fx+e\right)^{3}\cos\left(fx+e\right)}+\frac{8}{5\sin\left(fx+e\right)\cos(}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/5/sin(f*
x+e)^5/cos(f*x+e)-2/5/sin(f*x+e)^3/cos(f*x+e)+8/5/sin(f*x+e)/cos(f*x+e)-16/
5*cot(f*x+e)))
```

maxima [A] time = 0.34, size = 64, normalized size = 0.94

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+3\,b)\tan\left(fx+e\right)^{4}+5\,(2\,a+3\,b)\tan\left(fx+e\right)^{2}+3\,a+3\,b}{\tan\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 3*b)*tan(f*x + e)^4 + 5*(2*a + 3*b)*tan(
f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

mupad [**B**] time = 4.52, size = 60, normalized size = 0.88

$$\frac{b\tan\left(e+fx\right)}{f}-\frac{\left(a+3\,b\right)\,\tan\left(e+fx\right)^4+\left(\frac{2\,a}{3}+b\right)\,\tan\left(e+fx\right)^2+\frac{a}{5}+\frac{b}{5}}{f\tan\left(e+fx\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^6,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/5 + b/5 + tan(e + f*x)^2*((2*a)/3 + b) + tan(e + f*
x)^4*(a + 3*b))/(f*tan(e + f*x)^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.15 $\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx$

Optimal. Leaf size=97

$$-\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a^2-4*a*b+b^2)*cos(f*x+e)/f+2/3*a*(a-b)*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e)
^5/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$-\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -(((a^2 - 4*a*b + b^2)*Cos[e + f*x])/f) + (2*a*(a - b)*Cos[e + f*x]^3)/(3*f
) - (a^2*Cos[e + f*x]^5)/(5*f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e
+ f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)^2}{x^4}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(-4a+b)}{a^2}\right)+\frac{b^2}{x^4}+\frac{2(a-b)b}{x^2}-2a(a-b)x^2+a^2x^4\right)d\right.}{f}\\ &= -\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)}{f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}-\frac{a^2\cos^5(e}{5f}\end{aligned}$$

Mathematica [A] time = 0.61, size = 118, normalized size = 1.22

$$-\frac{\sec^3(e+fx)\left(24\left(22a^2-215ab+120b^2\right)\cos(2(e+fx))+12\left(7a^2-60ab+20b^2\right)\cos(4(e+fx))-16a^2\cos(\right.}{1920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -1/1920*((425*a^2 - 4400*a*b + 2000*b^2 + 24*(22*a^2 - 215*a*b + 120*b^2)*C
os[2*(e + f*x)] + 12*(7*a^2 - 60*a*b + 20*b^2)*Cos[4*(e + f*x)] - 16*a^2*Co
s[6*(e + f*x)] + 40*a*b*Cos[6*(e + f*x)] + 3*a^2*Cos[8*(e + f*x)])*Sec[e +
f*x]^3)/f
```

fricas [A] time = 0.50, size = 90, normalized size = 0.93

$$-\frac{3\,a^2\cos\left(fx+e\right)^8-10\left(a^2-ab\right)\cos\left(fx+e\right)^6+15\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-30\left(ab-b^2\right)\cos\left(fx+e\right)^2-}{15\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^8 - 10*(a^2 - a*b)*cos(f*x + e)^6 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e)^4 - 30*(a*b - b^2)*cos(f*x + e)^2 - 5*b^2)/(f*cos(f*
x + e)^3)
```

giac [B] time = 0.49, size = 446, normalized size = 4.60

$$2\left(\frac{5\left(6\,ab-5\,b^2+\frac{12\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{12\,b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{6\,ab\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{3\,b^2\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)^3}+\frac{8\,a^2-50\,ab+15\,b^2-\frac{40\,a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{220\,ab\left(\cos\left(fx+e\right)\right.}{\cos\left(fx+e\right)+1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/15*(5*(6*a*b - 5*b^2 + 12*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*
b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3 + (8*a^2 - 50*a*b + 15*b^2 - 40*a^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 220*a*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 320*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*b*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 30*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*
(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - 1)^5)/f
```

maple [A] time = 0.90, size = 155, normalized size = 1.60

$$\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+2ab\left(\frac{\sin^6\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)\right)+b^2\left(\frac{\sin^6}{3\cos}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x
+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*s
```

```
in(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*x+e)^4+4/3*sin(
f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.32, size = 89, normalized size = 0.92

$$-\frac{3\,a^2\cos\left(fx+e\right)^5-10\left(a^2-ab\right)\cos\left(fx+e\right)^3+15\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)-\frac{5\left(6\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2\right)}{\cos\left(fx+e\right)^3}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 10*(a^2 - a*b)*cos(f*x + e)^3 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e) - 5*(6*(a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

mupad [B] time = 4.30, size = 87, normalized size = 0.90

$$\frac{\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\,\left(2\,a\,b-2\,b^2\right)}{\cos\left(e+f\,x\right)^3}-\cos\left(e+f\,x\right)\,\left(a^2-4\,a\,b+b^2\right)-\frac{a^2\cos\left(e+f\,x\right)^5}{5}+\frac{2\,a\cos\left(e+f\,x\right)^3\,\left(a-b\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((b^2/3 + cos(e + f*x)^2*(2*a*b - 2*b^2))/cos(e + f*x)^3 - cos(e + f*x)*(a^
2 - 4*a*b + b^2) - (a^2*cos(e + f*x)^5)/5 + (2*a*cos(e + f*x)^3*(a - b))/3)
/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.16 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^3(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\cos^3(e+fx)}{3f}-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a*(a-2*b)*cos(f*x+e)/f+1/3*a^2*cos(f*x+e)^3/f+(2*a-b)*b*sec(f*x+e)/f+1/3*b
^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$\frac{a^2\cos^3(e+fx)}{3f}-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cos[e + f*x])/f) + (a^2*Cos[e + f*x]^3)/(3*f) + ((2*a - b)*b
*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2 \sin^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)\left(b+ax^2\right)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2}{}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 83, normalized size = 1.15

$$\frac{\sec^3(e+fx)\left(-3\left(11a^2-64ab+16b^2\right)\cos(2(e+fx))+a^2\cos(6(e+fx))-26a^2-6a(a-4b)\cos(4(e+fx))+16\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cos[2*(e + f*x)
] - 6*a*(a - 4*b)*Cos[4*(e + f*x)] + a^2*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/
(96*f)
```

fricas [A] time = 0.46, size = 67, normalized size = 0.93

$$\frac{a^2 \cos\left(fx+e\right)^6 - 3\left(a^2 - 2\,ab\right)\cos\left(fx+e\right)^4 + 3\left(2\,ab - b^2\right)\cos\left(fx+e\right)^2 + b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^6 - 3*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2*a*b - b^2)*
cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

giac [A] time = 0.40, size = 97, normalized size = 1.35

$$\frac{6\,ab\cos\left(fx+e\right)^2 - 3\,b^2\cos\left(fx+e\right)^2 + b^2}{3\,f\cos\left(fx+e\right)^3} + \frac{a^2 f^{11}\cos\left(fx+e\right)^3 - 3\,a^2 f^{11}\cos\left(fx+e\right) + 6\,abf^{11}\cos\left(fx+e\right)}{3\,f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 3*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 6*a*b*f^11*cos(f
*x + e))/f^12
```

maple [A] time = 0.89, size = 125, normalized size = 1.74

$$\frac{-\frac{a^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3} + 2ab\left(\frac{\sin^4\left(fx+e\right)}{\cos\left(fx+e\right)} + \left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)\right) + b^2\left(\frac{\sin^4\left(fx+e\right)}{3\cos\left(fx+e\right)^3} - \frac{\sin^4\left(fx+e\right)}{3\cos\left(fx+e\right)} - \cdots\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x)
```

```
[Out] 1/f*(-1/3*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2
+sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*sin(f*x+e)^4/cos(f*x+e)^3-1/3*sin(f*x+e
)^4/cos(f*x+e)-1/3*(2+sin(f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.32, size = 67, normalized size = 0.93

$$\frac{a^2\cos\left(fx+e\right)^3 - 3\left(a^2 - 2\,ab\right)\cos\left(fx+e\right) + \frac{3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2 + b^2}{\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^3 - 3*(a^2 - 2*a*b)*cos(f*x + e) + (3*(2*a*b - b^2)*c
os(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

mupad [B] time = 4.14, size = 66, normalized size = 0.92

$$\frac{\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\left(2\,a\,b-b^2\right)}{\cos\left(e+f\,x\right)^3} + \frac{a^2\cos\left(e+f\,x\right)^3}{3} - a\,\cos\left(e+f\,x\right)\,\left(a-2\,b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((b^2/3 + cos(e + f*x)^2*(2*a*b - b^2))/cos(e + f*x)^3 + (a^2*cos(e + f*x)^
3)/3 - a*cos(e + f*x)*(a - 2*b))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.17 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin(e+fx)\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a^2*cos(f*x+e)/f+2*a*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 270}

$$-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

```
[Out] -((a^2*Cos[e + f*x])/f) + (2*a*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*
f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 1.63

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(-3a^2\cos^4(e+fx)+6ab\cos^2(e+fx)+b^2\right)}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*(b^2 + 6*a*b*Cos[e + f*x]^2 - 3*a^2*Cos[e + f*x
]^4)*Sec[e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.58, size = 44, normalized size = 0.96

$$-\frac{3\,a^2\cos\left(fx+e\right)^4-6\,ab\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="fricas")
[Out] -1/3*(3*a^2*cos(f*x + e)^4 - 6*a*b*cos(f*x + e)^2 - b^2)/(f*cos(f*x + e)^3)
```

giac [A] time = 1.36, size = 47, normalized size = 1.02

$$-\frac{a^2\cos\left(fx+e\right)}{f}+\frac{6\,ab\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="giac")
[Out] -a^2*cos(f*x + e)/f + 1/3*(6*a*b*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

maple [A] time = 0.28, size = 42, normalized size = 0.91

$$\frac{\frac{\left(\sec^3\left(fx+e\right)\right)b^2}{3}+2ab\sec\left(fx+e\right)-\frac{a^2}{\sec\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x)
[Out] 1/f*(1/3*sec(f*x+e)^3*b^2+2*a*b*sec(f*x+e)-a^2/sec(f*x+e))
```

maxima [A] time = 0.32, size = 42, normalized size = 0.91

$$-\frac{3\,a^2\cos\left(fx+e\right)-\frac{6\,ab}{\cos\left(fx+e\right)}-\frac{b^2}{\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="maxima")
[Out] -1/3*(3*a^2*cos(f*x + e) - 6*a*b/cos(f*x + e) - b^2/cos(f*x + e)^3)/f
```

mupad [B] time = 0.06, size = 45, normalized size = 0.98

$$\frac{\frac{b^2}{3}+2\,a\,b\cos\left(e+f\,x\right)^2}{f\cos\left(e+f\,x\right)^3}-\frac{a^2\,\cos\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
[Out] (b^2/3 + 2*a*b*cos(e + f*x)^2)/(f*cos(e + f*x)^3) - (a^2*cos(e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x), x)
```

3.18 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=52

$$\frac{b(2a+b)\sec(e+fx)}{f}-\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a+b)^2*arctanh(cos(f*x+e))/f+b*(2*a+b)*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/
f
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 461, 207}

$$\frac{b(2a+b)\sec(e+fx)}{f}-\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cos[e + f*x]])/f) + (b*(2*a + b)*Sec[e + f*x])/f + (b^
2*Sec[e + f*x]^3)/(3*f)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx,x,\cos(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x\ldots\right.}{f}\\
&= -\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+f\ldots}{3f}
\end{aligned}$$

Mathematica [B] time = 0.51, size = 108, normalized size = 2.08

$$-\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(-3b(2a+b)\cos^2(e+fx)+3(a+b)^2\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.\right.\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cos[e + f*x]^2 + 3*(a +
b)^2*Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e
+ f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [B] time = 0.66, size = 101, normalized size = 1.94

$$-\frac{3\left(a^2+2ab+b^2\right)\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-3\left(a^2+2ab+b^2\right)\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\cos\left(fx\right.\right.}{6f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*
(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b
+ b^2)*cos(f*x + e)^2 - 2*b^2)/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-6*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))*a*b+6*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-6*a*b-4*b^2)*1/3/
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^3+(a^2+2*a*b+b^2)/4*ln(abs(1-co
s(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.78, size = 117, normalized size = 2.25

$$\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{2ab}{f\cos\left(fx+e\right)}+\frac{2ab\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{b^2}{3f\cos\left(fx+e\right)^3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*a^2*ln(csc(f*x+e)-cot(f*x+e))+2/f*a*b/cos(f*x+e)+2/f*a*b*ln(csc(f*x+e)-
cot(f*x+e))+1/3/f*b^2/cos(f*x+e)^3+1/f*b^2/cos(f*x+e)+1/f*b^2*ln(csc(f*x+e)
-cot(f*x+e))
```

maxima [A] time = 0.36, size = 82, normalized size = 1.58

$$-\frac{3\left(a^2+2ab+b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+2ab+b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right.}{\cos\left(fx+e\right)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 2*a*b + b^2)*l
og(cos(f*x + e) - 1) - 2*(3*(2*a*b + b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

mupad [B] time = 0.12, size = 53, normalized size = 1.02

$$\frac{\cos\left(e+f\,x\right)^2\,\left(b^2+2\,a\,b\right)+\frac{b^2}{3}}{f\,\cos\left(e+f\,x\right)^3}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,{\left(a+b\right)}^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x),x)
```

```
[Out] (cos(e + f*x)^2*(2*a*b + b^2) + b^2/3)/(f*cos(e + f*x)^3) - (atanh(cos(e +
f*x))*(a + b)^2)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x), x)
```

3.19 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=104

$$-\frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}-\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2}{}$$

```
[Out] -1/2*(a+b)*(a+5*b)*arctanh(cos(f*x+e))/f-1/6*(3*a^2+6*a*b+5*b^2)*cot(f*x+e)
*csc(f*x+e)/f+1/3*b*(6*a+5*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^2*sec(f*x+e)^
3/f
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4133, 462, 456, 453, 206}

$$-\frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}-\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + b)*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(2*f) - ((3*a^2 + 6*a*b + 5*b^2)
*Cot[e + f*x]*Csc[e + f*x])/(6*f) + (b*(6*a + 5*b)*Sec[e + f*x])/(3*f) + (b
^2*Csc[e + f*x]^2*Sec[e + f*x]^3)/(3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2\left(1-x^2\right)^2}\,dx, x, \cos(e+f\right.}{3f}$$

$$= -\frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b^2\csc^2(e+fx)\sec^3(e+}{3f}$$

$$= -\frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b(6a+5b)\sec(e+fx)}{3f}$$

$$= -\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx}{6f}$$

Mathematica [B] time = 6.59, size = 1021, normalized size = 9.82

$$\frac{\left(-a^2-2ba-b^2\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(b\sec^2(e+fx)+a\right)^2\cos^4(e+fx)}{2f(\cos(2e+2fx)a+a+2b)^2} + \frac{\left(a^2+2ba+b^2\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(b\sec^2(e+fx)\right.}{2f(\cos(2e+2fx)a+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-a^2 - 2*a*b - b^2)*Cos[e + f*x]^4*Csc[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*
x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) - (2*(a^2 + 6*a*b + 5*b^2)*
Cos[e + f*x]^4*Log[Cos[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*
b + a*Cos[2*e + 2*f*x])^2) + (2*(a^2 + 6*a*b + 5*b^2)*Cos[e + f*x]^4*Log[Si
n[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*b + a*Cos[2*e + 2*f*x
])^2) + (2*b*(12*a + 13*b)*Cos[e + f*x]^4*Sec[e]*(a + b*Sec[e + f*x]^2)^2)/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + ((a^2 + 2*a*b + b^2)*Cos[e + f*x]^
4*Sec[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^2) + (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*Sin[(f*x)/2])
/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*
x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(
b^2*Cos[e/2] + b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (2*Cos[e + f*
x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*Sin[(f*x)/2]))/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x
)/2] - Sin[e/2 + (f*x)/2])) - (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^
2*Sin[(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])
*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[
e + f*x]^2)^2*(b^2*Cos[e/2] - b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*
```

```
f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2)
- (2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*
Sin[(f*x)/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])*
(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

fricas [B] time = 0.61, size = 193, normalized size = 1.86

$$\frac{6\left(a^2+6\,ab+5\,b^2\right)\cos\left(fx+e\right)^4-4\left(6\,ab+5\,b^2\right)\cos\left(fx+e\right)^2-4\,b^2-3\left(\left(a^2+6\,ab+5\,b^2\right)\cos\left(fx+e\right)^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 4*(6*a*b + 5*b^2)*cos(f*x +
e)^2 - 4*b^2 - 3*((a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b
^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 6*a*b + 5*b^2)*
cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e
) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-10*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-12*(1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))*b*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
*a^2-b^2-2*b*a-a^2)*1/16/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+(-9*((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2-6*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^2*b*a+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2+12*(1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a-7*b^2-6*b*a)*1/3/((1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1)))-1)^3+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2+2*(1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))*a^2)/16+(5*b^2+6*b*a+a^2)/8*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+ex
p(1)))))
```

maple [B] time = 1.38, size = 195, normalized size = 1.88

$$-\frac{a^2\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}-\frac{ab}{f\sin\left(fx+e\right)^2\cos\left(fx+e\right)}+\frac{3ab}{f\cos\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*csc(f*x+e)*cot(f*x+e)+1/2/f*a^2*ln(csc(f*x+e)-cot(f*x+e))-1/f*a*
b/sin(f*x+e)^2/cos(f*x+e)+3/f*a*b/cos(f*x+e)+3/f*a*b*ln(csc(f*x+e)-cot(f*x+
e))+1/3/f*b^2/sin(f*x+e)^2/cos(f*x+e)^3-5/6/f*b^2/sin(f*x+e)^2/cos(f*x+e)+5
/2/f*b^2/cos(f*x+e)+5/2/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.33, size = 126, normalized size = 1.21

$$-\frac{3\left(a^2+6\,ab+5\,b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+6\,ab+5\,b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^2+6\,ab+5\,b^2\right)\cos(f\right.}{\cos(f}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 6*a*b + 5*b
^2)*log(cos(f*x + e) - 1) - 2*(3*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 2*(
6*a*b + 5*b^2)*cos(f*x + e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

mupad [B] time = 4.29, size = 96, normalized size = 0.92

$$\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\,\left(\frac{5\,b^2}{3}+2\,a\,b\right)-\cos\left(e+f\,x\right)^4\,\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right)}{f\,\left(\cos\left(e+f\,x\right)^3-\cos\left(e+f\,x\right)^5\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(a+b\right)\,\left(a+5\,b\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

```
[Out] (b^2/3 + cos(e + f*x)^2*(2*a*b + (5*b^2)/3) - cos(e + f*x)^4*(3*a*b + a^2/2
 + (5*b^2)/2))/(f*(cos(e + f*x)^3 - cos(e + f*x)^5)) - (atanh(cos(e + f*x))
*(a + b)*(a + 5*b))/(2*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.20 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=141

$$-\frac{\left(3a^2+30ab+35b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)\csc^3(e+fx)}{12f}+\frac{b(6a+7b)\sec(e+f}{3f}$$

```
[Out] -1/8*(3*a^2+30*a*b+35*b^2)*arctanh(cos(f*x+e))/f-1/24*(3*a+7*b)^2*cot(f*x+e
)*csc(f*x+e)/f-1/12*(3*a^2+6*a*b+7*b^2)*cot(f*x+e)*csc(f*x+e)^3/f+1/3*b*(6*
a+7*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^4*sec(f*x+e)^3/f
```

Rubi [A] time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 462, 456, 453, 206}

$$-\frac{\left(3a^2+30ab+35b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)\csc^3(e+fx)}{12f}+\frac{b(6a+7b)\sec(e+f}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)^2*C
ot[e + f*x]*Csc[e + f*x])/(24*f) - ((3*a^2 + 6*a*b + 7*b^2)*Cot[e + f*x]*Cs
c[e + f*x]^3)/(12*f) + (b*(6*a + 7*b)*Sec[e + f*x])/(3*f) + (b^2*Csc[e + f*
x]^4*Sec[e + f*x]^3)/(3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)^3} dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+7b)+3a^2x^2}{x^2\left(1-x^2\right)^3} dx, x, \cos(e+f$$

$$= -\frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)\csc^3(e+fx)}{12f} + \frac{b^2\csc^4(e+fx)\sec^3(e}{3f}$$

$$= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)c}{12f}$$

$$= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)c}{12f}$$

$$= -\frac{\left(3a^2+30ab+35b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(3a+7b)^2\cot(e+fx)c}{24f}$$

Mathematica [A] time = 1.85, size = 218, normalized size = 1.55

$$-\frac{\sec^4(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(\frac{1}{2}\left(105a^2+282ab+329b^2\right)(\cos(e+fx)+\cos(3(e+fx)))\csc^4(e+fx)+96}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/192*((b + a*Cos[e + f*x]^2)^2*((90*a^2 + 132*a*b - 102*b^2 + (6*a^2 + 60
*a*b + 70*b^2)*Cos[4*(e + f*x)] - 3*(3*a^2 + 30*a*b + 35*b^2)*Cos[6*(e + f*
x)])*Cot[e + f*x]*Csc[e + f*x]^3 + ((105*a^2 + 282*a*b + 329*b^2)*(Cos[e +
f*x] + Cos[3*(e + f*x)])*Csc[e + f*x]^4)/2 + 96*(3*a^2 + 30*a*b + 35*b^2)*C
os[e + f*x]^4*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e + f*x]
^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [B] time = 0.63, size = 286, normalized size = 2.03

$$\frac{6\left(3a^2+30ab+35b^2\right)\cos\left(fx+e\right)^6-10\left(3a^2+30ab+35b^2\right)\cos\left(fx+e\right)^4+16\left(6ab+7b^2\right)\cos\left(fx+e\right)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 30*a*b + 35*
b^2)*cos(f*x + e)^4 + 16*(6*a*b + 7*b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^
2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x +
 e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/
2) + 3*((3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b
^2)*cos(f*x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(-1/2*cos
(f*x + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-210*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2-180*((1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a-18*((1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1))))^2*a^2-24*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-32*(1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*a^2-b^2-2*b*a-a^2)*1/128/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2+
(-12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2-6*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*b*a+18*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2+
12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a-10*b^2-6*b*a)*1/3/((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))-1)^3+(32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*b^2+64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a+32*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+768*(1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))*b^2+1024*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a+256*(1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))*a^2)/4096+(35*b^2+30*b*a+3*a^2)/32*ln(abs(1-
cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 1.21, size = 264, normalized size = 1.87

$$-\frac{a^2\cot\left(fx+e\right)\left(\csc^3\left(fx+e\right)\right)}{4f}-\frac{3a^2\csc\left(fx+e\right)\cot\left(fx+e\right)}{8f}+\frac{3a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{}{2f\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/4/f*a^2*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a^2*csc(f*x+e)*cot(f*x+e)+3/8/f*a^
2*ln(csc(f*x+e)-cot(f*x+e))-1/2/f*a*b/sin(f*x+e)^4/cos(f*x+e)-5/4/f*a*b/sin
(f*x+e)^2/cos(f*x+e)+15/4/f*a*b/cos(f*x+e)+15/4/f*a*b*ln(csc(f*x+e)-cot(f*x
+e))-1/4/f*b^2/sin(f*x+e)^4/cos(f*x+e)^3+7/12/f*b^2/sin(f*x+e)^2/cos(f*x+e)
^3-35/24/f*b^2/sin(f*x+e)^2/cos(f*x+e)+35/8/f*b^2/cos(f*x+e)+35/8/f*b^2*ln(
csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.35, size = 165, normalized size = 1.17

$$-\frac{3\left(3a^2+30ab+35b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3a^2+30ab+35b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3a^2+30a\right.\right.}{}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 30*a*b + 35*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 30*a*
b + 35*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x
+ e)^6 - 5*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^4 + 8*(6*a*b + 7*b^2)*cos
(f*x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/
f
```

mupad [B] time = 4.42, size = 135, normalized size = 0.96

$$\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\,\left(\frac{7\,b^2}{3}+2\,a\,b\right)+\cos\left(e+f\,x\right)^6\,\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)-\cos\left(e+f\,x\right)^4\,\left(\frac{5\,a^2}{8}+\frac{25\,a\,b}{4}+\frac{175\,b^2}{24}\right)}{f\,\left(\cos\left(e+f\,x\right)^7-2\,\cos\left(e+f\,x\right)^5+\cos\left(e+f\,x\right)^3\right)}-\text{ata}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

```
[Out] (b^2/3 + cos(e + f*x)^2*(2*a*b + (7*b^2)/3) + cos(e + f*x)^6*((15*a*b)/4 +
(3*a^2)/8 + (35*b^2)/8) - cos(e + f*x)^4*((25*a*b)/4 + (5*a^2)/8 + (175*b^2
)/24))/(f*(cos(e + f*x)^3 - 2*cos(e + f*x)^5 + cos(e + f*x)^7)) - (atanh(co
s(e + f*x))*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/8))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.21 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx$

Optimal. Leaf size=148

$$-\frac{\left(a^2-12ab+12b^2\right)\tan(e+fx)}{6f}-\frac{\left(3a^2-36ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x\left(a^2-12ab+8b^2\right)+\frac{a^2\sin^6}{}$$

```
[Out] 5/16*(a^2-12*a*b+8*b^2)*x-1/16*(3*a^2-36*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)/f
+1/24*a*(a-12*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*(a^2-12*a*b+12*b^2)*tan(f*x+
e)/f+1/6*a^2*sin(f*x+e)^6*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 463, 455, 1814, 1153, 203}

$$-\frac{\left(a^2-12ab+12b^2\right)\tan(e+fx)}{6f}-\frac{\left(3a^2-36ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x\left(a^2-12ab+8b^2\right)+\frac{a^2\sin^6}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] (5*(a^2 - 12*a*b + 8*b^2)*x)/16 - ((3*a^2 - 36*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(a - 12*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((
a^2 - 12*a*b + 12*b^2)*Tan[e + f*x])/(6*f) + (a^2*Sin[e + f*x]^6*Tan[e + f*
x])/(6*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{x^6\left(7a^2-6(a+b)^2-6b^2x^2\right)}{\left(1+x^2\right)^3}\,dx, x, \tan(e\right.}{6f}$$

$$= \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} + -$$

$$= -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx}{24f}$$

$$= -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx}{24f}$$

$$= -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx}{24f}$$

$$= \frac{5}{16}\left(a^2-12ab+8b^2\right)x - \frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} +$$

Mathematica [B] time = 1.35, size = 499, normalized size = 3.37

$$\frac{\sec(e)\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(360fx\left(a^2-12ab+8b^2\right)\cos(2e+fx)+360fx\left(a^2-12ab+8b^2\right)\cos(f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e]*Sec[e + f*x]^3*(360*(a^2 - 12*a*b + 8*b^2)
*f*x*Cos[f*x] + 360*(a^2 - 12*a*b + 8*b^2)*f*x*Cos[2*e + f*x] + 120*a^2*f*x
*Cos[2*e + 3*f*x] - 1440*a*b*f*x*Cos[2*e + 3*f*x] + 960*b^2*f*x*Cos[2*e + 3
*f*x] + 120*a^2*f*x*Cos[4*e + 3*f*x] - 1440*a*b*f*x*Cos[4*e + 3*f*x] + 960*
```

```
b^2*f*x*Cos[4*e + 3*f*x] - 81*a^2*Sin[f*x] + 3444*a*b*Sin[f*x] - 3168*b^2*S
in[f*x] - 81*a^2*Sin[2*e + f*x] - 1164*a*b*Sin[2*e + f*x] + 2208*b^2*Sin[2*
e + f*x] - 109*a^2*Sin[2*e + 3*f*x] + 2076*a*b*Sin[2*e + 3*f*x] - 1936*b^2*
Sin[2*e + 3*f*x] - 109*a^2*Sin[4*e + 3*f*x] + 540*a*b*Sin[4*e + 3*f*x] - 14
4*b^2*Sin[4*e + 3*f*x] - 21*a^2*Sin[4*e + 5*f*x] + 156*a*b*Sin[4*e + 5*f*x]
 - 48*b^2*Sin[4*e + 5*f*x] - 21*a^2*Sin[6*e + 5*f*x] + 156*a*b*Sin[6*e + 5*
f*x] - 48*b^2*Sin[6*e + 5*f*x] + 6*a^2*Sin[6*e + 7*f*x] - 12*a*b*Sin[6*e +
7*f*x] + 6*a^2*Sin[8*e + 7*f*x] - 12*a*b*Sin[8*e + 7*f*x] - a^2*Sin[8*e + 9
*f*x] - a^2*Sin[10*e + 9*f*x]))/(768*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.56, size = 131, normalized size = 0.89

$$\frac{15\left(a^2-12\,ab+8\,b^2\right)fx\cos\left(fx+e\right)^3-\left(8\,a^2\cos\left(fx+e\right)^8-2\left(13\,a^2-12\,ab\right)\cos\left(fx+e\right)^6+3\left(11\,a^2-36\,ab\right.\right.}{48\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a^2 - 12*a*b + 8*b^2)*f*x*cos(f*x + e)^3 - (8*a^2*cos(f*x + e)^8
- 2*(13*a^2 - 12*a*b)*cos(f*x + e)^6 + 3*(11*a^2 - 36*a*b + 8*b^2)*cos(f*x
+ e)^4 - 16*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 16*b^2)*sin(f*x + e))/(f*cos(f
*x + e)^3)
```

giac [A] time = 1.27, size = 197, normalized size = 1.33

$$\frac{16\,b^2\tan\left(fx+e\right)^3+96\,ab\tan\left(fx+e\right)-96\,b^2\tan\left(fx+e\right)+15\left(a^2-12\,ab+8\,b^2\right)\left(fx+e\right)-\frac{33\,a^2\tan\left(fx+e\right)^5-\cdots}{\cdots}}{48\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 96*a*b*tan(f*x + e) - 96*b^2*tan(f*x + e) + 1
5*(a^2 - 12*a*b + 8*b^2)*(f*x + e) - (33*a^2*tan(f*x + e)^5 - 108*a*b*tan(f
*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 192*a*b*tan(f*x
 + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) - 84*a*b*tan(f*x + e)
 + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

maple [A] time = 0.92, size = 199, normalized size = 1.34

$$\frac{a^2\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin^7\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x)
```

```
[Out] 1/f*(a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)+2*a*b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)
^3+15/8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e)+b^2*(1/3*sin(f*x+e)^7/cos(f
*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e))
```

maxima [A] time = 0.45, size = 164, normalized size = 1.11

$$\frac{16\,b^2\tan\left(fx+e\right)^3+15\left(a^2-12\,ab+8\,b^2\right)\left(fx+e\right)+96\left(ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(11\,a^2-36\,ab+8\,b^2\right)\tan\left(fx+e\right)^5+8\left(5\,a^2\cdots\right.}{\tan\left(fx+e\right)^6+3\cdots}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) + 96*(a*b
 - b^2)*tan(f*x + e) - (3*(11*a^2 - 36*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a
^2 - 24*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(5*a^2 - 28*a*b + 8*b^2)*tan(f*x +
e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.79, size = 163, normalized size = 1.10

$$x\left(\frac{5\,a^2}{16}-\frac{15\,a\,b}{4}+\frac{5\,b^2}{2}\right)-\frac{\left(\frac{11\,a^2}{16}-\frac{9\,a\,b}{4}+\frac{b^2}{2}\right)\tan\left(e+f\,x\right)^5+\left(\frac{5\,a^2}{6}-4\,a\,b+b^2\right)\tan\left(e+f\,x\right)^3+\left(\frac{5\,a^2}{16}-\frac{7\,a\,b}{4}+\frac{b^2}{2}\right)\cdots}{f\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((5*a^2)/16 - (15*a*b)/4 + (5*b^2)/2) - (tan(e + f*x)*((5*a^2)/16 - (7*a*
b)/4 + b^2/2) + tan(e + f*x)^3*((5*a^2)/6 - 4*a*b + b^2) + tan(e + f*x)^5*(
(11*a^2)/16 - (9*a*b)/4 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 +
 tan(e + f*x)^6 + 1)) + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(4*b^2 -
 2*b*(a + b)))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.22 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx$

Optimal. Leaf size=114

$$-\frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{a(a-8b)\sin(e+fx)\cos}{8f}$$

```
[Out] 1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*(a^2-8
*a*b+4*b^2)*tan(f*x+e)/f+1/4*a^2*sin(f*x+e)^4*tan(f*x+e)/f+1/3*b^2*tan(f*x+
e)^3/f
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 455, 1153, 203}

$$-\frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{a(a-8b)\sin(e+fx)\cos}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8
*f) - ((a^2 - 8*a*b + 4*b^2)*Tan[e + f*x])/(4*f) + (a^2*Sin[e + f*x]^4*Tan[
e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{x^4\left(5a^2-4(a+b)^2-4b^2x^2\right)}{\left(1+x^2\right)^2}\,dx,x,\tan(e\right.}{4f}$$

$$= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{\text{Su}}{}$$

$$= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{\text{Su}}{}$$

$$= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f} + \frac{a}{}$$

$$= \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-8a\right.}{}$$

Mathematica [A] time = 1.62, size = 153, normalized size = 1.34

$$\frac{\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3\cos^3(e+fx)\left(4fx\left(3a^2-24ab+8b^2\right)+a^2\sin(4(e+fx))-8a(a-2b)\sin(2\right.\right.}{24f(a\cos(2(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(32*b^2*Sec[e]*Sin[f*x] + 64*(3*a
- 2*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*Cos[e + f*x]^3*(4*(3*a^2 - 24*a
*b + 8*b^2)*f*x - 8*a*(a - 2*b)*Sin[2*(e + f*x)] + a^2*Sin[4*(e + f*x)]) +
32*b^2*Cos[e + f*x]*Tan[e]))/(24*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.65, size = 107, normalized size = 0.94

$$\frac{3\left(3a^2-24ab+8b^2\right)fx\cos\left(fx+e\right)^3+\left(6a^2\cos\left(fx+e\right)^6-3\left(5a^2-8ab\right)\cos\left(fx+e\right)^4+16\left(3ab-2b^2\right)\cos\right.}{24f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 - 24*a*b + 8*b^2)*f*x*cos(f*x + e)^3 + (6*a^2*cos(f*x + e)^6
 - 3*(5*a^2 - 8*a*b)*cos(f*x + e)^4 + 16*(3*a*b - 2*b^2)*cos(f*x + e)^2 + 8
*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [A] time = 0.40, size = 132, normalized size = 1.16

$$\frac{8\,b^2\tan\left(fx+e\right)^3+48\,ab\tan\left(fx+e\right)-24\,b^2\tan\left(fx+e\right)+3\left(3\,a^2-24\,ab+8\,b^2\right)\left(fx+e\right)-\frac{3\left(5\,a^2\tan\left(fx+e\right)^3-\cdots\right.}{\cdots}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 48*a*b*tan(f*x + e) - 24*b^2*tan(f*x + e) + 3*
(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) - 3*(5*a^2*tan(f*x + e)^3 - 8*a*b*tan(f*
x + e)^3 + 3*a^2*tan(f*x + e) - 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)
/f
```

maple [A] time = 0.90, size = 123, normalized size = 1.08

$$\frac{a^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin^5\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)-\frac{3f}{2}\cdots\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x)
```

```
[Out] 1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x
-3/2*e)+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

maxima [A] time = 0.45, size = 120, normalized size = 1.05

$$\frac{8\,b^2\tan\left(fx+e\right)^3+3\left(3\,a^2-24\,ab+8\,b^2\right)\left(fx+e\right)+24\left(2\,ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5\,a^2-8\,ab\right)\tan\left(fx+e\right)^3+\left(3\,a^2-8\cdots\right.\right.}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\cdots\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) + 24*(2*a
*b - b^2)*tan(f*x + e) - 3*((5*a^2 - 8*a*b)*tan(f*x + e)^3 + (3*a^2 - 8*a*b
)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.33, size = 116, normalized size = 1.02

$$x\left(\frac{3\,a^2}{8}-3\,a\,b+b^2\right)+\frac{\left(a\,b-\frac{5\,a^2}{8}\right)\,\tan\left(e+f\,x\right)^3+\left(a\,b-\frac{3\,a^2}{8}\right)\,\tan\left(e+f\,x\right)}{f\,\left(\tan\left(e+f\,x\right)^4+2\,\tan\left(e+f\,x\right)^2+1\right)}+\frac{b^2\,\tan\left(e+f\,x\right)^3}{3\,f}-\frac{\tan\left(e+f\,x\right)\,\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - 3*a*b + b^2) + (tan(e + f*x)*(a*b - (3*a^2)/8) + tan(e + f*x
)^3*(a*b - (5*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b^2*t
an(e + f*x)^3)/(3*f) - (tan(e + f*x)*(3*b^2 - 2*b*(a + b)))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.23 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx$

Optimal. Leaf size=73

$$\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{1}{2}ax(a-4b)+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a*(a-4*b)*x-1/2*a*(a-4*b)*tan(f*x+e)/f+1/2*a^2*sin(f*x+e)^2*tan(f*x+e)/
f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 459, 321, 203}

$$\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{1}{2}ax(a-4b)+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] (a*(a - 4*b)*x)/2 - (a*(a - 4*b)*Tan[e + f*x])/(2*f) + (a^2*Sin[e + f*x]^2*
Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
```

```
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{\text{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2-2b^2x^2\right)}{1+x^2}\,dx,x,\tan(e\right.}{2f}$$

$$=\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3(e+fx)}{3f}-\frac{(a(a-4b))\,\text{Subst}\left(\int\right.}{2}$$

$$=-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3(e+fx)}{3f}$$

$$=\frac{1}{2}a(a-4b)x-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+$$

Mathematica [A] time = 1.00, size = 126, normalized size = 1.73

$$-\frac{\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a\cos^3(e+fx)(a\sin(2(e+fx))-2fx(a-4b))-4b(6a-b)\sec(e)\sin(fx)\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] -1/3*((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(-4*b^2*Sec[e]*Sin[f*x] - 4*(
6*a - b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*a*Cos[e + f*x]^3*(-2*(a - 4*b
)*f*x + a*Sin[2*(e + f*x)]) - 4*b^2*Cos[e + f*x]*Tan[e]))/(f*(a + 2*b + a*C
os[2*(e + f*x)])^2)
```

fricas [A] time = 0.58, size = 81, normalized size = 1.11

$$\frac{3\left(a^2-4\,ab\right)fx\cos\left(fx+e\right)^3-\left(3\,a^2\cos\left(fx+e\right)^4-2\left(6\,ab-b^2\right)\cos\left(fx+e\right)^2-2\,b^2\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 4*a*b)*f*x*cos(f*x + e)^3 - (3*a^2*cos(f*x + e)^4 - 2*(6*a*b
- b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [A] time = 1.37, size = 72, normalized size = 0.99

$$\frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\left(a^2-4\,ab\right)\left(fx+e\right)-\frac{3\,a^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

maple [A] time = 0.52, size = 71, normalized size = 0.97

$$\frac{a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(\tan\left(fx+e\right)-fx-e\right)+\frac{b^2\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x)
```

```
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(tan(f*x+e)-f*x-e
)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

maxima [A] time = 0.44, size = 67, normalized size = 0.92

$$\frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\left(a^2-4\,ab\right)\left(fx+e\right)-\frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.46, size = 94, normalized size = 1.29

$$\frac{b^2\tan\left(e+f\,x\right)^3}{3\,f}-\frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{\tan\left(e+f\,x\right)\,\left(2\,b^2-2\,b\,\left(a+b\right)\right)}{f}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a-4\,b)}{2\left(2\,a\,b-\frac{a^2}{2}\right)}\right)\,\left(a-4\,b\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (a^2*sin(2*e + 2*f*x))/(4*f) - (tan(e + f*x)*(
2*b^2 - 2*b*(a + b)))/f - (a*atan((a*tan(e + f*x)*(a - 4*b))/(2*(2*a*b - a^
2/2)))*(a - 4*b))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

3.24 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] time = 0.36, size = 106, normalized size = 2.65

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a^2fx\cos^3(e+fx)+2b(3a+b)\sec(e)\sin(fx)\cos^2(e+fx)+b^2\tan(e)\cos\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 1.66, size = 58, normalized size = 1.45

$$\frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

giac [A] time = 0.22, size = 53, normalized size = 1.32

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

maple [A] time = 0.98, size = 48, normalized size = 1.20

$$\frac{a^2\left(fx+e\right)+2ab\tan\left(fx+e\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.34, size = 44, normalized size = 1.10

$$a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

mupad [B] time = 4.33, size = 42, normalized size = 1.05

$$\frac{\frac{b^2 \tan\left(e+f x\right)^3}{3} - \tan\left(e + f x\right) \left(b^2 - 2 b \left(a + b\right)\right) + a^2 f x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^2\left(e + f x\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

3.25 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{2b(a+b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a+b)^2*cot(f*x+e)/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 270}

$$\frac{2b(a+b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Cot[e + f*x])/f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e +
 f*x]^3)/(3*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx&=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\&=-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [B] time = 1.14, size = 109, normalized size = 2.18

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(\sin(fx)\cos^2(e+fx)\left(3(a+b)^2\csc(e)\cot(e+fx)+b(6a+5b)\sec(e)\right)+\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(b^2*Sec[e]*Sin[f*x] + Cos[e + f
*x]^2*(3*(a + b)^2*Cot[e + f*x]*Csc[e] + b*(6*a + 5*b)*Sec[e])*Sin[f*x] + b
^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.62, size = 71, normalized size = 1.42

$$-\frac{\left(3 a^{2}+12 a b+8 b^{2}\right) \cos \left(f x+e\right)^{4}-2 \left(3 a b+2 b^{2}\right) \cos \left(f x+e\right)^{2}-b^{2}}{3 f \cos \left(f x+e\right)^{3} \sin \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 2*b^2)*cos(f*x +
 e)^2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

giac [A] time = 0.31, size = 64, normalized size = 1.28

$$\frac{b^{2} \tan \left(f x+e\right)^{3}+6 a b \tan \left(f x+e\right)+6 b^{2} \tan \left(f x+e\right)-\frac{3 \left(a^{2}+2 a b+b^{2}\right)}{\tan \left(f x+e\right)}}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 6*b^2*tan(f*x + e) - 3*(a^2
+ 2*a*b + b^2)/tan(f*x + e))/f
```

maple [A] time = 0.90, size = 96, normalized size = 1.92

$$\frac{-a^{2} \cot \left(f x+e\right)+2 a b \left(\frac{1}{\sin \left(f x+e\right) \cos \left(f x+e\right)}-2 \cot \left(f x+e\right)\right)+b^{2} \left(\frac{1}{3 \sin \left(f x+e\right) \cos \left(f x+e\right)^{3}}+\frac{4}{3 \sin \left(f x+e\right) \cos \left(f x+e\right)}-\frac{8 \cot (f x}{3}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+b^2*(1/3/
sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

maxima [A] time = 0.36, size = 54, normalized size = 1.08

$$\frac{b^{2} \tan \left(f x+e\right)^{3}+6 \left(a b+b^{2}\right) \tan \left(f x+e\right)-\frac{3 \left(a^{2}+2 a b+b^{2}\right)}{\tan \left(f x+e\right)}}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*(a*b + b^2)*tan(f*x + e) - 3*(a^2 + 2*a*b + b^2
)/tan(f*x + e))/f
```

mupad [B] time = 4.40, size = 56, normalized size = 1.12

$$\frac{b^2\,{\mathrm{tan}\left(e+f\,x\right)}^3}{3\,f}-\frac{a^2+2\,a\,b+b^2}{f\,\mathrm{tan}\left(e+f\,x\right)}+\frac{2\,b\,\mathrm{tan}\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (2*a*b + a^2 + b^2)/(f*tan(e + f*x)) + (2*b*ta
n(e + f*x)*(a + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2 \csc^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

3.26 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{b(2a+3b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}-\frac{(a+b)(a+3b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a+b)*(a+3*b)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f+b*(2*a+3*b)*tan(f*x+
e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$\frac{b(2a+3b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}-\frac{(a+b)(a+3b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)*(a + 3*b)*Cot[e + f*x])/f) - ((a + b)^2*Cot[e + f*x]^3)/(3*f) +
(b*(2*a + 3*b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx&=\frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^2}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(b(2a+3b)+\frac{(a+b)^2}{x^4}+\frac{(a+b)(a+3b)}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\&=-\frac{(a+b)(a+3b)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}+\frac{b(2a+3b)\tan}{f}\end{aligned}$$

Mathematica [A] time = 1.36, size = 151, normalized size = 1.99

$$-\frac{\csc(2e)\csc^3(2(e+fx))\left(-3a^2\sin(2(e+fx))+a^2\sin(6(e+fx))+3a^2\sin(4e+2fx)+a^2\sin(4e+6fx)-6ab\,\text{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/6*(Csc[2*e]*Csc[2*(e + f*x)]^3*(8*a*(a + 2*b)*Sin[2*e] - 6*(a + 2*b)^2*S
in[2*f*x] - 3*a^2*Sin[2*(e + f*x)] - 6*a*b*Sin[2*(e + f*x)] + a^2*Sin[6*(e
+ f*x)] + 2*a*b*Sin[6*(e + f*x)] + 3*a^2*Sin[4*e + 2*f*x] + a^2*Sin[4*e + 6
*f*x] + 8*a*b*Sin[4*e + 6*f*x] + 8*b^2*Sin[4*e + 6*f*x]))/f
```

fricas [A] time = 0.71, size = 101, normalized size = 1.33

$$-\frac{2\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{6}-3\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}+6\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}}{3\left(f\cos\left(fx+e\right)^{5}-f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^6 - 3*(a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 + 6*(a*b + b^2)*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos
(f*x + e)^3)*sin(f*x + e))
```

giac [A] time = 0.80, size = 105, normalized size = 1.38

$$\frac{b^{2}\tan\left(fx+e\right)^{3}+6\,ab\tan\left(fx+e\right)+9\,b^{2}\tan\left(fx+e\right)-\frac{3\,a^{2}\tan\left(fx+e\right)^{2}+12\,ab\tan\left(fx+e\right)^{2}+9\,b^{2}\tan\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 12*a*b*tan(f*x + e)^2 + 9*b^2*tan(f*x + e)^2 + a^2 + 2*a*b
 + b^2)/tan(f*x + e)^3)/f
```

maple [A] time = 1.38, size = 144, normalized size = 1.89

$$\frac{a^{2}\left(-\frac{2}{3}-\frac{\left(\csc^{2}\left(fx+e\right)\right)}{3}\right)\cot\left(fx+e\right)+2ab\left(-\frac{1}{3\sin\left(fx+e\right)^{3}\cos\left(fx+e\right)}+\frac{4}{3\sin\left(fx+e\right)\cos\left(fx+e\right)}-\frac{8\cot\left(fx+e\right)}{3}\right)+b^{2}\left(\frac{\cdots}{3\sin\left(fx+e\cdots\right.}\cdots\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/3/sin(f*x+e)^3/cos(f*
x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e))+b^2*(1/3/sin(f*x+e)^3/cos(f*
x+e)^3-2/3/sin(f*x+e)^3/cos(f*x+e)+8/3/sin(f*x+e)/cos(f*x+e)-16/3*cot(f*x+e
)))
```

maxima [A] time = 0.34, size = 80, normalized size = 1.05

$$\frac{b^{2}\tan\left(fx+e\right)^{3}+3\left(2\,ab+3\,b^{2}\right)\tan\left(fx+e\right)-\frac{3\left(a^{2}+4\,ab+3\,b^{2}\right)\tan\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + 3*b^2)*tan(f*x + e) - (3*(a^2 + 4*a*b
+ 3*b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2)/tan(f*x + e)^3)/f
```

mupad [B] time = 4.45, size = 85, normalized size = 1.12

$$\frac{b^2\tan\left(e+f\,x\right)^3}{3\,f}-\frac{\frac{2\,a\,b}{3}+\tan\left(e+f\,x\right)^2\,\left(a^2+4\,a\,b+3\,b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\tan\left(e+f\,x\right)^3}+\frac{b\tan\left(e+f\,x\right)\,\left(2\,a+3\,b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/3 + tan(e + f*x)^2*(4*a*b + a^2 + 3*b
^2) + a^2/3 + b^2/3)/(f*tan(e + f*x)^3) + (b*tan(e + f*x)*(2*a + 3*b))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.27 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{\left(a^2+6ab+6b^2\right)\cot(e+fx)}{f}+\frac{2b(a+2b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}+$$

```
[Out] -(a^2+6*a*b+6*b^2)*cot(f*x+e)/f-2/3*(a+b)*(a+2*b)*cot(f*x+e)^3/f-1/5*(a+b)^
2*cot(f*x+e)^5/f+2*b*(a+2*b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$-\frac{\left(a^2+6ab+6b^2\right)\cot(e+fx)}{f}+\frac{2b(a+2b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x])/f) - (2*(a + b)*(a + 2*b)*Cot[e + f*
x]^3)/(3*f) - ((a + b)^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x
])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+b+bx^2\right)^2}{x^6}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2b(a+2b)+\frac{(a+b)^2}{x^6}+\frac{2(a+b)(a+2b)}{x^4}+\frac{a^2+6ab+6b^2}{x^2}+b^2x^2\right)dx\right.}{f}\\ &= -\frac{\left(a^2+6ab+6b^2\right)\cot(e+fx)}{f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}-\frac{(a}{}\end{aligned}$$

Mathematica [B] time = 1.57, size = 353, normalized size = 3.43

$$-\frac{\csc(e)\sec(e)\csc^5(e+fx)\sec^3(e+fx)\left(-32\left(2a^2+9ab+12b^2\right)\sin(2fx)-24a^2\sin(2(e+fx))+8a^2\sin(4(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/1920*(Csc[e]*Csc[e + f*x]^5*Sec[e]*Sec[e + f*x]^3*(20*a*(5*a + 12*b)*Sin
[2*e] - 32*(2*a^2 + 9*a*b + 12*b^2)*Sin[2*f*x] - 24*a^2*Sin[2*(e + f*x)] -
108*a*b*Sin[2*(e + f*x)] - 54*b^2*Sin[2*(e + f*x)] + 8*a^2*Sin[4*(e + f*x)]
 + 36*a*b*Sin[4*(e + f*x)] + 18*b^2*Sin[4*(e + f*x)] + 8*a^2*Sin[6*(e + f*x
)] + 36*a*b*Sin[6*(e + f*x)] + 18*b^2*Sin[6*(e + f*x)] - 4*a^2*Sin[8*(e + f
*x)] - 18*a*b*Sin[8*(e + f*x)] - 9*b^2*Sin[8*(e + f*x)] + 8*a^2*Sin[2*(e +
2*f*x)] + 96*a*b*Sin[2*(e + 2*f*x)] + 128*b^2*Sin[2*(e + 2*f*x)] + 40*a^2*S
in[4*e + 2*f*x] + 8*a^2*Sin[4*e + 6*f*x] + 96*a*b*Sin[4*e + 6*f*x] + 128*b^
2*Sin[4*e + 6*f*x] - 4*a^2*Sin[6*e + 8*f*x] - 48*a*b*Sin[6*e + 8*f*x] - 64*
b^2*Sin[6*e + 8*f*x]))/f
```

fricas [A] time = 0.62, size = 139, normalized size = 1.35

$$-\frac{8\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right.}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^8 - 20*(a^2 + 12*a*b + 16*b^2
)*cos(f*x + e)^6 + 15*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^4 - 10*(3*a*b +
4*b^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*
cos(f*x + e)^3)*sin(f*x + e))
```

giac [A] time = 0.39, size = 151, normalized size = 1.47

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+60\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan\left(fx+e\right)^4+90\,ab\tan\left(fx+e\right)^4+90\,b^2\tan\left(fx+e\right)^4+10\,a^2\tan\left(f\right.}{\tan\left(fx+e\right.}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 60*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 30*a*b*tan(f*x + e)^2 + 20*b^2*tan(f*x + e)^2 + 3*a^2 +
 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

maple [A] time = 1.69, size = 190, normalized size = 1.84

$$a^2\left(-\frac{8}{15}-\frac{\left(\csc^4\left(fx+e\right)\right)}{5}-\frac{4\left(\csc^2\left(fx+e\right)\right)}{15}\right)\cot\left(fx+e\right)+2ab\left(-\frac{1}{5\sin\left(fx+e\right)^5\cos\left(fx+e\right)}-\frac{2}{5\sin\left(fx+e\right)^3\cos\left(fx+e\right)}+\frac{8}{5\sin\left(fx+e\right)\cos\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/5/
sin(f*x+e)^5/cos(f*x+e)-2/5/sin(f*x+e)^3/cos(f*x+e)+8/5/sin(f*x+e)/cos(f*x+
e)-16/5*cot(f*x+e))+b^2*(-1/5/sin(f*x+e)^5/cos(f*x+e)^3+8/15/sin(f*x+e)^3/c
os(f*x+e)^3-16/15/sin(f*x+e)^3/cos(f*x+e)+64/15/sin(f*x+e)/cos(f*x+e)-128/1
5*cot(f*x+e)))
```

maxima [A] time = 0.34, size = 107, normalized size = 1.04

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,\left(ab+2\,b^2\right)\tan\left(fx+e\right)-\frac{15\,\left(a^2+6\,ab+6\,b^2\right)\tan\left(fx+e\right)^4+10\,\left(a^2+3\,ab+2\,b^2\right)\tan\left(fx+e\right)^2+3\,a^2+6\,ab+3\,b^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + 2*b^2)*tan(f*x + e) - (15*(a^2 + 6*a
*b + 6*b^2)*tan(f*x + e)^4 + 10*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

mupad [B] time = 4.80, size = 108, normalized size = 1.05

$$\frac{b^2\tan\left(e+f\,x\right)^3}{3\,f}-\frac{\frac{2\,a\,b}{5}+\tan\left(e+f\,x\right)^4\,\left(a^2+6\,a\,b+6\,b^2\right)+\frac{a^2}{5}+\frac{b^2}{5}+\tan\left(e+f\,x\right)^2\,\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)}{f\,\tan\left(e+f\,x\right)^5}+\frac{2\,b\,t}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/5 + tan(e + f*x)^4*(6*a*b + a^2 + 6*b
^2) + a^2/5 + b^2/5 + tan(e + f*x)^2*(2*a*b + (2*a^2)/3 + (4*b^2)/3))/(f*ta
n(e + f*x)^5) + (2*b*tan(e + f*x)*(a + 2*b))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.28 $\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{b}\,(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f}-\frac{(a+b)^2\cos(e+fx)}{a^3f}+\frac{(2a+b)\cos^3(e+fx)}{3a^2f}-\frac{\cos^5(e+fx)}{5af}$$

```
[Out] -(a+b)^2*cos(f*x+e)/a^3/f+1/3*(2*a+b)*cos(f*x+e)^3/a^2/f-1/5*cos(f*x+e)^5/a
/f+(a+b)^2*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(7/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4133, 461, 205}

$$\frac{(2a+b)\cos^3(e+fx)}{3a^2f}-\frac{(a+b)^2\cos(e+fx)}{a^3f}+\frac{\sqrt{b}\,(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f}-\frac{\cos^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(7/2)*f) - ((
a + b)^2*Cos[e + f*x])/(a^3*f) + ((2*a + b)*Cos[e + f*x]^3)/(3*a^2*f) - Cos
[e + f*x]^5/(5*a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^3} - \frac{(2a+b)x^2}{a^2} + \frac{x^4}{a} + \frac{-a^2b-2ab^2-b^3}{a^3(b+ax^2)}\right) dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a+b)^2\cos(e+fx)}{a^3 f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2 f} - \frac{\cos^5(e+fx)}{5af} + \frac{(b(a+b)^2)\,\text{Subst}}{}$$
$$= \frac{\sqrt{b}\,(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2} f} - \frac{(a+b)^2\cos(e+fx)}{a^3 f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2 f} - $$

Mathematica [C] time = 3.26, size = 425, normalized size = 4.34

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-75a^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) - 75a^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)
*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[
(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f
*x)/2]))/Sqrt[b]] + 15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)*ArcTan[((-Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]] - 75*a^3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 75
*a^3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 8*Sqrt[a]*S
qrt[b]*Cos[e + f*x]*(89*a^2 + 220*a*b + 120*b^2 - 4*a*(7*a + 5*b)*Cos[2*(e
+ f*x)] + 3*a^2*Cos[4*(e + f*x)]))*Sec[e + f*x]^2)/(1920*a^(7/2)*Sqrt[b]*f*
(a + b*Sec[e + f*x]^2))
```

fricas [A] time = 0.72, size = 229, normalized size = 2.34

$$\left[-\frac{6a^2\cos(fx+e)^5 - 10(2a^2+ab)\cos(fx+e)^3 - 15(a^2+2ab+b^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right.}{30a^3 f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*a^2*cos(f*x + e)^5 - 10*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2 +
2*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e
) - b)/(a*cos(f*x + e)^2 + b)) + 30*(a^2 + 2*a*b + b^2)*cos(f*x + e))/(a^3*
f), -1/15*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2
+ 2*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(a^2 + 2*a
*b + b^2)*cos(f*x + e))/(a^3*f)]
```

giac [B] time = 0.27, size = 373, normalized size = 3.81

$$-\frac{\frac{15\left(a^2b+2ab^2+b^3\right)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\left(8a^2+25ab+15b^2-\frac{40a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{110ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{60b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\cdots\right.}{\cdots}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*c
os(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^3) - 2*(8*a^2 + 25*a*b + 15*b^2 - 40
*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 110*a*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^3*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f
```

maple [B] time = 0.90, size = 183, normalized size = 1.87

$$-\frac{\cos^5(fx+e)}{5af}+\frac{2\left(\cos^3(fx+e)\right)}{3af}+\frac{\left(\cos^3(fx+e)\right)b}{3f\,a^2}-\frac{\cos(fx+e)}{af}-\frac{2b\cos(fx+e)}{f\,a^2}-\frac{\cos(fx+e)\,b^2}{f\,a^3}+\frac{b\arctan\cdots}{f\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/5*cos(f*x+e)^5/a/f+2/3*cos(f*x+e)^3/a/f+1/3/f/a^2*cos(f*x+e)^3*b-cos(f*x
+e)/a/f-2/f/a^2*b*cos(f*x+e)-1/f/a^3*cos(f*x+e)*b^2+1/f*b/a/(a*b)^(1/2)*arc
tan(a*cos(f*x+e)/(a*b)^(1/2))+2/f*b^2/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(
a*b)^(1/2))+1/f*b^3/a^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

maxima [A] time = 0.43, size = 102, normalized size = 1.04

$$\frac{\frac{15\left(a^2b+2ab^2+b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{3a^2\cos(fx+e)^5-5\left(2a^2+ab\right)\cos(fx+e)^3+15\left(a^2+2ab+b^2\right)\cos(fx+e)}{a^3}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b
)*a^3) - (3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 + 15*(a^2 +
 2*a*b + b^2)*cos(f*x + e))/a^3)/f
```

mupad [B] time = 4.31, size = 123, normalized size = 1.26

$$\frac{\cos(e+fx)^3\left(\frac{b}{3a^2}+\frac{2}{3a}\right)}{f}-\frac{\cos(e+fx)^5}{5af}-\frac{\cos(e+fx)\left(\frac{1}{a}+\frac{b\left(\frac{b}{a^2}+\frac{2}{a}\right)}{a}\right)}{f}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+fx)\,(a+b)^2}{a^2b+2ab^2+b^3}\right)(a+b)^2\cdots}{a^{7/2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (cos(e + f*x)^3*(b/(3*a^2) + 2/(3*a)))/f - cos(e + f*x)^5/(5*a*f) - (cos(e
+ f*x)*(1/a + (b*(b/a^2 + 2/a))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(
e + f*x)*(a + b)^2)/(2*a*b^2 + a^2*b + b^3))*(a + b)^2)/(a^(7/2)*f)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.29 $\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f}-\frac{(a+b)\cos(e+fx)}{a^2f}+\frac{\cos^3(e+fx)}{3af}$$

```
[Out] -(a+b)*cos(f*x+e)/a^2/f+1/3*cos(f*x+e)^3/a/f+(a+b)*arctan(cos(f*x+e)*a^(1/2
)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 459, 321, 205}

$$-\frac{(a+b)\cos(e+fx)}{a^2f}+\frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f}+\frac{\cos^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(5/2)*f) - ((a
+ b)*Cos[e + f*x])/(a^2*f) + Cos[e + f*x]^3/(3*a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= \frac{\cos^3(e+fx)}{3af} - \frac{(a+b)\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af}$$
$$= -\frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af} + \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx\right)}{a^2f}$$
$$= \frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f} - \frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af}$$

Mathematica [C] time = 1.39, size = 376, normalized size = 5.30

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(3\left(a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(
Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] +
 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] - Sqrt[a
+ b]*Tan[(e + f*x)/2])/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(
e + f*x)/2])/Sqrt[b]] + 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(-5*a - 6*b + a*Cos[
2*(e + f*x)]))*Sec[e + f*x]^2)/(48*a^(5/2)*Sqrt[b]*f*(a + b*Sec[e + f*x]^2)
)
```

fricas [A] time = 0.87, size = 154, normalized size = 2.17

$$\left[\frac{2\,a\cos\left(fx+e\right)^3+3\,(a+b)\sqrt{-\frac{b}{a}}\,\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right)-6\,(a+b)\cos\left(fx+e\right)}{6\,a^2f}, \frac{a\cos\left(fx+e\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*cos(f*x + e)^3 + 3*(a + b)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*
a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(a + b)*cos(f*x
+ e))/(a^2*f), 1/3*(a*cos(f*x + e)^3 + 3*(a + b)*sqrt(b/a)*arctan(a*sqrt(b/
a)*cos(f*x + e)/b) - 3*(a + b)*cos(f*x + e))/(a^2*f)]
```

giac [A] time = 0.25, size = 89, normalized size = 1.25

$$\frac{\left(ab+b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2f} + \frac{a^2f^5\cos\left(fx+e\right)^3-3\,a^2f^5\cos\left(fx+e\right)-3\,abf^5\cos\left(fx+e\right)}{3\,a^3f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) + 1/3*(a^2*f
^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) - 3*a*b*f^5*cos(f*x + e))/(a^3*f
^6)
```

maple [A] time = 0.93, size = 103, normalized size = 1.45

$$\frac{\cos^3\left(fx+e\right)}{3af}-\frac{\cos\left(fx+e\right)}{af}-\frac{b\cos\left(fx+e\right)}{f\,a^2}+\frac{b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{fa\sqrt{ab}}+\frac{b^2\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{f\,a^2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/3*cos(f*x+e)^3/a/f-cos(f*x+e)/a/f-1/f/a^2*b*cos(f*x+e)+1/f*b/a/(a*b)^(1/2
)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/f*b^2/a^2/(a*b)^(1/2)*arctan(a*cos(f*x
+e)/(a*b)^(1/2))
```

maxima [A] time = 0.44, size = 63, normalized size = 0.89

$$\frac{\frac{3\left(ab+b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{a\cos\left(fx+e\right)^3-3\,(a+b)\cos\left(fx+e\right)}{a^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (a*co
s(f*x + e)^3 - 3*(a + b)*cos(f*x + e))/a^2)/f
```

mupad [B] time = 0.12, size = 76, normalized size = 1.07

$$\frac{\cos\left(e+f\,x\right)^3}{3\,a\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{b}{a^2}+\frac{1}{a}\right)}{f}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos\left(e+f\,x\right)\,(a+b)}{b^2+a\,b}\right)\,(a+b)}{a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] cos(e + f*x)^3/(3*a*f) - (cos(e + f*x)*(b/a^2 + 1/a))/f + (b^(1/2)*atan((a^
(1/2)*b^(1/2)*cos(e + f*x)*(a + b))/(a*b + b^2))*(a + b))/(a^(5/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

[Out] -cos(f*x+e)/a/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/f

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 321, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(3/2)*f) - Cos[e + f*x]/(a*f)

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{af}+\frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx,x,\cos(e+fx)\right)}{af}\\
&= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}
\end{aligned}$$

Mathematica [C] time = 0.55, size = 329, normalized size = 7.00

$$\frac{(a\cos(2(e+fx))+a+2b)\left(-4\sqrt{a}\sqrt{b}\cos(e+fx)-a\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-a\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (((a + 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*
Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]] + (a + 4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - a*ArcTan[(Sqrt[
a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - a*ArcTan[(Sqrt[a] + Sqrt[a +
b]*Tan[(e + f*x)/2])/Sqrt[b]] - 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x])*(a + 2*b +
a*Cos[2*(e + f*x)]))/(8*a^(3/2)*Sqrt[b]*f*(b + a*Cos[e + f*x]^2))
```

fricas [A] time = 0.56, size = 118, normalized size = 2.51

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right)-2\cos\left(fx+e\right)}{2af}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos\left(fx+e\right)}{b}\right)-\cos\left(fx+e\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - 2*cos(f*x + e))/(a*f), (sqrt(b/a)*arctan(a*sqrt(b
/a)*cos(f*x + e)/b) - cos(f*x + e))/(a*f)]
```

giac [A] time = 0.54, size = 44, normalized size = 0.94

$$\frac{b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,af}-\frac{\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - cos(f*x + e)/(a*f)
```

maple [A] time = 0.40, size = 46, normalized size = 0.98

$$-\frac{b\arctan\left(\frac{\sec\left(fx+e\right)b}{\sqrt{ab}}\right)}{fa\sqrt{ab}}-\frac{1}{fa\sec\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/a*b/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2))-1/f/a/sec(f*x+e)
```

maxima [A] time = 0.42, size = 40, normalized size = 0.85

$$\frac{\frac{b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{\cos\left(fx+e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) - cos(f*x + e)/a)/f
```

mupad [B] time = 0.07, size = 39, normalized size = 0.83

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos\left(e+f\,x\right)}{\sqrt{b}}\right)}{a^{3/2}\,f}-\frac{\cos\left(e+f\,x\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(a^(3/2)*f) - cos(e + f*x)/(
a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.31 $\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}\,f(a+b)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+
b)/f/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 481, 206, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}\,f(a+b)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*f) - ArcT
anh[Cos[e + f*x]]/((a + b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f}$$
$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)f}$$

Mathematica [C] time = 0.76, size = 239, normalized size = 4.35

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\ldots}\right.}{}\right.}{}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] -
Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])/((a + b)*f)
```

fricas [A] time = 0.76, size = 156, normalized size = 2.84

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right)-\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2(a+b)f}, \frac{2\sqrt{\frac{b}{a}}\ \mathrm{arc}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x +
e) + 1/2))/((a + b)*f), 1/2*(2*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b)
 - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/((a + b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(1/(4*a+4*b)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))-
2*b*1/4/(a+b)/sqrt(a*b)*atan((-a*cos(f*x+exp(1))+b)/(sqrt(a*b)*cos(f*x+exp(
1))+sqrt(a*b))))
```

maple [A] time = 0.78, size = 76, normalized size = 1.38

$$\frac{b \arctan\left(\frac{a \cos\left(fx+e\right)}{\sqrt{ab}}\right)}{f\left(a+b\right)\sqrt{ab}}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{f\left(2a+2b\right)}-\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{f\left(2a+2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*b/(a+b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/f/(2*a+2*b)*ln(-
1+cos(f*x+e))-1/f/(2*a+2*b)*ln(1+cos(f*x+e))
```

maxima [A] time = 0.44, size = 64, normalized size = 1.16

$$\frac{\frac{2 b \arctan\left(\frac{a \cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\left(a+b\right)}-\frac{\log\left(\cos\left(fx+e\right)+1\right)}{a+b}+\frac{\log\left(\cos\left(fx+e\right)-1\right)}{a+b}}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - log(cos(f*x
 + e) + 1)/(a + b) + log(cos(f*x + e) - 1)/(a + b))/f
```

mupad [B] time = 0.21, size = 123, normalized size = 2.24

$$-\frac{\operatorname{atanh}\left(\frac{\cos\left(e+f\,x\right)\left(2\,a^3+2\,a\,b^2\right)-\frac{\cos\left(e+f\,x\right)\left(8\,a^5+8\,a^4\,b-8\,a^3\,b^2-8\,a^2\,b^3\right)}{4\,{\left(a+b\right)}^2}}{2\,a\,b\,\left(a+b\right)}\right)}{f\,\left(a+b\right)}-\frac{\operatorname{atanh}\left(\frac{\cos\left(e+f\,x\right)\,\sqrt{-a\,b}}{b}\right)\,\sqrt{-a\,b}}{f\,\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - atanh((cos(e + f*x)*(2*a*b^2 + 2*a^3) - (cos(e + f*x)*(8*a^4*b + 8*a^5 -
8*a^2*b^3 - 8*a^3*b^2))/(4*(a + b)^2))/(2*a*b*(a + b)))/(f*(a + b)) - (atan
h((cos(e + f*x)*(-a*b)^(1/2))/b)*(-a*b)^(1/2))/(f*(a*b + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.32 $\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^2}-\frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^2}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)}$$

```
[Out] -1/2*(a-b)*arctanh(cos(f*x+e))/(a+b)^2/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f+
arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/f
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 471, 522, 206, 205}

$$\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^2}-\frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^2}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^2*f) - ((
a - b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x])
/(2*(a + b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
```

```
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-ax^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^2 f} + \frac{(ab)\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2 f} - \frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^2 f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

Mathematica [C] time = 1.55, size = 371, normalized size = 4.31

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-8\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/16*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*
(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]
- 8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + a*Csc[(e + f*x)/2]^2 + b*Csc[(e + f*
x)/2]^2 + 4*a*Log[Cos[(e + f*x)/2]] - 4*b*Log[Cos[(e + f*x)/2]] - 4*a*Log[S
in[(e + f*x)/2]] + 4*b*Log[Sin[(e + f*x)/2]] - a*Sec[(e + f*x)/2]^2 - b*Sec
[(e + f*x)/2]^2)*Sec[e + f*x]^2)/((a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

fricas [A] time = 1.19, size = 327, normalized size = 3.80

$$\left[\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{-ab}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(a+b\right)\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\left(fx+e\right)^2\right.}{4\left(\left(a^2+2ab+b^2\right)f\cos\left(fx+e\right)^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*
b)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(a + b)*cos(f*x + e) - ((a
 - b)*cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*
x + e)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 - (a^2 + 2*a*b + b^2)*f), 1/4*(4*sqrt(a*b)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b)*cos(f*x + e)/b) + 2*(a + b)*cos(f*x + e) - ((a - b)*cos(f*
```

```
x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*x + e)^2 - a
 + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 -
 (a^2 + 2*a*b + b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))/(16*b+16*a)+(2*(1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*a-b-a)/(16*b^2+32*b*a+16*a^2)/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2*b
*a*1/4/(b^2+2*b*a+a^2)/sqrt(a*b)*atan((-a*cos(f*x+exp(1))+b)/(sqrt(a*b)*cos
(f*x+exp(1))+sqrt(a*b)))+(-b+a)/(8*b^2+16*b*a+8*a^2)*ln(abs(1-cos(f*x+exp(1
)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.94, size = 158, normalized size = 1.84

$$\frac{ab\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{f\left(a+b\right)^{2}\sqrt{ab}}+\frac{1}{f\left(4a+4b\right)\left(-1+\cos\left(fx+e\right)\right)}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)a}{4f\left(a+b\right)^{2}}-\frac{\ln\left(-1+\cos\left(fx+e\right)\right)b}{4f\left(a+b\right)^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*a*b/(a+b)^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/f/(4*a+4*b)/
(-1+cos(f*x+e))+1/4/f/(a+b)^2*ln(-1+cos(f*x+e))*a-1/4/f/(a+b)^2*ln(-1+cos(f
*x+e))*b+1/f/(4*a+4*b)/(1+cos(f*x+e))-1/4/f/(a+b)^2*ln(1+cos(f*x+e))*a+1/4/
f/(a+b)^2*ln(1+cos(f*x+e))*b
```

maxima [A] time = 0.45, size = 128, normalized size = 1.49

$$\frac{\frac{4\,ab\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{ab}}-\frac{(a-b)\log\left(\cos\left(fx+e\right)+1\right)}{a^{2}+2\,ab+b^{2}}+\frac{(a-b)\log\left(\cos\left(fx+e\right)-1\right)}{a^{2}+2\,ab+b^{2}}+\frac{2\,\cos\left(fx+e\right)}{(a+b)\cos\left(fx+e\right)^{2}-a-b}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b))
 - (a - b)*log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) + (a - b)*log(cos(f*x
+ e) - 1)/(a^2 + 2*a*b + b^2) + 2*cos(f*x + e)/((a + b)*cos(f*x + e)^2 - a
- b))/f
```

mupad [B] time = 4.91, size = 392, normalized size = 4.56

$$-\frac{2\,a\,\cos\left(e+f\,x\right)+2\,b\,\cos\left(e+f\,x\right)-a\,\ln\left(\cos\left(e+f\,x\right)-1\right)+a\,\ln\left(\cos\left(e+f\,x\right)+1\right)+b\,\ln\left(\cos\left(e+f\,x\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] -(2*a*cos(e + f*x) + 2*b*cos(e + f*x) - atan((a^3*cos(e + f*x)*(-a*b)^(1/2)
*1i + a*b^2*cos(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*
2i)/(a*b^3 + a^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i - a*log(cos(e + f*x) - 1)
+ a*log(cos(e + f*x) + 1) + b*log(cos(e + f*x) - 1) - b*log(cos(e + f*x) +
1) + cos(e + f*x)^2*atan((a^3*cos(e + f*x)*(-a*b)^(1/2)*1i + a*b^2*cos(e +
f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*2i)/(a*b^3 + a^3*b +
 2*a^2*b^2))*(-a*b)^(1/2)*4i + a*log(cos(e + f*x) - 1)*cos(e + f*x)^2 - a*l
og(cos(e + f*x) + 1)*cos(e + f*x)^2 - b*log(cos(e + f*x) - 1)*cos(e + f*x)^
2 + b*log(cos(e + f*x) + 1)*cos(e + f*x)^2)/(4*a^2*f + 4*b^2*f - 4*a^2*f*co
s(e + f*x)^2 - 4*b^2*f*cos(e + f*x)^2 + 8*a*b*f - 8*a*b*f*cos(e + f*x)^2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.33 $\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=129

$$\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^3}-\frac{(3a^2-6ab-b^2)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^3}-\frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)}-\frac{(3a-b)\cot(e+}{8f(a}$$

```
[Out] -1/8*(3*a^2-6*a*b-b^2)*arctanh(cos(f*x+e))/(a+b)^3/f-1/8*(3*a-b)*cot(f*x+e)
*csc(f*x+e)/(a+b)^2/f-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f+a^(3/2)*arctan(co
s(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 471, 527, 522, 206, 205}

$$-\frac{(3a^2-6ab-b^2)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^3}-\frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)}-\frac{(3a-b)\cot(e}{8f(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^3*f) - ((
3*a^2 - 6*a*b - b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^3*f) - ((3*a - b)*Co
t[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4
*(a + b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^3\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-3ax^2}{\left(1-x^2\right)^2\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b(5a+b)-a(3a-b}{\left(1-x^2\right)\left(b+ax^2\right.}\right.}{8(a+}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\left(a^2b\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\right.}{(a+b)}$$

$$= \frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3 f} - \frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}(\cos(e+fx))}{8(a+b)^3 f} - \frac{(3a-b)\cot(e+}{8(a-}$$

Mathematica [C] time = 4.70, size = 549, normalized size = 4.26

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-64a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+l}\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/128*((a + 2*b + a*Cos[2*(e + f*x)])*(-64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
] - 64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + 6*a^2*Csc[(e + f*x)/2]^2 + 4*a*b*
Csc[(e + f*x)/2]^2 - 2*b^2*Csc[(e + f*x)/2]^2 + a^2*Csc[(e + f*x)/2]^4 + 2*
a*b*Csc[(e + f*x)/2]^4 + b^2*Csc[(e + f*x)/2]^4 + 24*a^2*Log[Cos[(e + f*x)/
2]] - 48*a*b*Log[Cos[(e + f*x)/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] - 24*a^2*L
og[Sin[(e + f*x)/2]] + 48*a*b*Log[Sin[(e + f*x)/2]] + 8*b^2*Log[Sin[(e + f*
x)/2]] - 6*a^2*Sec[(e + f*x)/2]^2 - 4*a*b*Sec[(e + f*x)/2]^2 + 2*b^2*Sec[(e
 + f*x)/2]^2 - a^2*Sec[(e + f*x)/2]^4 - 2*a*b*Sec[(e + f*x)/2]^4 - b^2*Sec[
(e + f*x)/2]^4)*Sec[e + f*x]^2)/((a + b)^3*f*(a + b*Sec[e + f*x]^2))
```

fricas [B] time = 0.84, size = 693, normalized size = 5.37

$$\left[\frac{2\left(3a^{2}+2ab-b^{2}\right)\cos\left(fx+e\right)^{3}+8\left(a\cos\left(fx+e\right)^{4}-2a\cos\left(fx+e\right)^{2}+a\right)\sqrt{-ab}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{-ab}\,\cos}{a\cos\left(fx+e\right)^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3 + 8*(a*cos(f*x + e)^4 - 2*a*c
os(f*x + e)^2 + a)*sqrt(-a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*b)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) -
 ((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x +
e)^2 + 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b -
 b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a
*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*
cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*f), 1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3
 + 16*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*sqrt(a*b)*arctan(sqrt(a*b
)*cos(f*x + e)/b) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) - ((3*a^2 - 6*a*b
- b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*
a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b - b^2)*cos(f*x + e
)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a*b - b^2)*log(-1/
2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+32*((1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*a)/(4096*b^2+8192*b*a+4096*a^2)+(6*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^2*b^2+36*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a-18*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))*b*a-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-b^2-2*b*a-a^2)/(1
28*b^3+384*b^2*a+384*b*a^2+128*a^3)/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
))^2-2*b*a^2*1/4/(b^3+3*b^2*a+3*b*a^2+a^3)/sqrt(a*b)*atan((-a*cos(f*x+exp(1
))+b)/(sqrt(a*b)*cos(f*x+exp(1))+sqrt(a*b)))+(-b^2-6*b*a+3*a^2)/(32*b^3+96*
b^2*a+96*b*a^2+32*a^3)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.96, size = 296, normalized size = 2.29

$$\frac{a^{2}b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{f\left(a+b\right)^{3}\sqrt{ab}}-\frac{1}{2f\left(8a+8b\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}+\frac{3a}{16f\left(a+b\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}-\frac{}{16f\left(a+b\right)^{2}\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*a^2*b/(a+b)^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-1/2/f/(8*a+8
*b)/(-1+cos(f*x+e))^2+3/16/f/(a+b)^2/(-1+cos(f*x+e))*a-1/16/f/(a+b)^2/(-1+c
os(f*x+e))*b+3/16/f/(a+b)^3*ln(-1+cos(f*x+e))*a^2-3/8/f/(a+b)^3*ln(-1+cos(f
*x+e))*a*b-1/16/f/(a+b)^3*ln(-1+cos(f*x+e))*b^2+1/2/f/(8*a+8*b)/(1+cos(f*x+
e))^2+3/16/f/(a+b)^2/(1+cos(f*x+e))*a-1/16/f/(a+b)^2/(1+cos(f*x+e))*b-3/16/
f/(a+b)^3*ln(1+cos(f*x+e))*a^2+3/8/f/(a+b)^3*ln(1+cos(f*x+e))*a*b+1/16/f/(a
+b)^3*ln(1+cos(f*x+e))*b^2
```

maxima [B] time = 0.46, size = 231, normalized size = 1.79

$$\frac{\frac{16\,a^{2}b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{ab}}-\frac{\left(3\,a^{2}-6\,ab-b^{2}\right)\log\left(\cos\left(fx+e\right)+1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{\left(3\,a^{2}-6\,ab-b^{2}\right)\log\left(\cos\left(fx+e\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(\left(3\,a-b\right)\cos\left(fx+e\right)^{3}-\left(5\,a+b\right)\cdots\right.}{\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(a^{2}+2\,ab+b^{2}\right)\cdots}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*sqrt(a*b)) - (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2
*b + 3*a*b^2 + b^3) + (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) - 1)/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) + 2*((3*a - b)*cos(f*x + e)^3 - (5*a + b)*cos(f*x +
e))/((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e
)^2 + a^2 + 2*a*b + b^2))/f
```

mupad [B] time = 7.90, size = 870, normalized size = 6.74

$$\frac{3\,a^{2}\cos\left(e+f\,x\right)^{3}-b^{2}\cos\left(e+f\,x\right)-5\,a^{2}\cos\left(e+f\,x\right)-b^{2}\cos\left(e+f\,x\right)^{3}-3\,a^{2}\,\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)+b^{2}\,\mathrm{ata}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] (atan((a^5*cos(e + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(
1/2)*12i + a^3*b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a
^3*b)^(1/2)*1i + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5
+ 12*a^3*b^4 + 30*a^4*b^3 + 28*a^5*b^2))*(-a^3*b)^(1/2)*8i - 5*a^2*cos(e +
f*x) - b^2*cos(e + f*x) + 3*a^2*cos(e + f*x)^3 - b^2*cos(e + f*x)^3 - 3*a^2
*atanh(cos(e + f*x)) + b^2*atanh(cos(e + f*x)) - atan((a^5*cos(e + f*x)*(-a
^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*cos(e +
f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^4*b*cos(
e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*a^4*b^3 +
 28*a^5*b^2))*cos(e + f*x)^2*(-a^3*b)^(1/2)*16i + atan((a^5*cos(e + f*x)*(-
a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*cos(e +
 f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^4*b*cos
(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*a^4*b^3
+ 28*a^5*b^2))*cos(e + f*x)^4*(-a^3*b)^(1/2)*8i - 6*a*b*cos(e + f*x) + 6*a^
2*cos(e + f*x)^2*atanh(cos(e + f*x)) - 3*a^2*cos(e + f*x)^4*atanh(cos(e + f
*x)) - 2*b^2*cos(e + f*x)^2*atanh(cos(e + f*x)) + b^2*cos(e + f*x)^4*atanh(
cos(e + f*x)) + 2*a*b*cos(e + f*x)^3 + 6*a*b*atanh(cos(e + f*x)) - 12*a*b*c
os(e + f*x)^2*atanh(cos(e + f*x)) + 6*a*b*cos(e + f*x)^4*atanh(cos(e + f*x)
))/(8*a^3*f + 8*b^3*f - 16*a^3*f*cos(e + f*x)^2 + 8*a^3*f*cos(e + f*x)^4 -
16*b^3*f*cos(e + f*x)^2 + 8*b^3*f*cos(e + f*x)^4 + 24*a*b^2*f + 24*a^2*b*f
- 48*a*b^2*f*cos(e + f*x)^2 - 48*a^2*b*f*cos(e + f*x)^2 + 24*a*b^2*f*cos(e
+ f*x)^4 + 24*a^2*b*f*cos(e + f*x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{b}\,(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4 f}+\frac{(3a+2b)\sin(e+fx)\cos^3(e+fx)}{8a^2 f}-\frac{\left(11a^2+18ab+8b^2\right)\sin(e+fx)\cos(e+$$

```
[Out] 1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*x/a^4-1/16*(11*a^2+18*a*b+8*b^2)*cos(
f*x+e)*sin(f*x+e)/a^3/f+1/8*(3*a+2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos
(f*x+e)^3*sin(f*x+e)^3/a/f-(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2
))*b^(1/2)/a^4/f
```

Rubi [A] time = 0.34, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4132, 470, 578, 527, 522, 203, 205}

$$-\frac{\left(11a^2+18ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3 f}+\frac{x\left(30a^2b+5a^3+40ab^2+16b^3\right)}{16a^4}-\frac{\sqrt{b}\,(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4 f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*x)/(16*a^4) - (Sqrt[b]*(a + b)^(5/2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*f) - ((11*a^2 + 18*a*b +
 8*b^2)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((3*a + 2*b)*Cos[e + f*x]^3
*Sin[e + f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-3(2a+b)x^2)}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{3(a+b)(3}{}\right.}{}$$

$$= -\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx}{8a^2f}$$

$$= -\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx}{8a^2f}$$

$$= \frac{(5a^3+30a^2b+40ab^2+16b^3)x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} - \frac{(11a^2+18a}{}$$

Mathematica [C] time = 4.09, size = 357, normalized size = 2.15

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{b(\cos(e)-i\sin(e))^4}\left(3a^3(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)+2\sqrt{b}\sqrt{a+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*Sqrt[b]*(9*a^4 + 136*a^3*
b + 384*a^2*b^2 + 384*a*b^3 + 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2
*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4
]*(3*a^3*(9*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + 2*Sqrt[b]
*Sqrt[a + b]*(-12*a^3*e + 60*a^3*f*x + 360*a^2*b*f*x + 480*a*b^2*f*x + 192*
b^3*f*x - 3*a*(15*a^2 + 32*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a + 2*
b)*Sin[4*(e + f*x)] - a^3*Sin[6*(e + f*x)]))))/(768*a^4*Sqrt[b]*Sqrt[a + b]
*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.75, size = 428, normalized size = 2.58

$$\left[\frac{3\left(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3\right)fx+12\left(a^2+2\,ab+b^2\right)\sqrt{-ab-b^2}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+\right.}{a^2\cos\left(fx\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 12*(a^2 + 2*a*b + b^2
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x + e)^3 + 3*(1
1*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), 1/48*(3*(5
*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 24*(a^2 + 2*a*b + b^2)*sqrt(a*b
+ b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x +
 e)*sin(f*x + e))) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x +
 e)^3 + 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)
]
```

giac [A] time = 0.26, size = 250, normalized size = 1.51

$$\frac{\frac{3\left(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3\right)\left(fx+e\right)}{a^4}-\frac{48\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4}-\frac{33\,a^2\tan\left(fx+e\right)^5+54\,ab\tan\left(fx+e\right)^5+}{}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 - 48*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*
tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^4) - (33*a^2*tan(f*x + e)
^5 + 54*a*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3
+ 96*a*b*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 42*
a*b*tan(f*x + e) + 24*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

maple [B] time = 0.85, size = 460, normalized size = 2.77

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{fa\sqrt{\left(a+b\right)b}}-\frac{3b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^2\sqrt{\left(a+b\right)b}}-\frac{3b^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^3\sqrt{\left(a+b\right)b}}-\frac{b^4\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^4\sqrt{\left(a+b\right)b}}-\frac{9\left(\tan^5\left(fx+\right.\right.}{8f\,a^2\left(1+\tan^2\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f*b^2/a^2/(
(a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f*b^3/a^3/((a+b)*b)^(
1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^4/a^4/((a+b)*b)^(1/2)*arcta
n(tan(f*x+e)*b/((a+b)*b)^(1/2))-9/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b
-1/2/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b^2-11/16/f/a/(tan(f*x+e)^2+1)^3
*tan(f*x+e)^5-2/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b-1/f/a^3/(tan(f*x+e)
^2+1)^3*tan(f*x+e)^3*b^2-5/6/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-5/16/f/a/(
tan(f*x+e)^2+1)^3*tan(f*x+e)-7/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b-1/2/
f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b^2+15/8/f/a^2*arctan(tan(f*x+e))*b+5/2
/f/a^3*arctan(tan(f*x+e))*b^2+1/f/a^4*arctan(tan(f*x+e))*b^3+5/16/f/a*arcta
n(tan(f*x+e))
```

maxima [A] time = 0.43, size = 209, normalized size = 1.26

$$-\frac{\frac{3\left(11a^2+18ab+8b^2\right)\tan\left(fx+e\right)^5+8\left(5a^2+12ab+6b^2\right)\tan\left(fx+e\right)^3+3\left(5a^2+14ab+8b^2\right)\tan\left(fx+e\right)}{a^3\tan\left(fx+e\right)^6+3a^3\tan\left(fx+e\right)^4+3a^3\tan\left(fx+e\right)^2+a^3}-\frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)\left(fx+e\right)}{a^4}+\cdots}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/48*((3*(11*a^2 + 18*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*
b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 14*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x
 + e)^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) - 3*(5*a^3 + 3
0*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 + 48*(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

mupad [B] time = 5.71, size = 1448, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((25*b^3*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*
b^3 - 5*a^4*b^2)^(1/2))/(128*((227*a*b^5)/128 + (217*b^6)/128 + (119*a^2*b^
4)/128 + (25*a^3*b^3)/128 + (13*b^7)/(16*a) + (5*b^8)/(32*a^2))) + (11*b^4*
tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2
)^(1/2))/(32*((217*a*b^6)/128 + (13*b^7)/16 + (227*a^2*b^5)/128 + (119*a^3*
b^4)/128 + (25*a^4*b^3)/128 + (5*b^8)/(32*a))) + (5*b^5*tan(e + f*x)*(- 5*a
*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2)^(1/2))/(32*((13*a
*b^7)/16 + (5*b^8)/32 + (217*a^2*b^6)/128 + (227*a^3*b^5)/128 + (119*a^4*b^
4)/128 + (25*a^5*b^3)/128)))*(-b*(a + b)^5)^(1/2))/(a^4*f) - (atan((((((51
2*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) - (tan(e
 + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3
*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) -
(tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^
4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^
3*5i + b^3*16i)*1i)/(32*a^4) - (((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*
b^3 + 320*a^11*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)
*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*
b*30i + a^3*5i + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816*a*b^8 + 512*b^9 +
 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/
(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i)*1i)/(32*a^4))/((992*a
*b^10 + 128*b^11 + 3344*a^2*b^9 + 6380*a^3*b^8 + 7496*a^4*b^7 + 5511*a^5*b^
6 + 2445*a^6*b^5 + 585*a^7*b^4 + 55*a^8*b^3)/(128*a^9) + (((((512*a^8*b^5 +
 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) - (tan(e + f*x)*(20
```

```
48*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(409
6*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) - (tan(e + f*
x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 183
6*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*
16i))/(32*a^4) + (((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11
*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i +
a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i
 + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7
+ 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*
b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4)))*(a*b^2*40i + a^2*b*30i
+ a^3*5i + b^3*16i)*1i)/(16*a^4*f) - ((tan(e + f*x)*(14*a*b + 5*a^2 + 8*b^2
))/(16*a^3) + (tan(e + f*x)^3*(12*a*b + 5*a^2 + 6*b^2))/(6*a^3) + (tan(e +
f*x)^5*(18*a*b + 11*a^2 + 8*b^2))/(16*a^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e
+ f*x)^4 + tan(e + f*x)^6 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.35 $\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{b}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}-\frac{(5a+4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{x\left(3a^2+12ab+8b^2\right)}{8a^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

```
[Out] 1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/
4*cos(f*x+e)^3*sin(f*x+e)/a/f-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))*b^(1/2)/a^3/f
```

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 203, 205}

$$\frac{x\left(3a^2+12ab+8b^2\right)}{8a^3}-\frac{\sqrt{b}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}-\frac{(5a+4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a^3*f) - ((5*a + 4*b)*Cos[e + f*x]*Sin[e +
f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{a+b+(b-4(a+b))x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+4b)}{(1+x^2)(a}\right.}{}$$

$$= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\left(b(a+b)^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(3a^2+12ab+8b^2\right)x}{8a^3} - \frac{\sqrt{b}\,(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f} - \frac{(5a+4b)\cos(e+fx)\sin(}{8a^2f}$$

Mathematica [C] time = 1.94, size = 303, normalized size = 2.59

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{b(\cos(e)-i\sin(e))^4}\left(\sqrt{b}\sqrt{a+b}\left(a^2\sin(4(e+fx))-2a^2e+12a^2fx-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*b +
 64*a*b^2 + 32*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4]*(a^2*(3*a + 2*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*a^2*e
+ 12*a^2*f*x + 48*a*b*f*x + 32*b^2*f*x - 8*a*(a + b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)]))))/(64*a^3*Sqrt[b]*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*
Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.57, size = 332, normalized size = 2.84

$$\left[\frac{\left(3a^2+12ab+8b^2\right)fx + 2\sqrt{-ab-b^2}\,(a+b)\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4 - 2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2 + 4\left(\left(a+2b\right)\cos\left(fx+e\right)^3 - b\cos\right.}{a^2\cos\left(fx+e\right)^4 + 2ab\cos\left(fx+e\right)^2 + b^2}\right.}{8a^3f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 12*a*b + 8*b^2)*f*x + 2*sqrt(-a*b - b^2)*(a + b)*log(((a^2 +
 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a +
 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)
/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^2*cos(f*x + e)^3
 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*((3*a^2 + 12*a*
b + 8*b^2)*f*x + 4*sqrt(a*b + b^2)*(a + b)*arctan(1/2*((a + 2*b)*cos(f*x +
e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^2*cos(f*x + e
)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

giac [A] time = 0.26, size = 160, normalized size = 1.37

$$\frac{\frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} - \frac{8(a^2b+2ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^3} - \frac{5a\tan(fx+e)^3+4b\tan(fx+e)^3+3a\tan(fx+e)+4b\tan}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a^3 - 8*(a^2*b + 2*a*b^2 + b^3)*(pi
*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))
/(sqrt(a*b + b^2)*a^3) - (5*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 3*a*tan
(f*x + e) + 4*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

maple [B] time = 1.02, size = 260, normalized size = 2.22

$$-\frac{b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{fa\sqrt{(a+b)\,b}}-\frac{2b^2\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f\,a^2\sqrt{(a+b)\,b}}-\frac{b^3\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f\,a^3\sqrt{(a+b)\,b}}-\frac{\left(\tan^3\left(fx+e\right)\right)b}{2f\,a^2\left(1+\tan^2\left(fx+e\right)\right)^2}-\frac{5\left(\tan\right.}{8fa\left(1+\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*b^2/a^2/(
(a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^3/a^3/((a+b)*b)^(
1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(
f*x+e)^3*b-5/8/f/a/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3-3/8/f/a/(tan(f*x+e)^2+1)
^2*tan(f*x+e)-1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)*b+3/2/f/a^2*arctan(ta
n(f*x+e))*b+1/f/a^3*arctan(tan(f*x+e))*b^2+3/8/f/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 137, normalized size = 1.17

$$-\frac{\frac{(5a+4b)\tan(fx+e)^3+(3a+4b)\tan(fx+e)}{a^2\tan(fx+e)^4+2a^2\tan(fx+e)^2+a^2} - \frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} + \frac{8(a^2b+2ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(((5*a + 4*b)*tan(f*x + e)^3 + (3*a + 4*b)*tan(f*x + e))/(a^2*tan(f*x
+ e)^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a
^3 + 8*(a^2*b + 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt
((a + b)*b)*a^3))/f
```

mupad [B] time = 4.58, size = 494, normalized size = 4.22

$$\frac{\operatorname{atanh}\left(\frac{9\,b^3\tan\left(e+f\,x\right)\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{32\left(\frac{13\,a\,b^4}{16}+\frac{25\,b^5}{32}+\frac{9\,a^2\,b^3}{32}+\frac{b^6}{4\,a}\right)}+\frac{b^4\tan\left(e+f\,x\right)\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{4\left(\frac{9\,a^3\,b^3}{32}+\frac{13\,a^2\,b^4}{16}+\frac{25\,a\,b^5}{32}+\frac{b^6}{4}\right)}\right)\sqrt{-b\,(a+b)^3}}{a^3\,f}-\frac{\frac{\tan\left(e+f\,x\right)(3\,a+4\,b)}{8\,a^2}+\frac{\mathrm{ta}}{}}{f\,\left(\tan\left(e+f\,x\right)^4+2\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((9*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(32
*((13*a*b^4)/16 + (25*b^5)/32 + (9*a^2*b^3)/32 + b^6/(4*a))) + (b^4*tan(e +
 f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(4*((25*a*b^5)/32 + b^6/
4 + (13*a^2*b^4)/16 + (9*a^3*b^3)/32)))*(-b*(a + b)^3)^(1/2))/(a^3*f) - ((t
an(e + f*x)*(3*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(5*a + 4*b))/(8*a^2))/(f
*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((159*b^3*tan(e + f*x))/(2
56*((27*a*b^2)/256 + (159*b^3)/256 + (75*b^4)/(64*a) + (29*b^5)/(32*a^2) +
b^6/(4*a^3))) + (75*b^4*tan(e + f*x))/(64*((159*a*b^3)/256 + (75*b^4)/64 +
(27*a^2*b^2)/256 + (29*b^5)/(32*a) + b^6/(4*a^2))) + (29*b^5*tan(e + f*x))/
(32*((75*a*b^4)/64 + (29*b^5)/32 + (159*a^2*b^3)/256 + (27*a^3*b^2)/256 + b
^6/(4*a))) + (b^6*tan(e + f*x))/(4*((29*a*b^5)/32 + b^6/4 + (75*a^2*b^4)/64
 + (159*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(e + f*x))/(256*((27
*b^2)/256 + (159*b^3)/(256*a) + (75*b^4)/(64*a^2) + (29*b^5)/(32*a^3) + b^6
/(4*a^4))))*(a*b*12i + a^2*3i + b^2*8i)*1i)/(8*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.36 $\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f}+\frac{x(a+2b)}{2a^2}-\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

```
[Out] 1/2*(a+2*b)*x/a^2-1/2*cos(f*x+e)*sin(f*x+e)/a/f-arctan(b^(1/2)*tan(f*x+e)/(
a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/f
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 471, 522, 203, 205}

$$-\frac{\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f}+\frac{x(a+2b)}{2a^2}-\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/(2*a^2) - (Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(a^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx,x,\tan(e+fx)\right)}{2af}$$
$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{a^2f} + \frac{(a+2b)}{}$$
$$= \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

Mathematica [C] time = 0.84, size = 245, normalized size = 3.22

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}} - \frac{-\frac{(a^2+8ab+8b^2)(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2e+fx)-(a}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{f\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}}{a^2}\right.}{16\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f) - (-4*(a + 2*b)*x - ((a^2 + 8*a*b +
 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a
*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
- I*Sin[2*e]))/(Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*a*Cos[2*f
*x]*Sin[2*e])/f + (2*a*Cos[2*e]*Sin[2*f*x])/f)/a^2))/(16*(a + b*Sec[e + f*x
]^2))
```

fricas [A] time = 0.53, size = 257, normalized size = 3.38

$$\left[\frac{2(a+2b)fx - 2a\cos(fx+e)\sin(fx+e) + \sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4 - 2(3ab+4b^2)\cos(fx+e)^2 + 4((a+2b)}{a^2\cos(fx+e)^4 + 2ab\cos(}\right.}{4a^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + 2*b)*f*x - 2*a*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b - b^2)*lo
g(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
+ 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^2*f), 1/2*
```

```
((a + 2*b)*f*x - a*cos(f*x + e)*sin(f*x + e) + sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))))
/(a^2*f)]
```

giac [A] time = 1.45, size = 97, normalized size = 1.28

$$\frac{\frac{(fx+e)(a+2b)}{a^2} - \frac{2\left(\pi\left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\sqrt{ab+b^2}}{a^2} - \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - 2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))*sqrt(a*b + b^2)/a^2 - tan(f*x + e)/((t
an(f*x + e)^2 + 1)*a))/f
```

maple [A] time = 0.95, size = 124, normalized size = 1.63

$$-\frac{b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{fa\sqrt{(a+b)b}}-\frac{b^2\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{fa^2\sqrt{(a+b)b}}-\frac{\tan(fx+e)}{2fa\left(1+\tan^2(fx+e)\right)}+\frac{\arctan(\tan(fx+e))}{2fa}+\frac{\arctan(\tan(}{f a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^2/a^2/(
(a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a*tan(f*x+e)/(tan
(f*x+e)^2+1)+1/2/f/a*arctan(tan(f*x+e))+1/f/a^2*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.45, size = 77, normalized size = 1.01

$$\frac{\frac{(fx+e)(a+2b)}{a^2} - \frac{\tan(fx+e)}{a\tan(fx+e)^2+a} - \frac{2\left(ab+b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - tan(f*x + e)/(a*tan(f*x + e)^2 + a) - 2*(a*b
 + b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2))/f
```

mupad [B] time = 4.45, size = 111, normalized size = 1.46

$$\frac{\mathrm{atanh}\left(\frac{\sin(e+fx)\sqrt{-b^2-ab}}{a\cos(e+fx)+b\cos(e+fx)}\right)\sqrt{-b^2-ab}-a\left(\frac{\sin(2e+2fx)}{4}-\frac{\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)}{2}\right)+b\,\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((sin(e + f*x)*(- a*b - b^2)^(1/2))/(a*cos(e + f*x) + b*cos(e + f*x))
)*(- a*b - b^2)^(1/2) - a*(sin(2*e + 2*f*x)/4 - atan(sin(e + f*x)/cos(e + f
*x))/2) + b*atan(sin(e + f*x)/cos(e + f*x)))/(a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e + fx)}{a + b\sec^2(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.37 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sec^2(e+fx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\ &= \frac{x}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx,x,\cot(e+fx)\right)}{af}\\ &= \frac{x}{a}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [C] time = 0.29, size = 182, normalized size = 4.04

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+b(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e}{}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.50, size = 231, normalized size = 5.13

$$\left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\,\sin\left(fx\right.}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right.}{4\,af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

giac [A] time = 0.76, size = 68, normalized size = 1.51

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

maple [A] time = 1.00, size = 48, normalized size = 1.07

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{fa\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a*arctan(
tan(f*x+e))
```

maxima [A] time = 0.45, size = 44, normalized size = 0.98

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\sqrt{\left(a+b\right)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

mupad [B] time = 4.56, size = 460, normalized size = 10.22

$$\frac{x}{a}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\tan\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2-\frac{\tan\left(e+f\,x\right)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\tan\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2+\frac{\tan\left(e+f\,x\right)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{\cdots}}{4\left(a^2+b\,a\right)}\right.}{2\left(a^2+b\,a\right)}\right.}{a^2+b\,a}}{\frac{\left(2\,b^3\tan\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2-\frac{\tan\left(e+f\,x\right)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\tan\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2+\frac{\tan\left(e+f\,x\right)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{\cdots}}{4\left(a^2+b\,a\right)}\right.}{2\left(a^2+b\,a\right)}\right.}{a^2+b\,a}}\right.\cdots}{f\,\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

3.38 $\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}$$

```
[Out] -cot(f*x+e)/(a+b)/f-arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(3
/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4132, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(3/2)*f)) -
 Cot[e + f*x]/((a + b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)}{(a+b)f}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{(a+b)f}\\ &= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}\end{aligned}$$

Mathematica [C] time = 0.64, size = 189, normalized size = 3.50

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\,\csc(e)\sin(fx)\sqrt{b(\cos(e)-i\sin(e))^4}\,\csc(e+fx)+b(\cos(2e)\right.}{2f(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}\,\left(a+b\sec^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Csc[e
]*Csc[e + f*x]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*Sin[f*x]))/(2*(a + b)^(3/2)*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.59, size = 271, normalized size = 5.02

$$\left[\frac{\sqrt{-\frac{b}{a+b}}\,\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\,\sin\left(fx+e\right)+\right.}{a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b^2}\right.}{4\,(a+b)f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b +
 b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^
4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/((a + b)*f*
sin(f*x + e)), 1/2*(sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x
+ e))/((a + b)*f*sin(f*x + e))]
```

giac [A] time = 0.75, size = 74, normalized size = 1.37

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,(a+b)}+\frac{1}{(a+b)\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*(a + b)) + 1/((a + b)*tan(f*x + e)))/f
```

maple [A] time = 0.83, size = 54, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\,(a+b)\,\sqrt{(a+b)\,b}}-\frac{1}{f\,(a+b)\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/(a+b)
/tan(f*x+e)
```

maxima [A] time = 0.44, size = 50, normalized size = 0.93

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,(a+b)}+\frac{1}{(a+b)\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)) + 1/((
a + b)*tan(f*x + e)))/f
```

mupad [B] time = 4.28, size = 46, normalized size = 0.85

$$-\frac{\cot\left(e+f\,x\right)}{f\,(a+b)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{f\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - cot(e + f*x)/(f*(a + b)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/(a + b)^(
1/2)))/(f*(a + b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.39 $\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}-\frac{a\cot(e+fx)}{f(a+b)^2}$$

```
[Out] -a*cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f-a*arctan(b^(1/2)*tan(f*x+e
)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 453, 325, 205}

$$-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}-\frac{a\cot(e+fx)}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(5/2)*f))
 - (a*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$
$$= -\frac{a\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)^2 f}$$
$$= -\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{a\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

Mathematica [C] time = 2.10, size = 226, normalized size = 2.97

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{1}{4}\sqrt{a+b}\,\csc(e)\sqrt{b(\cos(e)-i\sin(e))^4}\,\csc^3(e+fx)((b-2a)\sin(2e+3fx}{6f(a+b)^{5/2}\sqrt{b(\cos(e)-i\,\text{si}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*a*b*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(6*a*Sin[f*x] - 3*b*Sin
[2*e + f*x] + (-2*a + b)*Sin[2*e + 3*f*x]))/4))/(6*(a + b)^(5/2)*f*(a + b*S
ec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [B] time = 0.61, size = 397, normalized size = 5.22

$$\left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2 - a\right)\sqrt{-\frac{b}{a+b}}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4 - 2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2 + 4\left(\left(a^2+3\right.\right.}{a^2\cos\left(fx+e\right)^4+2}\right.}{12\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2 - \left(a^2+2\,ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2))*sin(f*x + e) - 12*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos
(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), -1/6*(2*(2*a - b)*cos(f
*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*
cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 6*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*
a*b + b^2)*f)*sin(f*x + e))]
```

giac [A] time = 0.96, size = 107, normalized size = 1.41

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)ab}{\left(a^2+2\,ab+b^2\right)\sqrt{ab+b^2}} + \frac{3\,a\tan\left(fx+e\right)^2+a+b}{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*a*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*a*tan(f*x + e)^2
 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

maple [A] time = 0.96, size = 74, normalized size = 0.97

$$-\frac{ab\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^{2}\sqrt{\left(a+b\right)b}}-\frac{1}{3f\left(a+b\right)\tan\left(fx+e\right)^{3}}-\frac{a}{f\left(a+b\right)^{2}\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*a*b/(a+b)^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/3/f
/(a+b)/tan(f*x+e)^3-1/f*a/(a+b)^2/tan(f*x+e)
```

maxima [A] time = 0.42, size = 82, normalized size = 1.08

$$-\frac{\frac{3\,ab\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{(a+b)b}}+\frac{3\,a\tan\left(fx+e\right)^{2}+a+b}{\left(a^{2}+2\,ab+b^{2}\right)\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqr
t((a + b)*b)) + (3*a*tan(f*x + e)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x +
 e)^3))/f
```

mupad [B] time = 4.31, size = 80, normalized size = 1.05

$$-\frac{\frac{1}{3\,(a+b)}+\frac{a\tan\left(e+f\,x\right)^{2}}{(a+b)^{2}}}{f\tan\left(e+f\,x\right)^{3}}-\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\left(a^{2}+2\,a\,b+b^{2}\right)}{(a+b)^{5/2}}\right)}{f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - (1/(3*(a + b)) + (a*tan(e + f*x)^2)/(a + b)^2)/(f*tan(e + f*x)^3) - (a*b^
(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(f*(a
 + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{4}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.40 $\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{f(a+b)^3}-\frac{\cot^5(e+fx)}{5f(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3f(a+b)^2}$$

```
[Out] -a^2*cot(f*x+e)/(a+b)^3/f-1/3*(2*a+b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(f*x+e)
^5/(a+b)/f-a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4132, 461, 205}

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{f(a+b)^3}-\frac{\cot^5(e+fx)}{5f(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(7/2)*f
)) - (a^2*Cot[e + f*x])/((a + b)^3*f) - ((2*a + b)*Cot[e + f*x]^3)/(3*(a +
b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)x^6} + \frac{2a+b}{(a+b)^2x^4} + \frac{a^2}{(a+b)^3x^2} - \frac{a^2b}{(a+b)^3\left(a+b+bx^2\right)}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a^2\cot(e+fx)}{(a+b)^3f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{\left(a^2b\right)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,d\right.}{(a+b)^3}$$

$$= -\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f} - \frac{a^2\cot(e+fx)}{(a+b)^3f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx}{5(a+b)f}$$

Mathematica [C] time = 1.72, size = 318, normalized size = 3.03

$$\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\,\csc(e)\sqrt{b(\cos(e)-i\sin(e))^4}\,\csc^5(e+fx)\left(10\left(8a^2+b^2\right)\sin(f\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(240*a^2*b*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^5*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(10*(8*a^2 + b^2)*Si
n[f*x] - 30*b*(3*a + b)*Sin[2*e + f*x] - 40*a^2*Sin[2*e + 3*f*x] + 30*a*b*S
in[2*e + 3*f*x] + 10*b^2*Sin[2*e + 3*f*x] + 15*a*b*Sin[4*e + 3*f*x] + 8*a^2
*Sin[4*e + 5*f*x] - 9*a*b*Sin[4*e + 5*f*x] - 2*b^2*Sin[4*e + 5*f*x])))/(480
*(a + b)^(7/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [B] time = 0.53, size = 587, normalized size = 5.59

$$\left[-\frac{4\left(8a^2-9ab-2b^2\right)\cos\left(fx+e\right)^5 - 20\left(4a^2-3ab-b^2\right)\cos\left(fx+e\right)^3 - 15\left(a^2\cos\left(fx+e\right)^4 - 2a^2\cos\left(f\right.\right.}{60\left(\left(a^3+3a^2b+3ab^2+b^3\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 3*a*b - b^2)
*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*co
s(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)), -1/30*(2*(
8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 3*a*b - b^2)*cos(f*x +
e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b/(a + b))
*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*
sin(f*x + e)))*sin(f*x + e) + 30*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b + 3*a*b
```

```
^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x +
e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e))]
```

giac [A] time = 0.30, size = 180, normalized size = 1.71

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)a^2b}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{15a^2\tan\left(fx+e\right)^4+10a^2\tan\left(fx+e\right)^2+15ab\tan\left(fx+e\right)^2+5b^2\tan\left(fx+e\right)^2+3a^2+6ab+3b^2}{\left(a^3+3a^2b+3ab^2+b^3\right)\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*a^2*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (15
*a^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 15*a*b*tan(f*x + e)^2 + 5*b^2
*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*t
an(f*x + e)^5))/f
```

maple [A] time = 1.01, size = 116, normalized size = 1.10

$$-\frac{a^2b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^3\sqrt{\left(a+b\right)b}}-\frac{1}{5f\left(a+b\right)\tan\left(fx+e\right)^5}-\frac{a^2}{f\left(a+b\right)^3\tan\left(fx+e\right)}-\frac{2a}{3f\left(a+b\right)^2\tan\left(fx+e\right)^3}-\frac{}{3f\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*a^2*b/(a+b)^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/5
/f/(a+b)/tan(f*x+e)^5-1/f*a^2/(a+b)^3/tan(f*x+e)-2/3/f/(a+b)^2/tan(f*x+e)^3
*a-1/3/f/(a+b)^2/tan(f*x+e)^3*b
```

maxima [A] time = 0.44, size = 137, normalized size = 1.30

$$-\frac{\frac{15a^2b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{15a^2\tan\left(fx+e\right)^4+5\left(2a^2+3ab+b^2\right)\tan\left(fx+e\right)^2+3a^2+6ab+3b^2}{\left(a^3+3a^2b+3ab^2+b^3\right)\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*a^2*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sqrt((a + b)*b)) + (15*a^2*tan(f*x + e)^4 + 5*(2*a^2 + 3*a*b +
 b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*tan(f*x + e)^5))/f
```

mupad [B] time = 5.05, size = 112, normalized size = 1.07

$$-\frac{\frac{1}{5\,(a+b)}+\frac{\tan\left(e+f\,x\right)^2\,(2\,a+b)}{3\,(a+b)^2}+\frac{a^2\,\tan\left(e+f\,x\right)^4}{(a+b)^3}}{f\,\tan\left(e+f\,x\right)^5}-\frac{a^2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)}{f\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - (1/(5*(a + b)) + (tan(e + f*x)^2*(2*a + b))/(3*(a + b)^2) + (a^2*tan(e +
f*x)^4)/(a + b)^3)/(f*tan(e + f*x)^5) - (a^2*b^(1/2)*atan((b^(1/2)*tan(e +
f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(f*(a + b)^(7/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{b}\,(a+b)(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}-\frac{(a+b)^2}{2a^2bf\,(a\,c}$$

```
[Out] -1/2*(a+b)*(3*a+7*b)*cos(f*x+e)/a^4/f+1/6*(a+b)*(3*a+7*b)*cos(f*x+e)^3/a^3/
b/f-1/5*cos(f*x+e)^5/a^2/f-1/2*(a+b)^2*cos(f*x+e)^5/a^2/b/f/(b+a*cos(f*x+e)
^2)+1/2*(a+b)*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/
f
```

Rubi [A] time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4133, 463, 459, 302, 205}

$$-\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(a\cos^2(e+fx)+b\right)}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{\sqrt{b}\,(a+b)(3a+7b)\,\mathrm{t}}{2a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(a + b)*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(
9/2)*f) - ((a + b)*(3*a + 7*b)*Cos[e + f*x])/(2*a^4*f) + ((a + b)*(3*a + 7*
b)*Cos[e + f*x]^3)/(6*a^3*b*f) - Cos[e + f*x]^5/(5*a^2*f) - ((a + b)^2*Cos[
e + f*x]^5)/(2*a^2*b*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4\left(1-x^2\right)^2}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(-2a^2+5(a+b)^2-2abx^2\right)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^2bf}$$

$$= -\frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)} + \frac{((a+b)(3a+7b))\,\text{Subst}\left(\int \frac{x^4}{b+ax^2}\right.}{2a^2bf}$$

$$= -\frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)} + \frac{((a+b)(3a+7b))\,\text{Subst}\left(\int\left(-\frac{b}{a^2}\right.\right.}{2}$$

$$= -\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf} - \frac{\cos^5(e+fx)}{5a^2f} -$$

$$= \frac{\sqrt{b}\,(a+b)(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b}{}$$

Mathematica [C] time = 6.30, size = 454, normalized size = 2.82

$$\frac{-\frac{45a^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}} - \frac{45a^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{15\left(3a^4+384a^2b^2+1280ab^3+896b^4\right)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a}\right.}{}\right.}{b^{3/}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a]
 - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2)
 + (15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[
a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/
2) - (45*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3
/2) - (45*a^4*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(
3/2) - (16*Sqrt[a]*Cos[e + f*x]*(150*a^3 + 1436*a^2*b + 2960*a*b^2 + 1680*b
^3 + a*(125*a^2 + 688*a*b + 560*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*a + 14*b)
*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])
)/(3840*a^(9/2)*f)
```

fricas [A] time = 0.70, size = 405, normalized size = 2.52

$$\left[-\frac{12\,a^3\cos\left(fx+e\right)^7-4\left(10\,a^3+7\,a^2b\right)\cos\left(fx+e\right)^5+20\left(3\,a^3+10\,a^2b+7\,ab^2\right)\cos\left(fx+e\right)^3-15\left(3\,a^2b+10\right.}{60\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*cos(f*x + e)^7 - 4*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 20*(3
*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3
+ (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x +
 e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(3*a^
2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/
30*(6*a^3*cos(f*x + e)^7 - 2*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 10*(3*a^3
+ 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3 + (3*
a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(
f*x + e)/b) + 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x
+ e)^2 + a^4*b*f)]
```

giac [B] time = 1.41, size = 545, normalized size = 3.39

$$-\frac{\frac{15\left(3\,a^2b+10\,ab^2+7\,b^3\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\,\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{30\left(a^2b+2\,ab^2+b^3+\frac{a^2b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{b^3\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)}{\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{b\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)a^4}-\frac{4\left(8\,a^2+50\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a
*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^4) + 30*(a^2*b + 2*a*b^2 + b^3
+ a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1))/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^4) - 4*(8*a^2 + 50
*a*b + 45*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 220*a*b*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - 180*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 270*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 180*b^2*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x + e) - 1)^4/(cos
(f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^4*(
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

maple [A] time = 0.95, size = 276, normalized size = 1.71

$$-\frac{\cos^5\left(fx+e\right)}{5a^2f}+\frac{2\left(\cos^3\left(fx+e\right)\right)}{3a^2f}+\frac{2\left(\cos^3\left(fx+e\right)\right)b}{3f\,a^3}-\frac{\cos\left(fx+e\right)}{a^2f}-\frac{4\cos\left(fx+e\right)b}{f\,a^3}-\frac{3\cos\left(fx+e\right)b^2}{f\,a^4}-\frac{}{2f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

[Out] -1/5*cos(f*x+e)^5/a^2/f+2/3*cos(f*x+e)^3/a^2/f+2/3/f/a^3*cos(f*x+e)^3*b-cos(f*x+e)/a^2/f-4/f/a^3*cos(f*x+e)*b-3/f/a^4*cos(f*x+e)*b^2-1/2/f*b/a^2*cos(f*x+e)/(b+a*cos(f*x+e)^2)-1/f*b^2/a^3*cos(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*b^3/a^4*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2/f*b/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+5/f*b^2/a^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+7/2/f*b^3/a^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))

maxima [A] time = 0.53, size = 148, normalized size = 0.92

$$-\frac{\frac{15\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)}{a^5\cos\left(fx+e\right)^2+a^4b}-\frac{15\left(3a^2b+10ab^2+7b^3\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{2\left(3a^2\cos\left(fx+e\right)^5-10\left(a^2+ab\right)\cos\left(fx+e\right)^3+15\left(a^2+4ab+3b^2\right)\cos\left(fx+e\right)\right)}{a^4}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] -1/30*(15*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)/(a^5*cos(f*x + e)^2 + a^4*b) - 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2*(3*a^2*cos(f*x + e)^5 - 10*(a^2 + a*b)*cos(f*x + e)^3 + 15*(a^2 + 4*a*b + 3*b^2)*cos(f*x + e))/a^4)/f

mupad [B] time = 0.16, size = 195, normalized size = 1.21

$$\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{2\,b}{3\,a^3}+\frac{2}{3\,a^2}\right)}{f}-\frac{\cos\left(e+f\,x\right)^5}{5\,a^2\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{1}{a^2}-\frac{b^2}{a^4}+\frac{2\,b\,\left(\frac{2\,b}{a^3}+\frac{2}{a^2}\right)}{a}\right)}{f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{a^2\,b}{2}+a\,b^2+\frac{b^3}{2}\right)}{f\,\left(a^5\,\cos\left(e+f\,x\right)^2+b\,a^4\right)}+\dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)

[Out] (cos(e + f*x)^3*((2*b)/(3*a^3) + 2/(3*a^2)))/f - cos(e + f*x)^5/(5*a^2*f) - (cos(e + f*x)*(1/a^2 - b^2/a^4 + (2*b*((2*b)/a^3 + 2/a^2))/a))/f - (cos(e + f*x)*(a*b^2 + (a^2*b)/2 + b^3/2))/(f*(a^4*b + a^5*cos(e + f*x)^2)) + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(a + b)*(3*a + 7*b))/(10*a*b^2 + 3*a^2*b + 7*b^3))*(a + b)*(3*a + 7*b))/(2*a^(9/2)*f)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)

[Out] Timed out

3.42 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f}-\frac{b(a+b)\cos(e+fx)}{2a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+2b)\cos(e+fx)}{a^3f}+\frac{\cos^3(e+fx)}{3a^2f}$$

```
[Out] -(a+2*b)*cos(f*x+e)/a^3/f+1/3*cos(f*x+e)^3/a^2/f-1/2*b*(a+b)*cos(f*x+e)/a^3
/f/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1
/2)/a^(7/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 455, 1153, 205}

$$-\frac{b(a+b)\cos(e+fx)}{2a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+2b)\cos(e+fx)}{a^3f}+\frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f}+\frac{\cos^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(7/2)*f)
- ((a + 2*b)*Cos[e + f*x])/(a^3*f) + Cos[e + f*x]^3/(3*a^2*f) - (b*(a + b)*
Cos[e + f*x])/(2*a^3*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^3f}$$

$$= -\frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)dx, x, \text{co}\right.}{2a^3f}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{(b(3a+5l}{}$$

$$= \frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(}{2a^3f}$$

Mathematica [C] time = 3.47, size = 403, normalized size = 3.54

$$\frac{-\frac{9a^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}} - \frac{9a^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{3\left(3a^3+192ab^2+320b^3\right)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin}\right.}{}\right.}{b^{3/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*(3*a^3 + 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) + (3*(3*a^3 + 1
92*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] - Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] + Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (32*Sqrt[a]*Cos[e + f*x]*(9
*a^2 + 56*a*b + 60*b^2 + 4*a*(2*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))/(384*a^(7/2)*f)
```

fricas [A] time = 0.58, size = 297, normalized size = 2.61

$$\left[\frac{4a^2\cos\left(fx+e\right)^5 - 4\left(3a^2+5ab\right)\cos\left(fx+e\right)^3 + 3\left(\left(3a^2+5ab\right)\cos\left(fx+e\right)^2+3ab+5b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a}{}\right.}{12\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*cos(f*x + e)^5 - 4*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2
+ 5*a*b)*cos(f*x + e)^2 + 3*a*b + 5*b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2
+ 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(3*a*b + 5*b
^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), 1/6*(2*a^2*cos(f*x + e)
```

```
^5 - 2*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2 + 5*a*b)*cos(f*x + e)^2 +
 3*a*b + 5*b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(3*a*b + 5
*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

giac [A] time = 0.64, size = 143, normalized size = 1.25

$$\frac{\left(3\,ab+5\,b^{2}\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}f}-\frac{\frac{ab\cos\left(fx+e\right)}{f}+\frac{b^{2}\cos\left(fx+e\right)}{f}}{2\left(a\cos\left(fx+e\right)^{2}+b\right)a^{3}}+\frac{a^{4}f^{11}\cos\left(fx+e\right)^{3}-3\,a^{4}f^{11}\cos\left(fx+e\right)-6\,a^{3}bf^{11}}{3\,a^{6}f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1/
2*(a*b*cos(f*x + e)/f + b^2*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)*a^3) +
1/3*(a^4*f^11*cos(f*x + e)^3 - 3*a^4*f^11*cos(f*x + e) - 6*a^3*b*f^11*cos(f
*x + e))/(a^6*f^12)
```

maple [A] time = 0.91, size = 165, normalized size = 1.45

$$\frac{\cos^{3}\left(fx+e\right)}{3a^{2}f}-\frac{\cos\left(fx+e\right)}{a^{2}f}-\frac{2\cos\left(fx+e\right)b}{f\,a^{3}}-\frac{b\cos\left(fx+e\right)}{2f\,a^{2}\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}-\frac{b^{2}\cos\left(fx+e\right)}{2f\,a^{3}\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}+\frac{3b\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/3*cos(f*x+e)^3/a^2/f-cos(f*x+e)/a^2/f-2/f/a^3*cos(f*x+e)*b-1/2/f*b/a^2*co
s(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*b^2/a^3*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2
/f*b/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+5/2/f*b^2/a^3/(a*b)^(
1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

maxima [A] time = 0.43, size = 104, normalized size = 0.91

$$-\frac{\frac{3\left(ab+b^{2}\right)\cos\left(fx+e\right)}{a^{4}\cos\left(fx+e\right)^{2}+a^{3}b}-\frac{3\left(3\,ab+5\,b^{2}\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}-\frac{2\left(a\cos\left(fx+e\right)^{3}-3\,(a+2\,b)\cos\left(fx+e\right)\right)}{a^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a*b + b^2)*cos(f*x + e)/(a^4*cos(f*x + e)^2 + a^3*b) - 3*(3*a*b +
5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*(a*cos(f*x + e)
^3 - 3*(a + 2*b)*cos(f*x + e))/a^3)/f
```

mupad [B] time = 0.14, size = 130, normalized size = 1.14

$$\frac{\cos\left(e+f\,x\right)^{3}}{3\,a^{2}\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{2\,b}{a^{3}}+\frac{1}{a^{2}}\right)}{f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{b^{2}}{2}+\frac{a\,b}{2}\right)}{f\,\left(a^{4}\cos\left(e+f\,x\right)^{2}+b\,a^{3}\right)}+\frac{\sqrt{b}\ \mathrm{atan}\left(\frac{\sqrt{a}\ \sqrt{b}\ \cos\left(e+f\,x\right)\,(3\,a+5\,b)}{5\,b^{2}+3\,a\,b}\right)\,(3\,a+5}{2\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] cos(e + f*x)^3/(3*a^2*f) - (cos(e + f*x)*((2*b)/a^3 + 1/a^2))/f - (cos(e +
f*x)*((a*b)/2 + b^2/2))/(f*(a^3*b + a^4*cos(e + f*x)^2)) + (b^(1/2)*atan((a
```

```
^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 5*b))/(3*a*b + 5*b^2))*(3*a + 5*b))/(2*a
^(7/2)*f)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\left(a\cos^2(e+fx)+b\right)}$$

```
[Out] -3/2*cos(f*x+e)/a^2/f+1/2*cos(f*x+e)^3/a/f/(b+a*cos(f*x+e)^2)+3/2*arctan(co
s(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 288, 321, 205}

$$\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\left(a\cos^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(5/2)*f) - (3*Cos[e
 + f*x])/(2*a^2*f) + Cos[e + f*x]^3/(2*a*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2af}$$

$$= -\frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+\right.}{2a^2f}$$

$$= \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] time = 3.14, size = 393, normalized size = 4.68

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(-\frac{a^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}} - \frac{a^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{(a^2+24b^2)\tan\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*(((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] - I*Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a
] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2
) + ((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] - Sqrt
[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] + Sqrt[a
 + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x]*(a + 3
*b + a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))*Sec[e + f*x]^4)/(
64*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.47, size = 201, normalized size = 2.39

$$\left[-\frac{4a\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right) + 6b\cos\left(fx+e\right)}{4\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}, -\frac{2a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/a)*log(-(a*cos
(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 6*
b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), -1/2*(2*a*cos(f*x + e)^3
- 3*(a*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 3
*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

giac [A] time = 0.49, size = 76, normalized size = 0.90

$$\frac{3\,b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2f}-\frac{\cos\left(fx+e\right)}{a^2f}-\frac{b\cos\left(fx+e\right)}{2\left(a\cos\left(fx+e\right)^2+b\right)a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - cos(f*x + e)/(a^
2*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 + b)*a^2*f)
```

maple [A] time = 0.62, size = 75, normalized size = 0.89

$$-\frac{b\sec\left(fx+e\right)}{2f\,a^2\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\sec\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\,a^2\sqrt{ab}}-\frac{1}{f\,a^2\sec\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)-3/2/f*b/a^2/(a*b)^(1/2)*arctan(s
ec(f*x+e)*b/(a*b)^(1/2))-1/f/a^2/sec(f*x+e)
```

maxima [A] time = 0.45, size = 70, normalized size = 0.83

$$-\frac{\frac{b\cos\left(fx+e\right)}{a^3\cos\left(fx+e\right)^2+a^2b}-\frac{3\,b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,\cos\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*cos(f*x + e)/(a^3*cos(f*x + e)^2 + a^2*b) - 3*b*arctan(a*cos(f*x +
e)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*cos(f*x + e)/a^2)/f
```

mupad [B] time = 4.56, size = 72, normalized size = 0.86

$$\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos\left(e+f\,x\right)}{\sqrt{b}}\right)}{2\,a^{5/2}\,f}-\frac{b\,\cos\left(e+f\,x\right)}{2\,f\,\left(a^3\cos\left(e+f\,x\right)^2+b\,a^2\right)}-\frac{\cos\left(e+f\,x\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (3*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(2*a^(5/2)*f) - (b*cos(e +
 f*x))/(2*f*(a^2*b + a^3*cos(e + f*x)^2)) - cos(e + f*x)/(a^2*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.44 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}f(a+b)^2}-\frac{b\cos(e+fx)}{2af(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)^2}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)^2/f-1/2*b*cos(f*x+e)/a/(a+b)/f/(b+a*cos(f*x+e)^2
)+1/2*(3*a+b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2/f
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4133, 470, 522, 206, 205}

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}f(a+b)^2}-\frac{b\cos(e+fx)}{2af(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(3/2)*(a +
b)^2*f) - ArcTanh[Cos[e + f*x]]/((a + b)^2*f) - (b*Cos[e + f*x])/(2*a*(a +
b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
```

```
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^2 f} + \frac{(b(3a+b)}{}$$

$$= \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^2 f} - \frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e\right.}$$

Mathematica [C] time = 1.27, size = 384, normalized size = 3.88

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\sqrt{b}(3a+b)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+c}{\sqrt{b}}\right)}{a^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*b*(a + b))/a + (Sqrt[b]
*(3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/a^
(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[e + f*x])/a^(3/2) - 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/
2]]*Sec[e + f*x] + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Sin[(e + f*x)/2]]*S
ec[e + f*x]))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 1.04, size = 390, normalized size = 3.94

$$\left[\frac{\left(\left(3a^2+ab\right)\cos\left(fx+e\right)^2+3ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right)-2\left(ab+b^2\right)\cos\left(fx+e\right)-2\left(\right.}{4\left(\left(a^4+2a^3b+a^2b^2\right)f\cos\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f
*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(a
*b + b^2)*cos(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a*b)*log(1/2*cos(f*x + e)
+ 1/2) + 2*(a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e) + 1/2))/((a^4 +
 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f), 1/2*
(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*
cos(f*x + e)/b) - (a*b + b^2)*cos(f*x + e) - (a^2*cos(f*x + e)^2 + a*b)*log
(1/2*cos(f*x + e) + 1/2) + (a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e)
 + 1/2))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 +
 a*b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(1/(4*a^2+8*a*b+4*b^2)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+
exp(1))))+(-3*a*b-b^2)*1/4/(a^3+2*a^2*b+a*b^2)/sqrt(a*b)*atan((-a*cos(f*x+e
xp(1))+b)/(sqrt(a*b)*cos(f*x+exp(1))+sqrt(a*b)))+((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1)))*a*b-(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-a*b-b^2)/(2*
a^3+4*a^2*b+2*a*b^2)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a+b))
```

maple [A] time = 1.19, size = 172, normalized size = 1.74

$$-\frac{b\cos\left(fx+e\right)}{2f\left(a+b\right)^{2}\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}-\frac{b^{2}\cos\left(fx+e\right)}{2f\left(a+b\right)^{2}a\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}+\frac{3b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{2f\left(a+b\right)^{2}\sqrt{ab}}+\frac{b^{2}\arctan}{2f\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/(a+b)^2*cos(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*b^2/(a+b)^2/a*cos(f*x+
e)/(b+a*cos(f*x+e)^2)+3/2/f*b/(a+b)^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)
^(1/2))+1/2/f*b^2/(a+b)^2/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/
2/f/(a+b)^2*ln(-1+cos(f*x+e))-1/2/f/(a+b)^2*ln(1+cos(f*x+e))
```

maxima [A] time = 0.43, size = 138, normalized size = 1.39

$$-\frac{\frac{b\cos\left(fx+e\right)}{a^{2}b+ab^{2}+\left(a^{3}+a^{2}b\right)\cos\left(fx+e\right)^{2}}-\frac{\left(3\,ab+b^{2}\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{3}+2\,a^{2}b+ab^{2}\right)\sqrt{ab}}+\frac{\log\left(\cos\left(fx+e\right)+1\right)}{a^{2}+2\,ab+b^{2}}-\frac{\log\left(\cos\left(fx+e\right)-1\right)}{a^{2}+2\,ab+b^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*cos(f*x + e)/(a^2*b + a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2) - (3*a*
b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b
)) + log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) - log(cos(f*x + e) - 1)/(a^2
 + 2*a*b + b^2))/f
```

mupad [B] time = 5.95, size = 2188, normalized size = 22.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*
a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b
 + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 +
 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 1
6*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^
3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^
4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 +
 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) - ((3*a + b)*(-a^3*b
)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a
^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b
^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^
2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((5*a*b^2)/2 + 3*a^2*b + b^3/2)/(a
*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x
)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a
+ b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*
b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)
^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b
^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b +
a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)
*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b +
a^3)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^
4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a
 + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*
b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/
(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))*(3*a + b)*(
-a^3*b)^(1/2)*1i)/(2*f*(2*a^4*b + a^5 + a^3*b^2)) - (atan((((((2*a^6*b + 2*
a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3
*a^2*b^2)) - (cos(e + f*x)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 3
2*a^5*b^3 + 32*a^6*b^2))/(8*(a + b)^2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(2*(a +
 b)^2) + (cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2)*1i)/(4*(a*b^2 +
2*a^2*b + a^3)))/(a + b)^2 - ((((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b
^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) + (cos(e + f*x)*(48
*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a
 + b)^2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(2*(a + b)^2) - (cos(e + f*x)*(6*a*b^
3 + 4*a^4 + b^4 + 9*a^2*b^2)*1i)/(4*(a*b^2 + 2*a^2*b + a^3)))/(a + b)^2)/((
((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a
^3*b + a^4 + 3*a^2*b^2)) - (cos(e + f*x)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 -
48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a + b)^2*(a*b^2 + 2*a^2*b + a^3)
))/(2*(a + b)^2) + (cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(4*(a
*b^2 + 2*a^2*b + a^3)))/(a + b)^2 - ((5*a*b^2)/2 + 3*a^2*b + b^3/2)/(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2) + (((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*
b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) + (cos(e + f*x)*(4
8*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(
a + b)^2*(a*b^2 + 2*a^2*b + a^3)))/(2*(a + b)^2) - (cos(e + f*x)*(6*a*b^3 +
 4*a^4 + b^4 + 9*a^2*b^2))/(4*(a*b^2 + 2*a^2*b + a^3)))/(a + b)^2))*1i)/(f*
(a + b)^2) - (b*cos(e + f*x))/(2*a*f*(a + b)*(b + a*cos(e + f*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.45 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{(a-b)\cos(e+fx)}{2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}f(a+b)^3}-\frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^3}-\frac{\cot(e+f}{2f(a+b)\left(a\right.}$$

```
[Out] -1/2*(a-3*b)*arctanh(cos(f*x+e))/(a+b)^3/f+1/2*(a-b)*cos(f*x+e)/(a+b)^2/f/(
b+a*cos(f*x+e)^2)-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*
(3*a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 470, 527, 522, 206, 205}

$$\frac{(a-b)\cos(e+fx)}{2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}f(a+b)^3}-\frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^3}-\frac{\cot(e+f}{2f(a+b)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*Sqrt[a]*(a +
b)^3*f) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^3*f) + ((a - b)*Cos[
e + f*x])/(2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (Cot[e + f*x]*Csc[e + f*
x])/(2*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-a+2b)x^2}{\left(1-x^2\right)\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-4b^2+\cdots}{\left(1-x^2\cdots\right.}\right.}{\cdots}$$

$$= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-3b)\,\text{Subst}\cdots}{\cdots}$$

$$= \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}\,(a+b)^3f} - \frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^3f} + \frac{(a-b)\cos\cdots}{2(a+b)^2f\left(b+\cdots\right.}$$

Mathematica [C] time = 1.94, size = 468, normalized size = 3.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)\sec^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(a\cos(2(e+fx))+a+2b)-4(a-3\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(-8*b*(a + b) - (4*Sqrt[b]*(
-3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/Sqr
t[a] - (4*Sqrt[b]*(-3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(
```

```
Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]
)*Sec[e + f*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[(e + f
*x)/2]^2*Sec[e + f*x] - 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[
(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log
[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Se
c[(e + f*x)/2]^2*Sec[e + f*x]))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.59, size = 698, normalized size = 4.75

$$\left[\frac{2\left(a^2-b^2\right)\cos\left(fx+e\right)^3-\left(\left(3\,a^2-ab\right)\cos\left(fx+e\right)^4-\left(3\,a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^2-3\,ab+b^2\right)\sqrt{-\frac{b}{a}}\,\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 - ((3*a^2 - a*b)*cos(f*x + e)^4 - (3*a^2
 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(-b/a)*log((a*cos(f*x + e
)^2 - 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 4*(a*b + b
^2)*cos(f*x + e) - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*co
s(f*x + e)^2 - a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*co
s(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2
*cos(f*x + e) + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4
 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*f), 1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 + 2*((3*a^2 - a*b)*cos
(f*x + e)^4 - (3*a^2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(b/a)
*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 4*(a*b + b^2)*cos(f*x + e) - ((a^2 -
3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b
 + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3
 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))/(16*b^2+32*b*a+16*a^2)
+(6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^2+4*((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1))))^3*b*a-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^
2-15*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+2*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*b*a+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2
-24*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-20*(1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1)))*b*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-3*b^2-6*b
*a-3*a^2)/(48*b^3+144*b^2*a+144*b*a^2+48*a^3)/(((1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1))))^3*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a+2*((1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*a+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+(1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1)))*a)+(-3*b+a)/(8*b^3+24*b^2*a+24*b*a^2+8*a^3)*ln(abs(1-cos(f
*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(b^2-3*b*a)*1/4/(b^3+3*b^2*a+3*b*a^2+a^
3)/sqrt(a*b)*atan((-a*cos(f*x+exp(1))+b)/(sqrt(a*b)*cos(f*x+exp(1))+sqrt(a*
b))))
```

maple [A] time = 1.31, size = 250, normalized size = 1.70

$$-\frac{b\cos\left(fx+e\right)a}{2f\left(a+b\right)^{3}\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}-\frac{b^{2}\cos\left(fx+e\right)}{2f\left(a+b\right)^{3}\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}+\frac{3b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)a}{2f\left(a+b\right)^{3}\sqrt{ab}}-\frac{b^{2}\arctan\left(\frac{a\cos}{}\right.}{2f\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/(a+b)^3*cos(f*x+e)/(b+a*cos(f*x+e)^2)*a-1/2/f*b^2/(a+b)^3*cos(f*x+
e)/(b+a*cos(f*x+e)^2)+3/2/f*b/(a+b)^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)
^(1/2))*a-1/2/f*b^2/(a+b)^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/
4/f/(a+b)^2/(-1+cos(f*x+e))+1/4/f/(a+b)^3*ln(-1+cos(f*x+e))*a-3/4/f/(a+b)^3
*ln(-1+cos(f*x+e))*b+1/4/f/(a+b)^2/(1+cos(f*x+e))-1/4/f/(a+b)^3*ln(1+cos(f*
x+e))*a+3/4/f/(a+b)^3*ln(1+cos(f*x+e))*b
```

maxima [A] time = 0.45, size = 231, normalized size = 1.57

$$-\frac{\frac{(a-3b)\log\left(\cos\left(fx+e\right)+1\right)}{a^{3}+3a^{2}b+3ab^{2}+b^{3}}-\frac{(a-3b)\log\left(\cos\left(fx+e\right)-1\right)}{a^{3}+3a^{2}b+3ab^{2}+b^{3}}-\frac{2\left(3ab-b^{2}\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)\sqrt{ab}}-\frac{2\left(\left(a-b\right)\cos\left(fx+e\right)^{3}+2b\cos\left(fx+\right.\right.}{\left(a^{3}+2a^{2}b+ab^{2}\right)\cos\left(fx+e\right)^{4}-a^{2}b-2ab^{2}-b^{3}-\left(a^{3}+a^{2}b\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a - 3*b)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a
- 3*b)*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(3*a*b - b
^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(
a*b)) - 2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))/((a^3 + 2*a^2*b + a*b
^2)*cos(f*x + e)^4 - a^2*b - 2*a*b^2 - b^3 - (a^3 + a^2*b - a*b^2 - b^3)*co
s(f*x + e)^2))/f
```

mupad [B] time = 5.65, size = 1845, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - ((cos(e + f*x)^3*(a - b))/(2*(2*a*b + a^2 + b^2)) + (b*cos(e + f*x))/(2*a
*b + a^2 + b^2))/(f*(b - a*cos(e + f*x)^4 + cos(e + f*x)^2*(a - b))) - (log
(cos(e + f*x) - 1)*(b/(a + b)^3 - 1/(4*(a + b)^2)))/f - (log(cos(e + f*x) +
 1)*(a - 3*b))/(4*f*(a + b)^3) - (atan((((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4
 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b
^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^
4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^
6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1/2)*(3*a
 - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b
^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a
*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b +
a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) +
((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^
2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)^(1/2)*((4*a^8
*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7
*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2
) + (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 -
80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^
```

```
3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))
)*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b
^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(((3*a*b^4)/4 - (3*a^4*b)/4 - (13*a^2*b^3
)/4 + (13*a^3*b^2)/4)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*
b^3 + 15*a^4*b^2) + ((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6
*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (
(-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 +
 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*
a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16
*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 14
4*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4
 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))*(
3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) - ((-a*b)^(1/2)*((cos(e +
 f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3
*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3
*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*
b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) + (cos(e + f*x)*(-a*b
)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^
5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a
^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2
*b^2))))*(-a*b)^(1/2)*(3*a - b)*1i)/(2*f*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.46 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=197

$$-\frac{3\left(a^2-6ab+b^2\right)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^4}+\frac{3a(a-3b)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{3\sqrt{a}\sqrt{b}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2f(a+b)^4}-$$

```
[Out] -3/8*(a^2-6*a*b+b^2)*arctanh(cos(f*x+e))/(a+b)^4/f+3/8*a*(a-3*b)*cos(f*x+e)
/(a+b)^3/f/(b+a*cos(f*x+e)^2)-1/8*(a-5*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(
b+a*cos(f*x+e)^2)-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f/(b+a*cos(f*x+e)^2)+3/
2*(a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^4/f
```

Rubi [A] time = 0.25, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 470, 527, 522, 206, 205}

$$-\frac{3\left(a^2-6ab+b^2\right)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^4}+\frac{3a(a-3b)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{3\sqrt{a}\sqrt{b}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2f(a+b)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*(a +
b)^4*f) - (3*(a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^4*f) + (
3*a*(a - 3*b)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) - ((a -
5*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (C
ot[e + f*x]*Csc[e + f*x]^3)/(4*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^3\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-a+4b)x^2}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+}{4(a+b)f\left(\right.}$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+}{4(a+b)f\left(\right.}$$

$$= \frac{3\sqrt{a}\,(a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4f} - \frac{3\left(a^2-6ab+b^2\right)\tanh^{-1}(\cos(e+fx))}{8(a+b)^4f} + \cdots$$

Mathematica [C] time = 2.44, size = 450, normalized size = 2.28

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-24\left(a^2-6ab+b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)(a\cos(2(e+fx))+a+2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
```

```
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
]*(a + 2*b + a*Cos[2*(e + f*x)]) + 96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a + b)*(11*a^2 + 43*a*b - 4*b^2 + 4
*(2*a^2 - 5*a*b + 5*b^2)*Cos[2*(e + f*x)] - 3*a*(a - 3*b)*Cos[4*(e + f*x)])
*Cot[e + f*x]*Csc[e + f*x]^3 - 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2*(e
 + f*x)])*Log[Cos[(e + f*x)/2]] + 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2
*(e + f*x)])*Log[Sin[(e + f*x)/2]])*Sec[e + f*x]^4)/(256*(a + b)^4*f*(a + b
*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.66, size = 1202, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*
a*b^2 + 5*b^3)*cos(f*x + e)^3 - 12*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3
*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b
^2)*sqrt(-a*b)*log((a*cos(f*x + e)^2 - 2*sqrt(-a*b)*cos(f*x + e) - b)/(a*co
s(f*x + e)^2 + b)) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e) - 3*((a^3 - 6
*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x
 + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x
+ e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^
2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x
+ e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*
x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(
f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f), 1/16*(6*(a
^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*a*b^2 + 5*b
^3)*cos(f*x + e)^3 + 24*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3*a*b + b^2)
*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a*
b)*arctan(sqrt(a*b)*cos(f*x + e)/b) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x +
 e) - 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b
^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^
2 - 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4
 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^
2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2*a+(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*b*a^2-b^2*a-b*a^2)/(2*b^4+8*b^3*a+12*b^2*a^2+8*b*
a^3+2*a^4)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*
```

```
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+(-18*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^2*b^2+108*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*
a-18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+8*(1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1)))*b^2-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-b^2-2
*b*a-a^2)/(128*b^4+512*b^3*a+768*b^2*a^2+512*b*a^3+128*a^4)/((1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1))))^2+(32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
2*b^2+64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a+32*((1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1))))^2*a^2-256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*b^2+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2)/(4096*b^4+16384*b^3*
a+24576*b^2*a^2+16384*b*a^3+4096*a^4)+(3*b^2*a-3*b*a^2)*1/4/(b^4+4*b^3*a+6*
b^2*a^2+4*b*a^3+a^4)/sqrt(a*b)*atan((-a*cos(f*x+exp(1))+b)/(sqrt(a*b)*cos(f
*x+exp(1))+sqrt(a*b)))+(3*b^2-18*b*a+3*a^2)/(32*b^4+128*b^3*a+192*b^2*a^2+1
28*b*a^3+32*a^4)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.97, size = 390, normalized size = 1.98

$$-\frac{a^2 b\cos\left(fx+e\right)}{2f\left(a+b\right)^4\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}-\frac{a\,b^2\cos\left(fx+e\right)}{2f\left(a+b\right)^4\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}+\frac{3a^2 b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{2f\left(a+b\right)^4\sqrt{ab}}-\frac{3a\,b^2\,\mathrm{arct}}{2f\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*b/(a+b)^4*cos(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*a*b^2/(a+b)^4*cos(
f*x+e)/(b+a*cos(f*x+e)^2)+3/2/f*a^2*b/(a+b)^4/(a*b)^(1/2)*arctan(a*cos(f*x+
e)/(a*b)^(1/2))-3/2/f*a*b^2/(a+b)^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(
1/2))-1/16/f/(a+b)^2/(-1+cos(f*x+e))^2+3/16/f/(a+b)^3/(-1+cos(f*x+e))*a-5/1
6/f/(a+b)^3/(-1+cos(f*x+e))*b+3/16/f/(a+b)^4*ln(-1+cos(f*x+e))*a^2-9/8/f/(a
+b)^4*ln(-1+cos(f*x+e))*a*b+3/16/f/(a+b)^4*ln(-1+cos(f*x+e))*b^2+1/16/f/(a+
b)^2/(1+cos(f*x+e))^2+3/16/f/(a+b)^3/(1+cos(f*x+e))*a-5/16/f/(a+b)^3/(1+cos
(f*x+e))*b-3/16/f/(a+b)^4*ln(1+cos(f*x+e))*a^2+9/8/f/(a+b)^4*ln(1+cos(f*x+e
))*a*b-3/16/f/(a+b)^4*ln(1+cos(f*x+e))*b^2
```

maxima [B] time = 0.43, size = 369, normalized size = 1.87

$$-\frac{\frac{3\left(a^2-6\,ab+b^2\right)\log\left(\cos\left(fx+e\right)+1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{3\left(a^2-6\,ab+b^2\right)\log\left(\cos\left(fx+e\right)-1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{24\left(a^2b-ab^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\sqrt{ab}}-\frac{\cdots}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\cos\left(f\right.}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4) - 3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) - 1)/(a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 24*(a^2*b - a*b^2)*arctan(a*cos(f*x +
e)/sqrt(a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - 2*(
3*(a^2 - 3*a*b)*cos(f*x + e)^5 - (5*a^2 - 14*a*b + 5*b^2)*cos(f*x + e)^3 -
3*(3*a*b - b^2)*cos(f*x + e))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x
+ e)^6 - (2*a^4 + 5*a^3*b + 3*a^2*b^2 - a*b^3 - b^4)*cos(f*x + e)^4 + a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*
cos(f*x + e)^2))/f
```

mupad [B] time = 9.33, size = 4338, normalized size = 22.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*
a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 2
0*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*
b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b
^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9
+ b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 +
36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11
- 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 +
 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4
 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^
3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b)
*3i)/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((cos(
e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32
*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)) -
(3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 -
6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*
b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3
*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (3*cos(e + f*
x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^
8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^
3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*b^
5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*a*
b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b)*3i)/(4*(4*a*b^3 + 4*a^3*b
+ a^4 + b^4 + 6*a^2*b^2)))/(((27*a^7*b)/64 + (81*a^3*b^5)/64 - (297*a^4*b^4
)/32 + (189*a^5*b^3)/16 - (135*a^6*b^2)/32)/(9*a*b^8 + 9*a^8*b + a^9 + b^9
+ 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7
*b^2) - (3*(-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 3
96*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4
+ 20*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a
^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^
7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a
^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3
 + 36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^
11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^
5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b +
a^4 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3
*b^3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a -
 b))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (3*(-a*b)^(1/2)*((co
s(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(
32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))
- (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2
- 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^
9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a
^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (3*cos(e +
f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*
b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*
b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*
b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*
a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b))/(4*(4*a*b^3 + 4*a^3*b +
 a^4 + b^4 + 6*a^2*b^2))))*(-a*b)^(1/2)*(a - b)*3i)/(2*f*(4*a*b^3 + 4*a^3*b
 + a^4 + b^4 + 6*a^2*b^2)) - ((3*cos(e + f*x)^5*(3*a*b - a^2))/(8*(3*a*b^2
+ 3*a^2*b + a^3 + b^3)) + (3*b*cos(e + f*x)*(3*a - b))/(8*(3*a*b^2 + 3*a^2*
b + a^3 + b^3)) + (cos(e + f*x)^3*(5*a^2 - 14*a*b + 5*b^2))/(8*(a + b)*(2*a
*b + a^2 + b^2)))/(f*(b - cos(e + f*x)^4*(2*a - b) + a*cos(e + f*x)^6 + cos
(e + f*x)^2*(a - 2*b))) - (atan((((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3
*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4
+ 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^
7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) - (co
```

```
s(e + f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))
*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168
*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*
(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3
/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) + (cos(e + f
*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a
*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16
*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*1i - ((((9*a^11*
b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b
^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 +
9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 +
 84*a^6*b^3 + 36*a^7*b^2) + (cos(e + f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a +
 b)^3) + (3*b^2)/(2*(a + b)^4))*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 179
2*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 716
8*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4
+ 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b
^2)/(2*(a + b)^4)) - (cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a
^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20
*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/
(2*(a + b)^4))*1i)/(((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a
^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3
 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^
6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) - (cos(e + f*x)*(3
/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*(1792*a^10*b
 + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 35
84*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*b^5 + 6*
a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^
2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) + (cos(e + f*x)*(9*a^7 -
108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*
b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a + b)^2) -
 (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) - ((27*a^7*b)/64 + (81*a^3*b^
5)/64 - (297*a^4*b^4)/32 + (189*a^5*b^3)/16 - (135*a^6*b^2)/32)/(9*a*b^8 +
9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 +
 84*a^6*b^3 + 36*a^7*b^2) + ((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)
/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114
*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 8
4*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (cos(e +
 f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*(179
2*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*
b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*(6*a*
b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*
(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) - (cos(e + f*x)*(
9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5
+ 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16*(a +
 b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))))*(3i/(8*(a + b)^2) -
 (b*3i)/(a + b)^3 + (b^2*3i)/(a + b)^4))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.47 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{b}\,(a+b)^{3/2}(3a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f}+\frac{(9a+8b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)}-\frac{b\left(19a^2+52ab+32b^2\right)\tan(e+}{16a^4f\left(a+b\tan^2(e+fx)+b\right.}$$

```
[Out] 1/16*(5*a^3+60*a^2*b+120*a*b^2+64*b^3)*x/a^5-1/2*(a+b)^(3/2)*(3*a+8*b)*arct
an(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f-1/48*(33*a^2+82*a*b+48*b^2
)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(9*a+8*b)*cos(f*x+e
)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f
/(a+b+b*tan(f*x+e)^2)-1/16*b*(19*a^2+52*a*b+32*b^2)*tan(f*x+e)/a^4/f/(a+b+b
*tan(f*x+e)^2)
```

Rubi [A] time = 0.43, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4132, 470, 578, 527, 522, 203, 205}

$$-\frac{b\left(19a^2+52ab+32b^2\right)\tan(e+fx)}{16a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(33a^2+82ab+48b^2\right)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x\left(60a^2b+5a^3+120ab^2+\right.}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*x)/(16*a^5) - (Sqrt[b]*(a + b)^(3/
2)*(3*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*f) - ((33
*a^2 + 82*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan
[e + f*x]^2)) + ((9*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b
+ b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Ta
n[e + f*x]^2)) - (b*(19*a^2 + 52*a*b + 32*b^2)*Tan[e + f*x])/(16*a^4*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(b-6(a+b))x^2\right)}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{(a+b}{}\right.}{}$$

$$= -\frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+f}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+f}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+f}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)x}{16a^5} - \frac{\sqrt{b}\,(a+b)^{3/2}(3a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f} - \frac{(}{}$$

Mathematica [C] time = 23.51, size = 2738, normalized size = 10.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/512*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Se
c[e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3
- 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x])
+ a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*
e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a
*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b +
 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a +
2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(4096*a^3*
(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x
]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) -
 (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(2
048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2
*Sec[e + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(
3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e
 + f*x)]))))/(2048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[
```

```
2*e + 2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2
*b^3 - 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Si
n[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4])) + (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^
4)*f*x*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f
*x] + 80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 76
8*a^2*b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2
*e] + 34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640
*a*b^4*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] -
 318*a^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 1
2*a^4*b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin
[2*(e + 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x]
- 256*a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[
6*e + 4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] +
2*a^4*b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6
*f*x] + 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])))/(2048
*a^4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x]
)^2*Sec[e + f*x]^4*(-(((a^6 - 48*a^5*b - 1200*a^4*b^2 - 6400*a^3*b^3 - 1344
0*a^2*b^4 - 12288*a*b^5 - 4096*b^6)*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a +
 b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b
*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x
])]*Cos[2*e])/(8*a^5*b*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I
/8)*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]
]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*S
in[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*b*Sqrt[a + b]*f
*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/(a + b)) - (Sec[2*e]*(-960*a^5*b*f*x*Co
s[2*e] - 10944*a^4*b^2*f*x*Cos[2*e] - 44544*a^3*b^3*f*x*Cos[2*e] - 83712*a^
2*b^4*f*x*Cos[2*e] - 73728*a*b^5*f*x*Cos[2*e] - 24576*b^6*f*x*Cos[2*e] - 48
0*a^5*b*f*x*Cos[2*f*x] - 4512*a^4*b^2*f*x*Cos[2*f*x] - 13248*a^3*b^3*f*x*Co
s[2*f*x] - 15360*a^2*b^4*f*x*Cos[2*f*x] - 6144*a*b^5*f*x*Cos[2*f*x] - 480*a
^5*b*f*x*Cos[4*e + 2*f*x] - 4512*a^4*b^2*f*x*Cos[4*e + 2*f*x] - 13248*a^3*b
^3*f*x*Cos[4*e + 2*f*x] - 15360*a^2*b^4*f*x*Cos[4*e + 2*f*x] - 6144*a*b^5*f
*x*Cos[4*e + 2*f*x] - 3*a^6*Sin[2*e] - 156*a^5*b*Sin[2*e] - 1500*a^4*b^2*Si
n[2*e] - 5760*a^3*b^3*Sin[2*e] - 10560*a^2*b^4*Sin[2*e] - 9216*a*b^5*Sin[2*
e] - 3072*b^6*Sin[2*e] + 3*a^6*Sin[2*f*x] + 366*a^5*b*Sin[2*f*x] + 3000*a^4
*b^2*Sin[2*f*x] + 8400*a^3*b^3*Sin[2*f*x] + 9600*a^2*b^4*Sin[2*f*x] + 3840*
a*b^5*Sin[2*f*x] + 216*a^5*b*Sin[4*e + 2*f*x] + 1800*a^4*b^2*Sin[4*e + 2*f*
x] + 5040*a^3*b^3*Sin[4*e + 2*f*x] + 5760*a^2*b^4*Sin[4*e + 2*f*x] + 2304*a
*b^5*Sin[4*e + 2*f*x] + 76*a^5*b*Sin[2*e + 4*f*x] + 460*a^4*b^2*Sin[2*e + 4
*f*x] + 768*a^3*b^3*Sin[2*e + 4*f*x] + 384*a^2*b^4*Sin[2*e + 4*f*x] + 76*a^
5*b*Sin[6*e + 4*f*x] + 460*a^4*b^2*Sin[6*e + 4*f*x] + 768*a^3*b^3*Sin[6*e +
 4*f*x] + 384*a^2*b^4*Sin[6*e + 4*f*x] - 16*a^5*b*Sin[4*e + 6*f*x] - 48*a^4
*b^2*Sin[4*e + 6*f*x] - 32*a^3*b^3*Sin[4*e + 6*f*x] - 16*a^5*b*Sin[8*e + 6*
f*x] - 48*a^4*b^2*Sin[8*e + 6*f*x] - 32*a^3*b^3*Sin[8*e + 6*f*x] + 4*a^5*b*
Sin[6*e + 8*f*x] + 4*a^4*b^2*Sin[6*e + 8*f*x] + 4*a^5*b*Sin[10*e + 8*f*x] +
 4*a^4*b^2*Sin[10*e + 8*f*x]))/(24*a^5*b*(a + b)*f*(a + 2*b + a*Cos[2*e + 2
*f*x]))))/(512*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.68, size = 674, normalized size = 2.52

$$\left[\frac{3\left(5\,a^{4}+60\,a^{3}b+120\,a^{2}b^{2}+64\,ab^{3}\right)fx\cos\left(fx+e\right)^{2}+3\left(5\,a^{3}b+60\,a^{2}b^{2}+120\,ab^{3}+64\,b^{4}\right)fx+6\left(3\,a^{2}b+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3
*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 6*(3*a^2*b + 11*a*b^2 +
8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e)
+ b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - (8*a^4*cos(f*x
+ e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*
b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*si
n(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), 1/48*(3*(5*a^4 + 60*a^3*b + 1
20*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b^2 + 120*a
*b^3 + 64*b^4)*f*x + 12*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 11*a^2*b + 8
*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^4*cos(f*x + e)^7
 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*b^2)*c
os(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*sin(f*x
+ e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f)]
```

giac [A] time = 0.38, size = 311, normalized size = 1.16

$$\frac{\frac{3\left(5a^3+60a^2b+120ab^2+64b^3\right)(fx+e)}{a^5}-\frac{24\left(3a^3b+14a^2b^2+19ab^3+8b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{24\left(a^2b\tan(fx+e)+2ab^2\tan(\right.}{\left(b\tan(fx+e)^2+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 - 24*(3*a^3*b
 + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^5) - 24*(a^2*b*tan
(f*x + e) + 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a
 + b)*a^4) - (33*a^2*tan(f*x + e)^5 + 108*a*b*tan(f*x + e)^5 + 72*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 192*a*b*tan(f*x + e)^3 + 144*b^2*tan(f*
x + e)^3 + 15*a^2*tan(f*x + e) + 84*a*b*tan(f*x + e) + 72*b^2*tan(f*x + e))
/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

maple [B] time = 0.95, size = 555, normalized size = 2.08

$$-\frac{b\tan\left(fx+e\right)}{2a^2f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{b^2\tan\left(fx+e\right)}{f\,a^3\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{b^3\tan\left(fx+e\right)}{2f\,a^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan}{2f\,a^2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)-1/f*b^2/a^3*tan(f*x+e)/(a+b+b*
tan(f*x+e)^2)-1/2/f*b^3/a^4*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a^2/((a
+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-7/f*b^2/a^3/((a+b)*b)^(1/
2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-19/2/f*b^3/a^4/((a+b)*b)^(1/2)*arct
an(tan(f*x+e)*b/((a+b)*b)^(1/2))-4/f*b^4/a^5/((a+b)*b)^(1/2)*arctan(tan(f*x
+e)*b/((a+b)*b)^(1/2))-9/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b-3/2/f/a^
4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b^2-11/16/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*
x+e)^5-4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b-3/f/a^4/(tan(f*x+e)^2+1)^3
*tan(f*x+e)^3*b^2-5/6/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-5/16/f/a^2/(tan
(f*x+e)^2+1)^3*tan(f*x+e)-7/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b-3/2/f/a
^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b^2+15/4/f/a^3*arctan(tan(f*x+e))*b+15/2/f
/a^4*arctan(tan(f*x+e))*b^2+4/f/a^5*arctan(tan(f*x+e))*b^3+5/16/f/a^2*arcta
n(tan(f*x+e))
```

maxima [A] time = 0.46, size = 302, normalized size = 1.13

$$-\frac{\frac{3\left(19a^2b+52ab^2+32b^3\right)\tan\left(fx+e\right)^7+\left(33a^3+253a^2b+516ab^2+288b^3\right)\tan\left(fx+e\right)^5+\left(40a^3+319a^2b+564ab^2+288b^3\right)\tan\left(fx+e\right)^3+3\left(5a^3+41a^2\cdots\right.}{a^4b\tan\left(fx+e\right)^8+\left(a^5+4a^4b\right)\tan\left(fx+e\right)^6+a^5+a^4b+3\left(a^5+2a^4b\right)\tan\left(fx+e\right)^4+\left(3a^5+4a^4b\right)\tan\left(fx+e\right)^2}\cdots}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*((3*(19*a^2*b + 52*a*b^2 + 32*b^3)*tan(f*x + e)^7 + (33*a^3 + 253*a^2
*b + 516*a*b^2 + 288*b^3)*tan(f*x + e)^5 + (40*a^3 + 319*a^2*b + 564*a*b^2
+ 288*b^3)*tan(f*x + e)^3 + 3*(5*a^3 + 41*a^2*b + 68*a*b^2 + 32*b^3)*tan(f*
x + e))/(a^4*b*tan(f*x + e)^8 + (a^5 + 4*a^4*b)*tan(f*x + e)^6 + a^5 + a^4*
b + 3*(a^5 + 2*a^4*b)*tan(f*x + e)^4 + (3*a^5 + 4*a^4*b)*tan(f*x + e)^2) -
3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 + 24*(3*a^3*b + 14*
a^2*b^2 + 19*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a
 + b)*b)*a^5))/f
```

mupad [B] time = 6.66, size = 1461, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atanh((75*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(2
56*((211*a*b^4)/128 + (811*b^5)/256 + (75*a^2*b^3)/256 + (41*b^6)/(16*a) +
(3*b^7)/(4*a^2))) + (17*b^4*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b
^2)^(1/2))/(16*((811*a*b^5)/256 + (41*b^6)/16 + (211*a^2*b^4)/128 + (75*a^3
*b^3)/256 + (3*b^7)/(4*a))) + (3*b^5*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4
- 3*a^2*b^2)^(1/2))/(4*((41*a*b^6)/16 + (3*b^7)/4 + (811*a^2*b^5)/256 + (21
1*a^3*b^4)/128 + (75*a^4*b^3)/256)))*(-b*(a + b)^3)^(1/2)*(3*a + 8*b))/(2*a
^5*f) - (atan((((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^
2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^
2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i
+ b^3*64i))/(32*a^5) - (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^
7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))
*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5) - (((((8*a^10*b^5
 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 + (tan(e + f*x)*(20
48*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(
4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) + (tan(e
+ f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*
b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3
*5i + b^3*64i)*1i)/(32*a^5))/((376*a*b^10 + 64*b^11 + 937*a^2*b^9 + (10285*
a^3*b^8)/8 + (33701*a^4*b^7)/32 + (8333*a^5*b^6)/16 + (38085*a^6*b^5)/256 +
 (2765*a^7*b^4)/128 + (285*a^8*b^3)/256)/a^12 + (((((8*a^10*b^5 + 17*a^11*b
^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3
+ 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*
(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) - (tan(e + f*x)*(3481
6*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a
^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64
i))/(32*a^5) + (((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^
2)/4)/a^12 + (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^
2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i
+ b^3*64i))/(32*a^5) + (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^
7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))
*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5)))*(a*b^2*120i + a^2*
b*60i + a^3*5i + b^3*64i)*1i)/(16*a^5*f) - ((tan(e + f*x)*(68*a*b^2 + 41*a^
```

```
2*b + 5*a^3 + 32*b^3))/(16*a^4) + (tan(e + f*x)^5*(516*a*b^2 + 253*a^2*b +
33*a^3 + 288*b^3))/(48*a^4) + (tan(e + f*x)^3*(564*a*b^2 + 319*a^2*b + 40*a
^3 + 288*b^3))/(48*a^4) + (b*tan(e + f*x)^7*(52*a*b + 19*a^2 + 32*b^2))/(16
*a^4))/(f*(a + b + tan(e + f*x)^2*(3*a + 4*b) + tan(e + f*x)^4*(3*a + 6*b)
+ b*tan(e + f*x)^8 + tan(e + f*x)^6*(a + 4*b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.48 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=191

$$-\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f}-\frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b\tan^2(e+fx)+b\right)}-\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)}+$$

```
[Out] 3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))*b^(1/2)*(a+b)^(1/2)/a^4/f-1/8*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+
b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-3/8*
b*(3*a+4*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.26, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 203, 205}

$$\frac{3x\left(a^2+8ab+8b^2\right)}{8a^4}-\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f}-\frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b\tan^2(e+fx)+b\right)}-\frac{(5a+6b)\sin(e}{8a^2f\left(a+b\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*f) - ((5*a + 6*b)*Cos[e + f*
x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) - (3*b*(3*a + 4*b)*Tan[e + f*
x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-5b)x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3(a\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b\cdots}{8a^3f\left(a+b+\cdots\right.}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b\cdots}{8a^3f\left(a+b+\cdots\right.}$$

$$= \frac{3\left(a^2+8ab+8b^2\right)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{(5a+6b)\cos(e\cdots}{8a^2f\left(a+b+\cdots\right.}$$

Mathematica [C] time = 13.24, size = 1105, normalized size = 5.79

$$-\frac{(\cos(2e+2fx)a+a+2b)^2\left(16x+\frac{\left(-a^3+6ba^2+24b^2a+16b^3\right)\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\cdots\right.}{256a^2\left(b\sec^2(e+fx)+a\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Se
c[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b
)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x
)]))))/(1024*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e +
2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 -
 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]))
+ (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^4)*f*x
*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f*x] +
80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 768*a^2*
b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2*e] +
34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640*a*b^4
*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] - 318*a
^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 12*a^4*
b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin[2*(e
+ 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x] - 256*
a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[6*e +
4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] + 2*a^4*
b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6*f*x]
+ 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)]))))/(1024*a^4*b
*(a + b)*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.62, size = 522, normalized size = 2.73

$$\left[\frac{3\left(a^{3}+8\,a^{2}b+8\,ab^{2}\right)fx\cos\left(fx+e\right)^{2}+3\left(a^{2}b+8\,ab^{2}+8\,b^{3}\right)fx+3\left(\left(a^{2}+2\,ab\right)\cos\left(fx+e\right)^{2}+ab+2\,b^{2}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 +
 8*b^3)*f*x + 3*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^3*
cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*b^2)*c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), 1/8*(3*(a^3 +
8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)*f*x + 6
*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) +
 (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*
a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

giac [A] time = 0.35, size = 204, normalized size = 1.07

$$\frac{\frac{3\left(a^{2}+8\,ab+8\,b^{2}\right)\left(fx+e\right)}{a^{4}}-\frac{12\left(a^{2}b+3\,ab^{2}+2\,b^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\sqrt{ab+b^{2}}\,a^{4}}-\frac{4\left(ab\tan\left(fx+e\right)+b^{2}\tan\left(fx+e\right)\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)a^{3}}-\frac{5\,a\tan\left(fx+e\right)^{3}+\cdots}{\cdots}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 - 12*(a^2*b + 3*a*b^2 + 2*b^3)*(
pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)
))/(sqrt(a*b + b^2)*a^4) - 4*(a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*tan(
f*x + e)^2 + a + b)*a^3) - (5*a*tan(f*x + e)^3 + 8*b*tan(f*x + e)^3 + 3*a*t
an(f*x + e) + 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

maple [A] time = 0.89, size = 323, normalized size = 1.69

$$-\frac{b\tan\left(fx+e\right)}{2a^2f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2f\,a^2\sqrt{(a+b)\,b}}-\frac{9b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2f\,a^3\sqrt{(a+b)\,b}}-\frac{b^2\tan\left(fx+e\right)}{2f\,a^3\left(a+b+b\left(\tan^2\left(fx+\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a^2/((a+b)*b)^(1/2)*ar
ctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-9/2/f*b^2/a^3/((a+b)*b)^(1/2)*arctan(tan
(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f*b^2/a^3*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-3/
f*b^3/a^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a^3/(tan
(f*x+e)^2+1)^2*tan(f*x+e)^3*b-5/8/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3-3/8
/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)-1/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)*
b+3/f/a^3*arctan(tan(f*x+e))*b+3/f/a^4*arctan(tan(f*x+e))*b^2+3/8/f/a^2*arc
tan(tan(f*x+e))
```

maxima [A] time = 0.48, size = 205, normalized size = 1.07

$$-\frac{\frac{3\left(3ab+4b^2\right)\tan\left(fx+e\right)^5+\left(5a^2+24ab+24b^2\right)\tan\left(fx+e\right)^3+3\left(a^2+5ab+4b^2\right)\tan\left(fx+e\right)}{a^3b\tan\left(fx+e\right)^6+\left(a^4+3a^3b\right)\tan\left(fx+e\right)^4+a^4+a^3b+\left(2a^4+3a^3b\right)\tan\left(fx+e\right)^2}-\frac{3\left(a^2+8ab+8b^2\right)\left(fx+e\right)}{a^4}+\frac{12\left(a^2b+3ab^2+2b^3\right)\arctan\left(\cdots\right)}{\sqrt{(a+b)b}\,a^4}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((3*(3*a*b + 4*b^2)*tan(f*x + e)^5 + (5*a^2 + 24*a*b + 24*b^2)*tan(f*x
 + e)^3 + 3*(a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a^3*b*tan(f*x + e)^6 + (a^
4 + 3*a^3*b)*tan(f*x + e)^4 + a^4 + a^3*b + (2*a^4 + 3*a^3*b)*tan(f*x + e)^
2) - 3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 + 12*(a^2*b + 3*a*b^2 + 2*b^3)*a
rctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

mupad [B] time = 5.77, size = 435, normalized size = 2.28

$$\frac{3\,\mathrm{atanh}\left(\frac{27\,b^3\tan\left(e+f\,x\right)\sqrt{-b^2-a\,b}}{64\left(\frac{27\,a\,b^3}{64}+\frac{81\,b^4}{64}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\,b^4\tan\left(e+f\,x\right)\sqrt{-b^2-a\,b}}{32\left(\frac{27\,a^2\,b^3}{64}+\frac{81\,a\,b^4}{64}+\frac{27\,b^5}{32}\right)}\right)\left(a+2\,b\right)\sqrt{-b\,\left(a+b\right)}}{2\,a^4\,f}-\frac{\frac{3\tan\left(e+f\,x\right)^5\left(4\,b^2+3\,a\,b\right)}{8\,a^3}+\frac{\tan\left(e+f\cdots\right.}{\cdots}}{f\left(b\tan\left(e+f\,x\right)^6+\left(a+3\,b\right)\,\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (3*atanh((27*b^3*tan(e + f*x)*(- a*b - b^2)^(1/2))/(64*((27*a*b^3)/64 + (81
*b^4)/64 + (27*b^5)/(32*a))) + (27*b^4*tan(e + f*x)*(- a*b - b^2)^(1/2))/(3
2*((81*a*b^4)/64 + (27*b^5)/32 + (27*a^2*b^3)/64)))*(a + 2*b)*(-b*(a + b))^
```

```
(1/2))/(2*a^4*f) - (atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (243*b^
3)/(256*a) + (27*b^4)/(16*a^2) + (27*b^5)/(32*a^3))) + (243*b^3*tan(e + f*x
))/(256*((27*a*b^2)/256 + (243*b^3)/256 + (27*b^4)/(16*a) + (27*b^5)/(32*a^
2))) + (27*b^4*tan(e + f*x))/(16*((243*a*b^3)/256 + (27*b^4)/16 + (27*a^2*b
^2)/256 + (27*b^5)/(32*a))) + (27*b^5*tan(e + f*x))/(32*((27*a*b^4)/16 + (2
7*b^5)/32 + (243*a^2*b^3)/256 + (27*a^3*b^2)/256)))*(a*b*8i + a^2*1i + b^2*
8i)*3i)/(8*a^4*f) - ((3*tan(e + f*x)^5*(3*a*b + 4*b^2))/(8*a^3) + (tan(e +
f*x)^3*(24*a*b + 5*a^2 + 24*b^2))/(8*a^3) + (3*tan(e + f*x)*(5*a*b + a^2 +
4*b^2))/(8*a^3))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan(e + f*x)^6
+ tan(e + f*x)^4*(a + 3*b)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.49 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{b}\,(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f\sqrt{a+b}}+\frac{x(a+4b)}{2a^3}-\frac{b\tan(e+fx)}{a^2f\left(a+b\tan^2(e+fx)+b\right)}-\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] 1/2*(a+4*b)*x/a^3-1/2*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/f/(a+b)^(1/2)-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-b*
tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f\sqrt{a+b}}-\frac{b\tan(e+fx)}{a^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(a+4b)}{2a^3}-\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*x)/(2*a^3) - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/(a^2*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b-3bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a+b)}{(1+}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+4b)\,\text{Subst}}{}$$

$$= \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}\,(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} -$$

Mathematica [C] time = 11.31, size = 825, normalized size = 6.35

$$-\frac{(\cos(2e+2fx)a+a+2b)^2\left(16x+\frac{\left(-a^3+6ba^2+24b^2a+16b^3\right)\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{128a^2\left(b\sec^2(e+fx)+a\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/128*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[
```

```
e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3 -
128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
 - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a*C
os[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b + 4
8*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*
b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(256*a^3*(a
+ b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(
((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*S
qrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(128*b^
(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e
 + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2))
+ (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x
)]))))/(256*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.66, size = 441, normalized size = 3.39

$$\left[\frac{4\left(a^2+4ab\right)fx\cos\left(fx+e\right)^2+4\left(ab+4b^2\right)fx+\left(\left(3a^2+4ab\right)\cos\left(fx+e\right)^2+3ab+4b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8ab\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 4*(a*b + 4*b^2)*f*x + ((3*a^2 +
4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b
 + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a^2*
cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f*x + e)^2 +
a^3*b*f), 1/4*(2*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 2*(a*b + 4*b^2)*f*x + (
(3*a^2 + 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(b/(a + b))*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)
)) - 2*(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f
*x + e)^2 + a^3*b*f)]
```

giac [A] time = 0.31, size = 158, normalized size = 1.22

$$\frac{\frac{(fx+e)(a+4b)}{a^3}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3ab+4b^2\right)}{\sqrt{ab+b^2}\,a^3}-\frac{2b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)+2b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^4+a\tan\left(fx+e\right)^2+2b\tan\left(fx+e\right)^2+a+b\right)a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 4*b)/a^3 - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arcta
n(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a*b + 4*b^2)/(sqrt(a*b + b^2)*a^3) -
(2*b*tan(f*x + e)^3 + a*tan(f*x + e) + 2*b*tan(f*x + e))/((b*tan(f*x + e)^4
 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)*a^2))/f
```

maple [A] time = 0.82, size = 155, normalized size = 1.19

$$-\frac{b\tan\left(fx+e\right)}{2a^2f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2f\,a^2\sqrt{(a+b)\,b}}-\frac{2b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\,a^3\sqrt{(a+b)\,b}}-\frac{\tan\left(fx+e\right)}{2f\,a^2\left(1+\tan^2\left(fx+e\right)\right)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a^2/((a+b)*b)^(1/2)*ar
ctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*b^2/a^3/((a+b)*b)^(1/2)*arctan(tan(f
*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a^2*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a^2*ar
ctan(tan(f*x+e))+2/f/a^3*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.60, size = 126, normalized size = 0.97

$$-\frac{\frac{2\,b\tan\left(fx+e\right)^{3}+(a+2\,b)\tan\left(fx+e\right)}{a^{2}b\tan\left(fx+e\right)^{4}+a^{3}+a^{2}b+\left(a^{3}+2\,a^{2}b\right)\tan\left(fx+e\right)^{2}}-\frac{\left(fx+e\right)(a+4\,b)}{a^{3}}+\frac{\left(3\,ab+4\,b^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^{3}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*tan(f*x + e)^3 + (a + 2*b)*tan(f*x + e))/(a^2*b*tan(f*x + e)^4 +
 a^3 + a^2*b + (a^3 + 2*a^2*b)*tan(f*x + e)^2) - (f*x + e)*(a + 4*b)/a^3 +
(3*a*b + 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3
))/f
```

mupad [B] time = 5.21, size = 816, normalized size = 6.28

atar

$$-\frac{\frac{\tan\left(e+f\,x\right)\,(a+2\,b)}{2\,a^{2}}+\frac{b\tan\left(e+f\,x\right)^{3}}{a^{2}}}{f\,\left(b\tan\left(e+f\,x\right)^{4}+(a+2\,b)\,\tan\left(e+f\,x\right)^{2}+a+b\right)}-\frac{\mathrm{atan}\left(\frac{b^{2}\tan\left(e+f\,x\right)}{4\left(\frac{b^{2}}{4}+\frac{b^{3}}{a}\right)}+\frac{b^{3}\tan\left(e+f\,x\right)}{b^{3}+\frac{a\,b^{2}}{4}}\right)\,(a\,1\mathrm{i}+b\,4\mathrm{i})\,1\mathrm{i}}{2\,a^{3}\,f}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan(((((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^6*b^3
+ 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b
*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^
3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4) + (((tan(e +
 f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)/a^6
- (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(1/2))/
(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4))*((3*a
)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4))/((8*a*b^4 + 8*b^5 + (3*a^2*b
^3)/2)/a^6 + (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^
6*b^3 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 +
b)*(-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2
))/(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4) - (((tan(
e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)/a
^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(1/2
))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4))*((
3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^
(1/2)*2i)/(f*(a^3*b + a^4)) - (atan((b^2*tan(e + f*x))/(4*(b^2/4 + b^3/a))
+ (b^3*tan(e + f*x))/((a*b^2)/4 + b^3))*(a*1i + b*4i)*1i)/(2*a^3*f) - ((tan
```

```
(e + f*x)*(a + 2*b))/(2*a^2) + (b*tan(e + f*x)^3)/a^2)/(f*(a + b + b*tan(e
+ f*x)^4 + tan(e + f*x)^2*(a + 2*b)))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.50 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}dx=\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^2}dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)\left(a+b+bx^2\right)}dx,x,\tan(e+fx)\right)}{2a(a+b)f}$$

$$=-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}dx,x,\tan(e+fx)\right)}{a^2f}-\frac{(b(3a}{}$$

$$=\frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 1.98, size = 240, normalized size = 2.61

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(2x(a\cos(2(e+fx))+a+2b)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{f(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b(3a+2b)(c}{}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

fricas [B] time = 0.79, size = 435, normalized size = 4.73

$$\left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2-4ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\left(ab+b^2\right)fx+\left(\left(3a^2+2ab\right)\cos\left(fx+e\right)^2+3a\right.}{8\left(\left(a^4+a^3b\right)f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
```

```
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

giac [A] time = 0.23, size = 119, normalized size = 1.29

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab+2\,b^2\right)}{\left(a^3+a^2b\right)\sqrt{ab+b^2}}+\frac{b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)\left(a^2+ab\right)}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

maple [A] time = 0.90, size = 127, normalized size = 1.38

$$-\frac{b\tan\left(fx+e\right)}{2a\left(a+b\right)f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2fa\left(a+b\right)\sqrt{\left(a+b\right)b}}-\frac{b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^2\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\tan\left(fx+\right.\right.}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a/(a+b)/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^2/a^2/(a+b)/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.48, size = 106, normalized size = 1.15

$$-\frac{\frac{b\tan\left(fx+e\right)}{a^3+2\,a^2b+ab^2+\left(a^2b+ab^2\right)\tan\left(fx+e\right)^2}+\frac{\left(3\,ab+2\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\left(a^3+a^2b\right)\sqrt{\left(a+b\right)b}}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

mupad [B] time = 6.35, size = 2056, normalized size = 22.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
```

```
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

3.51 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}-\frac{3\cot(e+fx)}{2f(a+b)^2}+\frac{\cot(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -3/2*cot(f*x+e)/(a+b)^2/f-3/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2
)/(a+b)^(5/2)/f+1/2*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}-\frac{3\cot(e+fx)}{2f(a+b)^2}+\frac{\cot(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*f)
 - (3*Cot[e + f*x])/(2*(a + b)^2*f) + Cot[e + f*x]/(2*(a + b)*f*(a + b + b*
Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{3\cot(e+fx)}{2(a+b)^2 f} + \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x,\right.}{2(a+b)^2 f}$$

$$= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2} f} - \frac{3\cot(e+fx)}{2(a+b)^2 f} + \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 2.22, size = 242, normalized size = 2.66

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{a(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + 2\csc(e)\sin(fx)\csc(e+fx)(a\cos(2(e+fx))\right.}{8f(a+b)^2\left(a+b\sec^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((3*b*ArcTan[(Sec[f*x]*(Cos[
2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e]
 - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2*(a + 2*b +
a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + (b*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a + b)^2*f*(a
 + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.64, size = 407, normalized size = 4.47

$$\left[-\frac{4(2a-b)\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\right.\right.}{a^2\cos\left(fx+e\right)^4+2}\right.}{8\left(\left(a^3+2a^2b+ab^2\right)f\cos\left(fx+e\right)^2+\left(a^2b+2ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/(a + b
))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*
sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*
cos(f*x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a - b
)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/(a + b))*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*s
```

```
in(f*x + e) + 6*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2
+ (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e))]
```

giac [A] time = 0.47, size = 133, normalized size = 1.46

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^2+2\,ab+b^2)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^2 +
 2*a + 2*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^2 + 2*
a*b + b^2)))/f
```

maple [A] time = 1.05, size = 86, normalized size = 0.95

$$-\frac{b\tan\left(fx+e\right)}{2f\left(a+b\right)^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{2f\left(a+b\right)^2\sqrt{(a+b)\,b}}-\frac{1}{f\left(a+b\right)^2\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/(a+b)^2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-3/2/f*b/(a+b)^2/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/(a+b)^2/tan(f*x+e)
```

maxima [A] time = 0.47, size = 117, normalized size = 1.29

$$-\frac{\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}+\frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{(a^2b+2\,ab^2+b^3)\tan(fx+e)^3+(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt(
(a + b)*b)) + (3*b*tan(f*x + e)^2 + 2*a + 2*b)/((a^2*b + 2*a*b^2 + b^3)*tan
(f*x + e)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

mupad [B] time = 4.40, size = 91, normalized size = 1.00

$$-\frac{\frac{1}{a+b}+\frac{3\,b\tan(e+f\,x)^2}{2\,(a+b)^2}}{f\left(b\tan\left(e+f\,x\right)^3+(a+b)\,\tan\left(e+f\,x\right)\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^2+2\,a\,b+b^2)}{(a+b)^{5/2}}\right)}{2\,f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - (1/(a + b) + (3*b*tan(e + f*x)^2)/(2*(a + b)^2))/(f*(b*tan(e + f*x)^3 + t
an(e + f*x)*(a + b))) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2
+ b^2))/(a + b)^(5/2)))/(2*f*(a + b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
[Out] Timed out
```

3.52 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{b}\,(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}-\frac{ab\tan(e+fx)}{2f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}-\frac{\cot^3(e+fx)}{3f(a+b)^2}-\frac{(a-b)\cot(e+fx)}{f(a+b)^3}$$

```
[Out] -(a-b)*cot(f*x+e)/(a+b)^3/f-1/3*cot(f*x+e)^3/(a+b)^2/f-1/2*(3*a-2*b)*arctan
(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f-1/2*a*b*tan(f*x+e)/(
a+b)^3/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.17, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 456, 1261, 205}

$$-\frac{\sqrt{b}\,(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}-\frac{ab\tan(e+fx)}{2f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}-\frac{\cot^3(e+fx)}{3f(a+b)^2}-\frac{(a-b)\cot(e+fx)}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b
)^(7/2)*f) - ((a - b)*Cot[e + f*x])/((a + b)^3*f) - Cot[e + f*x]^3/(3*(a +
b)^2*f) - (a*b*Tan[e + f*x])/(2*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4\left(a+b+bx^2\right)}\,dx, x, \tan(e-\right.}{2f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int\left(-\frac{2}{b(a+b)^2x^4} - \frac{2(a-b)}{b(a+b)^3x^2} + \frac{3a-}{(a+b)^3(a-}\right.\right.}{2f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{((3a-}{}$$

$$= -\frac{(3a-2b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2} f} - \frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{}{2(a+b)^}$$

Mathematica [C] time = 6.18, size = 303, normalized size = 2.46

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-3ab\sec(2e)\sin(2fx) - 2(a+b)\cot(e)\csc^2(e+fx)(a\cos(2(e+fx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-2*(a + b)*(a + 2*b + a*Cos
[2*(e + f*x)])*Cot[e]*Csc[e + f*x]^2 + (3*(3*a - 2*b)*b*ArcTan[(Sec[f*x]*(C
os[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2
*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 4*(a - 2*b
)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + 2*(a + b)*(
a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]^3*Sin[f*x] - 3*a*b*Sec[2*
e]*Sin[2*f*x] + 3*b*(a + 2*b)*Tan[2*e]))/(24*(a + b)^3*f*(a + b*Sec[e + f*x
]^2)^2)
```

fricas [B] time = 0.63, size = 663, normalized size = 5.39

$$\left[-\frac{4\left(4a^2-11ab\right)\cos\left(fx+e\right)^5 - 8\left(3a^2-8ab+4b^2\right)\cos\left(fx+e\right)^3 + 3\left(\left(3a^2-2ab\right)\cos\left(fx+e\right)^4 - \left(3a^2-5\right.\right.}{24\left(\left(a^4+3a^3b+3a^2b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^2 - 11*a*b)*cos(f*x + e)^5 - 8*(3*a^2 - 8*a*b + 4*b^2)*cos(f
*x + e)^3 + 3*((3*a^2 - 2*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos
(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*
cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) +
b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(
3*a*b - 2*b^2)*cos(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x + e)), -1/12*(2*(4*a^2 - 11*a*b)*cos(f*x
+ e)^5 - 4*(3*a^2 - 8*a*b + 4*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - 2*a*b)*cos(
f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(b
/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(
f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b - 2*b^2)*cos(f*x + e))/(((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b
^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x
 + e))]
```

giac [A] time = 0.46, size = 192, normalized size = 1.56

$$-\frac{\frac{3\,ab\tan\left(fx+e\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab-2\,b^2\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{2\left(3\,a\tan\left(fx+e\right)^2-3\,b\tan\left(fx+e\right)^2+a+b\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\tan\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*tan(f*x + e)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*tan(f*x + e)^2
 + a + b)) + 3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))*(3*a*b - 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*
b + b^2)) + 2*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a + b)/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

maple [A] time = 1.42, size = 160, normalized size = 1.30

$$-\frac{ab\tan\left(fx+e\right)}{2\left(a+b\right)^3f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)a}{2f\left(a+b\right)^3\sqrt{\left(a+b\right)b}}+\frac{b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^3\sqrt{\left(a+b\right)b}}-\frac{1}{3f\left(a+b\right)^2\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*a*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)-3/2/f/(a+b)^3*b/((a+b)*b
)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a+1/f/(a+b)^3*b^2/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/3/f/(a+b)^2/tan(f*x+e)^3-1/f/(a+
b)^3/tan(f*x+e)*a+1/f/(a+b)^3/tan(f*x+e)*b
```

maxima [A] time = 0.55, size = 193, normalized size = 1.57

$$-\frac{\frac{3\left(3\,ab-2\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{3\left(3\,ab-2\,b^2\right)\tan\left(fx+e\right)^4+2\left(3\,a^2+ab-2\,b^2\right)\tan\left(fx+e\right)^2+2\,a^2+4\,ab+2\,b^2}{\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)\tan\left(fx+e\right)^5+\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\tan\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)) + (3*(3*a*b - 2*b^2)*tan(f*x + e)^4 +
```

```
2*(3*a^2 + a*b - 2*b^2)*tan(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)/((a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*tan(f*x + e)^3))/f
```

mupad [B] time = 5.49, size = 141, normalized size = 1.15

$$-\frac{\frac{1}{3(a+b)}+\frac{\tan\left(e+f\,x\right)^2\,(3\,a-2\,b)}{3\,(a+b)^2}+\frac{b\tan\left(e+f\,x\right)^4\,(3\,a-2\,b)}{2\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+(a+b)\,\tan\left(e+f\,x\right)^3\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)\,(3\,a-2\,b)}{2\,f\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - (1/(3*(a + b)) + (tan(e + f*x)^2*(3*a - 2*b))/(3*(a + b)^2) + (b*tan(e +
f*x)^4*(3*a - 2*b))/(2*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) + b*tan(e + f
*x)^5)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))/(a + b)^(7/2))*(3*a - 2*b))/(2*f*(a + b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=188

$$-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5f(a+b)^4}-\frac{a\sqrt{b}\,(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{9/2}}-\frac{(10$$

```
[Out] -1/5*(5*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^4/f-1/15*(10*a+3*b)*cot(f*x+e)^3/(
a+b)^3/f-1/2*a*(3*a-4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+
b)^(9/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)-1/10*b*(5*a^2+2*b^
2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.26, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 462, 456, 1261, 205}

$$-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5f(a+b)^4}-\frac{a\sqrt{b}\,(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{9/2}}-\frac{(10$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a +
 b)^(9/2)*f) - ((5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(5*(a + b)^4*f) - ((10
*a + 3*b)*Cot[e + f*x]^3)/(15*(a + b)^3*f) - Cot[e + f*x]^5/(5*(a + b)*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/(10*(a + b)^4*
f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{10a+3b+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Su}}{}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Su}}{}$$

$$= -\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f} - \frac{\cot^5(e+fx}{5(a+b)f\left(a+b+b\,\text{ta}\right.}$$

$$= -\frac{a(3a-4b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f} - \frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\,\text{c}}{15(a+}$$

Mathematica [C] time = 3.22, size = 777, normalized size = 4.13

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\csc(e)\sec(2e)\csc^5(e+fx)\left(240a^3\sin(2e-fx)-240a^3\sin(2e+fx)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((960*a*(3*a - 4*b)*b*ArcTan
[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x
]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - Csc[e]*Csc[e + f*x]^5*Sec[2*e]*(10*a*(16*a^2 + 34*a*b + 123*b^2)*Sin[f*x
] - a*(16*a^2 - 223*a*b + 1336*b^2)*Sin[3*f*x] + 240*a^3*Sin[2*e - f*x] + 6
```

```
40*a^2*b*Sin[2*e - f*x] - 1460*a*b^2*Sin[2*e - f*x] + 240*b^3*Sin[2*e - f*x
] - 240*a^3*Sin[2*e + f*x] - 715*a^2*b*Sin[2*e + f*x] + 860*a*b^2*Sin[2*e +
 f*x] - 240*b^3*Sin[2*e + f*x] + 160*a^3*Sin[4*e + f*x] + 415*a^2*b*Sin[4*e
 + f*x] + 1830*a*b^2*Sin[4*e + f*x] + 165*a^2*b*Sin[2*e + 3*f*x] - 30*a*b^2
*Sin[2*e + 3*f*x] + 120*b^3*Sin[2*e + 3*f*x] - 16*a^3*Sin[4*e + 3*f*x] + 20
8*a^2*b*Sin[4*e + 3*f*x] - 1036*a*b^2*Sin[4*e + 3*f*x] + 180*a^2*b*Sin[6*e
+ 3*f*x] - 330*a*b^2*Sin[6*e + 3*f*x] + 120*b^3*Sin[6*e + 3*f*x] + 48*a^3*S
in[2*e + 5*f*x] - 268*a^2*b*Sin[2*e + 5*f*x] + 290*a*b^2*Sin[2*e + 5*f*x] -
 24*b^3*Sin[2*e + 5*f*x] + 48*a^3*Sin[6*e + 5*f*x] - 223*a^2*b*Sin[6*e + 5*
f*x] + 230*a*b^2*Sin[6*e + 5*f*x] - 24*b^3*Sin[6*e + 5*f*x] - 45*a^2*b*Sin[
8*e + 5*f*x] + 60*a*b^2*Sin[8*e + 5*f*x] - 16*a^3*Sin[4*e + 7*f*x] + 83*a^2
*b*Sin[4*e + 7*f*x] - 6*a*b^2*Sin[4*e + 7*f*x] - 15*a^2*b*Sin[6*e + 7*f*x]
- 16*a^3*Sin[8*e + 7*f*x] + 68*a^2*b*Sin[8*e + 7*f*x] - 6*a*b^2*Sin[8*e + 7
*f*x])))/(7680*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.75, size = 987, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(40*a^3 - 201*a
^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 29*a^2*b + 28*a*b^2)*
cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11*a^2*b +
 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b + 8*a*b^2)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 4*
a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*co
s(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*
cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*
f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f
*x + e)), -1/60*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 2*(40*a^3
 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 29*a^2*b + 28
*a*b^2)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11
*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b +
8*a*b^2)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)
^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3
*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)*sin(f*x + e))]
```

giac [A] time = 0.51, size = 263, normalized size = 1.40

$$-\frac{\frac{15\,a^2 b\tan\left(fx+e\right)}{\left(a^4+4\,a^3 b+6\,a^2 b^2+4\,ab^3+b^4\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{15\left(3\,a^2 b-4\,ab^2\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^4+4\,a^3 b+6\,a^2 b^2+4\,ab^3+b^4\right)\sqrt{ab+b^2}}+\frac{2\left(15\,a^2\tan\left(fx+e\right)^4-30\,ab\,\mathrm{t}\cdots\right.}{(\cdots}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*b*tan(f*x + e)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)) + 15*(3*a^2*b - 4*a*b^2)*(pi*floor((f*x + e)/pi
+ 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 4*a^3*b + 6
```

```
*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b + b^2)) + 2*(15*a^2*tan(f*x + e)^4 - 30*
a*b*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2
+ 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
5))/f
```

maple [A] time = 1.25, size = 189, normalized size = 1.01

$$-\frac{b a^2 \tan\left(fx+e\right)}{2f\left(a+b\right)^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b a^2 \arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2f\left(a+b\right)^4\sqrt{(a+b)\,b}}+\frac{2b^2 a \arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^4\sqrt{(a+b)\,b}}-\frac{1}{5f\left(a+b\right)^2\tan\left(fx \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b*a^2/(a+b)^4*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-3/2/f*b*a^2/(a+b)^4/((
a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+2/f*b^2*a/(a+b)^4/((a+b)
*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/5/f/(a+b)^2/tan(f*x+e)^5-1
/f*a^2/(a+b)^4/tan(f*x+e)+2/f*a/(a+b)^4/tan(f*x+e)*b-2/3/f*a/(a+b)^3/tan(f*
x+e)^3
```

maxima [A] time = 0.44, size = 268, normalized size = 1.43

$$-\frac{\frac{15\left(3a^2b-4ab^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{(a+b)b}}+\frac{15\left(3a^2b-4ab^2\right)\tan\left(fx+e\right)^6+10\left(3a^3-a^2b-4ab^2\right)\tan\left(fx+e\right)^4+6a^3+18a^2b+18ab^2+6b^3+2\left(10a^3+23a^2b \cdots\right.}{\left(a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5\right)\tan\left(fx+e\right)^7+\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\tan \cdots}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(3*a^2*b - 4*a*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a^2*b - 4*
a*b^2)*tan(f*x + e)^6 + 10*(3*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^4 + 6*a^3
 + 18*a^2*b + 18*a*b^2 + 6*b^3 + 2*(10*a^3 + 23*a^2*b + 16*a*b^2 + 3*b^3)*t
an(f*x + e)^2)/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x + e
)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e
)^5))/f
```

mupad [B] time = 6.31, size = 198, normalized size = 1.05

$$\frac{a\sqrt{b}\,\mathrm{atan}\left(\frac{a\sqrt{b}\tan\left(e+fx\right)(3a-4b)\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)}{(a+b)^{9/2}\left(4ab-3a^2\right)}\right)(3a-4b)}{2f\,(a+b)^{9/2}}-\frac{\frac{1}{5(a+b)}-\frac{\tan\left(e+fx\right)^4\left(4ab-3a^2\right)}{3(a+b)^3}+\frac{\tan\left(e+fx\right)^2(10a+\cdots}{15(a+b)^2}}{f\left(b\tan\left(e+fx\right)^7+(a+b)\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (a*b^(1/2)*atan((a*b^(1/2)*tan(e + f*x)*(3*a - 4*b)*(4*a*b^3 + 4*a^3*b + a^
4 + b^4 + 6*a^2*b^2))/((a + b)^(9/2)*(4*a*b - 3*a^2)))*(3*a - 4*b))/(2*f*(a
 + b)^(9/2)) - (1/(5*(a + b)) - (tan(e + f*x)^4*(4*a*b - 3*a^2))/(3*(a + b)
^3) + (tan(e + f*x)^2*(10*a + 3*b))/(15*(a + b)^2) - (b*tan(e + f*x)^6*(4*a
*b - 3*a^2))/(2*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b) + b*tan(e + f*x)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.54 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=214

$$-\frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(a\cos^2(e+fx)+b\right)}+\frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf}-\frac{\cos^5(e+fx)}{5a^3f}-\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(a\cos^2(e+fx)+b\right)^2}+\frac{\sqrt{}}{}$$

```
[Out] -1/2*(3*a^2+14*a*b+13*b^2)*cos(f*x+e)/a^5/f+1/12*(a+3*b)*(3*a+5*b)*cos(f*x+
e)^3/a^4/b/f-1/5*cos(f*x+e)^5/a^3/f-1/4*(a+b)^2*cos(f*x+e)^7/a^2/b/f/(b+a*c
os(f*x+e)^2)^2-1/8*b*(a+b)*(3*a+11*b)*cos(f*x+e)/a^5/f/(b+a*cos(f*x+e)^2)+1
/8*(15*a^2+70*a*b+63*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(11/
2)/f
```

Rubi [A] time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4133, 463, 455, 1810, 205}

$$-\frac{\left(3a^2+14ab+13b^2\right)\cos(e+fx)}{2a^5f}+\frac{\sqrt{b}\left(15a^2+70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f}-\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(a\cos^2(e+fx)+b\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])
/(8*a^(11/2)*f) - ((3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x])/(2*a^5*f) + ((a
+ 3*b)*(3*a + 5*b)*Cos[e + f*x]^3)/(12*a^4*b*f) - Cos[e + f*x]^5/(5*a^3*f)
- ((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*f*(b + a*Cos[e + f*x]^2)^2) - (b*(a +
 b)*(3*a + 11*b)*Cos[e + f*x])/(8*a^5*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6\left(1-x^2\right)^2}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^6\left(-4a^2+7(a+b)^2-4abx^2\right)}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx\right.}{4a^2bf}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-ab^2}{\cdots}\right.}{\cdots}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(4a\right.\right.}{\cdots}$$

$$= -\frac{\left(3a^2+14ab+13b^2\right)\cos(e+fx)}{2a^5f} + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} - \frac{\cos^5(e+}{5a^3f}$$

$$= \frac{\sqrt{b}\left(15a^2+70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} - \frac{\left(3a^2+14ab+13b^2\right)\cos(e+fx}{2a^5f}$$

Mathematica [C] time = 10.56, size = 1641, normalized size = 7.67

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-900*a^(11/2)*b^(3/2)*Cos[e
 + f*x] - 109000*a^(9/2)*b^(5/2)*Cos[e + f*x] - 936000*a^(7/2)*b^(7/2)*Cos[
e + f*x] - 2803072*a^(5/2)*b^(9/2)*Cos[e + f*x] - 3763200*a^(3/2)*b^(11/2)*
Cos[e + f*x] - 1935360*Sqrt[a]*b^(13/2)*Cos[e + f*x] - 900*a^(11/2)*b^(3/2)
*Cos[e + f*x]*Cos[2*(e + f*x)] + 900*a^(9/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b
+ a*Cos[2*(e + f*x)]) + 24000*a^(7/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Cos
[2*(e + f*x)]) + 43200*a^(5/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e +
 f*x)]) + 225*a^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 11520
0*a^2*b^3*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
```

```
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 537600*a*b^4*
ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(
f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*
x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 483840*b^5*ArcTan[((-Sq
rt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 225*a^5*ArcTan[((-Sqrt[a] + I*Sqrt[
a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
 Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b +
 a*Cos[2*(e + f*x)])^2 + 115200*a^2*b^3*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2
*(e + f*x)])^2 + 537600*a*b^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)
])^2 + 483840*b^5*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 225*a
^5*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Co
s[2*(e + f*x)])^2 - 225*a^5*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 19200*a^(5/2)*b^(5/2)*Cos[e]*C
os[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 20352*a^(9/2)*b^(5/2)*Cos[e + f*
x]*Cos[4*(e + f*x)] - 115712*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[4*(e + f*x)]
- 129024*a^(5/2)*b^(9/2)*Cos[e + f*x]*Cos[4*(e + f*x)] + 2048*a^(9/2)*b^(5/
2)*Cos[e + f*x]*Cos[6*(e + f*x)] + 4608*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[6*
(e + f*x)] - 384*a^(9/2)*b^(5/2)*Cos[e + f*x]*Cos[8*(e + f*x)] - 19200*a^(5
/2)*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] - 32496*a^(9/2
)*b^(5/2)*Csc[e + f*x]*Sin[4*(e + f*x)] - 252080*a^(7/2)*b^(7/2)*Csc[e + f*
x]*Sin[4*(e + f*x)] - 577024*a^(5/2)*b^(9/2)*Csc[e + f*x]*Sin[4*(e + f*x)]
- 403200*a^(3/2)*b^(11/2)*Csc[e + f*x]*Sin[4*(e + f*x)]))/(491520*a^(11/2)*
b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [A] time = 0.68, size = 579, normalized size = 2.71

$$\left[-\frac{48\,a^4\cos\left(fx+e\right)^9-16\left(10\,a^4+9\,a^3b\right)\cos\left(fx+e\right)^7+16\left(15\,a^4+70\,a^3b+63\,a^2b^2\right)\cos\left(fx+e\right)^5+50\left(15\,a^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(48*a^4*cos(f*x + e)^9 - 16*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 16*
(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 50*(15*a^3*b + 70*a^2*b^2
 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x
+ e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b
^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(
f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)
*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f
), -1/120*(24*a^4*cos(f*x + e)^9 - 8*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 8*
(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 25*(15*a^3*b + 70*a^2*b^2
 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x
+ e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b
^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(15*a
^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f
*cos(f*x + e)^2 + a^5*b^2*f)]
```

giac [B] time = 0.58, size = 837, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqr
t(a*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^5) + 30*(9*a^3*b + 33*a^2*b^
2 + 39*a*b^3 + 15*b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 49
*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 45*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^
3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2 + 45*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3 + 11*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 15*b^4*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2*a^5) - 16*(8*a^2 + 75*a*b + 90*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 330*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 360*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 480*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 540*b^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 360*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a
*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 90*b^2*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4)/(a^5*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/
f
```

maple [A] time = 1.02, size = 374, normalized size = 1.75

$$-\frac{\cos^5(fx+e)}{5a^3f}+\frac{2(\cos^3(fx+e))}{3a^3f}+\frac{(\cos^3(fx+e))b}{fa^4}-\frac{\cos(fx+e)}{a^3f}-\frac{6b\cos(fx+e)}{fa^4}-\frac{6\cos(fx+e)b^2}{fa^5}-\frac{}{8fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/5*cos(f*x+e)^5/a^3/f+2/3*cos(f*x+e)^3/a^3/f+1/f/a^4*cos(f*x+e)^3*b-cos(f
*x+e)/a^3/f-6/f/a^4*b*cos(f*x+e)-6/f/a^5*cos(f*x+e)*b^2-9/8/f*b/a^2/(b+a*co
s(f*x+e)^2)^2*cos(f*x+e)^3-13/4/f*b^2/a^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3
-17/8/f*b^3/a^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3-7/8/f*b^2/a^3/(b+a*cos(f*
x+e)^2)^2*cos(f*x+e)-11/4/f*b^3/a^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)-15/8/f*
b^4/a^5/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)+15/8/f*b/a^3/(a*b)^(1/2)*arctan(a*c
os(f*x+e)/(a*b)^(1/2))+35/4/f*b^2/a^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)
^(1/2))+63/8/f*b^3/a^5/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

maxima [A] time = 0.43, size = 204, normalized size = 0.95

$$-\frac{\frac{15\left((9a^3b+26a^2b^2+17ab^3)\cos(fx+e)^3+(7a^2b^2+22ab^3+15b^4)\cos(fx+e)\right)}{a^7\cos(fx+e)^4+2a^6b\cos(fx+e)^2+a^5b^2}-\frac{15(15a^2b+70ab^2+63b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^5}+\frac{8(3a^2\cos(fx}{}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*((9*a^3*b + 26*a^2*b^2 + 17*a*b^3)*cos(f*x + e)^3 + (7*a^2*b^2 +
 22*a*b^3 + 15*b^4)*cos(f*x + e))/(a^7*cos(f*x + e)^4 + 2*a^6*b*cos(f*x + e
)^2 + a^5*b^2) - 15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(a*cos(f*x + e)/sq
rt(a*b))/(sqrt(a*b)*a^5) + 8*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + 3*a*b)*cos(
f*x + e)^3 + 15*(a^2 + 6*a*b + 6*b^2)*cos(f*x + e))/a^5)/f
```

mupad [B] time = 4.51, size = 255, normalized size = 1.19

$$\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{b}{a^4}+\frac{2}{3\,a^3}\right)}{f}-\frac{\left(\frac{9\,a^3\,b}{8}+\frac{13\,a^2\,b^2}{4}+\frac{17\,a\,b^3}{8}\right)\,\cos\left(e+f\,x\right)^3+\left(\frac{7\,a^2\,b^2}{8}+\frac{11\,a\,b^3}{4}+\frac{15\,b^4}{8}\right)\,\cos\left(e+f\,x\right)}{f\,\left(a^7\,\cos\left(e+f\,x\right)^4+2\,a^6\,b\cos\left(e+f\,x\right)^2+a^5\,b^2\right)}-\frac{\cos\left(e\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (cos(e + f*x)^3*(b/a^4 + 2/(3*a^3)))/f - (cos(e + f*x)^3*((17*a*b^3)/8 + (9
*a^3*b)/8 + (13*a^2*b^2)/4) + cos(e + f*x)*((11*a*b^3)/4 + (15*b^4)/8 + (7*
a^2*b^2)/8))/(f*(a^5*b^2 + a^7*cos(e + f*x)^4 + 2*a^6*b*cos(e + f*x)^2)) -
cos(e + f*x)^5/(5*a^3*f) - (cos(e + f*x)*(1/a^3 - (3*b^2)/a^5 + (3*b*((3*b)
/a^4 + 2/a^3))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(70*a*b
+ 15*a^2 + 63*b^2))/(70*a*b^2 + 15*a^2*b + 63*b^3))*(70*a*b + 15*a^2 + 63*b
^2))/(8*a^(11/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.55 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f}+\frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+3b)\cos(e+f}{a^4f}$$

```
[Out] -(a+3*b)*cos(f*x+e)/a^4/f+1/3*cos(f*x+e)^3/a^3/f+1/4*b^2*(a+b)*cos(f*x+e)/a
^4/f/(b+a*cos(f*x+e)^2)^2-1/8*b*(9*a+13*b)*cos(f*x+e)/a^4/f/(b+a*cos(f*x+e)
^2)+5/8*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/f
```

Rubi [A] time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 455, 1814, 1153, 205}

$$\frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+3b)\cos(e+fx)}{a^4f}+\frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx}{\sqrt{b}}\right.}{8a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(9/2)*f
) - ((a + 3*b)*Cos[e + f*x])/(a^4*f) + Cos[e + f*x]^3/(3*a^3*f) + (b^2*(a +
 b)*Cos[e + f*x])/(4*a^4*f*(b + a*Cos[e + f*x]^2)^2) - (b*(9*a + 13*b)*Cos[
e + f*x])/(8*a^4*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{\left(b+ax^2\right)^2}\,dx, x, \cos(\right.}{4a^4f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-b^2(7a+11b)+8ab(}{b+a}\right.}{}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(8b(a+3b) - \right.\right.}{}$$

$$= -\frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b}{8a^4f\left(b+a\right.}$$

$$= \frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(}{4a^4f\left(\right.}$$

Mathematica [C] time = 9.78, size = 1153, normalized size = 7.49

$$\frac{(\cos(2(e+fx))a+a+2b)^3\sec^6(e+fx)\left(3\left(9a^4+1920b^3a+4480b^4\right)\tan^{-1}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan(}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(3*(9*a^4 + 1920*a*b^3 + 4
480*b^4)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin
[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2
]*Tan[(f*x)/2]))/Sqrt[b]] - (-27*a^(11/2)*Sqrt[b]*Cos[e + f*x] + 162*a^(9/2
)*b^(3/2)*Cos[e + f*x] + 10816*a^(7/2)*b^(5/2)*Cos[e + f*x] + 51552*a^(5/2)
*b^(7/2)*Cos[e + f*x] + 87424*a^(3/2)*b^(9/2)*Cos[e + f*x] + 53760*Sqrt[a]*
b^(11/2)*Cos[e + f*x] - 27*a^(11/2)*Sqrt[b]*Cos[e + f*x]*Cos[2*(e + f*x)] +
```

```
47936*a^(5/2)*b^(7/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 44800*a^(3/2)*b^(9/2
)*Cos[e + f*x]*Cos[2*(e + f*x)] + 27*a^(9/2)*Sqrt[b]*Cos[e + f*x]*(a + 2*b
+ a*Cos[2*(e + f*x)]) - 216*a^(7/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2
*(e + f*x)]) - 3600*a^(5/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*
x)]) - 5184*a^(3/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2
7*a^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]
*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*T
an[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 5760*a*b^3*ArcTan
[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2
] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2])
)/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 13440*b^4*ArcTan[((-Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(S
qrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a
 + 2*b + a*Cos[2*(e + f*x)])^2 + 27*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(
e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 27*a^4*ArcTan[(Sqr
t[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]
)^2 - 2304*a^(3/2)*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2
 + 1920*a^(7/2)*b^(5/2)*Cos[e + f*x]*Cos[4*(e + f*x)] + 3584*a^(5/2)*b^(7/2
)*Cos[e + f*x]*Cos[4*(e + f*x)] - 128*a^(7/2)*b^(5/2)*Cos[e + f*x]*Cos[6*(e
 + f*x)] + 2304*a^(3/2)*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin
[f*x] + 54*a^(9/2)*b^(3/2)*Csc[e + f*x]*Sin[4*(e + f*x)] + 3108*a^(7/2)*b^(
5/2)*Csc[e + f*x]*Sin[4*(e + f*x)])/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(245
76*a^(9/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [**A**] time = 0.68, size = 439, normalized size = 2.85

$$\left[\frac{16\,a^3\cos\left(fx+e\right)^7 - 16\left(3\,a^3+7\,a^2b\right)\cos\left(fx+e\right)^5 - 50\left(3\,a^2b+7\,ab^2\right)\cos\left(fx+e\right)^3 + 15\left(\left(3\,a^3+7\,a^2b\right)\cos \cdots\right.}{48\left(a^6f\cos\left(f \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*cos(f*x + e)^7 - 16*(3*a^3 + 7*a^2*b)*cos(f*x + e)^5 - 50*(3*
a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*cos(f*x + e)^4 + 3*
a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*co
s(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 3
0*(3*a*b^2 + 7*b^3)*cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x
 + e)^2 + a^4*b^2*f), 1/24*(8*a^3*cos(f*x + e)^7 - 8*(3*a^3 + 7*a^2*b)*cos(
f*x + e)^5 - 25*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*
cos(f*x + e)^4 + 3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sq
rt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 15*(3*a*b^2 + 7*b^3)*cos(f*x +
 e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 + a^4*b^2*f)]
```

giac [**A**] time = 0.50, size = 183, normalized size = 1.19

$$\frac{5\left(3\,ab+7\,b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4f} - \frac{\frac{9\,a^2b\cos\left(fx+e\right)^3}{f} + \frac{13\,ab^2\cos\left(fx+e\right)^3}{f} + \frac{7\,ab^2\cos\left(fx+e\right)}{f} + \frac{11\,b^3\cos\left(fx+e\right)}{f}}{8\left(a\cos\left(fx+e\right)^2+b\right)^2a^4} + \frac{a^6f^{17}\cos\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(3*a*b + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) - 1/
8*(9*a^2*b*cos(f*x + e)^3/f + 13*a*b^2*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x +
 e)/f + 11*b^3*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)^2*a^4) + 1/3*(a^6*f^
```

```
17*cos(f*x + e)^3 - 3*a^6*f^17*cos(f*x + e) - 9*a^5*b*f^17*cos(f*x + e))/(a
^9*f^18)
```

maple [A] time = 0.94, size = 231, normalized size = 1.50

$$\frac{\cos^3(fx+e)}{3a^3f}-\frac{\cos(fx+e)}{a^3f}-\frac{3b\cos(fx+e)}{f\,a^4}-\frac{9b\left(\cos^3(fx+e)\right)}{8f\,a^2\left(b+a\left(\cos^2(fx+e)\right)\right)^2}-\frac{13b^2\left(\cos^3(fx+e)\right)}{8f\,a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2}-\frac{}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/3*cos(f*x+e)^3/a^3/f-cos(f*x+e)/a^3/f-3/f/a^4*b*cos(f*x+e)-9/8/f*b/a^2/(b
+a*cos(f*x+e)^2)^2*cos(f*x+e)^3-13/8/f*b^2/a^3/(b+a*cos(f*x+e)^2)^2*cos(f*x
+e)^3-7/8/f*b^2/a^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)-11/8/f*b^3/a^4/(b+a*cos
(f*x+e)^2)^2*cos(f*x+e)+15/8/f*b/a^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^
(1/2))+35/8/f*b^2/a^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

maxima [A] time = 0.45, size = 149, normalized size = 0.97

$$-\frac{\frac{3\left(\left(9\,a^2b+13\,ab^2\right)\cos\left(fx+e\right)^3+\left(7\,ab^2+11\,b^3\right)\cos\left(fx+e\right)\right)}{a^6\cos\left(fx+e\right)^4+2\,a^5b\cos\left(fx+e\right)^2+a^4b^2}-\frac{15\left(3\,ab+7\,b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}-\frac{8\left(a\cos\left(fx+e\right)^3-3\,(a+3\,b)\cos\left(fx+e\right)\right)}{a^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*((9*a^2*b + 13*a*b^2)*cos(f*x + e)^3 + (7*a*b^2 + 11*b^3)*cos(f*x
+ e))/(a^6*cos(f*x + e)^4 + 2*a^5*b*cos(f*x + e)^2 + a^4*b^2) - 15*(3*a*b +
 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4) - 8*(a*cos(f*x + e
)^3 - 3*(a + 3*b)*cos(f*x + e))/a^4)/f
```

mupad [B] time = 0.17, size = 172, normalized size = 1.12

$$\frac{\cos\left(e+f\,x\right)^3}{3\,a^3\,f}-\frac{\left(\frac{9\,a^2\,b}{8}+\frac{13\,a\,b^2}{8}\right)\cos\left(e+f\,x\right)^3+\left(\frac{11\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)\cos\left(e+f\,x\right)}{f\,\left(a^6\cos\left(e+f\,x\right)^4+2\,a^5\,b\cos\left(e+f\,x\right)^2+a^4\,b^2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{3\,b}{a^4}+\frac{1}{a^3}\right)}{f}+\frac{5\,\sqrt{b}\,\mathrm{atar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] cos(e + f*x)^3/(3*a^3*f) - (cos(e + f*x)^3*((13*a*b^2)/8 + (9*a^2*b)/8) + c
os(e + f*x)*((7*a*b^2)/8 + (11*b^3)/8))/(f*(a^4*b^2 + a^6*cos(e + f*x)^4 +
2*a^5*b*cos(e + f*x)^2)) - (cos(e + f*x)*((3*b)/a^4 + 1/a^3))/f + (5*b^(1/2
)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 7*b))/(3*a*b + 7*b^2))*(3*a + 7
*b))/(8*a^(9/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=116

$$\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f}-\frac{15\cos(e+fx)}{8a^3f}+\frac{5\cos^3(e+fx)}{8a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{\cos^5(e+fx)}{4af\left(a\cos^2(e+fx)+b\right)^2}$$

```
[Out] -15/8*cos(f*x+e)/a^3/f+1/4*cos(f*x+e)^5/a/f/(b+a*cos(f*x+e)^2)^2+5/8*cos(f*
x+e)^3/a^2/f/(b+a*cos(f*x+e)^2)+15/8*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(
1/2)/a^(7/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 288, 321, 205}

$$\frac{5\cos^3(e+fx)}{8a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f}-\frac{15\cos(e+fx)}{8a^3f}+\frac{\cos^5(e+fx)}{4af\left(a\cos^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(7/2)*f) - (15*Cos
[e + f*x])/(8*a^3*f) + Cos[e + f*x]^5/(4*a*f*(b + a*Cos[e + f*x]^2)^2) + (5
*Cos[e + f*x]^3)/(8*a^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{4af}$$

$$= \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \text{c}\right.}{8a^2f}$$

$$= -\frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)} + \frac{(15b)}{}$$

$$= \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5}{8a^2f\left(b\right.}$$

Mathematica [C] time = 5.74, size = 656, normalized size = 5.66

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(15\left(a^3+64b^3\right)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{}\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(15*(a^3 + 64*b^3)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]] + 15*(a^3 + 64*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (Sqrt[a]*(24*a^4*Sqrt[b]*C
os[e + f*x] - 24*a^3*b^(3/2)*Cos[e + f*x] - 144*a^2*b^(5/2)*Cos[e + f*x] +
512*b^(9/2)*Cos[e + f*x] - 72*a^3*b^(3/2)*Cos[e + f*x]*Cos[2*(e + f*x)] - 2
4*a^3*Sqrt[b]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) + 72*a^2*b^(3/2)*
Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 1152*b^(7/2)*Cos[e + f*x]*(a
+ 2*b + a*Cos[2*(e + f*x)]) - 15*a^(5/2)*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[
(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 15*a^(5/2)*ArcTan
[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e +
f*x)])^2 - 512*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 5
12*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] + 6*a^4*Sqrt[b]
*Csc[e + f*x]*Sin[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4096*a
^(7/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [A] time = 0.65, size = 299, normalized size = 2.58

$$\left[-\frac{16\,a^2\cos\left(fx+e\right)^5 + 50\,ab\cos\left(fx+e\right)^3 + 30\,b^2\cos\left(fx+e\right) - 15\left(a^2\cos\left(fx+e\right)^4 + 2\,ab\cos\left(fx+e\right)^2 + b^2\right)}{16\left(a^5f\cos\left(fx+e\right)^4 + 2\,a^4bf\cos\left(fx+e\right)^2 + a^3b^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^2*cos(f*x + e)^5 + 50*a*b*cos(f*x + e)^3 + 30*b^2*cos(f*x + e)
 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/a)*log(-(a*
cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)))/
(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), -1/8*(8*a^2*
cos(f*x + e)^5 + 25*a*b*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) - 15*(a^2*cos(
f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f
*x + e)/b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

giac [A] time = 0.40, size = 97, normalized size = 0.84

$$\frac{15\,b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3f}-\frac{\cos\left(fx+e\right)}{a^3f}-\frac{\frac{9\,ab\cos\left(fx+e\right)^3}{f}+\frac{7\,b^2\cos\left(fx+e\right)}{f}}{8\left(a\cos\left(fx+e\right)^2+b\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 15/8*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - cos(f*x + e)/(a
^3*f) - 1/8*(9*a*b*cos(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a*cos(f*x + e
)^2 + b)^2*a^3)
```

maple [A] time = 0.58, size = 108, normalized size = 0.93

$$-\frac{7b^2\left(\sec^3\left(fx+e\right)\right)}{8f\,a^3\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^2}-\frac{9b\sec\left(fx+e\right)}{8f\,a^2\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^2}-\frac{15b\arctan\left(\frac{\sec\left(fx+e\right)b}{\sqrt{ab}}\right)}{8f\,a^3\sqrt{ab}}-\frac{1}{f\,a^3\sec\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a^3*b^2/(a+b*sec(f*x+e)^2)^2*sec(f*x+e)^3-9/8/f/a^2*b/(a+b*sec(f*x+e
)^2)^2*sec(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2))
-1/f/a^3/sec(f*x+e)
```

maxima [A] time = 0.45, size = 103, normalized size = 0.89

$$-\frac{\frac{9\,ab\cos\left(fx+e\right)^3+7\,b^2\cos\left(fx+e\right)}{a^5\cos\left(fx+e\right)^4+2\,a^4b\cos\left(fx+e\right)^2+a^3b^2}-\frac{15\,b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{8\cos\left(fx+e\right)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((9*a*b*cos(f*x + e)^3 + 7*b^2*cos(f*x + e))/(a^5*cos(f*x + e)^4 + 2*a
^4*b*cos(f*x + e)^2 + a^3*b^2) - 15*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqr
t(a*b)*a^3) + 8*cos(f*x + e)/a^3)/f
```

mupad [B] time = 0.15, size = 105, normalized size = 0.91

$$\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos\left(e+f\,x\right)}{\sqrt{b}}\right)}{8\,a^{7/2}\,f}-\frac{\frac{7\,b^2\,\cos\left(e+f\,x\right)}{8}+\frac{9\,a\,b\cos\left(e+f\,x\right)^3}{8}}{f\,\left(a^5\cos\left(e+f\,x\right)^4+2\,a^4\,b\cos\left(e+f\,x\right)^2+a^3\,b^2\right)}-\frac{\cos\left(e+f\,x\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (15*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(8*a^(7/2)*f) - ((7*b^2*c
os(e + f*x))/8 + (9*a*b*cos(e + f*x)^3)/8)/(f*(a^3*b^2 + a^5*cos(e + f*x)^4
 + 2*a^4*b*cos(e + f*x)^2)) - cos(e + f*x)/(a^3*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{b(7a+3b)\cos(e+fx)}{8a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}f(a+b)^3}-\frac{b\cos^3(e+fx)}{4af(a+b)\left(a\cos^2(e+fx)\right.}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)^3/f-1/4*b*cos(f*x+e)^3/a/(a+b)/f/(b+a*cos(f*x+e)
^2)^2-1/8*b*(7*a+3*b)*cos(f*x+e)/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)+1/8*(15*a
^2+10*a*b+3*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/(a+b)^3
/f
```

Rubi [A] time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4133, 470, 578, 522, 206, 205}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}f(a+b)^3}-\frac{b(7a+3b)\cos(e+fx)}{8a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b\cos^3(e+fx)}{4af(a+b)\left(a\cos^2(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/
(8*a^(5/2)*(a + b)^3*f) - ArcTanh[Cos[e + f*x]]/((a + b)^3*f) - (b*Cos[e +
f*x]^3)/(4*a*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) - (b*(7*a + 3*b)*Cos[e + f
*x])/(8*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^3f} - \frac{b\,c}{4a(a+b)f\,(}$$

Mathematica [C] time = 2.40, size = 447, normalized size = 2.90

$$\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(9a+5b)(a+b)(a\cos(2(e+fx))+a+2b)}{a^2} + \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\sec(e+fx)(a\cos}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a^2 - (2*
b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a^2 + (Sqrt[b]*(15*a^
2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
```

```
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e
 + f*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] + I
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqr
t[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a +
 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(5/2) - 8*(a + 2*b + a*Cos[2*(
e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 8*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x]))/(64*(a + b)^3*f*(a + b*Sec[e
 + f*x]^2)^3)
```

fricas [B] time = 0.81, size = 779, normalized size = 5.06

$$\left[-\frac{2\left(9\,a^3b+14\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^3-\left(\left(15\,a^4+10\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+15\,a^2b^2+10\,ab^3+3\,b^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 10*a
^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15*a^
3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)
^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^
2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 8*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*
x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2) - 8*(a^4*cos(f*x + e)^4 + 2
*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^
5)*f), -1/8*((9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 1
0*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15
*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)
*cos(f*x + e)/b) + (7*a^2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 4*(a^4*cos
(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2)
 - 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2
*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(1/(4*a^3+12*a^2*b+12*a*b^2+4*b^3)*ln(abs(1-cos(f*x+exp(1)))/ab
s(1+cos(f*x+exp(1))))+(-15*a^2*b-10*a*b^2-3*b^3)*1/4/(4*a^5+12*a^4*b+12*a^3
*b^2+4*a^2*b^3)/sqrt(a*b)*atan((-a*cos(f*x+exp(1))+b)/(sqrt(a*b)*cos(f*x+ex
p(1))+sqrt(a*b)))+(9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^3*b-((1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^2*b^2-13*((1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1))))^3*a*b^3-3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^4
-27*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3*b+9*((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^2*a^2*b^2-21*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
))^2*a*b^3-9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^4+27*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*a^3*b+13*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
```

```
))*a^2*b^2-23*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b^3-9*(1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))*b^4-9*a^3*b-21*a^2*b^2-15*a*b^3-3*b^4)/(8*a^5+24
*a^4*b+24*a^3*b^2+8*a^2*b^3)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a
+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*(1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a+b)^2)
```

maple [B] time = 0.93, size = 352, normalized size = 2.29

$$-\frac{9ba\left(\cos^3\left(fx+e\right)\right)}{8f\left(a+b\right)^3\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{7b^2\left(\cos^3\left(fx+e\right)\right)}{4f\left(a+b\right)^3\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{5b^3\left(\cos^3\left(fx+e\right)\right)}{8f\left(a+b\right)^3\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -9/8/f*b/(a+b)^3/(b+a*cos(f*x+e)^2)^2*a*cos(f*x+e)^3-7/4/f*b^2/(a+b)^3/(b+a
*cos(f*x+e)^2)^2*cos(f*x+e)^3-5/8/f*b^3/(a+b)^3/(b+a*cos(f*x+e)^2)^2/a*cos(
f*x+e)^3-7/8/f*b^2/(a+b)^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)-5/4/f*b^3/(a+b)^
3/(b+a*cos(f*x+e)^2)^2/a*cos(f*x+e)-3/8/f*b^4/(a+b)^3/(b+a*cos(f*x+e)^2)^2/
a^2*cos(f*x+e)+15/8/f*b/(a+b)^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)
)+5/4/f*b^2/(a+b)^3/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+3/8/f*b^
3/(a+b)^3/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/2/f/(a+b)^3*ln
(-1+cos(f*x+e))-1/2/f/(a+b)^3*ln(1+cos(f*x+e))
```

maxima [A] time = 0.43, size = 261, normalized size = 1.69

$$\frac{\frac{\left(15\,a^2b+10\,ab^2+3\,b^3\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{ab}}-\frac{\left(9\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^3+\left(7\,ab^2+3\,b^3\right)\cos\left(fx+e\right)}{a^4b^2+2\,a^3b^3+a^2b^4+\left(a^6+2\,a^5b+a^4b^2\right)\cos\left(fx+e\right)^4+2\,\left(a^5b+2\,a^4b^2+a^3b^3\right)\cos\left(fx+e\right)^2}-\frac{4\,\log\left(\cos\left(fx+e\right)\right.}{a^3+3\,a^2b+3\,ab^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)) - ((9*a^2*b + 5*a*b^2)*cos(f*x +
 e)^3 + (7*a*b^2 + 3*b^3)*cos(f*x + e))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + (a
^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^4 + 2*(a^5*b + 2*a^4*b^2 + a^3*b^3)*co
s(f*x + e)^2) - 4*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 4
*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/f
```

mupad [B] time = 8.60, size = 3557, normalized size = 23.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*
b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3
+ 6*a^5*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5
280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^
9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e +
 f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 25
6*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 230
4*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^
3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/
(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2)*1i)/(
```

16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2)*1i)/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))/((51*a*b^4 + 120*a^4*b + 9*b^5 + 139*a^2*b^3 + 185*a^3*b^2)/(32*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - ((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*1i)/(8*f*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) - ((cos(e + f*x)^3*(9*a*b + 5*b^2))/(8*a*(2*a*b + a^2 + b^2)) + (b*cos(e + f*x)*(7*a*b + 3*b^2))/(8*a^2*(2*a*b + a^2 + b^2)))/(f*(b^2 + a^2*cos(e + f*x)^4 + 2*a*b*cos(e + f*x)^2)) - (atan((((((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) + (cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*1i)/(2*(a + b)^3) - ((((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*1i)/(2*(a + b)^3))/((((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) + (cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 +

```
 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4
*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (51*a*b^4 + 120*a^4*b + 9*b^5 + 139
*a^2*b^3 + 185*a^3*b^2)/(32*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b
^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 +
 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/
(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7
*b^2)) + (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6
 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)
^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (cos
(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*
b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3)
))*1i)/(f*(a + b)^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

3.58 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=213

$$\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8af(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}f(a+b)^4}-\frac{b(2a-b)\cos(e+fx)}{4af(a+b)^2\left(a\cos^2(e+fx)+b\right.}$$

```
[Out] -1/2*(a-5*b)*arctanh(cos(f*x+e))/(a+b)^4/f-1/4*(2*a-b)*b*cos(f*x+e)/a/(a+b)
^2/f/(b+a*cos(f*x+e)^2)^2+1/8*(4*a^2-9*a*b-b^2)*cos(f*x+e)/a/(a+b)^3/f/(b+a
*cos(f*x+e)^2)-1/2*cos(f*x+e)*cot(f*x+e)^2/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/8
*(15*a^2-10*a*b-b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+
b)^4/f
```

Rubi [A] time = 0.31, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8af(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}f(a+b)^4}-\frac{b(2a-b)\cos(e+fx)}{4af(a+b)^2\left(a\cos^2(e+fx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8
*a^(3/2)*(a + b)^4*f) - ((a - 5*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^4*f) -
 ((2*a - b)*b*Cos[e + f*x])/(4*a*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) + ((
4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(8*a*(a + b)^3*f*(b + a*Cos[e + f*x]^2))
 - (Cos[e + f*x]*Cot[e + f*x]^2)/(2*(a + b)*f*(b + a*Cos[e + f*x]^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b+(-a+2b)x^2\right)}{\left(1-x^2\right)\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e}{2(a+b)f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e}{2(a+b)f}$$

$$= \frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f} - \frac{(a-5b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^4f} - \frac{}{4a(a}$$

Mathematica [C] time = 3.57, size = 532, normalized size = 2.50

$$\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\frac{\sqrt{b}\left(-15a^2+10ab+b^2\right)\sec(e+fx)(a\cos(2(e+fx))+a+2b)^2\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\right.\cdots}{\cdots}\right.\cdots}{a^{3/2}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a - (2*b*
(a + b)*(9*a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a - (Sqrt[b]*(-15*a^2 + 1
0*a*b + b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]
)/a^(3/2) - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (a + b)*(a + 2*b + a*Cos[2*(e
+ f*x)])^2*Csc[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 5*b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 5*b)*(a + 2*b +
a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(64*(a + b)^4*f*
(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.88, size = 1332, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + 2*(17*a^3*
b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 - ((15*a^4 - 10*a^3*b - a^2*
b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4
)*cos(f*x + e)^2)*sqrt(-b/a)*log((a*cos(f*x + e)^2 - 2*a*sqrt(-b/a)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 + 10*a*b^3 - b^4)*cos(f*
x + e) - 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*c
os(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 -
 (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b
 - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*
a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (
2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x
+ e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/8*((4*
a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a^3*b + 11*a^2*b^2
 - 5*a*b^3 + b^4)*cos(f*x + e)^3 + ((15*a^4 - 10*a^3*b - a^2*b^2)*cos(f*x +
 e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 15*a^2*
b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)
^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + (11*a^2*b^2 + 10*a*b^3 -
 b^4)*cos(f*x + e) - 2*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 1
0*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a
*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 - 5*a^3*b)*cos(
f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^
```

```
3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
+ 1/2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*
x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6
)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*
f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((10*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*a-b-a)/(16*b^4+64*b^3*a+96*b^2*a^2+64*b*a^3+16*a^
4)/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+(((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^3*b^4-13*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^3*a-5*((1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^2*a^2+9*((1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1))))^3*b*a^3+3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^4
-29*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^3*a+21*((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^2*b^2*a^2-27*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
)))^2*b*a^3+3*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^4-23*(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*b^3*a+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2
*a^2+27*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a^3+b^4-7*b^3*a-17*b^2*a^
2-9*b*a^3)/(8*b^4*a+32*b^3*a^2+48*b^2*a^3+32*b*a^4+8*a^5)/(((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a
+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))*a+b+a)^2+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))/(16*b^3+48*b^2
*a+48*b*a^2+16*a^3)+(-5*b+a)/(8*b^4+32*b^3*a+48*b^2*a^2+32*b*a^3+8*a^4)*ln(
abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(b^3+10*b^2*a-15*b*a^2)*1/4/
(4*b^4*a+16*b^3*a^2+24*b^2*a^3+16*b*a^4+4*a^5)/sqrt(a*b)*atan((-a*cos(f*x+e
xp(1))+b)/(sqrt(a*b)*cos(f*x+exp(1))+sqrt(a*b))))
```

maple [B] time = 1.16, size = 430, normalized size = 2.02

$$-\frac{9b\left(\cos^3\left(fx+e\right)\right)a^2}{8f\left(a+b\right)^4\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{5b^2\left(\cos^3\left(fx+e\right)\right)a}{4f\left(a+b\right)^4\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{b^3\left(\cos^3\left(fx+e\right)\right)}{8f\left(a+b\right)^4\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -9/8/f*b/(a+b)^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a^2-5/4/f*b^2/(a+b)^4/(b
+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a-1/8/f*b^3/(a+b)^4/(b+a*cos(f*x+e)^2)^2*co
s(f*x+e)^3-7/8/f*b^2/(a+b)^4/(b+a*cos(f*x+e)^2)^2*a*cos(f*x+e)-3/4/f*b^3/(a
+b)^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)+1/8/f*b^4/(a+b)^4/(b+a*cos(f*x+e)^2)^
2/a*cos(f*x+e)+15/8/f*b/(a+b)^4*a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/
2))-5/4/f*b^2/(a+b)^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-1/8/f*b^
3/(a+b)^4/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/4/f/(a+b)^3/(-1+
cos(f*x+e))+1/4/f/(a+b)^4*ln(-1+cos(f*x+e))*a-5/4/f/(a+b)^4*ln(-1+cos(f*x+e
))*b+1/4/f/(a+b)^3/(1+cos(f*x+e))-1/4/f/(a+b)^4*ln(1+cos(f*x+e))*a+5/4/f/(a
+b)^4*ln(1+cos(f*x+e))*b
```

maxima [B] time = 0.44, size = 399, normalized size = 1.87

$$-\frac{\frac{2\,(a-5\,b)\log\left(\cos\left(fx+e\right)+1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\,(a-5\,b)\log\left(\cos\left(fx+e\right)-1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(15\,a^2b-10\,ab^2-b^3\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\sqrt{ab}}-\frac{(4}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\cos\left(fx+e\right)^6}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(a - 5*b)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) - 2*(a - 5*b)*log(cos(f*x + e) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(a*cos(f*x + e)/sqrt(a*b)
)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((4*a^3 - 9
*a^2*b - a*b^2)*cos(f*x + e)^5 + (17*a^2*b - 6*a*b^2 + b^3)*cos(f*x + e)^3
+ (11*a*b^2 - b^3)*cos(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos
(f*x + e)^6 - a^4*b^2 - 3*a^3*b^3 - 3*a^2*b^4 - a*b^5 - (a^6 + a^5*b - 3*a^
4*b^2 - 5*a^3*b^3 - 2*a^2*b^4)*cos(f*x + e)^4 - (2*a^5*b + 5*a^4*b^2 + 3*a^
3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^2))/f
```

mupad [B] time = 6.92, size = 2728, normalized size = 12.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - ((cos(e + f*x)^3*(17*a^2*b - 6*a*b^2 + b^3))/(8*(a*b^3 + 3*a^3*b + a^4 +
3*a^2*b^2)) - (cos(e + f*x)^5*(9*a*b - 4*a^2 + b^2))/(8*(3*a*b^2 + 3*a^2*b
+ a^3 + b^3)) + (b^2*cos(e + f*x)*(11*a - b))/(8*(a*b^3 + 3*a^3*b + a^4 + 3
*a^2*b^2)))/(f*(b^2 - cos(e + f*x)^4*(2*a*b - a^2) + cos(e + f*x)^2*(2*a*b
- b^2) - a^2*cos(e + f*x)^6)) - (log(cos(e + f*x) - 1)*((3*b)/(2*(a + b)^4)
 - 1/(4*(a + b)^3)))/f - (log(cos(e + f*x) + 1)*(a - 5*b))/(4*f*(a + b)^4)
- (atan(((((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4
- 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a
^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2
*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7
*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10
 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7
*b^3 + 36*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(
1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a
^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*
(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 +
6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2)
)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(1
0*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*
b^2)) + (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 -
 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^
3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) - ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*
b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*
b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10
+ 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*
b^3 + 36*a^8*b^2) + (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1
792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^
6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(
4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6
*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))
/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10
*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b
```

```
^2)))/(((47*a*b^5)/32 - (15*a^5*b)/16 + (5*b^6)/64 + (21*a^2*b^4)/4 - (473*
a^3*b^3)/32 + (475*a^4*b^2)/64)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8 + 36*a^
3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^8*b^2) +
 (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^
3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 +
 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2
 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 2
94*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2
*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 3
6*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^1
1*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 -
 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b
 + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^
5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4
*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b -
15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((
cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^3*b^
3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*
a^4*b^3 + 15*a^5*b^2)) - ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2 + (
3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 294*a
^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8
 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^
8*b^2) + (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^11*b
+ 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 - 358
4*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b + a
^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 +
15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6
*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b - 15*a
^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2))))*(-a^3*b
)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(8*f*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b
^3 + 6*a^5*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=257

$$\frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}f(a+b)^5}$$

```
[Out] -3/8*(a^2-10*a*b+5*b^2)*arctanh(cos(f*x+e))/(a+b)^5/f+1/8*(a^2-9*a*b+2*b^2)
*cos(f*x+e)/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+3/8*(a^2-6*a*b+b^2)*cos(f*x+e)/(
a+b)^4/f/(b+a*cos(f*x+e)^2)-1/8*(a-7*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(b+
a*cos(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)^2+
3/8*(5*a^2-10*a*b+b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^5/f
/a^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$\frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}f(a+b)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(
8*Sqrt[a]*(a + b)^5*f) - (3*(a^2 - 10*a*b + 5*b^2)*ArcTanh[Cos[e + f*x]])/(
8*(a + b)^5*f) + ((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a
*Cos[e + f*x]^2)^2) + (3*(a^2 - 6*a*b + b^2)*Cos[e + f*x])/(8*(a + b)^4*f*(
b + a*Cos[e + f*x]^2)) - ((a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2
*f*(b + a*Cos[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*(
b + a*Cos[e + f*x]^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3\left(b+ax^2\right)^3}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b+(-a+4b)x^2\right)}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx,x,\cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(}{4(a+b)f}$$

$$= \frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\,\mathrm{c}}{8(a+b)^2f}$$

$$= \frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\,\mathrm{c}}{8(a+b)^2f}$$

$$= \frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}\,(a+b)^5f} - \frac{3\left(a^2-10ab+5b^2\right)\tanh^{-1}(\cos(e}{8(a+b)^5f}$$

Mathematica [C] time = 5.10, size = 549, normalized size = 2.14

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(-48\left(a^2-10ab+5b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)(a\cos(2(e+fx))+a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((48*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] + (48*Sqrt[b]*(5*a^2 -
10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] -
2*(a + b)*(30*a^3 + 112*a^2*b + 182*a*b^2 - 140*b^3 + (35*a^3 + 78*a^2*b -
93*a*b^2 + 224*b^3)*Cos[2*(e + f*x)] + 2*(a^3 - 8*a^2*b + 53*a*b^2 - 10*b^3
)*Cos[4*(e + f*x)] - 3*a^3*Cos[6*(e + f*x)] + 18*a^2*b*Cos[6*(e + f*x)] - 3
*a*b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 48*(a^2 - 10*a*b + 5
*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]] + 48*(a^2 - 10
*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]])*Sec[e
 + f*x]^6)/(1024*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 1.00, size = 1833, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26
*a^3*b + 26*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a
*b^3 + 19*b^4)*cos(f*x + e)^3 + 3*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e
)^8 - 2*(5*a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 3
0*a^3*b + 46*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^
3 + b^4 + 2*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-b
/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10*a^3*b + 5*a^2*b
^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)
^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*
b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x
+ e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^6 + (a^4
 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*b^2 - 10*
a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x + e)^2)*l
og(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*
b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a
^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b
+ 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^
5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/16*(6*(a
^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26*a^3*b + 26
*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a*b^3 + 19*b
^4)*cos(f*x + e)^3 + 6*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e)^8 - 2*(5*
a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 30*a^3*b + 4
6*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^3 + b^4 + 2
*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a
*sqrt(b/a)*cos(f*x + e)/b) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10
*a^3*b + 5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b
^3)*cos(f*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f
*x + e)^4 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 -
 5*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b +
5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f
*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4
 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*c
os(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*
a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b -
9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x +
e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(
f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7
)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
```

```
*pi/x/2)2/f*((32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^3+96*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2*a+96*((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1))))^2*b*a^2+32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3-512
*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3-768*(1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))*b^2*a+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3)/(4096*
b^6+24576*b^5*a+61440*b^4*a^2+81920*b^3*a^3+61440*b^2*a^4+24576*b*a^5+4096*
a^6)+(-30*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*b^4+84*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^6*b^2*a^2+48*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^6*b*a^3-6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*a^4-24*((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b^4+72*((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1))))^5*b^3*a-24*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b^2*a^2-
104*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b*a^3+16*((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^5*a^4+123*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))
^4*b^4-84*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^3*a-30*((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^4*b^2*a^2-84*((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^4*b*a^3-5*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^4+212*((1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^4-152*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^3*b^3*a-64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^2*
a^2+280*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b*a^3-20*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^3*a^4+108*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^2*b^4+40*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^3*a-224*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2*a^2-136*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^2*b*a^3+20*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^4+
12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^4+32*(1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1)))*b^3*a+24*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2*a^2-4*(
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^4-b^4-4*b^3*a-6*b^2*a^2-4*b*a^3-a^
4)/(128*b^5+640*b^4*a+1280*b^3*a^2+1280*b^2*a^3+640*b*a^4+128*a^5)/(((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^3*a+2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*((1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1))))^2*a+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)^2+(15*b^2-30*b*a+3*a^2)/(32*b^5+16
0*b^4*a+320*b^3*a^2+320*b^2*a^3+160*b*a^4+32*a^5)*ln(abs(1-cos(f*x+exp(1)))
/abs(1+cos(f*x+exp(1))))+(-3*b^3+30*b^2*a-15*b*a^2)*1/4/(4*b^5+20*b^4*a+40*
b^3*a^2+40*b^2*a^3+20*b*a^4+4*a^5)/sqrt(a*b)*atan((-a*cos(f*x+exp(1))+b)/(s
qrt(a*b)*cos(f*x+exp(1))+sqrt(a*b))))
```

maple [B] time = 1.24, size = 567, normalized size = 2.21

$$-\frac{9b\left(\cos^3\left(fx+e\right)\right)a^3}{8f\left(a+b\right)^5\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{3b^2\left(\cos^3\left(fx+e\right)\right)a^2}{4f\left(a+b\right)^5\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}+\frac{3b^3\left(\cos^3\left(fx+e\right)\right)a}{8f\left(a+b\right)^5\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -9/8/f*b/(a+b)^5/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a^3-3/4/f*b^2/(a+b)^5/(b
+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a^2+3/8/f*b^3/(a+b)^5/(b+a*cos(f*x+e)^2)^2*
cos(f*x+e)^3*a-7/8/f*b^2/(a+b)^5/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)*a^2-1/4/f*
b^3/(a+b)^5/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)*a+5/8/f*b^4/(a+b)^5/(b+a*cos(f*
x+e)^2)^2*cos(f*x+e)+15/8/f*b/(a+b)^5/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)
^(1/2))*a^2-15/4/f*b^2/(a+b)^5/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
*a+3/8/f*b^3/(a+b)^5/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-1/16/f/(a
+b)^3/(-1+cos(f*x+e))^2+3/16/f/(a+b)^4/(-1+cos(f*x+e))*a-9/16/f/(a+b)^4/(-1
+cos(f*x+e))*b+3/16/f/(a+b)^5*ln(-1+cos(f*x+e))*a^2-15/8/f/(a+b)^5*ln(-1+co
s(f*x+e))*a*b+15/16/f/(a+b)^5*ln(-1+cos(f*x+e))*b^2+1/16/f/(a+b)^3/(1+cos(f
*x+e))^2+3/16/f/(a+b)^4/(1+cos(f*x+e))*a-9/16/f/(a+b)^4/(1+cos(f*x+e))*b-3/
16/f/(a+b)^5*ln(1+cos(f*x+e))*a^2+15/8/f/(a+b)^5*ln(1+cos(f*x+e))*a*b-15/16
/f/(a+b)^5*ln(1+cos(f*x+e))*b^2
```

maxima [B] time = 0.44, size = 528, normalized size = 2.05

$$-\frac{\frac{3\left(a^2-10\,ab+5\,b^2\right)\log\left(\cos\left(fx+e\right)+1\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}-\frac{3\left(a^2-10\,ab+5\,b^2\right)\log\left(\cos\left(fx+e\right)-1\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}-\frac{6\left(5\,a^2b-10\,ab^2+b^3\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\sqrt{ab}}-\frac{\cdots}{\left(a^6+4\,a^5b+6\,a^4b^2\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x + e) + 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x
 + e) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 6*(5
*a^2*b - 10*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 + 5*a^4*b +
 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) - 2*(3*(a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^7 - (5*a^3 - 31*a^2*b + 31*a*b^2 - 5*b^3)*cos(f*x + e
)^5 - (19*a^2*b - 34*a*b^2 + 19*b^3)*cos(f*x + e)^3 - 12*(a*b^2 - b^3)*cos(
f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^8
 - 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cos(f*x +
e)^6 + a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6 + (a^6 - 9*a^4*b^2 -
 16*a^3*b^3 - 9*a^2*b^4 + b^6)*cos(f*x + e)^4 + 2*(a^5*b + 3*a^4*b^2 + 2*a^
3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cos(f*x + e)^2))/f
```

mupad [B] time = 9.60, size = 5613, normalized size = 21.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan(((((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^
3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 +
28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-a*b
)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9
- 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 21
0*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^1
0 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 4
95*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a
^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 -
 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b
^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b
 + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^
8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^
2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10
*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(16*(a*b^5 + 5*a^5*b +
a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (((cos(e + f*x)*(9*a*b^6 - 18
0*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2)
)/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4
 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54
*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^
8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11
 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*
a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b
^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 25
6*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 2304
0*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3
+ 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10
*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4
*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^
```

```
5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10
*a*b + b^2)*3i)/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^
4*b^2)))/(((135*a*b^7)/256 + (135*a^7*b)/256 - (1215*a^2*b^6)/128 + (13257*
a^3*b^5)/256 - (5913*a^4*b^4)/64 + (13257*a^5*b^3)/256 - (1215*a^6*b^2)/128
)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^
4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3
 + 66*a^10*b^2) - (3*((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*
b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b +
 a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2
)) + (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 -
 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*
a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12
 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 79
2*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*
b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 23
04*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6
+ 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*
b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7
*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6
*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10
*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 +
5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (3*((cos(e + f*x)*(
9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1
215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5
+ 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*b - 6*a
^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*
b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2
)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^
4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3
 + 66*a^10*b^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304
*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^
5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 192
00*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*
a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*
b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(
a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)
*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^
3 + 10*a^4*b^2))))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(8*f*(a*b^5 + 5*
a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) - (atan(((((6*a^13*b -
6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a
^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*
b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495
*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*
b^3 + 66*a^10*b^2) - (cos(e + f*x)*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4)
+ (3*b^2)/(a + b)^5)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10
 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8
*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(32*(8*a*b^7 + 8*a^
7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^
6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5) + (co
s(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800
*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 +
 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (
9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)*1i - (((6*a^13*b - 6*a^2*b^12 - 54*
a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8
*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11
+ 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a
^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^
2) + (cos(e + f*x)*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b
)^5)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9
```

```
- 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9
*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8
+ 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*
(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5) - (cos(e + f*x)*(9*a*
b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*
a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70
*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^
4) + (3*b^2)/(a + b)^5)*1i)/((((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a
^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b
^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^
12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6
*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (cos(e + f*
x)*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)*(2304*a^12*
b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8
- 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^1
0*b^3 + 8960*a^11*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 5
6*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*
b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5) + (cos(e + f*x)*(9*a*b^6 - 180*a^6*b
+ 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8
*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^
5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a
+ b)^5) - ((135*a*b^7)/256 + (135*a^7*b)/256 - (1215*a^2*b^6)/128 + (13257*
a^3*b^5)/256 - (5913*a^4*b^4)/64 + (13257*a^5*b^3)/256 - (1215*a^6*b^2)/128
)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^
4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3
 + 66*a^10*b^2) + (((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 4
50*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^
10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 +
 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*
a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) + (cos(e + f*x)*(3/(16*(
a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)*(2304*a^12*b + 256*a^1
3 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6
*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 896
0*a^11*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 +
 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a +
b)^4) + (3*b^2)/(a + b)^5) - (cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 1
80*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*
a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*
a^6*b^2)))*(3/(16*(a + b)^3) - (9*b)/(4*(a + b)^4) + (3*b^2)/(a + b)^5)))*(
3i/(8*(a + b)^3) - (b*9i)/(2*(a + b)^4) + (b^2*6i)/(a + b)^5))/f - ((cos(e
+ f*x)^3*(19*a^2*b - 34*a*b^2 + 19*b^3))/(8*(4*a*b^3 + 4*a^3*b + a^4 + b^4
+ 6*a^2*b^2)) + (cos(e + f*x)^5*(31*a*b^2 - 31*a^2*b + 5*a^3 - 5*b^3))/(8*(
4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (3*b^2*cos(e + f*x)*(a - b))/
(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - (3*a*cos(e + f*x)^7*(a^2
- 6*a*b + b^2))/(8*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))/(f*(cos(e
+ f*x)^4*(a^2 - 4*a*b + b^2) + b^2 + cos(e + f*x)^6*(2*a*b - 2*a^2) + cos(e
 + f*x)^2*(2*a*b - 2*b^2) + a^2*cos(e + f*x)^8))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.60 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=314

$$-\frac{5\sqrt{b}\sqrt{a+b}\,(a+4b)(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f}+\frac{(9a+10b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{5x(a+2b)\left(a^2+16\right.}{16a^6}$$

```
[Out] 5/16*(a+2*b)*(a^2+16*a*b+16*b^2)*x/a^6-5/8*(a+4*b)*(3*a+4*b)*arctan(b^(1/2)
*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^6/f-1/48*(33*a^2+110*a*b+80*
b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+1/24*(9*a+10*b)*cos
(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^3*sin(f*x+
e)^3/a/f/(a+b+b*tan(f*x+e)^2)^2-5/48*b*(9*a^2+32*a*b+24*b^2)*tan(f*x+e)/a^4
/f/(a+b+b*tan(f*x+e)^2)^2-5/16*b*(5*a^2+20*a*b+16*b^2)*tan(f*x+e)/a^5/f/(a+
b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.50, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4132, 470, 578, 527, 522, 203, 205}

$$-\frac{5b\left(5a^2+20ab+16b^2\right)\tan(e+fx)}{16a^5f\left(a+b\tan^2(e+fx)+b\right)}-\frac{5b\left(9a^2+32ab+24b^2\right)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\left(33a^2+110ab+80b^2\right)\sin(e+fx}{48a^3f\left(a+b\tan^2(e+fx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b)*(a^2 + 16*a*b + 16*b^2)*x)/(16*a^6) - (5*Sqrt[b]*Sqrt[a + b]*(
a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*f)
- ((33*a^2 + 110*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b
+ b*Tan[e + f*x]^2)^2) + ((9*a + 10*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2
*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(
a + b + b*Tan[e + f*x]^2)^2) - (5*b*(9*a^2 + 32*a*b + 24*b^2)*Tan[e + f*x])
/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (5*b*(5*a^2 + 20*a*b + 16*b^2)*T
an[e + f*x])/(16*a^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-6a-7b)x^2)}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= \frac{5(a+2b)\left(a^2+16ab+16b^2\right)x}{16a^6} - \frac{5\sqrt{b}\sqrt{a+b}\,(a+4b)(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+}{\sqrt{a+b}}\right.}{8a^6f}$$

Mathematica [C] time = 18.87, size = 1639, normalized size = 5.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(65536*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) - (15*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a
^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[
2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*S
in[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^
4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*
a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a
*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2
*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2
)))/(262144*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos
[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6 - 8*a^5*b + 12
0*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTan[(Se
c[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/
```

```
(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^4 + 32*a^3*b + 160
*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (256*a*Sin[2*(e + f*x)]
)/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2624*a^2*b^3 + 3200*a*b^4 + 128
0*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3
 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2
*b + a*Cos[2*(e + f*x)]))))/(65536*a^4*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*
b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-6144*(7*a^3 + 54*a^2*b + 120*a*b
^2 + 80*b^3)*x - (3*(3*a^8 - 64*a^7*b + 2240*a^6*b^2 + 53760*a^5*b^3 + 3136
00*a^4*b^4 + 802816*a^3*b^5 + 1032192*a^2*b^6 + 655360*a*b^7 + 163840*b^8)*
ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
 + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[
2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (12*(a^6 + 72*
a^5*b + 840*a^4*b^2 + 3584*a^3*b^3 + 6912*a^2*b^4 + 6144*a*b^5 + 2048*b^6)*
Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos
[2*(e + f*x)])^2) + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*((-I)*Cos[2*(e + f*x)
] + Sin[2*(e + f*x)]))/f + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*(I*Cos[2*(e +
f*x)] + Sin[2*(e + f*x)]))/f + (192*a^2*(a + 2*b)*((-6*I)*Cos[4*(e + f*x)]
- 6*Sin[4*(e + f*x)]))/f + ((1152*I)*a^2*(a + 2*b)*(Cos[4*(e + f*x)] + I*Si
n[4*(e + f*x)]))/f + (256*a^3*Sin[6*(e + f*x)])/f + (3*(3*a*(-a^7 + 22*a^6*
b + 1352*a^5*b^2 + 11312*a^4*b^3 + 37120*a^3*b^4 + 57856*a^2*b^5 + 43008*a*
b^6 + 12288*b^7)*Sec[2*e]*Sin[2*f*x] + (3*a^8 - 64*a^7*b - 4480*a^6*b^2 - 4
5696*a^5*b^3 - 196928*a^4*b^4 - 438272*a^3*b^5 - 528384*a^2*b^6 - 327680*a*
b^7 - 81920*b^8)*Tan[2*e]))/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])
)))/(393216*a^6*(a + b*Sec[e + f*x]^2)^3)
```

fricas [A] time = 0.63, size = 930, normalized size = 2.96

$$\left[\frac{30\left(a^5+18\,a^4b+48\,a^3b^2+32\,a^2b^3\right)fx\cos\left(fx+e\right)^4+60\left(a^4b+18\,a^3b^2+48\,a^2b^3+32\,ab^4\right)fx\cos\left(fx+e\right)^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(30*(a^5 + 18*a^4*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 6
0*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 30*(a^3
*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2
*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*
b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x +
 e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 2*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5
+ 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)
^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2
+ 20*a^2*b^3 + 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4
+ 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*f), 1/48*(15*(a^5 + 18*a^4*b + 48*a^3*
b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 30*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3
+ 32*a*b^4)*f*x*cos(f*x + e)^2 + 15*(a^3*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b
^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 +
16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqr
t(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos
(f*x + e)*sin(f*x + e))) - (8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 10*a^4*b)*co
s(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)^5 + 20*(6*a^4
*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2 + 20*a^2*b^3 +
 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*co
s(f*x + e)^2 + a^6*b^2*f)]
```

giac [A] time = 0.61, size = 372, normalized size = 1.18

$$\frac{15\left(a^3+18a^2b+48ab^2+32b^3\right)\left(fx+e\right)}{a^6}-\frac{30\left(3a^3b+19a^2b^2+32ab^3+16b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^6}-\frac{6\left(7a^2b^2\tan(fx+e)^3+23a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 - 30*(3*a^3*b +
 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^6) - 6*(7*a^2*b^2*t
an(f*x + e)^3 + 23*a*b^3*tan(f*x + e)^3 + 16*b^4*tan(f*x + e)^3 + 9*a^3*b*t
an(f*x + e) + 34*a^2*b^2*tan(f*x + e) + 41*a*b^3*tan(f*x + e) + 16*b^4*tan(
f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^5) - (33*a^2*tan(f*x + e)^5 + 162
*a*b*tan(f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 288*
a*b*tan(f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 126*a*b
*tan(f*x + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

maple [B] time = 1.28, size = 689, normalized size = 2.19

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8f\,a^3\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{23b^3\left(\tan^3\left(fx+e\right)\right)}{8f\,a^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(fx+e\right)}{8a^2f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-23/8/f*b^3/a^4/(a+b+b*ta
n(f*x+e)^2)^2*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-17
/4/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-41/8/f*b^3/a^4/(a+b+b*tan(f*
x+e)^2)^2*tan(f*x+e)-15/8/f*b/a^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b
)*b)^(1/2))-95/8/f*b^2/a^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1
/2))-20/f*b^3/a^5/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*
b^4/a^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-2/f*b^4/a^5/(a+b+b*tan(f*x+e)^2
)^2*tan(f*x+e)-10/f*b^4/a^6/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(
1/2))-27/8/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b-3/f/a^5/(tan(f*x+e)^2+1)
^3*tan(f*x+e)^5*b^2-11/16/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-6/f/a^4/(ta
n(f*x+e)^2+1)^3*tan(f*x+e)^3*b-6/f/a^5/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b^2-
5/6/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-5/16/f/a^3/(tan(f*x+e)^2+1)^3*tan
(f*x+e)-21/8/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b-3/f/a^5/(tan(f*x+e)^2+1)
^3*tan(f*x+e)*b^2+45/8/f/a^4*arctan(tan(f*x+e))*b+15/f/a^5*arctan(tan(f*x+e
))*b^2+10/f/a^6*arctan(tan(f*x+e))*b^3+5/16/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.45, size = 418, normalized size = 1.33

$$-\frac{15\left(5a^2b^2+20ab^3+16b^4\right)\tan\left(fx+e\right)^9+40\left(3a^3b+19a^2b^2+39ab^3+24b^4\right)\tan\left(fx+e\right)^7+\left(33a^4+470a^3b+1910a^2b^2+2880ab^3+1440b^4\right)\tan\left(fx+e\right)\cdots}{a^5b^2\tan\left(fx+e\right)^{10}+\left(2a^6b+5a^5b^2\right)\tan\left(fx+e\right)^8+a^7+2a^6b+a^5b^2+\left(a^7+8a^6b+10a^5b^2\right)\tan\left(fx+e\right)^6+\left(3a^7\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*((15*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*tan(f*x + e)^9 + 40*(3*a^3*b + 1
9*a^2*b^2 + 39*a*b^3 + 24*b^4)*tan(f*x + e)^7 + (33*a^4 + 470*a^3*b + 1910*
```

```
a^2*b^2 + 2880*a*b^3 + 1440*b^4)*tan(f*x + e)^5 + 40*(a^4 + 14*a^3*b + 46*a
^2*b^2 + 57*a*b^3 + 24*b^4)*tan(f*x + e)^3 + 15*(a^4 + 14*a^3*b + 41*a^2*b^
2 + 44*a*b^3 + 16*b^4)*tan(f*x + e))/(a^5*b^2*tan(f*x + e)^10 + (2*a^6*b +
5*a^5*b^2)*tan(f*x + e)^8 + a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 8*a^6*b + 10*a
^5*b^2)*tan(f*x + e)^6 + (3*a^7 + 12*a^6*b + 10*a^5*b^2)*tan(f*x + e)^4 + (
3*a^7 + 8*a^6*b + 5*a^5*b^2)*tan(f*x + e)^2) - 15*(a^3 + 18*a^2*b + 48*a*b^
2 + 32*b^3)*(f*x + e)/a^6 + 30*(3*a^3*b + 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*a
rctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^6))/f
```

mupad [B] time = 7.72, size = 2117, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (5*atan(((5*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176
000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - ((
(20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (tan(
e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*
5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*
(16*a*b + a^2 + 16*b^2))/(32*a^6) + (5*((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 92
5*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (
5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*
b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^
2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6))/((4750*a*b^10
 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*a^4*b^7)/32
+ (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4)/1024 + (187
5*a^8*b^3)/1024)/a^15 - (((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*
a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(1
28*a^10) - (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)
/a^15 - (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a
^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6
))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6) + (((tan(e + f*x)*(179200
*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 1230
0*a^5*b^4 + 925*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a
^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^1
3*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b
 + a^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^
6)))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(16*a^6*f) - ((5*tan(e + f*x)^3*(57
*a*b^3 + 14*a^3*b + a^4 + 24*b^4 + 46*a^2*b^2))/(6*a^5) + (5*tan(e + f*x)^7
*(39*a*b^3 + 3*a^3*b + 24*b^4 + 19*a^2*b^2))/(6*a^5) + (tan(e + f*x)^5*(288
0*a*b^3 + 470*a^3*b + 33*a^4 + 1440*b^4 + 1910*a^2*b^2))/(48*a^5) + (5*tan(
e + f*x)*(44*a*b^3 + 14*a^3*b + a^4 + 16*b^4 + 41*a^2*b^2))/(16*a^5) + (5*b
*tan(e + f*x)^9*(20*a*b^2 + 5*a^2*b + 16*b^3))/(16*a^5))/(f*(2*a*b + tan(e
+ f*x)^6*(8*a*b + a^2 + 10*b^2) + a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2
) + b^2*tan(e + f*x)^10 + tan(e + f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e +
f*x)^4*(12*a*b + 3*a^2 + 10*b^2))) + (atan((((a + 4*b)*(3*a + 4*b)*(-b*(a +
 b))^(1/2)*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 1760
00*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - (5*
(a + 4*b)*((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a
^15 - (5*tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)
*(-b*(a + b))^(1/2))/(2048*a^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6))
*5i)/(16*a^6) + ((a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e + f*x)*(1
79200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 +
 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) + (5*(a + 4*b)*((20*a^12*b^5 + 35
*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (5*tan(e + f*x)*(2048*
a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2))/(2048*a
^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6))*5i)/(16*a^6))/((4750*a*b^10
```

```
 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*a^4*b^7)/32
+ (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4)/1024 + (187
5*a^8*b^3)/1024)/a^15 - (5*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e
 + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800
*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - (5*(a + 4*b)*((20*a^1
2*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (5*tan(e + f
*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2
))/(2048*a^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6)))/(16*a^6) + (5*(a
 + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 92
5*a^6*b^3))/(128*a^10) + (5*(a + 4*b)*((20*a^12*b^5 + 35*a^13*b^4 + (65*a^1
4*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (5*tan(e + f*x)*(2048*a^12*b^3 + 1024*a^1
3*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2))/(2048*a^16))*(3*a + 4*b)*(
-b*(a + b))^(1/2))/(16*a^6)))/(16*a^6)))*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))
^(1/2)*5i)/(8*a^6*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=238

$$-\frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{(5a+8b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\sqrt{b}\left(5a^2+\right.}{}$$

```
[Out] 3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctan(b^(1/2)*tan(
f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f/(a+b)^(1/2)-1/8*(5*a+8*b)*cos(f*x+e)*sin(
f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*
tan(f*x+e)^2)^2-1/8*b*(7*a+12*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2-3/
2*b*(a+2*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.34, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 470, 527, 522, 203, 205}

$$-\frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f\sqrt{a+b}}+\frac{3x\left(a^2+12ab+16b^2\right)}{8a^5}-\frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{b(7a}{8a^3f\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*f) - ((5*a
 + 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) +
 (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(7
*a + 12*b)*Tan[e + f*x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*(a +
 2*b)*Tan[e + f*x])/(2*a^4*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-7b)x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a}{8a^3f\,(a+}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a}{8a^3f\,(a+}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a}{8a^3f\,(a+}$$

$$= \frac{3\left(a^2+12ab+16b^2\right)x}{8a^5} - \frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}\,f} - \frac{(5a+}{8a^2}$$

Mathematica [C] time = 25.01, size = 2469, normalized size = 10.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(16384*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
 + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b
]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))
/(16384*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b + a*Cos[2*e + 2*f
*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 64
0*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Si
n[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]
*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Se
c[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*
(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 2
56*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 51
2*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a
^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2
*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*
Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e
] + 2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1
536*b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin
[2*f*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Si
n[2*f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4
*b^2*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[
2*(e + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a
^4*b^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[
4*e + 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^
2))/(65536*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Co
s[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Si
n[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(
Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^
3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e
+ 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (
9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e]
)/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^2*(a + b)^2*f*(a + b*Sec
[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(
a + 2*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4
+ 3072*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e +
f*x)])^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^
2 + 2624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24
*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6
)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(16384*a^4*(
a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*(768*(7*a^2 + 32*a*b + 32*b^2)*x + (3*(a^7 - 14*a^6*b + 336*a^5*b^2 + 5600
*a^4*b^3 + 22400*a^3*b^4 + 37632*a^2*b^5 + 28672*a*b^6 + 8192*b^7)*ArcTan[(
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(
b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*(a^5 + 50*a^4*b + 4
00*a^3*b^2 + 1120*a^2*b^3 + 1280*a*b^4 + 512*b^5)*Sec[2*e]*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) - ((768
*I)*a*(a + 2*b)*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/f + ((768*I)*a*(a
+ 2*b)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/f + (128*a^2*Sin[4*(e + f*x
```

```
)])/f + (a*(3*a^6 - 44*a^5*b - 1900*a^4*b^2 - 10880*a^3*b^3 - 23360*a^2*b^4
 - 21504*a*b^5 - 7168*b^6)*Sec[2*e]*Sin[2*f*x] + (-3*a^7 + 42*a^6*b + 2192*
a^5*b^2 + 16480*a^4*b^3 + 51200*a^3*b^4 + 77824*a^2*b^5 + 57344*a*b^6 + 163
84*b^7)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])))))/(32768
*a^5*(a + b*Sec[e + f*x]^2)^3)
```

fricas [A] time = 0.60, size = 803, normalized size = 3.37

$$\left[\frac{12\left(a^{4}+12\,a^{3}b+16\,a^{2}b^{2}\right)fx\cos\left(fx+e\right)^{4}+24\left(a^{3}b+12\,a^{2}b^{2}+16\,ab^{3}\right)fx\cos\left(fx+e\right)^{2}+12\left(a^{2}b^{2}+12\,ab^{3}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 24*(a^3*b + 12
*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 12*a*b^3 + 16*b^4)*
f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^2 + 20*a*
b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-b/
(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(
f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
+ e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)) + 4*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b)*cos(f*x +
e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2*a*b^3)*cos(
f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 +
a^5*b^2*f), 1/16*(6*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 12*(
a^3*b + 12*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2 + 12*a*b^3 +
 16*b^4)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^
2 + 20*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b
)*cos(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2
*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f
*x + e)^2 + a^5*b^2*f)]
```

giac [A] time = 0.51, size = 325, normalized size = 1.37

$$\frac{\frac{3\left(a^{2}+12\,ab+16\,b^{2}\right)\left(fx+e\right)}{a^{5}}-\frac{3\left(5\,a^{2}b+20\,ab^{2}+16\,b^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\sqrt{ab+b^{2}}\,a^{5}}-\frac{12\,ab^{2}\tan\left(fx+e\right)^{7}+24\,b^{3}\tan\left(fx+e\right)^{7}+19\,a^{2}b\tan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 - 3*(5*a^2*b + 20*a*b^2 + 16*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^5) - (12*a*b^2*tan(f*x + e)^7 + 24*b^3*tan(f*x +
 e)^7 + 19*a^2*b*tan(f*x + e)^5 + 72*a*b^2*tan(f*x + e)^5 + 72*b^3*tan(f*x
+ e)^5 + 5*a^3*tan(f*x + e)^3 + 46*a^2*b*tan(f*x + e)^3 + 108*a*b^2*tan(f*x
 + e)^3 + 72*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 27*a^2*b*tan(f*x + e
) + 48*a*b^2*tan(f*x + e) + 24*b^3*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2*a^4))/f
```

maple [A] time = 1.18, size = 423, normalized size = 1.78

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8f\,a^3\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{3b^3\left(\tan^3\left(fx+e\right)\right)}{2f\,a^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(fx+e\right)}{8a^2f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{}{8f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-3/2/f*b^3/a^4/(a+b+b*tan
(f*x+e)^2)^2*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-21/
8/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-3/2/f*b^3/a^4/(a+b+b*tan(f*x+
e)^2)^2*tan(f*x+e)-15/8/f*b/a^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*
b)^(1/2))-15/2/f*b^2/a^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2
))-6/f*b^3/a^5/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/2/f/a
^4/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-5/8/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e
)^3-3/8/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)-3/2/f/a^4/(tan(f*x+e)^2+1)^2*ta
n(f*x+e)*b+9/2/f/a^4*arctan(tan(f*x+e))*b+6/f/a^5*arctan(tan(f*x+e))*b^2+3/
8/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 299, normalized size = 1.26

$$-\frac{\frac{12\left(ab^2+2b^3\right)\tan\left(fx+e\right)^7+\left(19a^2b+72ab^2+72b^3\right)\tan\left(fx+e\right)^5+\left(5a^3+46a^2b+108ab^2+72b^3\right)\tan\left(fx+e\right)^3+3\left(a^3+9a^2b+16ab^2+8b^3\right)\tan\left(fx+e\right)}{a^4b^2\tan\left(fx+e\right)^8+2\left(a^5b+2a^4b^2\right)\tan\left(fx+e\right)^6+a^6+2a^5b+a^4b^2+\left(a^6+6a^5b+6a^4b^2\right)\tan\left(fx+e\right)^4+2\left(a^6+3a^5b+2a^4b^2\right)\tan\left(fx+e\right)^2}-}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((12*(a*b^2 + 2*b^3)*tan(f*x + e)^7 + (19*a^2*b + 72*a*b^2 + 72*b^3)*t
an(f*x + e)^5 + (5*a^3 + 46*a^2*b + 108*a*b^2 + 72*b^3)*tan(f*x + e)^3 + 3*
(a^3 + 9*a^2*b + 16*a*b^2 + 8*b^3)*tan(f*x + e))/(a^4*b^2*tan(f*x + e)^8 +
2*(a^5*b + 2*a^4*b^2)*tan(f*x + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + (a^6 + 6*a
^5*b + 6*a^4*b^2)*tan(f*x + e)^4 + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x +
e)^2) - 3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 + 3*(5*a^2*b + 20*a*b^2 + 1
6*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^5))/f
```

mupad [B] time = 7.19, size = 1317, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4
 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2
)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(
20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a
*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^
2 + 16*b^2)*3i)/(16*(a^5*b + a^6)) + (((tan(e + f*x)*(4608*a*b^6 + 2304*b^7
 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3*((12*a^10*b^4
+ 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*a^10*b^3 + 128*
a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a
^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b
*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6)))/((540*a*b
^7 + 216*b^8 + (999*a^2*b^6)/2 + (837*a^3*b^5)/4 + (1215*a^4*b^4)/32 + (135
*a^5*b^3)/64)/a^12 - (3*((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^
```

```
5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3
+ (3*a^12*b^2)/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*
(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a +
 b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2
)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)) + (3*((tan(e + f*x)*(4608*a
*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3
*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*
a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256
*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5
*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)
)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(8*f*(a^5*b + a^6)) -
(atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (81*b^3)/(64*a) + (27*b^4)
/(16*a^2))) + (81*b^3*tan(e + f*x))/(64*((27*a*b^2)/256 + (81*b^3)/64 + (27
*b^4)/(16*a))) + (27*b^4*tan(e + f*x))/(16*((81*a*b^3)/64 + (27*b^4)/16 + (
27*a^2*b^2)/256)))*(a*b*12i + a^2*1i + b^2*16i)*3i)/(8*a^5*f) - ((tan(e + f
*x)^5*(72*a*b^2 + 19*a^2*b + 72*b^3))/(8*a^4) + (tan(e + f*x)^3*(108*a*b^2
+ 46*a^2*b + 5*a^3 + 72*b^3))/(8*a^4) + (3*tan(e + f*x)*(16*a*b^2 + 9*a^2*b
 + a^3 + 8*b^3))/(8*a^4) + (3*b*tan(e + f*x)^7*(a*b + 2*b^2))/(2*a^4))/(f*(
2*a*b + tan(e + f*x)^4*(6*a*b + a^2 + 6*b^2) + a^2 + b^2 + tan(e + f*x)^6*(
2*a*b + 4*b^2) + b^2*tan(e + f*x)^8 + tan(e + f*x)^2*(6*a*b + 2*a^2 + 4*b^2
)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
[Out] Timed out
```

3.62 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=184

$$\frac{x(a+6b)}{2a^4}-\frac{b(11a+12b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3b\tan(e+fx)}{4a^2f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-}}{8a^4f(a+b)^{3/2}}$$

```
[Out] 1/2*(a+6*b)*x/a^4-1/8*(15*a^2+40*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b
)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan
(f*x+e)^2)^2-3/4*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+12*b
)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.28, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{3/2}}-\frac{b(11a+12b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3b\tan(e+fx)}{4a^2f\left(a+b\tan^2(e+fx)-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 6*b)*x)/(2*a^4) - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*f) - (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*a^2*f*
(a + b + b*Tan[e + f*x]^2)^2) - (b*(11*a + 12*b)*Tan[e + f*x])/(8*a^3*(a +
b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b-5bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(}{}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a}{8a^3(a+b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a}{8a^3(a+b)f}$$

$$= \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} - \frac{\cos(e+fx)\sin}{2af\left(a+b+b\tan\right.}$$

Mathematica [C] time = 17.64, size = 1915, normalized size = 10.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^(5/2)*f*(a + b*Sec[e
 + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
+ 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]
```

```
*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e
 + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/
(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])
^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*
b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*
x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Co
s[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*
e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a +
 b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a
^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^
4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b
^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x
*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[
6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] +
2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*
b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f
*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*
f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2
*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e
 + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b
^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e
+ 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/
(4096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e
])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 6
4*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f
*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5
 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^
2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(4096*b^2*(a + b)^2*f*(a + b*Sec[e +
f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2
*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 307
2*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a
+ 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]
)^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2
624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*
b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan
[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(8192*a^4*(a + b*
Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.70, size = 815, normalized size = 4.43

$$\left[\frac{16\left(a^4+7a^3b+6a^2b^2\right)fx\cos\left(fx+e\right)^4+32\left(a^3b+7a^2b^2+6ab^3\right)fx\cos\left(fx+e\right)^2+16\left(a^2b^2+7ab^3+6b^4\right)f}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 7*a^
2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 7*a*b^3 + 6*b^4)*f*x +
((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a +
```

```
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)) - 4*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^3*b + 18*a^2*b^2)
*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7
 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^
2 + a^4*b^3)*f), 1/16*(8*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 1
6*(a^3*b + 7*a^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 7*a*b^3 +
 6*b^4)*f*x + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 40*a*b^3 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^
3*b + 18*a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*si
n(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x
 + e)^2 + (a^5*b^2 + a^4*b^3)*f)]
```

giac [A] time = 0.86, size = 219, normalized size = 1.19

$$-\frac{\frac{(15a^2b+40ab^2+24b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}}+\frac{7ab^2\tan(fx+e)^3+8b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+17ab^2\tan(fx+e)+8b\cdots}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) +
(7*a*b^2*tan(f*x + e)^3 + 8*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) + 17*
a*b^2*tan(f*x + e) + 8*b^3*tan(f*x + e))/((a^4 + a^3*b)*(b*tan(f*x + e)^2 +
 a + b)^2) - 4*(f*x + e)*(a + 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)^2 +
1)*a^3))/f
```

maple [A] time = 0.96, size = 314, normalized size = 1.71

$$-\frac{7b^2\left(\tan^3(fx+e)\right)}{8fa^2\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2(a+b)}-\frac{b^3\left(\tan^3(fx+e)\right)}{fa^3\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2(a+b)}-\frac{9b\tan(fx+e)}{8a^2f\left(a+b+b\left(\tan^2(f\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a^2*b^2/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)^3-1/f/a^3*b^3/(a+b+b
*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e
)^2)^2-1/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/a^2*b/(a+b)/((a
+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-5/f/a^3*b^2/(a+b)/((a+b)*
b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f/a^4*b^3/(a+b)/((a+b)*b)^(
1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a^3*tan(f*x+e)/(tan(f*x+e)^
2+1)+1/2/f/a^3*arctan(tan(f*x+e))+3/f/a^4*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.44, size = 272, normalized size = 1.48

$$-\frac{\frac{(15a^2b+40ab^2+24b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+a^4b)\sqrt{(a+b)b}}+\frac{(11ab^2+12b^3)\tan(fx+e)^5+(17a^2b+40ab^2+24b^3)\tan(fx+e)^3+(4a^3+21a^2b+29ab^2+\cdots}{(a^4b^2+a^3b^3)\tan(fx+e)^6+a^6+3a^5b+3a^4b^2+a^3b^3+(2a^5b+5a^4b^2+3a^3b^3)\tan(fx+e)^4+(a^6+5a^5b\cdots}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + a^4*b)*sqrt((a + b)*b)) + ((11*a*b^2 + 12*b^3)*tan(f*x + e)^5 + (1
7*a^2*b + 40*a*b^2 + 24*b^3)*tan(f*x + e)^3 + (4*a^3 + 21*a^2*b + 29*a*b^2
+ 12*b^3)*tan(f*x + e))/((a^4*b^2 + a^3*b^3)*tan(f*x + e)^6 + a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3 + (2*a^5*b + 5*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^4 +
 (a^6 + 5*a^5*b + 7*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^2) - 4*(f*x + e)*(a +
 6*b)/a^4)/f
```

mupad [B] time = 7.83, size = 2628, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4
 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b^
4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e +
 f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*
b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i +
b*6i)*1i)/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 +
 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^5
 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^
2) + (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^
3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a
^4))*(a*1i + b*6i)*1i)/(4*a^4))/(((297*a*b^6)/4 + 27*b^7 + (279*a^2*b^5)/4
+ (805*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b^2) - (((tan
(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^
3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^
10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a*1i +
 b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*a^
4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6i))/(4*a^4
) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 +
241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^5 + (29*a^9*b^4)/
2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*
x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2
))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6
i))/(4*a^4)))*(a*1i + b*6i)*1i)/(2*a^4*f) - ((tan(e + f*x)*(17*a*b + 4*a^2
+ 12*b^2))/(8*a^3) + (tan(e + f*x)^5*(11*a*b^2 + 12*b^3))/(8*a^3*(a + b)) +
 (b*tan(e + f*x)^3*(40*a*b + 17*a^2 + 24*b^2))/(8*a^3*(a + b)))/(f*(2*a*b +
 tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4*(2*a*b +
 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((-b*(a + b)^3)^(1/2)*((tan(e + f*x
)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*
(2*a^7*b + a^8 + a^6*b^2)) - ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^
4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e +
 f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a
^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a
^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*
b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2)*1i)/(16*(3*a^6*
b + a^7 + a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(326
4*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7
*b + a^8 + a^6*b^2)) + ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^4)/2 +
 (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*
(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a^9*b^4
 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b +
 a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*b + a^
7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2)*1i)/(16*(3*a^6*b + a^
7 + a^4*b^3 + 3*a^5*b^2)))/(((297*a*b^6)/4 + 27*b^7 + (279*a^2*b^5)/4 + (80
```

```
5*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b^2) - ((-b*(a + b
)^3)^(1/2)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*
b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b*(a + b)^3)^(1/2)*
((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^
11 + a^9*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^
2)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7
*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^
2 + 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 +
 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2
)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241
*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b*(a + b)^3)^(1/2)*((6*a^8*b
^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*
b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a
^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8
+ a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^
2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))
/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))))*(-b*(a + b)^3)^(1/2)*(40*a*b
+ 15*a^2 + 24*b^2)*1i)/(8*f*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.63 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b\tan(e+f}{4af(a+b)\left(a+b\tan^2(\right.}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+}{4af(a+b)\left(a+b\tan^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+\right.}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(\right.}$$

Mathematica [C] time = 5.62, size = 332, normalized size = 2.31

$$\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b\left(\left(9a^2+28ab+16b^2\right)\sin(2e)-3a(3a+2b)\sin(2fx)\right)(a\cos(2(e+fx))+a+2b)}{f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{b\left(15a^2+20ab+\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
```

```
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.59, size = 819, normalized size = 5.69

$$\left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos\left(fx+e\right)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos\left(fx+e\right)^2+32\left(a^2b^2+2ab^3+b^4\right)fx+\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

giac [A] time = 0.32, size = 205, normalized size = 1.42

$$-\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7ab^2\tan\left(fx+e\right)^3+4b^3\tan\left(fx+e\right)^3+9a^2b\tan\left(fx+e\right)+13ab^2\tan\left(fx+e\right)+4b^3\tan\left(\cdots\right.}{\left(a^4+2a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

maple [B] time = 0.97, size = 321, normalized size = 2.23

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8fa\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{b^3\left(\tan^3\left(fx+e\right)\right)}{2f\,a^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{\cdots}{8a\left(a+b\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a*b^2/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/2/f/a^2*
b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a/
(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/2/f/a^2*b^2/(a+b+b*tan(f*x+e)^2)^2/(a+b)*t
an(f*x+e)-15/8/f/a*b/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((
a+b)*b)^(1/2))-5/2/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x
+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b^3/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(t
an(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^3*arctan(tan(f*x+e))
```

maxima [**A**] time = 0.43, size = 231, normalized size = 1.60

$$-\frac{\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{(a+b)b}}+\frac{\left(7\,ab^{2}+4\,b^{3}\right)\tan\left(fx+e\right)^{3}+\left(9\,a^{2}b+13\,ab^{2}+4\,b^{3}\right)\tan\left(fx+e\right)}{a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}+\left(a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4}\right)\tan\left(fx+e\right)^{4}+2\,\left(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4}\right)\tan}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

mupad [**B**] time = 8.61, size = 3271, normalized size = 22.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a
^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b
^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 +
a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b
^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e +
f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(
64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6
 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9
*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7
+ 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2
))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3)
 - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^
4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a
```

```
^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*
x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2
*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6
 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^
8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3
+ 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)
*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3
+ 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*
a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^
5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^
2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))
 + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*
a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6
*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b
+ a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1
/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 +
3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4
 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7
*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^
(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105
*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e
 + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(
32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*
a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^
6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b
 + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b
+ a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b
^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b
^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*
(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^
5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))))*(-b*(a + b)^5)^(1/2)*(20*a*b +
15*a^2 + 8*b^2)*1i)/(8*f*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
+ 10*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

3.64 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} \, dx$

Optimal. Leaf size=124

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{15\cot(e+fx)}{8f(a+b)^3}+\frac{5\cot(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\cot(e+fx)}{4f(a+b)\left(a+b\tan^2(e+f\right.}$$

```
[Out] -15/8*cot(f*x+e)/(a+b)^3/f-15/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/(a+b)^(7/2)/f+1/4*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+5/8*cot(f*x
+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 290, 325, 205}

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{15\cot(e+fx)}{8f(a+b)^3}+\frac{5\cot(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\cot(e+fx)}{4f(a+b)\left(a+b\tan^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*f
) - (15*Cot[e + f*x])/(8*(a + b)^3*f) + Cot[e + f*x]/(4*(a + b)*f*(a + b +
b*Tan[e + f*x]^2)^2) + (5*Cot[e + f*x])/(8*(a + b)^2*f*(a + b + b*Tan[e + f
*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{15\,\text{Su}}{}$$

$$= -\frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2\right.}$$

$$= -\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}$$

Mathematica [C] time = 6.82, size = 987, normalized size = 7.96

$$\frac{(\cos(2e+2fx)a+a+2b)^3\left(\frac{15b\tan^{-1}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-2b\sin(fx)+a\sin(2e+fx))\right)}{64\sqrt{a+b}\,f\sqrt{b\cos(4e)-ib\sin(4e)}}\right.}{(a+b)^3\left(b\sec^2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((15*b*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Cos[2*e])/(64*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin
[4*e]]) - (((15*I)/64)*b*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Co
s[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I
*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])
/(Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e
+ f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]
*Sec[e + f*x]^6*(-32*a^4*Sin[f*x] - 64*a^3*b*Sin[f*x] + 22*a^2*b^2*Sin[f*x]
 + 80*a*b^3*Sin[f*x] + 16*b^4*Sin[f*x] + 32*a^4*Sin[3*f*x] + 46*a^3*b*Sin[3
*f*x] - 54*a^2*b^2*Sin[3*f*x] - 8*a*b^3*Sin[3*f*x] - 48*a^4*Sin[2*e - f*x]
- 128*a^3*b*Sin[2*e - f*x] - 106*a^2*b^2*Sin[2*e - f*x] + 80*a*b^3*Sin[2*e
- f*x] + 16*b^4*Sin[2*e - f*x] + 48*a^4*Sin[2*e + f*x] + 146*a^3*b*Sin[2*e
+ f*x] + 182*a^2*b^2*Sin[2*e + f*x] + 80*a*b^3*Sin[2*e + f*x] + 16*b^4*Sin[
2*e + f*x] - 32*a^4*Sin[4*e + f*x] - 82*a^3*b*Sin[4*e + f*x] - 54*a^2*b^2*S
in[4*e + f*x] - 80*a*b^3*Sin[4*e + f*x] - 16*b^4*Sin[4*e + f*x] - 8*a^4*Sin
[2*e + 3*f*x] + 18*a^3*b*Sin[2*e + 3*f*x] + 54*a^2*b^2*Sin[2*e + 3*f*x] + 8
*a*b^3*Sin[2*e + 3*f*x] + 32*a^4*Sin[4*e + 3*f*x] + 73*a^3*b*Sin[4*e + 3*f*
x] + 24*a^2*b^2*Sin[4*e + 3*f*x] + 8*a*b^3*Sin[4*e + 3*f*x] - 8*a^4*Sin[6*e
 + 3*f*x] - 9*a^3*b*Sin[6*e + 3*f*x] - 24*a^2*b^2*Sin[6*e + 3*f*x] - 8*a*b^
3*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] - 9*a^3*b*Sin[2*e + 5*f*x] - 2*
a^2*b^2*Sin[2*e + 5*f*x] + 9*a^3*b*Sin[4*e + 5*f*x] + 2*a^2*b^2*Sin[4*e + 5
```

```
*f*x] + 8*a^4*Sin[6*e + 5*f*x]))/(512*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^3)
```

fricas [B] time = 0.61, size = 615, normalized size = 4.96

$$\left[-\frac{4\left(8a^2-9ab-2b^2\right)\cos\left(fx+e\right)^5+20\left(5ab-b^2\right)\cos\left(fx+e\right)^3-15\left(a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2\right.}{32\left(\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)f\cos\left(fx+e\right)^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 20*(5*a*b - b^2)*cos(f*x
 + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/(a +
 b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
+ e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e)
)*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x + e))
, -1/16*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 10*(5*a*b - b^2)*cos(f*
x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/(a +
 b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
 e)*sin(f*x + e)))*sin(f*x + e) + 30*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a
*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x +
 e))]
```

giac [A] time = 0.50, size = 184, normalized size = 1.48

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{7b^2\tan(fx+e)^3+9ab\tan(fx+e)+9b^2\tan(fx+e)}{\left(a^3+3a^2b+3ab^2+b^3\right)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8}{\left(a^3+3a^2b+3ab^2+b^3\right)\tan(fx+e)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (7*b^2*t
an(f*x + e)^3 + 9*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2) + 8/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*tan(f*x + e)))/f
```

maple [A] time = 1.07, size = 157, normalized size = 1.27

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8f\left(a+b\right)^3\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9ab\tan\left(fx+e\right)}{8\left(a+b\right)^3f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b^2\tan\left(fx+\right.}{8f\left(a+b\right)^3\left(a+b+b\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/(a+b)^3*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8*a*b*tan(f*x+e)/(
a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-9/8/f/(a+b)^3*b^2/(a+b+b*tan(f*x+e)^2)^2*ta
```

```
n(f*x+e)-15/8/f/(a+b)^3*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/
2))-1/f/(a+b)^3/tan(f*x+e)
```

maxima [B] time = 0.44, size = 219, normalized size = 1.77

$$-\frac{\frac{15\,b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{15\,b^2\tan\left(fx+e\right)^4+25\left(ab+b^2\right)\tan\left(fx+e\right)^2+8\,a^2+16\,ab+8\,b^2}{\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\tan\left(fx+e\right)^5+2\left(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5\right)\tan\left(fx+e\right)^3+\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3\cdots\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sqrt((a + b)*b)) + (15*b^2*tan(f*x + e)^4 + 25*(a*b + b^2)*tan(f*x
+ e)^2 + 8*a^2 + 16*a*b + 8*b^2)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan
(f*x + e)^5 + 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x + e
)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e
)))/f
```

mupad [B] time = 5.11, size = 146, normalized size = 1.18

$$-\frac{\frac{1}{a+b}+\frac{25\,b\tan\left(e+f\,x\right)^2}{8\,(a+b)^2}+\frac{15\,b^2\tan\left(e+f\,x\right)^4}{8\,(a+b)^3}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(2\,b^2+2\,a\,b\right)+\tan\left(e+f\,x\right)\,\left(a^2+2\,a\,b+b^2\right)+b^2\tan\left(e+f\,x\right)^5\right)}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\,(\cdots}{(a\cdots}\right.}{8\,f\,(a+b)^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(a + b) + (25*b*tan(e + f*x)^2)/(8*(a + b)^2) + (15*b^2*tan(e + f*x)^4
)/(8*(a + b)^3))/(f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b +
 a^2 + b^2) + b^2*tan(e + f*x)^5)) - (15*b^(1/2)*atan((b^(1/2)*tan(e + f*x)
*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(8*f*(a + b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.65 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{5\sqrt{b}\,(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}-\frac{b(7a-4b)\tan(e+fx)}{8f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{ab\tan(e+fx)}{4f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] -(a-2*b)*cot(f*x+e)/(a+b)^4/f-1/3*cot(f*x+e)^3/(a+b)^3/f-5/8*(3*a-4*b)*arct
an(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/f-1/4*a*b*tan(f*x+e)
/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-1/8*(7*a-4*b)*b*tan(f*x+e)/(a+b)^4/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.25, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 456, 1259, 1261, 205}

$$-\frac{5\sqrt{b}\,(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}-\frac{b(7a-4b)\tan(e+fx)}{8f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{ab\tan(e+fx)}{4f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a +
 b)^(9/2)*f) - ((a - 2*b)*Cot[e + f*x])/((a + b)^4*f) - Cot[e + f*x]^3/(3*(
a + b)^3*f) - (a*b*Tan[e + f*x])/(4*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^
2) - ((7*a - 4*b)*b*Tan[e + f*x])/(8*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e\right.}{4f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)} - \text{Sub}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)} - \text{Sub}$$

$$= -\frac{(a-2b)\cot(e+fx)}{(a+b)^4f} - \frac{\cot^3(e+fx)}{3(a+b)^3f} - \frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8(a}$$

$$= -\frac{5(3a-4b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4f} - \frac{\cot^3(e+fx)}{3(a+b)^3f} - \frac{}{4(a+}$$

Mathematica [C] time = 4.05, size = 994, normalized size = 6.06

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^6(e+fx)\left(\frac{480(3a-4b)b\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2(e+fx))a+a+2b)^2(\cos}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((480*(3*a - 4*b)*b*ArcTan[(
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*
```

```
x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - (Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(4*(44*a^4 + 122*a^3*b + 63*a^2*b^2 + 12
6*a*b^3 + 36*b^4)*Sin[f*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1208*a*b^3
 + 48*b^4)*Sin[3*f*x] + 224*a^4*Sin[2*e - f*x] + 576*a^3*b*Sin[2*e - f*x] +
 124*a^2*b^2*Sin[2*e - f*x] - 2184*a*b^3*Sin[2*e - f*x] + 144*b^4*Sin[2*e -
 f*x] - 224*a^4*Sin[2*e + f*x] - 657*a^3*b*Sin[2*e + f*x] - 538*a^2*b^2*Sin
[2*e + f*x] + 984*a*b^3*Sin[2*e + f*x] + 144*b^4*Sin[2*e + f*x] + 176*a^4*S
in[4*e + f*x] + 569*a^3*b*Sin[4*e + f*x] + 666*a^2*b^2*Sin[4*e + f*x] + 170
4*a*b^3*Sin[4*e + f*x] - 144*b^4*Sin[4*e + f*x] + 48*a^4*Sin[2*e + 3*f*x] +
 111*a^3*b*Sin[2*e + 3*f*x] + 360*a^2*b^2*Sin[2*e + 3*f*x] + 312*a*b^3*Sin[
2*e + 3*f*x] - 48*b^4*Sin[2*e + 3*f*x] - 96*a^4*Sin[4*e + 3*f*x] - 152*a^3*
b*Sin[4*e + 3*f*x] + 146*a^2*b^2*Sin[4*e + 3*f*x] - 728*a*b^3*Sin[4*e + 3*f
*x] - 48*b^4*Sin[4*e + 3*f*x] + 48*a^4*Sin[6*e + 3*f*x] + 192*a^3*b*Sin[6*e
 + 3*f*x] + 558*a^2*b^2*Sin[6*e + 3*f*x] - 168*a*b^3*Sin[6*e + 3*f*x] + 48*
b^4*Sin[6*e + 3*f*x] + 16*a^4*Sin[2*e + 5*f*x] - 598*a^2*b^2*Sin[2*e + 5*f*
x] + 48*a*b^3*Sin[2*e + 5*f*x] + 72*a^3*b*Sin[4*e + 5*f*x] + 150*a^2*b^2*Si
n[4*e + 5*f*x] - 48*a*b^3*Sin[4*e + 5*f*x] + 16*a^4*Sin[6*e + 5*f*x] + 27*a
^3*b*Sin[6*e + 5*f*x] - 388*a^2*b^2*Sin[6*e + 5*f*x] + 45*a^3*b*Sin[8*e + 5
*f*x] - 60*a^2*b^2*Sin[8*e + 5*f*x] + 16*a^4*Sin[4*e + 7*f*x] - 83*a^3*b*Si
n[4*e + 7*f*x] + 6*a^2*b^2*Sin[4*e + 7*f*x] + 27*a^3*b*Sin[6*e + 7*f*x] - 6
*a^2*b^2*Sin[6*e + 7*f*x] + 16*a^4*Sin[8*e + 7*f*x] - 56*a^3*b*Sin[8*e + 7*
f*x]))/a))/(6144*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.65, size = 1009, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(24*a^3 - 134*a^
2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 32*a*b^2 + 16*b^3
)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (3*a^3 - 10*a^2*b
 + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b - 11*a*b^2 + 4*b^3)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 60*(3*a*b^2 - 4*
b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*co
s(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5
)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5
 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6
)*f)*sin(f*x + e)), -1/48*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 -
 2*(24*a^3 - 134*a^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b
- 32*a*b^2 + 16*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6
- (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b
- 11*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*c
os(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))))*sin(f*x +
 e) - 30*(3*a*b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a
^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3
 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3
 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))]
```

giac [A] time = 0.67, size = 275, normalized size = 1.68

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab-4\,b^2\right)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\sqrt{ab+b^2}}+\frac{3\left(7\,ab^2\tan\left(fx+e\right)^3-4\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)+5\,ab^2\tan\left(fx+e\right)-4\,b^3\tan\left(f\right.\right.}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*(3*a*b - 4*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
sqrt(a*b + b^2)) + 3*(7*a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 9*a^2
*b*tan(f*x + e) + 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(3*a*tan(f
*x + e)^2 - 6*b*tan(f*x + e)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*tan(f*x + e)^3))/f
```

maple [B] time = 1.23, size = 306, normalized size = 1.87

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)a}{8f\left(a+b\right)^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{b^3\left(\tan^3\left(fx+e\right)\right)}{2f\left(a+b\right)^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(fx+e\right)a^2}{8f\left(a+b\right)^4\left(a+b+b\left(\tan^2\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/(a+b)^4*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a+1/2/f/(a+b)^4*b^3/
(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f/(a+b)^4*b/(a+b+b*tan(f*x+e)^2)^2*
tan(f*x+e)*a^2-5/8/f/(a+b)^4*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)*a+1/2/f/
(a+b)^4*b^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/(a+b)^4*b/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a+5/2/f/(a+b)^4*b^2/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/3/f/(a+b)^3/tan(f*x+e)^3-1/f/(a+b)
^4/tan(f*x+e)*a+2/f/(a+b)^4/tan(f*x+e)*b
```

maxima [B] time = 0.44, size = 323, normalized size = 1.97

$$-\frac{\frac{15\left(3ab-4b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{(a+b)b}}+\frac{15\left(3ab^2-4b^3\right)\tan\left(fx+e\right)^6+25\left(3a^2b-ab^2-4b^3\right)\tan\left(fx+e\right)^4+8a^3+24a^2b+24ab^2+8b^3}{\left(a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6\right)\tan\left(fx+e\right)^7+2\left(a^5b+5a^4b^2+10a^3b^3+10a^2b^4+5ab^5+b^6\right)\tan\left(fx+e\right)^5+(a^6}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*a*b - 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a*b^2 - 4*b^3)
*tan(f*x + e)^6 + 25*(3*a^2*b - a*b^2 - 4*b^3)*tan(f*x + e)^4 + 8*a^3 + 24*
a^2*b + 24*a*b^2 + 8*b^3 + 8*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*tan(f*x +
e)^2)/((a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(f*x + e)^7 + 2
*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*tan(f*x + e)
^5 + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
*tan(f*x + e)^3))/f
```

mupad [B] time = 6.88, size = 207, normalized size = 1.26

$$-\frac{\frac{1}{3\,(a+b)}+\frac{25\tan\left(e+f\,x\right)^4\left(3\,a\,b-4\,b^2\right)}{24\,(a+b)^3}+\frac{\tan\left(e+f\,x\right)^2\left(3\,a-4\,b\right)}{3\,(a+b)^2}+\frac{5\tan\left(e+f\,x\right)^6\left(3\,a\,b^2-4\,b^3\right)}{8\,(a+b)^4}}{f\left(\tan\left(e+f\,x\right)^3\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^5\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^7\right)}-\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\left(\right.}{}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(3*(a + b)) + (25*tan(e + f*x)^4*(3*a*b - 4*b^2))/(24*(a + b)^3) + (ta
n(e + f*x)^2*(3*a - 4*b))/(3*(a + b)^2) + (5*tan(e + f*x)^6*(3*a*b^2 - 4*b^
3))/(8*(a + b)^4))/(f*(tan(e + f*x)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)^5*
(2*a*b + 2*b^2) + b^2*tan(e + f*x)^7)) - (5*b^(1/2)*atan((b^(1/2)*tan(e + f
*x)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))/(a + b)^(9/2))*(3*a - 4*b)
)/(8*f*(a + b)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.66 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{b}\left(15a^2-40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{11/2}}-\frac{b\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{40f(a+b)^5\left(a+b\tan^2(e+fx)+b\right)}-\frac{b\left(5a^2+4b^2\right)\tan(e+}{20f(a+b)^4\left(a+b\tan^2(e+\right.}$$

```
[Out] -1/5*(5*a^2-20*a*b+2*b^2)*cot(f*x+e)/(a+b)^5/f-1/15*(10*a+b)*cot(f*x+e)^3/(
a+b)^4/f-1/8*(15*a^2-40*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b
^(1/2)/(a+b)^(11/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/20*
b*(5*a^2+4*b^2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^2-1/40*b*(35*a^2-
40*a*b+24*b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.37, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 462, 456, 1259, 1261, 205}

$$-\frac{\sqrt{b}\left(15a^2-40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{11/2}}-\frac{b\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{40f(a+b)^5\left(a+b\tan^2(e+fx)+b\right)}-\frac{b\left(5a^2+4b^2\right)\tan(e+}{20f(a+b)^4\left(a+b\tan^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/(8*(a + b)^(11/2)*f) - ((5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(5*(a +
 b)^5*f) - ((10*a + b)*Cot[e + f*x]^3)/(15*(a + b)^4*f) - Cot[e + f*x]^5/(5
*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Tan[e + f*x])
/(20*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(35*a^2 - 40*a*b + 24*b
^2)*Tan[e + f*x])/(40*(a + b)^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{10a+b+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx\right.}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} -$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} -$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} -$$

$$= -\frac{\left(5a^2-20ab+2b^2\right)\cot(e+fx)}{5(a+b)^5f} - \frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4f} - \frac{\cot^5(e+}{5(a+b)f\left(a+b+\right.}$$

$$= -\frac{\sqrt{b}\left(15a^2-40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f} - \frac{\left(5a^2-20ab+2b^2\right)\cot(e+fx)}{5(a+b)^5f}$$

Mathematica [C] time = 5.77, size = 479, normalized size = 1.98

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(8\left(8a^2-59ab+23b^2\right)\csc(e)\sin(fx)\csc(e+fx)(a\cos(2(e+fx))+a+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-8*(4*a - 11*b)*(a + b)*(a
+ 2*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^2 - 24*(a + b)^2*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^4 + (15*b*(15*a^2 - 40*a*b + 8
*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4]) + 8*(8*a^2 - 59*a*b + 23*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])
^2*Csc[e]*Csc[e + f*x]*Sin[f*x] + 8*(4*a - 11*b)*(a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x] + 24*(a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*x] - 60*b^2*(a + b)*Sec[2*e]
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]) + 15*b*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[2*e]*((9*a^2 + 16*a*b - 8*b^2)*Sin[2*e] + 3*a*(-3*a + 2*b)*Sin[2*f*x]))
)/(960*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.67, size = 1423, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*x + e)^9 - 4*(160*a^4 -
 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)^7 + 4*(120*a^4 - 1205*
a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b
- 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 40*a^3*b
 + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b + 48*a^2*b^2 - 8*a*b^3)
*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2 - 72*a*b^3 + 8*b^4)*cos
(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*a^3*b - 55*a^2*b^2 + 48
*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 6
0*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b
+ 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*
b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*
x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*f)*sin(f*x + e)), -1/240*(2*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*
x + e)^9 - 2*(160*a^4 - 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)
^7 + 2*(120*a^4 - 1205*a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x
 + e)^5 + 10*(75*a^3*b - 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 -
 15*((15*a^4 - 40*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b
+ 48*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2
- 72*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*
a^3*b - 55*a^2*b^2 + 48*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f
```

```
*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(
((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*co
s(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b
^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^
4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*sin(f*x + e))]
```

giac [A] time = 1.09, size = 382, normalized size = 1.58

$$-\frac{\frac{15\left(15\,a^2b-40\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\sqrt{ab+b^2}}+\frac{15\left(7\,a^2b^2\tan(fx+e)^3-8\,ab^3\tan(fx+e)^3+9\,a^3b\tan(fx+e)+a^2b^2\tan(fx+e\right.}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\left(b\tan(fx+e)^2+a+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b + b^2)) + 15*(7*a^2*b^2*tan(f*x + e)^
3 - 8*a*b^3*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) + a^2*b^2*tan(f*x + e) -
8*a*b^3*tan(f*x + e))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan
(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 5*a*b*tan(f*x
 + e)^2 - 5*b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e)^5))/f
```

maple [A] time = 1.18, size = 411, normalized size = 1.70

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)a^2}{8f\left(a+b\right)^5\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{b^3\left(\tan^3\left(fx+e\right)\right)a}{f\left(a+b\right)^5\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(fx+e\right)a}{8f\left(a+b\right)^5\left(a+b+b\left(\tan^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+1/f*b^3/(a+b)^5/
(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a-9/8/f*b/(a+b)^5/(a+b+b*tan(f*x+e)^2)^
2*tan(f*x+e)*a^3-1/8/f*b^2/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^2+1/
f*b^3/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)*a-15/8/f*b/(a+b)^5/((a+b)*b
)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a^2+5/f*b^2/(a+b)^5/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a-1/f*b^3/(a+b)^5/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/5/f/(a+b)^3/tan(f*x+e)^5-2/3/f/(a+
b)^4/tan(f*x+e)^3*a+1/3/f/(a+b)^4/tan(f*x+e)^3*b-1/f/(a+b)^5/tan(f*x+e)*a^2
+4/f/(a+b)^5/tan(f*x+e)*a*b-1/f/(a+b)^5/tan(f*x+e)*b^2
```

maxima [A] time = 0.45, size = 434, normalized size = 1.79

$$-\frac{\frac{15\left(15\,a^2b-40\,ab^2+8\,b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\sqrt{(a+b)b}}+\frac{15\left(15\,a^2b^2-40\,ab^3+8\,b^4\right)\tan(fx+e)^8+25\left(15\,a^3b-25\,a^2b^2-32\,ab^3+8\,b^4\right)\tan(fx+e)^6+8\left(15\right.}{\left(a^5b^2+5\,a^4b^3+10\,a^3b^4+10\,a^2b^5+5\,ab^6+b^7\right)\tan(fx+e)^9+2\left(a^6b+6\,a^5b^2+15\right.}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt((a + b
```

```
)*b)) + (15*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*tan(f*x + e)^8 + 25*(15*a^3*b -
 25*a^2*b^2 - 32*a*b^3 + 8*b^4)*tan(f*x + e)^6 + 8*(15*a^4 - 10*a^3*b - 57*
a^2*b^2 - 24*a*b^3 + 8*b^4)*tan(f*x + e)^4 + 24*a^4 + 96*a^3*b + 144*a^2*b^
2 + 96*a*b^3 + 24*b^4 + 8*(10*a^4 + 31*a^3*b + 33*a^2*b^2 + 13*a*b^3 + b^4)
*tan(f*x + e)^2)/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*tan(f*x + e)^9 + 2*(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15
*a^2*b^5 + 6*a*b^6 + b^7)*tan(f*x + e)^7 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*tan(f*x + e)^5))/f
```

mupad [B] time = 7.49, size = 267, normalized size = 1.10

$$-\frac{\frac{1}{5\,(a+b)}+\frac{\tan\left(e+f\,x\right)^2\,(10\,a+b)}{15\,(a+b)^2}+\frac{5\tan\left(e+f\,x\right)^6\left(15\,a^2\,b-40\,a\,b^2+8\,b^3\right)}{24\,(a+b)^4}+\frac{\tan\left(e+f\,x\right)^4\left(15\,a^2-40\,a\,b+8\,b^2\right)}{15\,(a+b)^3}+\frac{\tan\left(e+f\,x\right)^8\left(15\,a^2\,b^2-40\,a\,b^3+8\,b^4\right)}{8\,(a+b)^5}}{f\left(\tan\left(e+f\,x\right)^5\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^7\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(5*(a + b)) + (tan(e + f*x)^2*(10*a + b))/(15*(a + b)^2) + (5*tan(e +
f*x)^6*(15*a^2*b - 40*a*b^2 + 8*b^3))/(24*(a + b)^4) + (tan(e + f*x)^4*(15*
a^2 - 40*a*b + 8*b^2))/(15*(a + b)^3) + (tan(e + f*x)^8*(8*b^4 - 40*a*b^3 +
 15*a^2*b^2))/(8*(a + b)^5))/(f*(tan(e + f*x)^5*(2*a*b + a^2 + b^2) + tan(e
 + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) - (b^(1/2)*atan((b^(1/2)*t
an(e + f*x)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(a +
 b)^(11/2))*(15*a^2 - 40*a*b + 8*b^2))/(8*f*(a + b)^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.67 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^5(e+fx)\,dx$

Optimal. Leaf size=139

$$\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e}{f}$$

```
[Out] 2/15*(5*a+b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a^2/f-1/5*cos(f*x+e)^5*(
a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(
1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4134, 462, 451, 277, 217, 206}

$$\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (2*(5*a + b)*Cos[e + f*x]^3*(a
+ b*Sec[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]
^2)^(3/2))/(5*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a+bx^2}}{x^6}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(-2(5a+b)+5ax^2\right)\sqrt{a+bx^2}}{x^4}\right.}{5af}$$

$$= \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\,\text{se}\right.}{5af}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2\right.}{15a^2f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2\right.}{15a^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a}{}$$

Mathematica [A] time = 0.85, size = 152, normalized size = 1.09

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{2\left(a\cos^2(e+fx)+b\right)^{5/2}}{5a^2} - \frac{2(2a+b)\left(a\cos^2(e+fx)+b\right)^{3/2}}{3a^2} + 2\sqrt{a\cos^2(e+fx)+b} - 2\sqrt{b}\,\text{tanh}\right.}{\sqrt{2}f\sqrt{a\cos(2e+2fx)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

```
[Out] -((Cos[e + f*x]*(-2*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]] + 2
*Sqrt[b + a*Cos[e + f*x]^2] - (2*(2*a + b)*(b + a*Cos[e + f*x]^2)^(3/2))/(3
*a^2) + (2*(b + a*Cos[e + f*x]^2)^(5/2))/(5*a^2))*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]))
```

fricas [A] time = 0.88, size = 295, normalized size = 2.12

$$\left[\frac{15\,a^2\sqrt{b}\,\log\left(\frac{a\cos\left(fx+e\right)^2+2\,\sqrt{b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(3\,a^2\cos\left(fx+e\right)^5-\left(10\,a^2-ab\right)\cos\left(fx+e\right)^3+\right.}{30\,a^2 f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(3*a^2*cos
(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f), -1/15*(15*
a^2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e)/b) + (3*a^2*cos(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15
*a^2 - 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-64*(1/240*(-15*b*(-sqrt(a+b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-
2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^9+165*b*sqrt(
a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*t
an(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1))
)^2+a+b))^8-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1))
)^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+
exp(1)))^2+a+b))^7*(-320*a^2+540*b^2-100*a*b)-sqrt(a+b)*(-sqrt(a+b)*tan(1/2
*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2
*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^6*(-640*a^2-66
0*b^2+1660*a*b)-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp
(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(
f*x+exp(1)))^2+a+b))^5*(832*a^3-30*b^3-2460*a*b^2+1250*a^2*b)-(-sqrt(a+b)*t
an(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)
))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-2880*a
^5+135*b^5+700*a*b^4-1430*a^2*b^3+380*a^3*b^2+3335*a^4*b)-sqrt(a+b)*(-sqrt(
a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+
exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4*
(2560*a^3+810*b^3+900*a*b^2-3910*a^2*b)-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2
+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+
exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(320*a^4-660*b^4+900*a*b^3+3
140*a^2*b^2-3220*a^3*b)-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(
a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)
))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*(-3200*a^4+60*b^4-1500*a*b^3+980*a
^2*b^2+3180*a^3*b)-sqrt(a+b)*(-768*a^5-45*b^5-100*a*b^4+450*a^2*b^3-1028*a^
3*b^2+1539*a^4*b))/(2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*
tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))
```

```
^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sq
rt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp
(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2+3*a-b)^5+1/32*b*atan(1/2*(-sqrt
(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*ta
n(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))
^2+a+b))/sqrt(-b))/sqrt(-b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 4.58, size = 1840, normalized size = 13.24

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/30/f*(-1+cos(f*x+e))^2*(-4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b
^(3/2)*(a+b)^(3/2)+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*b^(1/2)*(a+b)^(3/2)*4^(1/2)*a^2+15*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*
b^(5/2)*4^(1/2)*a^2-15*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*4^(1/2
)*a^2+10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*(a+b)^(3/2)*a+
15*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*4^(1/2)*a^3-15*ln(-2*(-1+c
os(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)
/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*4^(1/2)*a^3-15*cos(f*x+e)*arctanh(1/8*(-
1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*(a+b)^(3/2)*4^(1/2
)*a^2*b-4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)^4*b^(3/2)*
(a+b)^(3/2)-16*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)^3*b^(
3/2)*(a+b)^(3/2)-24*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)^
2*b^(3/2)*(a+b)^(3/2)-16*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*
x+e)*b^(3/2)*(a+b)^(3/2)+16*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos
(f*x+e)^4*b^(1/2)*(a+b)^(3/2)*a-56*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3
/2)*cos(f*x+e)^3*b^(1/2)*(a+b)^(3/2)*a-84*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(3/2)*cos(f*x+e)^2*b^(1/2)*(a+b)^(3/2)*a+15*cos(f*x+e)*ln(-4*(-1+cos(f
*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin
(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*4^(1/2)*a^2-15*cos(f*x+e)*ln(-2*(-1+cos(f*x+
e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*
x+e)^2/(a+b)^(1/2))*b^(5/2)*4^(1/2)*a^2-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^(3/2)*(a+b)^(3/2)*4^(1/2)*a+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*(a+b)^(3/2)*4^(1/2)*a^2-20*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(3/2)*cos(f*x+e)*b^(1/2)*(a+b)^(3/2)*a+15*cos(f*x+e)*ln(-4*(-1+
cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(
1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b
)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*4^(1/2)*a^3-15*cos(f*x+e)*ln(-2*(-1+cos
(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/s
in(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*4^(1/2)*a^3-15*arctanh(1/8*(-1+cos(f*x+e))
*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*(a+b)^(3/2)*4^(1/2)*a^2*b+6*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)^6*b^(1/2)*(a+b)^(3/2)*a
+24*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)^5*b^(1/2)*(a+b)^
(3/2)*a)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4/b^(1/2)/(a+b)^(3/2)/a^2
```

maxima [A] time = 0.43, size = 171, normalized size = 1.23

$$\frac{\frac{20\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3}{a} - 30\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right) - 15\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\sqrt{b}}\right) - \frac{2\left(3\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)\cdots\right.}{\cdots}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(20*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 30*sqrt(a + b/cos(
f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 2
*(3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a + b/cos(f*x + e)^2)^
(3/2)*b*cos(f*x + e)^3)/a^2)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.68 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^3(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] 1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a
+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4134, 451, 277, 217, 206}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a + b*Sec[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
```

```
&& IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e\right.}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx\right.}{3af}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx\right.}{3af}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\text{c}}{}$$

Mathematica [A] time = 0.38, size = 120, normalized size = 1.20

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{a\cos^2(e+fx)+b}\left(a\cos^2(e+fx)-3a+b\right)+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+}}{\sqrt{b}}\right.\right.}{3af\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(3*a*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[
b]] + Sqrt[b + a*Cos[e + f*x]^2]*(-3*a + b + a*Cos[e + f*x]^2))*Sqrt[a + b*
Sec[e + f*x]^2])/(3*a*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.85, size = 232, normalized size = 2.32

$$\left[\frac{3a\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)+2\left(a\cos(fx+e)^3-(3a-b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e}{\cos(fx+e}}}{6af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(a*cos(f*x + e
)^3 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f), -1/3*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/b) - (a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-16*(1/12*(-3*b*(-sqrt(a+b)*tan(1/2*
(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*
a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^5-(-sqrt(a+b)*t
an(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)
))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-48*a^3
-9*b^3+6*a*b^2+15*a^2*b)-sqrt(a+b)*(12*a-9*b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(
1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2
*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4-(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*
tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(16*a^2+6*b^2-1
8*a*b)-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+ex
p(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*
(f*x+exp(1)))^2+a+b))^2*(-24*a^2+6*b^2+6*a*b)-sqrt(a+b)*(-20*a^3+3*b^3-6*a*
b^2+19*a^2*b))/(2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(
1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2
*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a
*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1))
)^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2+3*a-b)^3+1/8*b*atan(1/2*(-sqrt(a+b)
*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2
*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+
b))/sqrt(-b))/sqrt(-b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 1.84, size = 1525, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/6/f*(-1+cos(f*x+e))^2*(2*cos(f*x+e)^4*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(3/2)+8*cos(f*x+e)^3*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+6*cos(f*x+e)^2*(a+b)^(3/2)*b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+3*cos(f*x+e)*4^(1/2)*b^(5/2)*ln(-2*(-1
+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^
(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+
b)/sin(f*x+e)^2/(a+b)^(1/2))*a-3*cos(f*x+e)*4^(1/2)*b^(5/2)*ln(-4*(-1+cos(f
*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin
(f*x+e)^2/(a+b)^(1/2))*a-6*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*arcta
nh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+
e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b-4*cos
(f*x+e)*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+3*c
os(f*x+e)*4^(1/2)*b^(3/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2-3*cos(f
*x+e)*4^(1/2)*b^(3/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+3*(a+b)^(3/
2)*4^(1/2)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+3*4^(1/2)*b^
(5/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
```

```
s(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-3*4^(1/2)*b^(5/2)*ln(-4*(-1+c
os(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)
/sin(f*x+e)^2/(a+b)^(1/2))*a-3*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*(-1+cos(f*x
+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b-4*(a+b)^(3/2)*b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+3*4^(1/2)*b^(3/2)*ln(-2*(-1+cos(
f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/si
n(f*x+e)^2/(a+b)^(1/2))*a^2-3*4^(1/2)*b^(3/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b
)^(1/2))*a^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4/b^(1/2)/a/(a+b)^(3/2)
```

maxima [A] time = 0.42, size = 116, normalized size = 1.16

$$\frac{\frac{2\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3}{a} - 6\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right) - 3\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\sqrt{b}}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 6*sqrt(a + b/cos(f*x
 + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x +
 e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.69 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(
a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{4134, 277, 217, 206\}$

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+\right.}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+f}{\sqrt{a+b\sec^2(}}\right.}{f}$$
$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [A] time = 0.13, size = 98, normalized size = 1.48

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)+b}}{\sqrt{b}}\right)-\sqrt{a\cos^2(e+fx)+b}\right)}{f\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]]
- Sqrt[b + a*Cos[e + f*x]^2])*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.84, size = 182, normalized size = 2.76

$$\left[-\frac{2\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^2}\right)}{2f}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos}{\cos}}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b)
*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*arctan(sqrt(-b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-8*(1/2*(a*(-sqrt(a+b)*tan(1/2*(f*x+
exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan
(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))+a*sqrt(a+b))/(2*sqrt
(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*
tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)
))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))
^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+e
xp(1)))^2+a+b))^2+3*a-b)+1/4*b*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2
+sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan
(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b))*
sign(cos(f*x+exp(1)))/f
```

maple [A] time = 0.53, size = 93, normalized size = 1.41

$$-\frac{\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{fa\sec\left(fx+e\right)}+\frac{b\sec\left(fx+e\right)\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{fa}+\frac{\sqrt{b}\,\ln\left(\sec\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sec^2\left(fx+\right.\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)+1/f/a*b*sec(f*x+e)*(a+b*sec(f*x+
e)^2)^(1/2)+1/f*b^(1/2)*ln(sec(f*x+e)*b^(1/2)+(a+b*sec(f*x+e)^2)^(1/2))
```

maxima [A] time = 0.42, size = 88, normalized size = 1.33

$$-\frac{2\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\sqrt{b}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a + b/c
os(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x
+ e) + sqrt(b))))/f
```

mupad [B] time = 6.77, size = 87, normalized size = 1.32

$$-\frac{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{f}-\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{a}\,\cos\left(e+f\,x\right)}\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,1{}\mathrm{i}}{\sqrt{a}\,f\,\sqrt{\frac{b}{a\,{\cos\left(e+f\,x\right)}^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] - (cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2))/f - (b^(1/2)*asin((b^(1/2)*1i
)/(a^(1/2)*cos(e + f*x)))*(a + b/cos(e + f*x)^2)^(1/2)*1i)/(a^(1/2)*f*(b/(a
*cos(e + f*x)^2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\,\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

3.70 $\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-arctanh(sec(
f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4134, 402, 217, 206, 377, 207}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (S
qrt[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
```

```
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sec(e+fx)\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a+bx^2}}\right.}{f}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\right.}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.13, size = 119, normalized size = 1.45

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{b}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)\right)}{f\sqrt{a\cos(2e+2fx)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(Sqrt[b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a
+ b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]])*Cos[e + f*x]*Sqrt
[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

fricas [A] time = 0.78, size = 496, normalized size = 6.05

$$\left[\frac{\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+2b\right)}{\cos\left(fx+e\right)^2-1}\right)+\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\cdots}{\cos\left(fx+e\right)^2}\right.}{2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) +
sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*(2*sqrt(-a - b)*arct
an(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a
 + b)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
```

```
 b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/f,
 (sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b)) - sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)-2*(1/4
*sqrt(a+b)*ln(abs(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1
/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*
b*tan(1/2*(f*x+exp(1)))^2+a+b)))-1/4*sqrt(a+b)*ln(abs(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1
)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)))+b*atan
(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(
1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f
*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b)+sqrt(a+b)*(a+b)*ln(abs(-(a+b)*(-sqrt
(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x
+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-s
qrt(a+b)*(a-b)))/(-4*a-4*b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 2.01, size = 688, normalized size = 8.39

$$-\frac{\sqrt{4}\,\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)\left(\ln\left(-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\cos\left(fx+e\right)\sqrt{a+b}+\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\sqrt{a+b}-a\cos(fx+e\right.}{\sin\left(fx+e\right)^2\sqrt{a+b}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(ln(-2*(-
1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)
^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)
+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)-2*b^(3/2)*ln(-4*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))+2*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4
^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
```

```
4^(1/2))*(a+b)^(1/2)*b-ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^(1/2)-ln(-
4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(
f*x+e)))*b^(1/2)*a-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(3/2))*(-1+cos(f*x+e))/sin(f*x+e)^2/((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/b^(1/2)/(a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \csc\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^2\left(e + fx\right)} \csc\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

3.71 $\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f\sqrt{a+b}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+2*b)*
arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)-1/2*
cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4134, 467, 523, 217, 206, 377, 207}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f\sqrt{a+b}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*Sqrt[a + b]*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2
*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
```

```
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+2bx^2}{\left(-1+x^2\right)\sqrt{a+}}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\right.}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.}{f}$$

$$= \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}\,f} - \text{co}$$

Mathematica [A] time = 0.43, size = 163, normalized size = 1.31

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{b}}\right)-\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+}}{\sqrt{a+b}}\right.\right.}{\sqrt{2}\,f(a+b)\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[b]*(a + b)*ArcTanh[Sqrt[a
+ b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(a + 2*b)*ArcTanh[Sqrt[a + b
 - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2*Sqrt[a + b - a*S
in[e + f*x]^2]))/(Sqrt[2]*(a + b)*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.72, size = 867, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 -
2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
+ 2*b)/(cos(f*x + e)^2 - 1)) + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*l
og((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a + b)*
f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a
+ b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^2 - (a + b)*f), -1/4*(4*((a + b)*cos(f*x + e)^2 - a - b)
*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/b) - 2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) - ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x +
e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 2*((a
 + b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(-1/16*sqrt(a
*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1))
)^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)+2*(-1/16*(-(a-b)*(-sqrt(a+b)*tan(1/2*(
f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a
*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-sqrt(a+b)*(a+b))
/((-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(
1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2
+a+b))^2-a-b)-1/2*b*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+
sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+e
xp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b)-1/8*(-a-2*b)
*atan((-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*
tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)
))^2+a+b))/sqrt(-a-b))/sqrt(-a-b)+sqrt(a+b)*(-a-2*b)*ln(abs((a+b)*(-sqrt(a+
b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+ex
p(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))+sqrt
(a+b)*(a-b)))/(-16*a-16*b)))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 1.72, size = 3015, normalized size = 24.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(-2*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+4*cos(f*x+e)^2*(a+b)^(3/2)*4^(1/2)*b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-4*cos(f*x+e)^2*(a+b)^(3/
2)*4^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(
1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^
(1/2))*a*b-4^(1/2)*b^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3-5*4^(1
/2)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)+b)/(-1+cos(f*x+e)))*a+4*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*(-1+cos(f*x+e)
)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2+4*cos(f*x+e)^2*4^(1/2)*b^(
7/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))-2*cos(f*x+e)^2*4^(1/2)*b^(7/2)*l
n(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))-4*4^(1/2)*b^(3/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2-4^(
1/2)*b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+b)/(-1+cos(f*x+e)))*a^3-4*4^(1/2)*b^(7/2)*ln(-4*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))-8*cos(f*x+e)^2*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(3/2)-16*cos(f*x+e)*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(3/2)+2*(a+b)^(3/2)*4^(1/2)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+3*4^(1/2)*b^(5/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-
8*4^(1/2)*b^(5/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-4*4^(1/2)*b^(3/2)
*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*c
os(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2-8*(a+b)^(3/2)*b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+2*cos(f*x+e)^2*4^(1/2)*b^(7/2)*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)
))+5*cos(f*x+e)^2*4^(1/2)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a+8*cos(f*x+e)^2*4^(1/2)*b^(
5/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-3*cos(f*x+e)^2*4^(1/2)*b^(5/2)
*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*c
os(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-4*cos(f*x+e)^2*(a+b)^(3/2)*4^(1/2)
*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/si
n(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2
+4*cos(f*x+e)^2*4^(1/2)*b^(3/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2+4*cos(f*x+e)^2*4^(1/2)*b^(
3/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+cos(f*x+e)^2*4^(1/2)*b^(1/2)
```

```
*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1
+cos(f*x+e)))*a^3+cos(f*x+e)^2*4^(1/2)*b^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+
b)^(1/2))*a^3-2*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*b^(3/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)+2*4^(1/2)*b^(7/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1
/2))-2*4^(1/2)*b^(7/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-2*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+4*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*(-1+
cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b)*cos(f*x+e)*((b+
a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)/sin(f*x+e)^4/(a+b)^(5/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a} \csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^{2}{\left(e + f x \right)}} \csc^{3}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.72 $\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=183

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\,\mathrm{se}}}{4f}$$

```
[Out] -1/8*(3*a^2+12*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))/(a+b)^(3/2)/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*
b^(1/2)/f-1/8*(3*a+4*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b
)/f-1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.22, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4134, 467, 578, 523, 217, 206, 377, 207}

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\,\mathrm{se}}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]])/(8*(a + b)^(3/2)*f) - ((3*a + 4*b)*Cot[e + f*x]*Csc[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqr
t[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+4bx^2\right)}{\left(-1+x^2\right)^2\sqrt{a+b}}\right.}{4}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\,\mathrm{c}}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\,\mathrm{c}}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\,\mathrm{c}}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+f}{\sqrt{a+b\sec^2(e+f}}\right.}{8(a+b)^{3/2}f}$$

Mathematica [A] time = 1.38, size = 198, normalized size = 1.08

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(-\left(3a^2+12ab+8b^2\right)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)+8\sqrt{b}(a+b)^2\tanh^{-1}\right.}{4\sqrt{2}f(a+b)^2\sqrt{a\cos(2(e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(8*Sqrt[b]*(a + b)^2*ArcTanh[Sqrt[
a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(3*a^2 + 12*a*b + 8*b^2)*A
rcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2
*(3*a + 4*b + 2*(a + b)*Csc[e + f*x]^2)*Sqrt[a + b - a*Sin[e + f*x]^2]))/(4
*Sqrt[2]*(a + b)^2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

fricas [B] time = 1.30, size = 1476, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2
)*cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e
)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4
- 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*
cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3
 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f
*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b +
 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/(a + b)) + 4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2
*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*cos
(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5
*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(16*((a^2 + 2*a*b + b^2)*cos(f*x
 + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*
cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^
2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
+ a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^
3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*
f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b
+ 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/(a + b)) - 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 -
2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(s
qrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a
^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*
x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(2*(-(-8192*a
-8192*b)*tan(1/2*(f*x+exp(1)))^2/(-2097152*a-2097152*b)-(-73728*a-90112*b)/
(-2097152*a-2097152*b))*sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1
)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)+2*((-(-s
qrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(
f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)
)*(-6*a^3+4*b^3+4*a*b^2-6*a^2*b)-(4*a^2-6*b^2)*(-sqrt(a+b)*tan(1/2*(f*x+exp
(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/
2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3-sqrt(a+b)*(-sqrt(a+b)
*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(
1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*(3*a
^2+7*b^2+10*a*b)-sqrt(a+b)*(-5*a^3-5*b^3-15*a*b^2-15*a^2*b))/((-sqrt(a+b)*t
an(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)
))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a-b)^2
/(-64*a-64*b)-1/2*b*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+
sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+e
xp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b)+1/2*(-3*a^2-
8*b^2-12*a*b)*atan((-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+
exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/
2*(f*x+exp(1)))^2+a+b))/sqrt(-a-b))/sqrt(-a-b)/(-16*a-16*b)+(-3*a^2-8*b^2-1
2*a*b)*ln(abs((a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x
+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1
/2*(f*x+exp(1)))^2+a+b))+sqrt(a+b)*(a-b)))/sqrt(a+b)/(-64*a-64*b)))*sign(co
s(f*x+exp(1)))/f
```

maple [B] time = 1.90, size = 9758, normalized size = 53.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\, \csc\left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}{\sin\left(e+f\,x\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.73 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^6(e+fx)\,dx$

Optimal. Leaf size=240

$$-\frac{(a-b)(5a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16a^2f}+\frac{\left(5a^3-15a^2b-5ab^2-b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16a^{5/2}f}$$

```
[Out] 1/16*(5*a^3-15*a^2*b-5*a*b^2-b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+
e)^2)^(1/2))/a^(5/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))*b^(1/2)/f-1/16*(a-b)*(5*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^
(1/2)/a^2/f-1/24*(5*a-b)*cos(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)
/a/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.38, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4132, 467, 578, 523, 217, 206, 377, 203}

$$\frac{\left(-15a^2b+5a^3-5ab^2-b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}-\frac{(a-b)(5a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

```
[Out] ((5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f - ((a - b)*(5*a + b)*Cos[e + f*x]*
Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a^2*f) - ((5*a - b)*Cos[e
+ f*x]*Sin[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) - (Cos[e + f
*x]*Sin[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{(1+x^2)^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4(5(a+}{(1+x^2)^3}\right.}{}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} - \frac{\cos(e+}{}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a}{}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a}{}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a}{}$$

$$= -\frac{\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\right.}{}$$

Mathematica [F] time = 8.91, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6, x]
```

fricas [A] time = 4.91, size = 1715, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/384*(96*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
 e)^4) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 - 2*
(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), 1/384
```

```
*(192*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
 b^2)*sin(f*x + e))) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f
*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^
2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a^3*f), 1/192*(48*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) - 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8
*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*co
s(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos
(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))
) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 -
 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*f), 1/192*(96*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(5*a^3 - 15*a^2*b -
 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^
3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a}\,\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

maple [C] time = 3.36, size = 2665, normalized size = 11.10

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/48/f*(-30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-10*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*cos(f*x+e)^4*a^2*b-40*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
cos(f*x+e)^3*a^2*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b
^2+40*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b+((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+33*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*cos(f*x+e)*a^2*b-14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)*a*b^2+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
```

```
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+10*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^3*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^3*sin(f*x+e)+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7
*a^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3-26*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3+26*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3+33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^3*a^3-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*
a^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-33*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+90*2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
a^2*b*sin(f*x+e)+30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-15*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-96*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e))
*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*sin(f*x+e)/(-1+cos(f*x+
e))/(b+a*cos(f*x+e)^2)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.74 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^4(e+fx)\,dx$

Optimal. Leaf size=181

$$\frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(}}{4f}$$

```
[Out] 1/8*(3*a^2-6*a*b-b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(3/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
-1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*cos(f
*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4132, 467, 578, 523, 217, 206, 377, 203}

$$\frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/f - ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b +
b*Tan[e + f*x]^2])/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
```

```
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2(\cdots}{(1+x\cdots}\right.}{\cdots}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e\cdots}{\cdots}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e\cdots}{\cdots}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e\cdots}{\cdots}$$

$$= \frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+\cdots}{\sqrt{a+b+b\tan^2\cdots}}\right.}{f}$$

Mathematica [F] time = 5.35, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4, x]
```

fricas [A] time = 1.83, size = 1565, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(16*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f)
, 1/64*(32*a^2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e
)^2 + b^2)*sin(f*x + e))) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3
 - (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(a^2*f), 1/32*(8*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x
+ e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
 + 8*b^2)/cos(f*x + e)^4) - (3*a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*
x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) +
4*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), 1/32*(16*a^2*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - (3*
a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)
*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
- 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*a^2*cos(f*x + e)^3 - (5*a^
2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

maple [C] time = 1.49, size = 1939, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*(2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2+6*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a^2*sin(f*x+e)-12*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-2*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*b^2*sin(f*x+e)+16*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^2*sin(f*x+e)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4
*a^2-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2+3*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b+5*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^2*a^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*c
os(f*x+e)^2*a*b-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b+((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2+5*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*
x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+
a*cos(f*x+e)^2)/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{4}\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\,\sin^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

3.75 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^2(e+fx)\,dx$

Optimal. Leaf size=123

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+a
rctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*cos(f*x
+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4132, 467, 523, 217, 206, 377, 203}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+b+}{\left(1+x^2\right)\sqrt{a}}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}\,f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \text{co}$$

Mathematica [C] time = 5.80, size = 432, normalized size = 3.51

$$\frac{e^{-i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{2e^{2i(e+fx)}\left(-i(a-b)\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+i\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]*(I*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a*f*x - 2*b*f*
```

```
x - I*(a - b)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*(a - b)*Log[a + a*E^((2
*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*Sqrt[a]*Sqrt[b]*Log[((-(Sqrt[b]*
(-1 + E^((2*I)*(e + f*x)))) + I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*f)/(2*b*(1 + E^((2*I)*(e + f*x))))]))/(Sqrt[a]*Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e +
f*x]^2])/(4*Sqrt[2]*E^(I*(e + f*x))*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

fricas [B] time = 1.09, size = 1417, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) - sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) - 4*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/(a*f), -1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)*sin(f*x + e) - 8*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*(a - b)*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*f), -1/8*(4*a*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b
)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 +
 (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*
x + e)^2)*sin(f*x + e))) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4))/(a*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*a*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/(a*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx+e\right)^2 + a} \sin \left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

maple [C] time = 1.18, size = 1290, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*(-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)+2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-4*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)
+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*cos(f*x+e)*sin(f*x
+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)
^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a}\,\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{2}\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,\sin^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.76 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+f}$$
$$= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.91, size = 1227, normalized size = 15.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
```

```
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 4.74, size = 588, normalized size = 7.44

$$-\frac{\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin\left(fx+e\right)}, \sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^{2}+6ab-b^{2}}{\left(a+b\right)^{2}}}\right)a+\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin\left(fx+e\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*(EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a+EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-2*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a)*cos(f*x+e)*sin(f*x+e)^2*((b+a*co
s(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)/(-1+
cos(f*x+e))/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^{2}{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

3.77 $\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = $\{4132, 277, 217, 206\}$

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \text{t}\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a+l}}\right.}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{f}$$

Mathematica [C] time = 0.20, size = 61, normalized size = 0.90

$$-\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, (b*Sin[e + f*x]^2)/(a + b -
 a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

fricas [B] time = 0.67, size = 306, normalized size = 4.50

$$\left[\frac{\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin(f}{4f\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f
*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*si
n(f*x + e) - 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f
*sin(f*x + e)), 1/2*(sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f
*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(fx+e)^2+a}\,\csc(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

maple [C] time = 2.06, size = 1003, normalized size = 14.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b-2*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*si
n(f*x+e)*cos(f*x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f
*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(b+a*cos(f*x+e)
^2)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.33, size = 50, normalized size = 0.74

$$\frac{\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - sqrt(b*tan(f*x + e)^2 +
a + b)/tan(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.78 $\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)/(
a+b)/f
```

Rubi [A] time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4132, 451, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b
+ b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+b+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan\right.}{3(a+b)f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan\right.}{3(a+b)f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{f}$$

Mathematica [C] time = 7.22, size = 285, normalized size = 2.71

$$-\frac{\sqrt{2}\cot(e+fx)\csc^2(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}\left(\frac{4b\tan^2(e+fx)\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)^2\sqrt{\frac{a+b}{}}}{(a+b)^2}\right.}{3f\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a\,c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[2]*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (
a*Sin[e + f*x]^2)/(a + b))*((4*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e +
f*x]^2)/(a + b))]*Sec[e + f*x]^2*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a +
b - a*Sin[e + f*x]^2)^2*Tan[e + f*x]^2)/(a + b)^2 + (a + b + 2*a*Sin[e + f*
x]^2)*(Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + ArcSin[Sqrt[-((b*Tan[e + f*x]
^2)/(a + b))]]*Sqrt[-((b*Tan[e + f*x]^2)/(a + b))])))/(f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e
+ f*x]^2])
```

fricas [B] time = 1.04, size = 436, normalized size = 4.15

$$\left[\frac{3\left((a+b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{b}\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}}{\cos\left(fx+e\right)^4}\right.}{12\left((a+b)f\cos\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*
cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 +
 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e
)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), 1/6*(3*((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(
f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x + e)
^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\,\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

maple [C] time = 1.86, size = 3847, normalized size = 36.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*(-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^3*sin(
f*x+e)*a*b-3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*b^2+6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^3*sin(f*x+e)*a*b+6*cos(f*x+e)^3*
sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(
```

```
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x
+e)*a*b+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+3*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b+3*cos(f*x+e)*sin(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-6*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*c
os(f*x+e)*sin(f*x+e)*a*b-6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+2*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2+3*cos(f*x+e)^4*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-6*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^2*sin
```

```
(f*x+e)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2-2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+3*cos(f*x+e)^2*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f
*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)^3/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)
```

maxima [A] time = 0.35, size = 82, normalized size = 0.78

$$\frac{3\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)-\frac{3\sqrt{b\tan\left(fx+e\right)^2+a+b}}{\tan\left(fx+e\right)}-\frac{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 3*sqrt(b*tan(f*x +
 e)^2 + a + b)/tan(f*x + e) - (b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan
(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.79 $\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5f(a+b)}-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e\right.}{15f(a+b)^2}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a+4*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)
^2)^(3/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4132, 462, 451, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5f(a+b)}-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e\right.}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (2*(5*a + 4*b)*Cot[e +
f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(15*(a + b)^2*f) - (Cot[e + f*x]^5
*(a + b + b*Tan[e + f*x]^2)^(3/2))/(5*(a + b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2\sqrt{a+b+bx^2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\,(2(5a+4b)\cdots}{x^4}\right.}{5(a\cdots}$$

$$= -\frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2 f} - \frac{\cot^5(e+fx)\left(a\cdots\right.}{5\cdots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b\cdots\right.}{15(a+b)^2 \cdots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b\cdots\right.}{15(a+b)^2 \cdots}$$

$$= \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \cdots$$

Mathematica [C] time = 7.98, size = 422, normalized size = 2.83

$$\frac{\sqrt{2}\,e^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{i\left(8a^2\left(-6e^{2i(e+fx)}+16e^{4i(e+fx)}-6e^{6i(e+fx)}+e^{8i(e+fx)}+1\right)+ab\left(-136e^{2i(e+fx)}+\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]*(((-I)*(8*a^2*(1 - 6*E^((2*I)*(e + f*x)) + 16*E^
```

```
((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x)) + E^((8*I)*(e + f*x))) + b^2*(15
- 80*E^((2*I)*(e + f*x)) + 178*E^((4*I)*(e + f*x)) - 80*E^((6*I)*(e + f*x))
 + 15*E^((8*I)*(e + f*x))) + a*b*(25 - 136*E^((2*I)*(e + f*x)) + 318*E^((4*
I)*(e + f*x)) - 136*E^((6*I)*(e + f*x)) + 25*E^((8*I)*(e + f*x)))))/((a + b
)^2*(-1 + E^((2*I)*(e + f*x)))^5) - (15*Sqrt[b]*Log[(-4*Sqrt[b]*(-1 + E^((2
*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) +
a*(1 + E^((2*I)*(e + f*x)))^2])*Sqrt[a + b*Sec[e + f*x]^2])/(15*f*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]])
```

fricas [B] time = 2.47, size = 656, normalized size = 4.40

$$\left[\frac{15\left(\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{b}\,\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(\ldots}{\ldots}\right)}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x
+ e)^5 - (20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*
b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*
a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2
 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/30*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e
)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*ar
ctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*
sin(f*x + e) - 2*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x + e)^5 - (20*a^2 + 59*a
*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*b^2)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x +
e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)*sin(
f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

maple [C] time = 1.84, size = 8587, normalized size = 57.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.34, size = 143, normalized size = 0.96

$$\frac{15\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right) - \frac{15\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{\tan\left(fx+e\right)} - \frac{10\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}}{(a+b)\tan\left(fx+e\right)^{3}} + \frac{2\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}{(a+b)^{2}\tan\left(fx+e\right)^{3}} - \frac{3\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}}{(a+b)\tan\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 15*sqrt(b*tan(f*
x + e)^2 + a + b)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a +
b)*tan(f*x + e)^3) + 2*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)^2*tan(f*
x + e)^3) - 3*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan(f*x + e)^5))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}{\sin\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.80 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=196

$$\frac{b(3a-4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e\right.}{3af}$$

```
[Out] -1/3*(3*a-4*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+2/3*cos(f*x+e)^3*(a+
b*sec(f*x+e)^2)^(5/2)/a/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(5/2)/a/f+1/2
*(3*a-4*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1
/2*(3*a-4*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4134, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a-4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 4*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 4*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (2*C
os[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f) - (Cos[e + f*x]^5*(a +
b*Sec[e + f*x]^2)^(5/2))/(5*a*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(-10a+5ax^2\right)\left(a+bx^2\right)^{3/2}}{x^4}\right.}{5af}$$

$$= \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+\right.}{5af}$$

$$= -\frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+\right.}{3a}$$

$$= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(\right.}{3a}$$

$$= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(\right.}{3a}$$

$$= \frac{(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b}}{2af}$$

Mathematica [A] time = 1.31, size = 188, normalized size = 0.96

$$\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-5(3a-4b)\left(\sqrt{-a\sin^2(e+fx)+a+b}\left(-a\sin^2(e+fx)+a+4b\right)-3b^{\cdots}\right.\right.}{15bf(a\cos(2(e+fx))\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-6*b*(a + b - a*Sin[
e + f*x]^2)^(5/2))/a + 15*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2) -
 5*(3*a - 4*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]
+ Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(15*b*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

fricas [A] time = 1.28, size = 351, normalized size = 1.79

$$\left[-\frac{15\left(3\,a^2-4\,ab\right)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^2-2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)+2\left(6\,a^2\cos\left(fx+e\right)^6-4\left(5\right.\right.}{60\,af\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 4*a*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(
f*x + e)^2) + 2*(6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4 +
2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 4*a*b)*sqrt(-
b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
/b)*cos(f*x + e) + (6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4
 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)4*(1/2*(sqrt(a+b)*(2
*b^3-a*b^2+a^2*b)*sign(cos(f*x+exp(1)))+(2*b^2+a*b)*(-sqrt(a+b)*tan(1/2*(f*
x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*t
an(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*sign(cos(f*x+exp
(1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*
tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)
))^2+a+b))*(-2*b^3-3*a*b^2+3*a^2*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-2*b^2
+3*a*b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+
b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(
```

```
1)))^2+a+b))^2*sign(cos(f*x+exp(1))))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*
x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*t
an(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-
2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^2-1/
15*(-sqrt(a+b)*(-768*a^6-15*b^6+90*a*b^5-690*a^2*b^4+2200*a^3*b^3-4103*a^4*
b^2+3654*a^5*b)*sign(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+s
qrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+ex
p(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(320*a^5+420*b^5-1260*a*b^4-12
60*a^2*b^3+10620*a^3*b^2-3560*a^4*b)*sign(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(
1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^
4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^5*(832*a^4-
210*b^4+1680*a*b^3-5370*a^2*b^2-60*a^3*b)*sign(cos(f*x+exp(1)))-(-sqrt(a+b)
*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(
1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^7*(-32
0*a^3-300*b^3+1020*a*b^2+120*a^2*b)*sign(cos(f*x+exp(1)))-(-15*b^2+30*a*b)*
(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/
2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a
+b))^9*sign(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan
(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+
2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-2880*a^6+105*b^6-510*a*b^5+2430*a^2*b^4
-3160*a^3*b^3-3695*a^4*b^2+11150*a^5*b)*sign(cos(f*x+exp(1)))-sqrt(a+b)*(-s
qrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(
f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)
)^2*(-3200*a^5-300*b^5+1380*a*b^4-3420*a^2*b^3-1620*a^3*b^2+11800*a^4*b)*si
gn(cos(f*x+exp(1)))-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*ta
n(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2
+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4*(2560*a^4-210*b^4+5670*a^2*b^2-9420*a^
3*b)*sign(cos(f*x+exp(1)))-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sq
rt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp
(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^6*(-640*a^3+420*b^3-2100*a*b^2+30
00*a^2*b)*sign(cos(f*x+exp(1)))-sqrt(a+b)*(105*b^2-330*a*b)*(-sqrt(a+b)*tan
(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))
^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^8*sign(cos
(f*x+exp(1))))/(2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(
1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2
*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a
*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1))
)^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2+3*a-b)^5-1/4*(-4*b^2+3*a*b)*sign(co
s(f*x+exp(1)))*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(
a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)
))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b))/f
```

maple [B] time = 1.77, size = 2537, normalized size = 12.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x)
```

```
[Out] 1/60/f*(-1+cos(f*x+e))^3*(60*cos(f*x+e)^2*b^(13/2)*ln(-2*(-1+cos(f*x+e))*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2
/(a+b)^(1/2))*a-60*cos(f*x+e)^2*b^(13/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/
2))*a+45*cos(f*x+e)^2*b^(5/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5-45*
cos(f*x+e)^2*b^(5/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
```

```
e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5+6*cos(f*x+e)
^2*b^(7/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-15*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*(a+b)^(7/2)*a-15*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)*(a+b)^(7/2)*a^2+315*cos(f*x+e)^2*b
^(9/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+6*cos(f*x+e)^3*b^(7/2)*(a+
b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+225*cos(f*x+e)^2*b^(11
/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-
a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2-225*cos(f*x+e)^2*b^(11/2)*ln(
-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2-315*cos(f*x+e)^2*b^(9/2)*ln(-4*(-1+c
os(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)
/sin(f*x+e)^2/(a+b)^(1/2))*a^3+195*cos(f*x+e)^2*b^(7/2)*ln(-2*(-1+cos(f*x+e
))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x
+e)^2/(a+b)^(1/2))*a^4-195*cos(f*x+e)^2*b^(7/2)*ln(-4*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))*a^4-45*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos
(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^3*b+15*cos(f*x+e)^2*(a+b)^(7/2)*ar
ctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f
*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^2*b^
2+60*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/
2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*4^(1/2))*a*b^3-8*cos(f*x+e)^5*b^(3/2)*(a+b)^(7/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+12*cos(f*x+e)^4*b^(5/2)*(a+b)^(7/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-20*a^3*b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)^5-8*a^2*b^(3/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)^4-74*b^(5/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)^3*a-20*c
os(f*x+e)^4*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^3-50*cos(f*x+e)^3*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^2-74*cos(f*x+e)^2*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a+30*cos(f*x+e)^3*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^3-50*a^2*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)^2-15*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)*a+30*cos(f*x+e)^2*b^(1/2)*(a+b)^(
7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3-15*a^2*b^(3/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)+6*a^2*b^(3/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)^7+6*a^3*b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(f*x+e)^7
+6*a^2*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*cos(
f*x+e)^6+12*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)
*cos(f*x+e)^5*a+6*cos(f*x+e)^6*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a^3)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
*4^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/a/b^(1/2)
/(a+b)^(9/2)
```

maxima [A] time = 0.43, size = 277, normalized size = 1.41

$$-\frac{\frac{12\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a}-40\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3+60\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a\cos\left(fx+e\right)-120\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/a - 40*(a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e
) - 120*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) - 30*sqrt(a + b/cos(f*x +
 e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 45*a*
sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b
/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 60*b^(3/2)*log((sqrt(a + b/cos(
f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e
) + sqrt(b))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.81 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=162

$$\frac{b(3a-2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a-2b)\cos(e+fx)\left(a+b\right.}{3af}$$

```
[Out] -1/3*(3*a-2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+1/3*cos(f*x+e)^3*(a+
b*sec(f*x+e)^2)^(5/2)/a/f+1/2*(3*a-2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec
(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*(3*a-2*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(
1/2)/a/f
```

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4134, 453, 277, 195, 217, 206}

$$\frac{b(3a-2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a-2b)\cos(e+fx)\left(a+b\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 2*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 2*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a-2b)\,\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}\,d\ldots\right.}{3af}$$

$$= -\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\ldots\right.}{3af}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(\ldots\right.}{3a\ldots}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(\ldots\right.}{3a\ldots}$$

$$= \frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec(e+fx)\sqrt{a+\ldots}}{2af}$$

Mathematica [A] time = 0.76, size = 164, normalized size = 1.01

$$\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(3\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)^{5/2}-(3a-2b)\left(\sqrt{-a\sin^2(e+f\ldots}\right.\right.}{3bf(a\cos(2(e+fx))+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(3*Sec[e + f*x]^2*(a +
 b - a*Sin[e + f*x]^2)^(5/2) - (3*a - 2*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b -
 a*Sin[e + f*x]^2]/Sqrt[b]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*S
in[e + f*x]^2))))/(3*b*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

fricas [A] time = 0.85, size = 278, normalized size = 1.72

$$\left[-\frac{3\,(3\,a-2\,b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^{2}-2\,\sqrt{b}\,\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}\right)-2\left(2\,a\cos\left(fx+e\right)^{4}-2\,(3\,a-4\right.}{12\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x +
 e)^2) - 2*(2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/6*(3*(3*a - 2*b
)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b)*cos(f*x + e) - (2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)
^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)4*(1/2*(sqrt(a+b)*(2*b^3-a*b^2+a^2*b)*sign(cos
(f*x+exp(1)))+(2*b^2+a*b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/
2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b
*tan(1/2*(f*x+exp(1)))^2+a+b))^3*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*
(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*
a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-2*b^3-3*a*b^2
+3*a^2*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-2*b^2+3*a*b)*(-sqrt(a+b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-
2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*sign(cos(f*
x+exp(1))))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/
2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b
*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*t
an(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^
2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^2+1/3*(sqrt(a+b)*(-20*a^4+6*a*
b^3-24*a^2*b^2+46*a^3*b)*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp
(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/
2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(16*a^3+12*a*b^2-36*a
^2*b)*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(
1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2
*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-48*a^4-18*a*b^3+12*a^2*b^2+78*a^3*b)*sig
n(cos(f*x+exp(1)))+6*a*b*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2
*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*
```

```
tan(1/2*(f*x+exp(1)))^2+a+b))^5*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-sqrt(a+b)
*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(
1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*(-24
*a^3+12*a*b^2+36*a^2*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(12*a^2-18*a*b)*(-s
qrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(
f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)
)^4*sign(cos(f*x+exp(1))))/(2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2
+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+
exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1
)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*
(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2+3*a-b)^3-1/4*(-2*b^2+3*
a*b)*sign(cos(f*x+exp(1)))*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqr
t(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2
*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b))/f
```

maple [B] time = 1.43, size = 1913, normalized size = 11.81

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x)
```

```
[Out] -1/12/f*(-1+cos(f*x+e))^3*(-6*cos(f*x+e)^2*b^(13/2)*ln(-2*(-1+cos(f*x+e))*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^
2/(a+b)^(1/2))-6*cos(f*x+e)^2*b^(7/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))
*a^3+6*cos(f*x+e)^2*b^(7/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+8*cos
(f*x+e)^3*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+8
*cos(f*x+e)^2*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)+3*cos(f*x+e)*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)+3*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-6*
cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*
cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(1/2)*4^(1/2))*b^3-18*cos(f*x+e)^2*b^(11/2)*ln(-2*(-1+cos(f*x+e))*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/
(a+b)^(1/2))*a+18*cos(f*x+e)^2*b^(11/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2
))*a-18*cos(f*x+e)^2*b^(9/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+18*c
os(f*x+e)^2*b^(9/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+6*cos(f*x+e)^
2*b^(13/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))+3*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(5/2)*(a+b)^(7/2)+9*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(
1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^
2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^2*b+3*cos(
f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(
f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b^(1/2)*4^(1/2))*a*b^2+2*cos(f*x+e)^5*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*cos(f*x+e)^4*b^(3/2)*(a+b)^(7/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+2*cos(f*x+e)^4*b^(1/2)*(a+b)^(7/2)*((b
```

```
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*cos(f*x+e)^3*b^(3/2)*(a+b)^(
7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-6*cos(f*x+e)^3*b^(1/2)*(
a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*cos(f*x+e)^2*b
^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-6*cos(f*x+
e)^2*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+3*
cos(f*x+e)*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a+2*cos(f*x+e)^5*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/b^(1/2)/(a+b)^(9/2)
```

maxima [A] time = 0.44, size = 250, normalized size = 1.54

$$\frac{4\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-12\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a\cos\left(fx+e\right)+12\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos\left(fx+e\right)+\frac{6\sqrt{\cdots}}{\left(a+\cdots\right.}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a + b/cos(f*x
 + e)^2)*a*cos(f*x + e) + 12*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) + 6*
sqrt(a + b/cos(f*x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x
 + e)^2 - b) - 9*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - s
qrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*b^(3/2)*lo
g((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x +
 e)^2)*cos(f*x + e) + sqrt(b))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.82 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

```
[Out] -cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*a*arctanh(sec(f*x+e)*b^(1/2)/(a+
b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/
f
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4134, 277, 195, 217, 206}

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*f) + (3*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f) - (Cos[e + f*x]*(
a + b*Sec[e + f*x]^2)^(3/2))/f
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
```

```
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,\right.}{f}$$

$$= \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a+b\sec^2(e+f\right.}{f}$$

$$= \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a+b\sec^2(e+f\right.}{f}$$

$$= \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx}}{2f}$$

Mathematica [C] time = 0.60, size = 73, normalized size = 0.73

$$-\frac{a\cos(e+fx)(a\cos(2(e+fx))+a+2b)^2\sqrt{a+b\sec^2(e+fx)}\;{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{a\cos^2(e+fx)}{b}+1\right)}{20b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] -1/20*(a*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Hypergeometric2F1[2,
 5/2, 7/2, 1 + (a*Cos[e + f*x]^2)/b]*Sqrt[a + b*Sec[e + f*x]^2])/(b^2*f)
```

fricas [A] time = 0.87, size = 231, normalized size = 2.31

$$\left[\frac{3\,a\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(2\,a\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\,f\cos\left(fx+e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(
2*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos
(f*x + e)), -1/2*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + (2*a*cos(f*x + e)^2 - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)4*((a^2*sqrt(a+b)*sign(cos(f*x+exp(1)))+a^2*(-sqrt(a+b)*tan(1/2*(f*x+exp
(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/
2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*sign(cos(f*x+exp(1))))/
(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1
)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*
x+exp(1)))^2+a+b))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+
exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/
2*(f*x+exp(1)))^2+a+b))^2-3*a+b)+1/2*(sqrt(a+b)*(2*b^3-a*b^2+a^2*b)*sign(co
s(f*x+exp(1)))+(2*b^2+a*b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1
/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*
b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2
*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2
*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-2*b^3-3*a*b^
2+3*a^2*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-2*b^2+3*a*b)*(-sqrt(a+b)*tan(1
/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4
-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*sign(cos(f
*x+exp(1))))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1
/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*
b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*
tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))
^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^2-3/4*a*b*sign(cos(f*x+exp(1)
))*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1/2*(f
*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan
(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-b))/sqrt(-b))/f
```

maple [A] time = 0.36, size = 121, normalized size = 1.21

$$-\frac{\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{fa\sec\left(fx+e\right)}+\frac{b\sec\left(fx+e\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec\left(fx+e\right)\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{2f}+3a\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(5/2)+1/f/a*b*sec(f*x+e)*(a+b*sec(f*x+
e)^2)^(3/2)+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f+3/2/f*a*b^(1/2)*ln(
sec(f*x+e)*b^(1/2)+(a+b*sec(f*x+e)^2)^(1/2))
```

maxima [A] time = 0.43, size = 142, normalized size = 1.42

$$-\frac{4\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,a\cos\left(fx+e\right)-\frac{2\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,ab\cos\left(fx+e\right)}{\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)\cos\left(fx+e\right)^{2}-b}+3\,a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+\sqrt{b}}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e) - 2*sqrt(a + b/cos(f*x +
e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 3*a*sq
rt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/c
os(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

mupad [B] time = 6.11, size = 61, normalized size = 0.61

$$-\frac{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\,{\cos\left(e+f\,x\right)}^2}\right)}{f\,{\left(\frac{b}{a\,{\cos\left(e+f\,x\right)}^2}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] -(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -b
/(a*cos(e + f*x)^2)))/(f*(b/(a*cos(e + f*x)^2) + 1)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

3.83 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

```
[Out] -(a+b)^(3/2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f+1/2
*(3*a+2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1
/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4134, 416, 523, 217, 206, 377, 207}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/f + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{-1+x^2}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a+b)+b(3a+2b)x^2}{\left(-1+x^2\right)\sqrt{a+bx^2}}dx,\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a+bx}}\right.}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\right.}{f}$$

$$= \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec}{\sqrt{a+b\sec^2}}\right.}{f}$$

Mathematica [A] time = 0.55, size = 171, normalized size = 1.40

$$\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\,b\sqrt{a\cos(2(e+fx))+a+2b}+2\sqrt{b}\,(3a+2b)\cos^2(e+fx)\tanh^{-1}\left(\frac{\sqrt{-a\sin}}{}\right.\right.}{2\sqrt{2}\,f\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[b]*(3*a + 2*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos
[e + f*x]^2 - 4*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a
 + b]]*Cos[e + f*x]^2 + Sqrt[2]*b*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])*Sec[e
 + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e
+ f*x)]])
```

fricas [A] time = 0.86, size = 715, normalized size = 5.86

$$\left[\frac{2\,(a+b)^{\frac{3}{2}}\cos\left(fx+e\right)\log\left(\frac{2\left(a\cos\left(fx+e\right)^{2}-2\,\sqrt{a+b}\,\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)+a+2\,b\right)}{\cos\left(fx+e\right)^{2}-1}\right)+(3\,a+2\,b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(-\right.}{4\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/co
s(f*x + e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
 + e)), 1/4*(4*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) + (3*a + 2*b)*s
qrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*((3*a + 2*b)*sqrt
(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b)*cos(f*x + e) - (a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2
*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
 2*b)/(cos(f*x + e)^2 - 1)) - b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(f*cos(f*x + e)), 1/2*(2*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) - (
3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Evaluation time
: 1.52Error: Bad Argument Type
```

maple [B] time = 1.58, size = 2563, normalized size = 21.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*4^(1/2)*cos(f*x+e)*(-1+cos(f
*x+e))^3*(-cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(13/2)-15*cos(f*x+e)^2*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*
b^(9/2)*a^2-6*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(11/2)*a-cos(f*x+e)^2*ln(-4*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e))
)*b^(1/2)*a^6-cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(1/2)*a^
6-6*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(3/2)*a^5-6*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+
e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*
x+e)^2/(a+b)^(1/2))*b^(3/2)*a^5-6*cos(f*x+e)^2*ln(-4*(-1+cos(f*x+e))*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+
b)^(1/2))*b^(5/2)*a^4-15*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(5/2)*a^4-9*cos(f*x+e)
^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*a^4-20*cos(f*x+e)^2*ln(-4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*
x+e)))*b^(7/2)*a^3+cos(f*x+e)^2*b^(13/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/
2))-20*cos(f*x+e)^2*b^(7/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+cos(f
*x+e)*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b^(3/
2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+2*cos(f*x+e)^2
*(a+b)^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4
^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
4^(1/2))*b^3+6*cos(f*x+e)^2*b^(11/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*
a-12*cos(f*x+e)^2*b^(11/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+9*cos(f*
x+e)^2*b^(9/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2-24*cos(f*x+e)^2*b^
(9/2)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2-2*cos(f*x+e)^2*b^(13/2)*ln(
```

```
-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))+((b+a*csc(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(5/2)*(a+b)^(7/2)+3*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(-1+cos(f*x+
e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^2*b+5*cos(f*x+e)^2*(a+b)^(
7/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2
)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))
*a*b^2+cos(f*x+e)*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a)/sin(f*x+e)^6/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/b^(1/2)/
(a+b)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.84 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}\,(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+}{\sqrt{a+b\sec^2(e+}\right.}{2f}$$

```
[Out] -1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+1/2*(3*a+4*b)*arctanh
(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+4*b)*arctanh
(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+b*sec(f*x+e
)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4134, 467, 528, 523, 217, 206, 377, 207}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}\,(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+}{\sqrt{a+b\sec^2(e+}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqr
t[a + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2
])/f - (Cot[e + f*x]*Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(-1+x^2\right)^2}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\,(}{-1+}\right.}{}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\,\text{se}\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\,\text{se}\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\,\text{se}\right.}{2f}$$

$$= \frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{\sqrt{a+b}\,(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+}}\right.}{2f}$$

Mathematica [A] time = 1.42, size = 202, normalized size = 1.25

$$-\frac{\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a\cos(2(e+fx))+a+2b}\,((a+2b)\cos(2(e+fx))+a)-\sqrt{b}\right.}{4\sqrt{2}f\sqrt{a\cos(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(Csc[e + f*x]^2*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*Sqrt[
a + 2*b + a*Cos[2*(e + f*x)]]*(a + (a + 2*b)*Cos[2*(e + f*x)]) - Sqrt[b]*(3
*a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Sin[2*(e + f*x)]^
2 + Sqrt[a + b]*(a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b
]]*Sin[2*(e + f*x)]^2))/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

fricas [A] time = 1.00, size = 984, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2
*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + ((3*a + 4*b)*cos(f*x +
e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e
)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(2*((a + 4*b)*cos(f*x +
e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + ((3*a + 4*b)*cos(f
*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqr
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*
x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*a + 4*b)*co
s(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 4*b)*cos(f*x + e
)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(
a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/
(cos(f*x + e)^2 - 1)) - 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/2*(((a +
4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - ((3
*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((a + 2*b)
*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*
x + e)^3 - f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
```

```
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Evaluation time: 1.72Unable to divide, perhaps due to rounding error%%%
{%%{[%%%{16384,[5,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[6]%
%%}+%%%{%%%{98304,[6,4]%%%}+%%%{98304,[5,5]%%%},[5]%%%}+%%%{%%{[%%%{245760,
[6,4]%%%}+%%%{49152,[5,5]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%}
,[4]%%%}+%%%{%%%{327680,[7,4]%%%}+%%%{-131072,[6,5]%%%}+%%%{-458752,[5,6]%%
%},[3]%%%}+%%%{%%{[%%%{245760,[7,4]%%%}+%%%{-688128,[6,5]%%%}+%%%{-147456,[
5,6]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[2]%%%}+%%%{%%%{9830
4,[8,4]%%%}+%%%{-491520,[7,5]%%%}+%%%{294912,[6,6]%%%}+%%%{884736,[5,7]%%%}
,[1]%%%}+%%%{%%{[%%%{16384,[8,4]%%%}+%%%{-147456,[7,5]%%%}+%%%{442368,[6,6]
%%%}+%%%{-442368,[5,7]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[0
]%%%} / %%%{%%{[%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+
%%%{-1,[0,1]%%%}]%%},[6]%%%}+%%%{%%%{-6,[2,0]%%%}+%%%{-12,[1,1]%%%}+%%%{-6,
[0,2]%%%},[5]%%%}+%%%{%%{[%%%{-15,[2,0]%%%}+%%%{-18,[1,1]%%%}+%%%{-3,[0,2]%
%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[4]%%%}+%%%{%%%{-20,[3,0]
%%%}+%%%{-12,[2,1]%%%}+%%%{36,[1,2]%%%}+%%%{28,[0,3]%%%},[3]%%%}+%%%{%%{[%%
%{-15,[3,0]%%%}+%%%{27,[2,1]%%%}+%%%{51,[1,2]%%%}+%%%{9,[0,3]%%%},0]:[1,0,%
%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[2]%%%}+%%%{%%%{-6,[4,0]%%%}+%%%{24,[3
,1]%%%}+%%%{12,[2,2]%%%}+%%%{-72,[1,3]%%%}+%%%{-54,[0,4]%%%},[1]%%%}+%%%{%%
{[%%%{-1,[4,0]%%%}+%%%{8,[3,1]%%%}+%%%{-18,[2,2]%%%}+%%%{27,[0,4]%%%},0]:[1
,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.82, size = 5178, normalized size = 32.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.85 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\left(a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}-\frac{3(a+2b)\csc^2(e+fx)\,\mathrm{s}}{}$$

[Out] -1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*(a+2*b)*arctanh (sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-3/8*(a^2+8*a*b+8*b^ 2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)+3 /8*(a+4*b)*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+2*b)*csc(f*x+e)^2*s ec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.34, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4134, 467, 577, 582, 523, 217, 206, 377, 207}

$$-\frac{3\left(a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}-\frac{3(a+2b)\csc^2(e+fx)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] (3*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x] ^2]])/(2*f) - (3*(a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/S qrt[a + b*Sec[e + f*x]^2]])/(8*Sqrt[a + b]*f) + (3*(a + 4*b)*Sec[e + f*x]*S qrt[a + b*Sec[e + f*x]^2])/(8*f) - (3*(a + 2*b)*Csc[e + f*x]^2*Sec[e + f*x] *Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a + b*Se c[e + f*x]^2)^(3/2))/(4*f)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(-1+x^2\right)^3}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b\ldots}}{(-\ldots}\right.}{\ldots}$$

$$= -\frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+f\ldots}{\ldots}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\,\text{s}\ldots}{\ldots}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\,\text{s}\ldots}{\ldots}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\,\text{s}\ldots}{\ldots}$$

$$= \frac{3\sqrt{b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{3\left(a^2+8ab+8b^2\right)\tanh^{-1}\left(\ldots\right.}{8\sqrt{a+b}\,f}$$

Mathematica [A] time = 3.35, size = 262, normalized size = 1.20

$$-\frac{\sec(e+fx)\left(a\cos^2(e+fx)+b\right)\sqrt{a+b\sec^2(e+fx)}\left(3b\sqrt{a+b}\left(a^2+8ab+8b^2\right)\cos^2(e+fx)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(\ldots}}{\sqrt{\ldots}}\right.\right.}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((b + a*Cos[e + f*x]^2)*(-12*b^(3/2)*(a^2 + 3*a*b + 2*b^2)*ArcTanh[Sqr
t[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos[e + f*x]^2 + 3*b*Sqrt[a + b]*(a^2
+ 8*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Cos[e
+ f*x]^2 + (b*(a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a + 4*b + 8*(a
 + 3*b)*Cos[2*(e + f*x)] - 3*(a + 4*b)*Cos[4*(e + f*x)])*Csc[e + f*x]^4)/(8
*Sqrt[2]))*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*(a + b)*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

fricas [A] time = 0.91, size = 1511, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos
(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + 3*a*b + 2*b^2)*cos(
f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)
```

```
*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 +
 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 +
 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos
(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*
((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e
)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + 6*((a^2
 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 +
 (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
 + e)^2) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b
^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*co
s(f*x + e)), -1/16*(24*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a
*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(-b)*a
rctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
- 3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x
+ e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 5*a*b + 4*b^2)*cos(f*x
+ e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*
f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*((a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^3 + (a^2 + 8*a*b + 8*b
^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 12*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*
cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*cos(f*x + e)/b) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2
+ 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3
 + (a + b)*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
```

```
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Evaluation ti
me: 2.54Unable to divide, perhaps due to rounding error%%%{%%%{262144,[6,6]
%%%},[12]%%%}+%%%{%%{[%%%{1572864,[6,6]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1
,[0,1]%%%}]%%},[11]%%%}+%%%{%%%{3145728,[7,6]%%%},[10]%%%}+%%%{%%{[%%%{5242
88,[7,6]%%%}+%%%{-12058624,[6,7]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%
%%}]%%},[9]%%%}+%%%{%%%{-7077888,[8,6]%%%}+%%%{-23592960,[7,7]%%%}+%%%{-393
2160,[6,8]%%%},[8]%%%}+%%%{%%{[%%%{-9437184,[8,6]%%%}+%%%{6291456,[7,7]%%%}
+%%%{40894464,[6,8]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[7]%%
%}+%%%{%%%{44040192,[8,7]%%%}+%%%{62914560,[7,8]%%%}+%%%{2097152,[6,9]%%%},
[6]%%%}+%%%{%%{[%%%{9437184,[9,6]%%%}+%%%{28311552,[8,7]%%%}+%%%{-47185920,
[7,8]%%%}+%%%{-66060288,[6,9]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}
]%%},[5]%%%}+%%%{%%%{7077888,[10,6]%%%}+%%%{-15728640,[9,7]%%%}+%%%{-896532
48,[8,8]%%%}+%%%{-53477376,[7,9]%%%}+%%%{13369344,[6,10]%%%},[4]%%%}+%%%{%%
{[%%%{-524288,[10,6]%%%}+%%%{-27262976,[9,7]%%%}+%%%{-3145728,[8,8]%%%}+%%%
{73400320,[7,9]%%%}+%%%{49807360,[6,10]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1
,[0,1]%%%}]%%},[3]%%%}+%%%{%%%{-3145728,[11,6]%%%}+%%%{-6291456,[10,7]%%%}+
%%%{31457280,[9,8]%%%}+%%%{50331648,[8,9]%%%}+%%%{-3145728,[7,10]%%%}+%%%{-
18874368,[6,11]%%%},[2]%%%}+%%%{%%{[%%%{-1572864,[11,6]%%%}+%%%{4718592,[10
,7]%%%}+%%%{9437184,[9,8]%%%}+%%%{-15728640,[8,9]%%%}+%%%{-33030144,[7,10]%
%%}+%%%{-14155776,[6,11]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},
[1]%%%}+%%%{%%%{-262144,[12,6]%%%}+%%%{1572864,[11,7]%%%}+%%%{-786432,[10,8
]%%%}+%%%{-7340032,[9,9]%%%}+%%%{2359296,[8,10]%%%}+%%%{14155776,[7,11]%%%}
+%%%{7077888,[6,12]%%%},[0]%%%} / %%%{%%{[%%%{1,[1,0]%%%}+%%%{1,[0,1]%%%},0
]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[12]%%%}+%%%{%%%{6,[2,0]%%%}+%
%%{12,[1,1]%%%}+%%%{6,[0,2]%%%},[11]%%%}+%%%{%%{[%%%{12,[2,0]%%%}+%%%{12,[1
,1]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[10]%%%}+%%%{%%%{2,[3
,0]%%%}+%%%{-42,[2,1]%%%}+%%%{-90,[1,2]%%%}+%%%{-46,[0,3]%%%},[9]%%%}+%%%{%
%{[%%%{-27,[3,0]%%%}+%%%{-117,[2,1]%%%}+%%%{-105,[1,2]%%%}+%%%{-15,[0,3]%%%
},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[8]%%%}+%%%{%%%{-36,[4,0]%%
%}+%%%{-48,[3,1]%%%}+%%%{168,[2,2]%%%}+%%%{336,[1,3]%%%}+%%%{156,[0,4]%%%},
[7]%%%}+%%%{%%{[%%%{168,[3,1]%%%}+%%%{408,[2,2]%%%}+%%%{248,[1,3]%%%}+%%%{8
,[0,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[6]%%%}+%%%{%%%{36
,[5,0]%%%}+%%%{180,[4,1]%%%}+%%%{72,[3,2]%%%}+%%%{-504,[2,3]%%%}+%%%{-684,[
1,4]%%%}+%%%{-252,[0,5]%%%},[5]%%%}+%%%{%%{[%%%{27,[5,0]%%%}+%%%{-33,[4,1]%
%%}+%%%{-402,[3,2]%%%}+%%%{-546,[2,3]%%%}+%%%{-153,[1,4]%%%}+%%%{51,[0,5]%%
%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[4]%%%}+%%%{%%%{-2,[6,0]%%
%}+%%%{-108,[5,1]%%%}+%%%{-222,[4,2]%%%}+%%%{152,[3,3]%%%}+%%%{738,[2,4]%%%
}+%%%{660,[1,5]%%%}+%%%{190,[0,6]%%%},[3]%%%}+%%%{%%{[%%%{-12,[6,0]%%%}+%%%
{-36,[5,1]%%%}+%%%{96,[4,2]%%%}+%%%{312,[3,3]%%%}+%%%{180,[2,4]%%%}+%%%{-84
,[1,5]%%%}+%%%{-72,[0,6]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},
[2]%%%}+%%%{%%%{-6,[7,0]%%%}+%%%{6,[6,1]%%%}+%%%{66,[5,2]%%%}+%%%{30,[4,3]%
%%}+%%%{-210,[3,4]%%%}+%%%{-366,[2,5]%%%}+%%%{-234,[1,6]%%%}+%%%{-54,[0,7]%
%%},[1]%%%}+%%%{%%{[%%%{-1,[7,0]%%%}+%%%{5,[6,1]%%%}+%%%{3,[5,2]%%%}+%%%{-3
1,[4,3]%%%}+%%%{-19,[3,4]%%%}+%%%{63,[2,5]%%%}+%%%{81,[1,6]%%%}+%%%{27,[0,7
]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0,1]%%%}]%%},[0]%%%} Error: Bad Argu
ment Value
```

maple [B] time = 2.13, size = 10199, normalized size = 46.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \csc \left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^5} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.86 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=298

$$-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}+\frac{\left(5a^2-40ab+3b^2\right)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan}}{48af}$$

```
[Out] 1/16*(5*a^3-45*a^2*b+15*a*b^2+b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/a^(3/2)/f+1/2*(3*a-5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan
(f*x+e)^2)^(1/2))*b^(1/2)/f-1/16*(5*a^2-26*a*b+b^2)*(a+b+b*tan(f*x+e)^2)^(1
/2)*tan(f*x+e)/a/f+1/48*(5*a^2-40*a*b+3*b^2)*sin(f*x+e)^2*(a+b+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)/a/f+1/24*(5*a-3*b)*sin(f*x+e)^4*(a+b+b*tan(f*x+e)^2)^(
1/2)*tan(f*x+e)/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.47, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4132, 467, 577, 578, 582, 523, 217, 206, 377, 203}

$$\frac{\left(-45a^2b+5a^3+15ab^2+b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] ((5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/(16*a^(3/2)*f) + ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - ((5*a^2 - 26*a*
b + b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a^2 -
40*a*b + 3*b^2)*Sin[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(48*a*f) + ((5*a - 3*b)*Sin[e + f*x]^4*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(24*f) - (Cos[e + f*x]*Sin[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(6*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+\ldots}{\ldots}$$

$$= \frac{\left(5a^2-40ab+3b^2\right)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af}$$

$$= -\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{\left(5a^2-40\ldots\right.}{\ldots}$$

$$= -\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{\left(5a^2-40\ldots\right.}{\ldots}$$

$$= -\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{\left(5a^2-40\ldots\right.}{\ldots}$$

$$= \frac{\left(5a^3-45a^2b+15ab^2+b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(3a-5b)\sqrt{b}\ldots}{\ldots}$$

Mathematica [F] time = 9.99, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6, x]
```

fricas [A] time = 17.85, size = 1855, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 48*(3*a^3 - 5*
a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*
b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqr
```

```
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(
f*x + e)^4) + 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4
 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), 1/384*(96*
(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(5*a^3 - 45*a^2*b + 15*a*b^2
+ b^3)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
 e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*
a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x
 + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), -1/
192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
 e) + 24*(3*a^3 - 5*a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*co
s(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a
^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f
*x + e)), -1/192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*cos(f*x + e) - 48*(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(8*a^3*
cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 -
 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \sin \left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

maple [C] time = 2.28, size = 3067, normalized size = 10.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x)
```

```
[Out] -1/48/f*(270*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
```

```
2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b+94*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+9*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b-68*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^3*a*b^2-9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*
a^2*b+68*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2-24*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2-94*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b-26*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^7*a^3+26*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^6*a^3+33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3-33*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3+8*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^9*a^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^8*a^3+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)
^3*b^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3-144*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a^2*b+240*cos(f*x+e)^2*sin(f*x+e)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*a*b^2-63*cos(f*x+e)^2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*a^2*b-75*cos(f*x+e)^2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*a*b^2-90*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b
^2+24*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-17*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b^2-22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^6*a^2*b+22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^7*a^2*b+17*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a*b^2+
3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*b
^3-30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3-6*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
```

```
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f
*x+e)*cos(f*x+e)^2*b^3+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*s
in(f*x+e)*cos(f*x+e)^2*a^3)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3
/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/a/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx + e\right)^{2} + a\right)^{\frac{3}{2}} \sin \left(fx + e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.87 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=217

$$\frac{3\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{3(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{3(a-b)\sin^2(e+fx)}{}$$

```
[Out] 3/8*(a^2-6*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
/a^(1/2)+3/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b
^(1/2)/f-3/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/8*(a-b)*sin(
f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/4*cos(f*x+e)*sin(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.34, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4132, 467, 577, 582, 523, 217, 206, 377, 203}

$$\frac{3\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{3(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{3(a-b)\sin^2(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 3*b)*Tan[e + f*x]*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(8*f) + (3*(a - b)*Sin[e + f*x]^2*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b +
 b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2}{}\right.}{}$$

$$= \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+}{}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+f}{}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+f}{}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+f}{}$$

$$= \frac{3\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b}\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{2f}$$

Mathematica [A] time = 5.31, size = 211, normalized size = 0.97

$$\frac{3\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(\frac{\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}} + 4\sqrt{b}(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)\right)}{2\sqrt{2}f(a\cos(2e+2fx)+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]])/Sqrt[a] + 4*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/
(2*Sqrt[2]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((-7*a + 26*b + (-6*a
+ 10*b)*Cos[2*(e + f*x)] + a*Cos[4*(e + f*x)])*Sqrt[a + b*Sec[e + f*x]^2]*T
an[e + f*x])/(32*f)
```

fricas [A] time = 6.16, size = 1667, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
```

```
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 24*(a^2 - a*b)*sqrt(b)*cos(f*x
+ e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*a^2*
cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), 1/64*(48*(a^2 - a
*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*
sin(f*x + e)))*cos(f*x + e) - 3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b
^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^
3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqr
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*
cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a^2 - 6
*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2
*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + 12*(a^2 - a*b)*sqrt(b)*co
s(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x
+ e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(
2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a
^2 - 6*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - 24*(a^2 - a*b)*sqrt
(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x
+ e)))*cos(f*x + e) - 4*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^
2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*c
os(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

maple [C] time = 1.80, size = 2309, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x)
```

```
[Out] 1/8/f*(2*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+6*sin(f*x
+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
```

```
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-36*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*sin(f*x+e)*cos(
f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+6*cos(f*x+e)^2
*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+9*cos(f*x+e
)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+24*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a*b-24*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*b^2-2*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-5*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2+7*cos(f*x+e)^5*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*cos(f*x+e)^4*a^2-7*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b+5*cos(f*x+e)
^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^2*a*b-5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*b^2+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-4*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/co
s(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

3.88 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=161

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}+\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)\cdot}}\right)}{2f}$$

```
[Out] 1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f
+1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)
/f+b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2*cos(f*x+e)*sin(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4132, 467, 528, 523, 217, 206, 377, 203}

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}+\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)\cdot}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^2\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f}+\frac{\text{Subst}\left(\int\right.}{}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cos(e+fx)\sin(e+fx)\left(\right.}{2}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cos(e+fx)\sin(e+fx)\left(\right.}{2}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cos(e+fx)\sin(e+fx)\left(\right.}{2}$$

$$=\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{(3a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+b+}}\right.}{2f}$$

Mathematica [C] time = 5.76, size = 493, normalized size = 3.06

$$\frac{e^{-i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{i\left(-1+e^{2i(e+fx)}\right)\left(a\left(1+e^{2i(e+fx)}\right)^2-4be^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2e^{2i(e+fx)}\left(-i\sqrt{a}\,(a-3b)\log\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((I*(-1 + E^((2*I)*(e + f*x)))*(-4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2))/(1 + E^((2*I)*(e + f*x)))^2 + (2*E^((2*I)*(e + f*x))*(
2*Sqrt[a]*(a - 3*b)*f*x - I*Sqrt[a]*(a - 3*b)*Log[a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] + I*Sqrt[a]*(a - 3*b)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
+ f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + 2*Sqrt[b]*(-3*a + b)*Log[((Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/(b*(-3*a +
 b)*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*E^(I*(e + f*
x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 2.07, size = 1535, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a)*(a - 3*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b
)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7
*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e)) + 2*(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(a*cos(f*x + e)^2 - b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e))
, 1/16*(4*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - sqrt(-a)*(a - 3*b)*cos(f*x +
e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(a*
cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
```

```
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) +
(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) + 4*(a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e) - 2*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(a*cos(f*x +
e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(
f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

maple [C] time = 1.26, size = 1583, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x)
```

```
[Out] 1/2/f*(6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a*b-2*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^2+2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-6*si
n(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
```

```
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^5*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)^4*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \sin \left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**2,x)
[Out] Timed out
```

3.89 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4128, 416, 523, 217, 206, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int\frac{(a+b)(2a+b)+b(3a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx,x,\tan(e\right.}{2f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}+\frac{a^2\,\text{Subst}\left(\int\frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+\right.}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}+\frac{a^2\,\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{f}$$

$$=\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e\ldots}{}$$

Mathematica [C] time = 4.99, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}\,e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+ia^{3/2}\log\ldots}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
```

```
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 1.40, size = 1457, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.47, size = 1557, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*(6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a*b+2*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^2-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b-cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b-((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*
cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

3.90 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=105

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right.}{f}$$

```
[Out] 3/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+
3/2*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-cot(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(3/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4132, 277, 195, 217, 206}

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*
x]^2]])/(2*f) + (3*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f) - (
Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/f
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^2}dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\,\text{Subst}\left(\int \sqrt{a+b\ldots}\right.}{\ldots}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b+b\,\text{t}\ldots\right.}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b+b\,\text{t}\ldots\right.}{f}$$

$$= \frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{3b\tan(e+fx)\sqrt{a+b+\ldots}}{2f}$$

Mathematica [C] time = 0.18, size = 64, normalized size = 0.61

$$-\frac{(a+b)\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\;{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{b\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x]*Hypergeometric2F1[-1/2, 2, 1/2, (b*Sin[e + f*x]^2)/
(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

fricas [A] time = 1.29, size = 370, normalized size = 3.52

$$\left[\frac{3\,(a+b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2\ldots}{\cos\left(fx+e\right)^2}}\ldots}{\cos\left(fx+e\right)^4}\right.\ldots}{8\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*
b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e)^2 - b)*sqrt
```

```
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e)), 1/4
*(3*(a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x +
 e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin
(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \csc\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

maple [C] time = 1.40, size = 2032, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(6*EllipticPi((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*cos(f*x+e)^3*sin(f*x+e)*a*b+6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2
-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*
a*b-3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*b^2+6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a*b+6*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
```

```
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a*b-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^4*a^2+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+3*cos(f*x+e)^2*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2)/sin(f*x+e)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)
```

maxima [A] time = 0.34, size = 98, normalized size = 0.93

$$\frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+3\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+3\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b\tan\left(fx+e\right)-\frac{2\left(b\tan\left(fx+e\right)\cdots\right.}{\tan\left(fx\cdots\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*b^(3/2)*arcsin
h(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(
f*x + e) - 2*(b*tan(f*x + e)^2 + a + b)^(3/2)/tan(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{3/2}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.91 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}\,(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan\right.}{3f(a+}$$

```
[Out] 1/2*(3*a+5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2
)/f+1/2*b*(3*a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*a+
5*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(5/2)/(a+b)/f
```

Rubi [A] time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4132, 453, 277, 195, 217, 206}

$$\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}\,(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan\right.}{3f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 5*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a
 + b)*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int\frac{(1+x^2)\left(a+b+bx^2\right)^{3/2}}{x^4}dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}+\frac{(3a+5b)\,\text{Subst}\left(\int\frac{(a}{}\right.}{3}$$

$$=-\frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}-\frac{\cot^3(e+fx)}{}$$

$$=\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3a+5b)\cot(e}{}$$

$$=\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3a+5b)\cot(e}{}$$

$$=\frac{\sqrt{b}\,(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+5b)\tan(e+fx)}{2(a}$$

Mathematica [C] time = 8.09, size = 135, normalized size = 0.78

$$-\frac{\csc(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}\left((a+b)\left((a+b)\csc^2(e+fx)+2a\right)\right.}{6f(a+b)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]*(a + b*Sec[e
 + f*x]^2)^(3/2)*((a + b)*(2*a + (a + b)*Csc[e + f*x]^2)*Hypergeometric2F1[
1, 2, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + 8*b*Hypergeometric2F1[2, 3, 3/2
, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec[e + f*x]^2)))/((a + b)^3*f)
```

fricas [A] time = 2.53, size = 472, normalized size = 2.74

$$\left[\frac{3\left((3\,a+5\,b)\cos\left(fx+e\right)^{3}-(3\,a+5\,b)\cos\left(fx+e\right)\right)\sqrt{b}\,\log\left(\frac{\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+4\left((a-b)\cos\left(fx\right.\right.}{\cos\left(fx+e\right.}\right.}{24\left(f\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*(
(4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*s
in(f*x + e)), 1/12*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e
))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*s
in(f*x + e)))*sin(f*x + e) - 2*((4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b
)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

maple [C] time = 1.57, size = 3925, normalized size = 22.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/6/f*(-9*cos(f*x+e)^5*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(
f*x+e)*a*b-15*cos(f*x+e)^5*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
sin(f*x+e)*b^2+18*cos(f*x+e)^5*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
```

```
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b+30*cos(f*x+e)
^5*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2-9*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a*b-15*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^2+18*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*si
n(f*x+e)*a*b+30*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2+9*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a*b+15*cos(f*x+e
)^3*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-18*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*cos(f*x+e)^3*sin(f*x+e)*a*b-30*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*b^2+4*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+15*co
s(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+9*cos(f*x+e)^2*sin(f
*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+15*cos(f*x+e)^2*s
in(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
```

```
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-18*EllipticPi
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*cos(f*x+e)^2*sin(f*x+e)*a*b-30*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*b^2-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2-16*cos(f*x
+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+15*cos(f*x+e)^4*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^2*a*b-20*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)^2/sin(f*x+e)^3/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.34, size = 243, normalized size = 1.41

$$\frac{9\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+\frac{6\,ab^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{a+b}+9\,b^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+\frac{6\,b^{\frac{5}{2}}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{a+b}+9\,\sqrt{b\tan\left(fx\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 6*a*b^(3/2)*arcs
inh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 9*b^(3/2)*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b)) + 6*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a
 + b) + 9*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 6*sqrt(b*tan(f*x
+ e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 6*(b*tan(f*x + e)^2 + a + b)^(3/
2)/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*tan(f*x + e
)) - 2*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.92 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=209

$$\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}\,(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\right.}{5f(a}$$

```
[Out] 1/2*(3*a+7*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2
)/f+1/2*b*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*a+
7*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-2/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(5/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(5/2)/(a+
b)/f
```

Rubi [A] time = 0.20, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4132, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}\,(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\right.}{5f(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 7*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 7*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*
(a + b)*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)*f
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}\left(10(a\right.}{x^4}\right.}{5}$$

$$= -\frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\right.}{5(a+b)}$$

$$= -\frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{2\cot^3(e+fx)\left(\right.}{}$$

$$= \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+}{}$$

$$= \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+}{}$$

$$= \frac{\sqrt{b}\,(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+7b)\tan(e+fx)\sqrt{a}}{2(a+b}$$

Mathematica [C] time = 10.33, size = 512, normalized size = 2.45

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{i\left(16a^2\left(1+e^{2i(e+fx)}\right)^2\left(-6e^{2i(e+fx)}+16e^{4i(e+fx)}-6e^{6i(e+fx)}+e^{8i(e+fx)}+1\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(16*a^2*(1 + E^((2*I)*(e + f*x)))^2*(1
- 6*E^((2*I)*(e + f*x)) + 16*E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x)) +
E^((8*I)*(e + f*x))) + b^2*(105 - 350*E^((2*I)*(e + f*x)) + 231*E^((4*I)*(e
 + f*x)) + 412*E^((6*I)*(e + f*x)) + 231*E^((8*I)*(e + f*x)) - 350*E^((10*I
)*(e + f*x)) + 105*E^((12*I)*(e + f*x))) + a*b*(115 - 402*E^((2*I)*(e + f*x
)) + 317*E^((4*I)*(e + f*x)) + 708*E^((6*I)*(e + f*x)) + 317*E^((8*I)*(e +
f*x)) - 402*E^((10*I)*(e + f*x)) + 115*E^((12*I)*(e + f*x)))))/((a + b)*(-1
 + E^((2*I)*(e + f*x)))^5*(1 + E^((2*I)*(e + f*x)))^2) - (15*Sqrt[b]*(3*a +
 7*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec
[e + f*x]^2)^(3/2))/(15*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

fricas [A] time = 10.59, size = 682, normalized size = 3.26

$$\left[\frac{15\left(\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^5-2\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^3+\left(3a^2+10ab+7b^2\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7
*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(b)*log((
(a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16
*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 245*b^2)*cos
(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a*b - 15*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^5 -
 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x + e)), 1/60*(
15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7*b^2)*co
s(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
e) - 2*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 2
45*b^2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a
*b - 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f
```

```
*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x +
e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

maple [C] time = 2.44, size = 8726, normalized size = 41.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.35, size = 273, normalized size = 1.31

$$\frac{45\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+\frac{60\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{a+b}+45\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+\frac{60\,b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{a+b}+45\,\sqrt{b\tan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 60*a*b^(3/2)*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 45*b^(3/2)*arcsinh(b*tan(f
*x + e)/sqrt((a + b)*b)) + 60*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b
))/(a + b) + 45*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 60*sqrt(b*t
an(f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 30*(b*tan(f*x + e)^2 + a
+ b)^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*ta
n(f*x + e)) - 20*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3)
- 6*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^5))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}{{\sin\left(e+f\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.93 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=123

$$\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f}-\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f}-\frac{\cos^5(e+\ldots}{}$$

```
[Out] -1/15*(15*a^2+20*a*b+8*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f+2/15*
(5*a+2*b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/5*cos(f*x+e)^5*(a+b
*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4134, 462, 453, 264}

$$-\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f}+\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f}-\frac{\cos^5(e\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((15*a^2 + 20*a*b + 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^
3*f) + (2*(5*a + 2*b)*Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^2*f)
 - (Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/(5*a*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-2(5a+2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} +$$

$$= -\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{}}{15a^2f}$$

Mathematica [A] time = 0.93, size = 93, normalized size = 0.76

$$-\frac{\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(3a^2\cos(4(e+fx))+89a^2-4a(7a+4b)\cos(2(e+fx))+144ab+64b^2\right.}{240a^3f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/240*((a + 2*b + a*Cos[2*(e + f*x)])*(89*a^2 + 144*a*b + 64*b^2 - 4*a*(7*
a + 4*b)*Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x])/(a^3*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.54, size = 87, normalized size = 0.71

$$-\frac{\left(3\,a^2\cos\left(fx+e\right)^5-2\left(5\,a^2+2\,ab\right)\cos\left(fx+e\right)^3+\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{15\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 2*(5*a^2 + 2*a*b)*cos(f*x + e)^3 + (15*a^2 +
20*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
a^3*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/15*((320*a+320*b)*(-tan((f*x+e
xp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-
2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^7+sqrt(a+b)*(640*
a-320*b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*t
an((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+
b))^6+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan(
(f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))
^5*(-832*a^2-960*b^2-2560*a*b)+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan
```

```
((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*ta
n((f*x+exp(1))/2)^2+a+b))*(2880*a^4-320*b^4-1920*a*b^3+4480*a^3*b)+sqrt(a+b
)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x
+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^4*(
-2560*a^2+960*b^2-1600*a*b)+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f
*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((
f*x+exp(1))/2)^2+a+b))^3*(-320*a^3+960*b^3+4160*a*b^2+2880*a^2*b)+sqrt(a+b)
*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+
exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2*(3
200*a^3-960*b^3+1280*a*b^2+5440*a^2*b)+sqrt(a+b)*(768*a^4+320*b^4-320*a*b^3
-832*a^2*b^2+576*a^3*b))/(2*sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqr
t(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*t
an((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*
tan((f*x+exp(1))/2)^2+a+b))^2+3*a-b)^5/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 1.96, size = 105, normalized size = 0.85

$$-\frac{\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)\left(3\left(\cos^4\left(fx+e\right)\right)a^2-10a^2\left(\cos^2\left(fx+e\right)\right)-4\left(\cos^2\left(fx+e\right)\right)ab+15a^2+20ab+8\right.}{15f\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-10*a^2*cos(f*x+e)^2-4*cos(f*
x+e)^2*a*b+15*a^2+20*a*b+8*b^2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos
(f*x+e)/a^3
```

maxima [A] time = 0.35, size = 162, normalized size = 1.32

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a}-\frac{10\left(\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)\right)}{a^2}+\frac{3\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5-10\left(a+\right.}{}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - 10*((a + b/cos(f*x +
e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a
^2 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a + b/cos(f*x + e
)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e
))/a^3)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

```
[Out] -1/3*(3*a+2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f+1/3*cos(f*x+e)^3*(
a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4134, 453, 264}

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(3*a^2*f) + (Cos[e +
 f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a+bx^2}}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}+\frac{(3a+2b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx,x,\sec(e+f\right.}{3af}\\
&= -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}+\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 64, normalized size = 0.86

$$\frac{\sec(e+fx)(a\cos(2(e+fx))-5a-4b)(a\cos(2(e+fx))+a+2b)}{12a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-5*a - 4*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f
*x])/(12*a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.50, size = 57, normalized size = 0.77

$$\frac{\left(a\cos\left(fx+e\right)^{3}-(3\,a+2\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(a^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
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(f)-sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2+b)/a/sign(cos(f*x+exp(1)))/sign(f))/a
))+(a+b)/a*(sqrt(b)/a*sign(cos(f*x+exp(1)))*sign(f)-sqrt(a*f^2*(-cos(f*x+ex
p(1))/f)^2+b)/a/sign(cos(f*x+exp(1)))/sign(f)))
```

maple [A] time = 1.49, size = 69, normalized size = 0.93

$$\frac{\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)\left(a\left(\cos^{2}\left(fx+e\right)\right)-3a-2b\right)}{3f\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{2}}}\ \cos\left(fx+e\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-3*a-2*b)/((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)/cos(f*x+e)/a^2
```

maxima [A] time = 0.34, size = 83, normalized size = 1.12

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - ((a + b/cos(f*x + e)^2)
^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^2)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

```
[Out] -cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4134, 264}

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}\end{aligned}$$

Mathematica [A] time = 0.11, size = 48, normalized size = 1.60

$$-\frac{\sec(e+fx)(a\cos(2e+2fx)+a+2b)}{2af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x])/(a*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

fricas [A] time = 0.51, size = 37, normalized size = 1.23

$$-\frac{\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)sqrt(b)/a/abs(f)*sign(cos(f*x+exp(1)))*sign(f)-sqrt(a*f^2*(-cos(f*x
+exp(1))/f)^2+b)/a/abs(f)/sign(cos(f*x+exp(1)))/sign(f)
```

maple [A] time = 0.36, size = 31, normalized size = 1.03

$$-\frac{\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{fa\sec\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.33, size = 28, normalized size = 0.93

$$-\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

mupad [B] time = 5.38, size = 46, normalized size = 1.53

$$-\frac{\cos\left(e+f\,x\right)\,\sqrt{\frac{a+2\,b+a\,\cos\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] -(cos(e + f*x)*((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2
))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.96 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4134, 377, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
b]*f))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}\,f}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 86, normalized size = 2.00

$$-\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\,\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)}{\sqrt{2}f\sqrt{a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + 2*b + a*Cos
[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a + b]*f*Sqrt[a + b*Sec[e + f*x]
^2]))
```

fricas [A] time = 0.63, size = 140, normalized size = 3.26

$$\left[\frac{\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a+b}f},\ \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a+b}\right)}{(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))/(sqrt(a + b)*f)
, sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b))/((a + b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/2*(
sqrt(a+b)*ln(abs((a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+ex
p(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+e
xp(1))/2)^2+a+b))+sqrt(a+b)*(a-b)))/(-2*a-2*b)+sqrt(a+b)*ln(abs(-tan((f*x+e
xp(1))/2)^2*sqrt(a+b)+sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)))/(2*a+2
*b)-sqrt(a+b)*ln(abs(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan(
```

```
(f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan
((f*x+exp(1))/2)^2+a+b)))/(2*a+2*b))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 1.74, size = 280, normalized size = 6.51

$$\frac{\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\left(\ln\left(-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\cos\left(fx+e\right)\sqrt{a+b}+\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\sqrt{a+b}-a\cos\left(fx+e\right)+b\right)}{\sin\left(fx+e\right)^2\sqrt{a+b}}\right)+\ln\left(-\frac{4\left(\sqrt{\frac{b+\cdots}{(1\cdots}}\right.}{\cdots}\right.\right.}{2f\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\left(-1+\cos\left(fx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2*(-1+cos(f*x+e))*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2
/(a+b)^(1/2))+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(-1+cos(f*x+e))))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^
(1/2)/cos(f*x+e)/(-1+cos(f*x+e))/(a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.97 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=87

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

```
[Out] -1/2*a*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4134, 471, 12, 377, 207}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*(a +
 b)^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a
 + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
```

```
&& IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{a}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e}{\sqrt{a+b\,\text{se}}}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

Mathematica [A] time = 0.99, size = 140, normalized size = 1.61

$$-\frac{a\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{(a+b)\csc^2(e+fx)}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right)}{2\sqrt{2}f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]*Sqrt[a + b - a*Sin[
e + f*x]^2]*(((a + b)*Csc[e + f*x]^2)/a + ArcTanh[Sqrt[1 - (a*Sin[e + f*x]^
2)/(a + b)]]/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))/(Sqrt[2]*(a + b)^2*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.62, size = 305, normalized size = 3.51

$$\left[\frac{2\,(a+b)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{a+b}\,\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+\right.\right.}{\cos\left(fx+e\right)^2-1}\right.}{4\left(\left(a^2+2ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2ab+b^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
(a*cos(f*x + e)^2 - a)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
```

```
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)
*f), 1/2*((a*cos(f*x + e)^2 - a)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a + b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/((a^2 + 2*a*b + b^2)*f*co
s(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Warning, integration of abs or sign assumes constant sign by interv
als (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Disconti
nuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Eval
uation time: 0.69Unable to divide, perhaps due to rounding error%%%{1,[4]%%
%}+%%%{%%%{-2,[1,0]%%%}+%%%{-2,[0,1]%%%},[2]%%%}+%%%{%%%{1,[2,0]%%%}+%%%{2,
[1,1]%%%}+%%%{1,[0,2]%%%},[0]%%%} / %%%{%%%{1,[1,0]%%%}+%%%{1,[0,1]%%%},[4]
%%%}+%%%{%%%{-2,[2,0]%%%}+%%%{-4,[1,1]%%%}+%%%{-2,[0,2]%%%},[2]%%%}+%%%{%%%
{1,[3,0]%%%}+%%%{3,[2,1]%%%}+%%%{3,[1,2]%%%}+%%%{1,[0,3]%%%},[0]%%%} Error:
 Bad Argument Value
```

maple [B] time = 1.86, size = 2199, normalized size = 25.28

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4/f*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2
/(a+b)^(1/2))*cos(f*x+e)^3*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*co
s(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a*b+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a^2+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a*b+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^2*a^
2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b
)^(1/2))*cos(f*x+e)^2*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f
*x+e)))*cos(f*x+e)^2*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*
x+e)))*cos(f*x+e)^2*a*b-2*cos(f*x+e)^2*(a+b)^(3/2)*a-((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^2-
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^
(1/2))*cos(f*x+e)*a*b-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e
)))*cos(f*x+e)*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))
*cos(f*x+e)*a*b-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f
*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin
(f*x+e)^2/(a+b)^(1/2))*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*
x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^
(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/(-1+cos(f*x+e)))*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x
+e)))*a*b-2*(a+b)^(3/2)*b)*sin(f*x+e)^2/(-1+cos(f*x+e))^2/cos(f*x+e)/((b+a*
cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(1+cos(f*x+e))^2/(a+b)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.98 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}-\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a}}{8f(a+b)^2}$$

```
[Out] -3/8*a^2*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/
2)/f-1/8*(5*a+2*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f
-1/4*cot(f*x+e)^3*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4134, 470, 527, 12, 377, 207}

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}-\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(8*
(a + b)^(5/2)*f) - ((5*a + 2*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(8*(a + b)^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(4*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{-a-2(2a+b)x^2}{\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, s\right.}{4(a+b)f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+f}{4(a}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+f}{4(a}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+f}{4(a}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+}}{8(a+b)^2f}$$

Mathematica [C] time = 0.19, size = 78, normalized size = 0.57

$$-\frac{a^2\sec(e+fx)(a\cos(2(e+fx))+a+2b)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)}{2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a^2*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[1/2, 3, 3/2, 1 -
 (a*Sin[e + f*x]^2)/(a + b)]*Sec[e + f*x])/((a + b)^3*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

fricas [A] time = 0.70, size = 491, normalized size = 3.56

$$\left[\frac{3\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\sqrt{a+b}\,\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\,\sqrt{a+b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)+a+2\,b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\,(3}{16\left(\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 + a*b)*cos(f
*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*f), 1/8*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-a
 - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/(a + b)) + (3*(a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos
(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/64*(2*
(-tan((f*x+exp(1))/2)^2*(2*a+2*b)/(-8*a^2-8*b^2-16*a*b)-(18*a+6*b)/(-8*a^2-
8*b^2-16*a*b))*sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan
((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+2*(((-tan((f*x+exp(1))/2)
^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((
f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))*(-6*a^3+4*a*b^2-2*a^2*b)+(
-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+ex
p(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^3*(4*a
^2-2*b^2-4*a*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x
+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*
x+exp(1))/2)^2+a+b))^2*(3*a^2+3*b^2+6*a*b)+sqrt(a+b)*(-5*a^3-b^3-7*a*b^2-11
*a^2*b))/((-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*
tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a
+b))^2-a-b)^2/(-a^2-b^2-2*a*b)+6*a^2*atan((-tan((f*x+exp(1))/2)^2*sqrt(a+b)
+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/
2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a-b))/sqrt(-a-b)/(-a^2-b^2-2*a*b
)+3*a^2*ln(abs(-sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x
+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*
x+exp(1))/2)^2+a+b))-a+b))/sqrt(a+b)/(-a^2-b^2-2*a*b)))/sign(tan((f*x+exp(1
))/2)^2-1)
```

maple [B] time = 1.92, size = 4983, normalized size = 36.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16/f*(-10*(a+b)^(5/2)*a*b-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2-6*cos(f*x+e)^5*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b-3*cos(f*x+e)^5
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-6*cos(f*x
+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/
(a+b)^(1/2))*a^3*b-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2-6*cos(f*x+e)^4*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b-3*cos(f*x+e)^4*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2+12*cos(f*x+e)^3*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1
/2))*a^3*b+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2
*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2+12*cos(f*x+e)^3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+6*cos(f*x+e)^3*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2+12*cos(f*x+e)^2*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^
3*b+6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+co
s(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/
2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/
sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2+12*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-6*cos(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b-3*cos(
```

```
f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2
/(a+b)^(1/2))*a^2*b^2-6*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/
2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/
(-1+cos(f*x+e)))*a^3*b-3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/(-1+cos(f*x+e)))*a^2*b^2+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+
b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/(-1+cos(f*x+e)))*a^4-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+b)/(-1+cos(f*x+e)))*a^4+6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4+6*cos(f*x+e)^2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-3*cos(f*x+e)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*
a^4-3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a
^4-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos
(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/s
in(f*x+e)^2/(a+b)^(1/2))*a^4-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+
cos(f*x+e)))*a^3*b-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)
))*a^2*b^2-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2
*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b-3*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2+2*(a+b
)^(5/2)*cos(f*x+e)^2*a*b-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b
)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+b)/(-1+cos(f*x+e)))*a^4+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-3*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-6*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
```

```
)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b-3*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2)
)*a^4+6*(a+b)^(5/2)*cos(f*x+e)^4*a^2-10*(a+b)^(5/2)*cos(f*x+e)^2*a^2-4*(a+b
)^(5/2)*b^2)/(-1+cos(f*x+e))^2/cos(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)
^(1/2)/(1+cos(f*x+e))^2/(a+b)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{5}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{5}\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.99 $\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=193

$$\frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{(9a+5b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24a^2f}-\frac{\left(33a^2+40ab+\right.}{}$$

```
[Out] 5/16*(a+b)^3*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f-1/48*(33*a^2+40*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1
/2)/a^3/f+1/24*(9*a+5*b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)
/a^2/f+1/6*cos(f*x+e)^3*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.28, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4132, 470, 578, 527, 12, 377, 203}

$$-\frac{\left(33a^2+40ab+15b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f}+\frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{(9a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (5*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(16*a^(7/2)*f) - ((33*a^2 + 40*a*b + 15*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqr
t[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + ((9*a + 5*b)*Cos[e + f*x]^3*Sin[e
 + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos[e + f*x]^3*Sin[e
+ f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)-2(3a+b)x^2\right)}{\left(1+x^2\right)^3\sqrt{a+b+bx^2}}\right.}{6af}$$

$$= \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(}{}$$

$$= -\frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+}{}$$

$$= -\frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+}{}$$

$$= -\frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+}{}$$

$$= \frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} - \frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+f}{48a^3f}$$

Mathematica [A] time = 1.39, size = 163, normalized size = 0.84

$$\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left(15(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\sqrt{a}\sin(e+fx)\sqrt{-a\sin^2(e+fx)}\right.}{48\sqrt{2}a^{7/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(15*(a + b)^3*ArcTan[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sq
rt[a + b - a*Sin[e + f*x]^2]*(15*(a + b)^2 + 10*a*(a + b)*Sin[e + f*x]^2 +
8*a^2*Sin[e + f*x]^4)))/(48*Sqrt[2]*a^(7/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 2.24, size = 639, normalized size = 3.31

$$\left[-\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14a^3b+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 - 2*(1
3*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 15*a*b^2)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/1
92*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^3*co
s(f*x + e)^5 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 1
5*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.76, size = 2425, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/48/f*sin(f*x+e)*(30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-14*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^3*a*b^2-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)*a^2*b-40*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2+2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b-45*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+30*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*
sin(f*x+e)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+14*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b+5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^6*a^3-26*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^4*a^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^3+26
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3-33*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^3+33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)^2*a^3-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(
f*x+e)*b^3+33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+40*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*a^3*sin(f*x+e)-15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*b^3*sin(f*x+e)+90*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+90*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*
b^2*sin(f*x+e)-45*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
```

```
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin
(f*x+e)+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/((b
+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^{6}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{6}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.100 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=135

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}-\frac{(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\cos}{}$$

```
[Out] 3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/4*c
os(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4132, 470, 527, 12, 377, 203}

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}-\frac{(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*a^(5/2)*f) - ((5*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+b)x^2}{\left(1+x^2\right)^2\sqrt{a+b+bx^2}}\,dx, x,\right.}{4af}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+\right.}{}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+\right.}{}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+\right.}{}$$

$$= \frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\,\text{t}}}{8a^2f}$$

Mathematica [A] time = 0.45, size = 145, normalized size = 1.07

$$\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left(3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\sqrt{a}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+}\right.}{8\sqrt{2}\,a^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(3*(a + b)^2*ArcTan[(Sqrt[
a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2]*(3*(a + b) + 2*a*Sin[e + f*x]^2)))/(8*Sqrt[2]*a
^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.94, size = 565, normalized size = 4.19

$$\left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 + 3*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
 a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x
+ e)^3 - (5*a^2 + 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.44, size = 1701, normalized size = 12.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*sin(f*x+e)*(2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^
2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2-3*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
```

```
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b^2*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+12*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^3*a*b+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)^2*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b-5*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)*b^2+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/((b+a*cos(f
*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.101 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=85

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

```
[Out] 1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4132, 471, 12, 377, 203}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{a+b}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+f}{2af}$$

Mathematica [A] time = 0.23, size = 125, normalized size = 1.47

$$\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left((a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\sqrt{a}\,\sin(e+fx)\sqrt{-a\sin^2(e+fx)}\right.}{2\sqrt{2}\,a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]*S
in[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqrt[a
+ b - a*Sin[e + f*x]^2]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [B] time = 0.71, size = 497, normalized size = 5.85

$$\left[-\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-a}\,(a+b)\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
```

```
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)))/(a^2*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.56, size = 1055, normalized size = 12.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*
x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos
(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.102 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4128, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [B] time = 0.07, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

fricas [B] time = 0.60, size = 408, normalized size = 10.46

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.47, size = 380, normalized size = 9.74

$$-\frac{\sqrt{2}\sqrt{\frac{i\sqrt{a}\sqrt{b}\cos(fx+e)-i\sqrt{a}\sqrt{b}+a\cos(fx+e)+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2\left(i\sqrt{a}\sqrt{b}\cos(fx+e)-i\sqrt{a}\sqrt{b}-a\cos(fx+e)-b\right)}{(1+\cos(fx+e))(a+b)}}\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e)\cdots}{\sin(\cdots}\right.\right.}{f\sqrt{\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}}\,\cos\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(
1/2)/cos(f*x+e)/(-1+cos(f*x+e))/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [B] time = 0.62, size = 992, normalized size = 25.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2
*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 +
 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f
*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*si
n(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a +
2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

3.103 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

```
[Out] -cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4132, 264}

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}\end{aligned}$$

Mathematica [A] time = 0.11, size = 55, normalized size = 1.67

$$-\frac{\csc(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/((a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.67, size = 47, normalized size = 1.42

$$-\frac{\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{(a+b)f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a + b)*f*sin(f*
x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [A] time = 1.70, size = 48, normalized size = 1.45

$$-\frac{\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{f\sin\left(fx+e\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)/(a+b)
```

maxima [A] time = 0.36, size = 33, normalized size = 1.00

$$-\frac{\sqrt{b\tan\left(fx+e\right)^2+a+b}}{(a+b)f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*f*tan(f*x + e))
```

mupad [B] time = 4.65, size = 74, normalized size = 2.24

$$-\frac{\left(2\sin\left(2e+2fx\right)+\sin\left(4e+4fx\right)\right)\sqrt{\frac{a+2b+a\cos\left(2e+2fx\right)}{\cos\left(2e+2fx\right)+1}}}{2f\sin\left(2e+2fx\right)^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] -((2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((a + 2*b + a*cos(2*e + 2*f*x))/(
cos(2*e + 2*f*x) + 1))^(1/2))/(2*f*sin(2*e + 2*f*x)^2*(a + b))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.104 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}-\frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

```
[Out] -1/3*(3*a+b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)
^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4132, 453, 264}

$$-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}-\frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{3(a+b)f}$$
$$= -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+}}{3(a+b)f}$$

Mathematica [A] time = 0.19, size = 74, normalized size = 0.95

$$\frac{\csc^3(e+fx)\sec(e+fx)(a\cos(2(e+fx))-2a-b)(a\cos(2(e+fx))+a+2b)}{6f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-2*a - b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x
]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.55, size = 96, normalized size = 1.23

$$-\frac{\left(2\,a\cos\left(fx+e\right)^3-(3\,a+b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b +
 b^2)*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [A] time = 1.64, size = 66, normalized size = 0.85

$$\frac{\left(2a\left(\cos^2\left(fx+e\right)\right)-3a-b\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)}{3f\sin\left(fx+e\right)^3(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*(2*a*cos(f*x+e)^2-3*a-b)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(
f*x+e)/sin(f*x+e)^3/(a+b)^2
```

maxima [A] time = 0.36, size = 96, normalized size = 1.23

$$-\frac{\frac{3\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan\left(fx+e\right)}-\frac{2\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}{(a+b)^{2}\tan\left(fx+e\right)}+\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan\left(fx+e\right)^{3}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 2*sqrt(b*ta
n(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2 +
a + b)/((a + b)*tan(f*x + e)^3))/f
```

mupad [B] time = 9.92, size = 123, normalized size = 1.58

$$-\frac{2\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^{2}}}\left(a\,1i-a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i-b\,e^{e\,2i+f\,x\,2i}\,2i\right)}{3\,f\,(a+b)^{2}\left(e^{e\,2i+f\,x\,2i}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(2*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*
x*1i)/2)^2)^(1/2)*(a*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*1i
 - b*exp(e*2i + f*x*2i)*2i))/(3*f*(a + b)^2*(exp(e*2i + f*x*2i) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{4}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.105 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=132

$$-\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)}-\frac{2(5a+3b)\,\text{co}}{}$$

```
[Out] -1/15*(15*a^2+10*a*b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f
-2/15*(5*a+3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f
*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4132, 462, 453, 264}

$$-\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)}-\frac{2(5a+3b)\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((15*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(1
5*(a + b)^3*f) - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*(
a + b)*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{2(5a+3b)+5(a+b)x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+\right.}{5(a+b)f}$$
$$= -\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2 f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan}}{5(a+b)f}$$
$$= -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3 f} - \frac{2(5a+3b)\cot^3(e+}{1}$$

Mathematica [A] time = 0.32, size = 100, normalized size = 0.76

$$-\frac{\csc^5(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^2\cos(4(e+fx))+8a^2-2a(3a+b)\cos(2(e+fx))+8\right.}{30f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(8*a^2 + 8*a*b + 3*b^2 - 2*a*(3*a + b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/((a
 + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 1.15, size = 171, normalized size = 1.30

$$-\frac{\left(8a^2\cos\left(fx+e\right)^5-4\left(5a^2+ab\right)\cos\left(fx+e\right)^3+\left(15a^2+10ab+3b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(}{\cos(}}}{15\left(\left(a^3+3a^2b+3ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3a^2b+3ab^2+b^3\right)f\cos\left(fx+e\right)^2+\left(a^3+3a^2b+3a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^2*cos(f*x + e)^5 - 4*(5*a^2 + a*b)*cos(f*x + e)^3 + (15*a^2 + 10
*a*b + 3*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [A] time = 1.68, size = 101, normalized size = 0.77

$$-\frac{\left(8\left(\cos^{4}\left(fx+e\right)\right)a^{2}-20a^{2}\left(\cos^{2}\left(fx+e\right)\right)-4\left(\cos^{2}\left(fx+e\right)\right)ab+15a^{2}+10ab+3b^{2}\right)\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{2}}}\ \cos\left(\right.}{15f\sin\left(fx+e\right)^{5}\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f*(8*cos(f*x+e)^4*a^2-20*a^2*cos(f*x+e)^2-4*cos(f*x+e)^2*a*b+15*a^2+1
0*a*b+3*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^
5/(a+b)^3
```

maxima [A] time = 0.35, size = 191, normalized size = 1.45

$$-\frac{\frac{15\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan\left(fx+e\right)}-\frac{20\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}{(a+b)^{2}\tan\left(fx+e\right)}+\frac{8\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b^{2}}{(a+b)^{3}\tan\left(fx+e\right)}+\frac{10\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan\left(fx+e\right)^{3}}-\frac{4\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}{(a+b)^{2}\tan\left(fx+e\right)^{3}}+\frac{3}{}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 20*sqrt(b
*tan(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x + e)
^2 + a + b)*b^2/((a + b)^3*tan(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a + b
)/((a + b)*tan(f*x + e)^3) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*b/((a + b)^2*
tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)^5)
)/f
```

mupad [B] time = 15.15, size = 723, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] (((32*a + 16*b)/(5*f*(6*a + 6*b)*(a*1i + b*1i)) + (32*a + 80*b)/(5*f*(6*a +
 6*b)*(a*1i + b*1i)))*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2
)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*
x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2)^2)^(1/2)*((a*(2*a + b)*32i)/(15*f*(a*1i + b*1i)^2*(8*
a + 8*b)) + (a*(2*a + 3*b)*32i)/(15*f*(a*1i + b*1i)^2*(8*a + 8*b)))*(2*exp(
e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^2*(exp(
e*2i + f*x*2i) + 1)) + ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)
/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*((96*a + 32*b)
/(5*f*(a*1i + b*1i)*(16*a + 16*b)) + (160*a + 160*b)/(5*f*(a*1i + b*1i)*(16
*a + 16*b)) + (256*a + 320*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b))))/((exp(e*2
i + f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i
)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x
*4i) + 1)*32i)/(f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*2i) + 1)*(10*a
 + 10*b)) - (8*a^2*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2
)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(15*f*(exp(e*2i +
f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)*(a*1i + b*1i)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{6}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.106 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{2b\left(15a^2+40ab+24b^2\right)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(15a^2+40ab+24b^2\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos^5}{5af\sqrt{a+b}}$$

```
[Out] -1/15*(15*a^2+40*a*b+24*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)+2/15
*(5*a+3*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/5*cos(f*x+e)^5/a/f
/(a+b*sec(f*x+e)^2)^(1/2)-2/15*b*(15*a^2+40*a*b+24*b^2)*sec(f*x+e)/a^4/f/(a
+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4134, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2+40ab+24b^2\right)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}+\frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(\frac{8b(5a+3b)}{a^2}+15\right)\cos(e+fx)}{15af\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos^5(e+}{5af\sqrt{a+b\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15 + (8*b*(5*a + 3*b))/a^2)*Cos[e + f*x])/(15*a*f*Sqrt[a + b*Sec[e + f*x
]^2]) + (2*(5*a + 3*b)*Cos[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]
) - Cos[e + f*x]^5/(5*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(15*a^2 + 40*a
*b + 24*b^2)*Sec[e + f*x])/(15*a^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a+3b)+5ax^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(-15a^2-8b(5a+3b)\right.}{}$$

$$= -\frac{\left(15a^2+8b(5a+3b)\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5}{5af\sqrt{a+b}}$$

$$= -\frac{\left(15a^2+8b(5a+3b)\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5}{5af\sqrt{a+b}}$$

Mathematica [B] time = 7.51, size = 432, normalized size = 2.53

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+2a+4b)(a\cos(2e+2fx)+a+2b)^{3/2}}{32a^2f\sqrt{a\cos(2(e+fx))+a+2b}\,\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\sec^3(e+fx)\left(-2a^2\cos(4(e+fx))+2\right.}{192a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(64*a*f*Sqrt[a + 2*
b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) + ((2*a + 4*b + a*Cos
[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(32*a^2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) - ((27*
a^2 + 128*a*b + 128*b^2 + 16*a*(a + 2*b)*Cos[2*(e + f*x)] - 2*a^2*Cos[4*(e
+ f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(192*a^3*f*Sq
rt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) - ((40*a^3 +
 336*a^2*b + 768*a*b^2 + 512*b^3 + a*(25*a^2 + 128*a*b + 128*b^2)*Cos[2*(e
+ f*x)] - 4*a^2*(a + 2*b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*(a + 2*b
 + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(320*a^4*f*Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.75, size = 136, normalized size = 0.80

$$-\frac{\left(3\,a^3\cos\left(fx+e\right)^7 - 2\left(5\,a^3+3\,a^2b\right)\cos\left(fx+e\right)^5 + \left(15\,a^3+40\,a^2b+24\,ab^2\right)\cos\left(fx+e\right)^3 + 2\left(15\,a^2b+4\right.\right.}{15\left(a^5f\cos\left(fx+e\right)^2+a^4bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^3*cos(f*x + e)^7 - 2*(5*a^3 + 3*a^2*b)*cos(f*x + e)^5 + (15*a^3
+ 40*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 2*(15*a^2*b + 40*a*b^2 + 24*b^3)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)
^2 + a^4*b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)2/f*(2*(-1/16*(-4*a^3*b^4*sign(tan((f*x+exp(1))/2)^2-1)-8*a^4*b^3*sign(t
an((f*x+exp(1))/2)^2-1)-4*a^5*b^2*sign(tan((f*x+exp(1))/2)^2-1))/a^7/b-1/16
*tan((f*x+exp(1))/2)^2*(-4*a^3*b^4*sign(tan((f*x+exp(1))/2)^2-1)-8*a^4*b^3*
sign(tan((f*x+exp(1))/2)^2-1)-4*a^5*b^2*sign(tan((f*x+exp(1))/2)^2-1))/a^7/
b)/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1)
)/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+2/15*((15*b^2+15*a*b)*(-tan((f*x+exp(
1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a
*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^9+sqrt(a+b)*(-105*b^
2-165*a*b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b
*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+
a+b))^8+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*ta
n((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b
))^7*(320*a^3+300*b^3+600*a*b^2+540*a^2*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^
2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f
*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^6*(640*a^3-420*b^3+120*a*b^
2+1020*a^2*b)+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^
4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)
^2+a+b))^5*(-832*a^4+210*b^4-1770*a*b^3-4890*a^2*b^2-3870*a^3*b)+(-tan((f*x
+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^
4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))*(2880*a^6-105*b
^6+915*a*b^5-1290*a^2*b^4-6890*a^3*b^3+4835*a^4*b^2+12295*a^5*b)+sqrt(a+b)*
(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+e
xp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^4*(-2
560*a^4+210*b^4+2430*a*b^3-90*a^2*b^2-7110*a^3*b)+(-tan((f*x+exp(1))/2)^2*s
qrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^3*(-320*a^5-420*b^5-720*a*b^4+
6840*a^2*b^3+11280*a^3*b^2+2540*a^4*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sq
rt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+e
xp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2*(3200*a^5+300*b^5-1200*a*b^4-
3960*a^2*b^3+8400*a^3*b^2+14060*a^4*b)+sqrt(a+b)*(768*a^6+15*b^6-225*a*b^5+
1350*a^2*b^4-1810*a^3*b^3-1477*a^4*b^2+2691*a^5*b))/a^3/(2*sqrt(a+b)*(-tan(
(f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))
/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-tan((f*x+
exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4
-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2+3*a-b)^5/sign(
tan((f*x+exp(1))/2)^2-1))
```

maple [B] time = 3.24, size = 35190, normalized size = 205.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.38, size = 250, normalized size = 1.46

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a^{2}}-\frac{10\left(\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}-6\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,b\cos\left(fx+e\right)\right)}{a^{3}}+\frac{15\,b}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,a^{2}\cos\left(fx+e\right)}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^3 + 15*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 30*b^2/(sqrt(a
+ b/cos(f*x + e)^2)*a^3*cos(f*x + e)) + 15*b^3/(sqrt(a + b/cos(f*x + e)^2)*
a^4*cos(f*x + e)) + 3*((a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a +
 b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b
^2*cos(f*x + e))/a^4)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.107 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/3*(3*a+4*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)+1/3*cos(f*x+e)^3/a
/f/(a+b*sec(f*x+e)^2)^(1/2)-2/3*b*(3*a+4*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e
)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4134, 453, 271, 191}

$$-\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a + 4*b)*Cos[e + f*x])/(3*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Cos[e +
f*x]^3/(3*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(3*a + 4*b)*Sec[e + f*x])/
(3*a^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a+4b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3af}$$

$$= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{(2b(3a+4b))\,\text{Subst}}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(3a+4b)\sec(e+}{3a^3f\sqrt{a+b\sec^2(e+}}$$

Mathematica [A] time = 3.40, size = 93, normalized size = 0.82

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^2(-\cos(4(e+fx)))+9a^2+8a(a+2b)\cos(2(e+fx))+64ab+64}{48a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/48*((a + 2*b + a*Cos[2*(e + f*x)])*(9*a^2 + 64*a*b + 64*b^2 + 8*a*(a + 2
*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^3)/(a^3*f*(a + b*
Sec[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.56, size = 98, normalized size = 0.86

$$\frac{\left(a^2\cos\left(fx+e\right)^5-\left(3\,a^2+4\,ab\right)\cos\left(fx+e\right)^3-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^5 - (3*a^2 + 4*a*b)*cos(f*x + e)^3 - 2*(3*a*b + 4*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^4*f*cos(f*x
+ e)^2 + a^3*b*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 1.90, size = 12782, normalized size = 112.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.36, size = 141, normalized size = 1.24

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^3}+\frac{3b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)}+\frac{3b^2}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,\cdots}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^3
+ 3*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 3*b^2/(sqrt(a + b/cos
(f*x + e)^2)*a^3*cos(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.108 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4134, 271, 191}

$$-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{af}\\
&= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 1.27, size = 64, normalized size = 1.03

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)(a\cos(2(e+fx))+a+4b)}{4a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 4*b + a*Cos[2*(e + f*x)])*Sec[e +
 f*x]^3)/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.51, size = 67, normalized size = 1.08

$$-\frac{\left(a\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{a^3f\cos\left(fx+e\right)^2+a^2bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(a*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(a^3*f*cos(f*x + e)^2 + a^2*b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(2*(1/8*b^2/a^2/s
ign(tan((f*x+exp(1))/2)^2-1)/b+1/8*b^2*tan((f*x+exp(1))/2)^2/a^2/sign(tan((
f*x+exp(1))/2)^2-1)/b)/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4
-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+(-tan((f*x+exp(1)
)/2)^2*sqrt(a+b)+sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/
2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/a/(2*sqrt(a+
b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*
x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-
tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2+3*a-b
)/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 0.29, size = 59, normalized size = 0.95

$$\frac{-\frac{1}{a\sec\left(fx+e\right)\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}-\frac{2b\sec\left(fx+e\right)}{a^2\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)-2*b/a^2*sec(f*x+e)/(a+b*sec(f
*x+e)^2)^(1/2))
```

maxima [A] time = 0.35, size = 57, normalized size = 0.92

$$-\frac{\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2}+\frac{b}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}a^2\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 + b/(sqrt(a + b/cos(f*x + e)^
2)*a^2*cos(f*x + e)))/f
```

mupad [B] time = 10.87, size = 155, normalized size = 2.50

$$-\frac{e^{-e\,1i-f\,x\,1i}\,\left(e^{e\,2i+f\,x\,2i}+1\right)\,\sqrt{a+\frac{b}{{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)}^2}}\,\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+8\,b\,e^{e\,2i+f\,x\,2i}\right)}{2\,a^2\,f\,\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] -(exp(- e*1i - f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i
)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*
4i + f*x*4i) + 8*b*exp(e*2i + f*x*2i)))/(2*a^2*f*(a + 2*a*exp(e*2i + f*x*2i
) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.109 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\sec(e+fx)}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f-b*s
ec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 382, 377, 207}

$$-\frac{b\sec(e+fx)}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
3/2)*f)) - (b*Sec[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{(a+b)f}$$

$$= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)f}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f} - \frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] time = 0.74, size = 113, normalized size = 1.41

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(a\sqrt{-a\sin^2(e+fx)+a+b}\,\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)+b\sqrt{a+b}\right)}{2af(a+b)^{3/2}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(b*Sqrt[a + b] + a*ArcT
anh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + b - a*Sin[e + f*x]
^2]))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.67, size = 344, normalized size = 4.30

$$\left[-\frac{2\left(ab+b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\left(a^2\cos\left(fx+e\right)^2+ab\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2}{\cos\left(fx+e\right)^2}}\right.}{\cos\left(fx+e\right)^2-1}\right.}{2\left(\left(a^4+2a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b+2a^2b^2+ab^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) - (a^2*cos(f*x + e)^2 + a*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqr
t(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b
)/(cos(f*x + e)^2 - 1)))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3
*b + 2*a^2*b^2 + a*b^3)*f), ((a^2*cos(f*x + e)^2 + a*b)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)
*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp
(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep^2-1)]Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Evaluati
on time: 1.06Error: Bad Argument Type
```

maple [B] time = 2.13, size = 1094, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(b+a*cos(f*x+e)^2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*
x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^2+((b+a*cos(f*x+e)^2)/(1+cos
```

```
(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a*b+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*a^2+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*a*b+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)
^(1/2))*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b+2*(a+b)^(3/2)*b)
/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(a+b)^(5/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.110 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=126

$$-\frac{3b\sec(e+fx)}{2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{5/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/2*(a-2*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)
^(5/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-3/2*b*s
ec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4134, 471, 527, 12, 377, 207}

$$-\frac{3b\sec(e+fx)}{2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{5/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])
/(2*(a + b)^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*Sqrt[a + b*
Sec[e + f*x]^2]) - (3*b*Sec[e + f*x])/(2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x
]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-2b)\,\mathrm{S}}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-2b)\,\mathrm{S}}{}$$

$$= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3}{2(a+b)^2}$$

Mathematica [C] time = 0.31, size = 97, normalized size = 0.77

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)\csc^2(e+fx)-(a-2b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\right)}{4f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Csc[e + f*x]^2 - (a - 2*b)*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]
^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.73, size = 547, normalized size = 4.34

$$\left[-\frac{\left(\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4-\left(a^2-3\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-ab+2\,b^2\right)\sqrt{a+b}\,\log\left(\frac{2\left(a\cos\left(fx+e\right)^2+2\,\sqrt{a+b}\sqrt{\frac{a\cos(}{\cos(}}\right.}{\cos\left(fx+e\right.}\right.}{4\left(\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-\left(a^4+2\,a^3b-2\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 2*a*b)*cos(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2
 - a*b + 2*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^
2 - 1)) - 2*((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)
^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f), 1/2*(((a^2 - 2*a*b)*cos(f*x +
e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-a - b)*arc
tan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(
a + b)) + ((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 +
 a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2
 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*(tan(
(f*x+exp(1))/2)^2*(-tan((f*x+exp(1))/2)^2*(131072*a*b^5*sign(tan((f*x+exp(1
))/2)^2-1)+524288*a^2*b^4*sign(tan((f*x+exp(1))/2)^2-1)+786432*a^3*b^3*sign
(tan((f*x+exp(1))/2)^2-1)+524288*a^4*b^2*sign(tan((f*x+exp(1))/2)^2-1)+1310
72*a^5*b*sign(tan((f*x+exp(1))/2)^2-1))/(-4194304*a*b^6-20971520*a^2*b^5-41
943040*a^3*b^4-41943040*a^4*b^3-20971520*a^5*b^2-4194304*a^6*b)-(1310720*a*
b^5*sign(tan((f*x+exp(1))/2)^2-1)+3670016*a^2*b^4*sign(tan((f*x+exp(1))/2)^
2-1)+3145728*a^3*b^3*sign(tan((f*x+exp(1))/2)^2-1)+524288*a^4*b^2*sign(tan(
(f*x+exp(1))/2)^2-1)-262144*a^5*b*sign(tan((f*x+exp(1))/2)^2-1))/(-4194304*
a*b^6-20971520*a^2*b^5-41943040*a^3*b^4-41943040*a^4*b^3-20971520*a^5*b^2-4
```

```
194304*a^6*b))-(1179648*a*b^5*sign(tan((f*x+exp(1))/2)^2-1)+3670016*a^2*b^4
*sign(tan((f*x+exp(1))/2)^2-1)+3932160*a^3*b^3*sign(tan((f*x+exp(1))/2)^2-1
)+1572864*a^4*b^2*sign(tan((f*x+exp(1))/2)^2-1)+131072*a^5*b*sign(tan((f*x+
exp(1))/2)^2-1))/(-4194304*a*b^6-20971520*a^2*b^5-41943040*a^3*b^4-41943040
*a^4*b^3-20971520*a^5*b^2-4194304*a^6*b))/sqrt(a*tan((f*x+exp(1))/2)^4+b*ta
n((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b
)+2*(((a-b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+
b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2
+a+b))+sqrt(a+b)*(a+b))/sqrt(a+b)/(sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a
+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1
))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2+sqrt(a+b)*(-a-b))/(-16*a-16*b)/si
gn(tan((f*x+exp(1))/2)^2-1)+(a-2*b)*ln(abs(-sqrt(a+b)*(-tan((f*x+exp(1))/2)
^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((
f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))-a+b))/sqrt(a+b)/(-16*a^2-1
6*b^2-32*a*b)/sign(tan((f*x+exp(1))/2)^2-1)+1/2*(-a+2*b)*atan((-tan((f*x+ex
p(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2
*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a-b))/sqrt(-
a-b)/(4*a^2+4*b^2+8*a*b)/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [B] time = 2.29, size = 3289, normalized size = 26.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f/(b+a*cos(f*x+e)^2)^2*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^3-2*cos(f*x+e)^2*(a+b)^(5/2)*a+4*cos
(f*x+e)^2*(a+b)^(5/2)*b+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f
*x+e)))*b^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3-((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1
/2))*a^3+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e
)^3*a^3-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+
e)^3*b^3+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)
^2/(a+b)^(1/2))*cos(f*x+e)^3*a^3-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-
a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*b^3+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a^3-2*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*b^3+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
```

```
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e
)^2*a^3-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e
)^2/(a+b)^(1/2))*cos(f*x+e)^2*b^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(
-1+cos(f*x+e)))*cos(f*x+e)*a^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-
1+cos(f*x+e)))*cos(f*x+e)*b^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^3+2*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*b^3
+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^2+3*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*
a*b^2-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)
^3*a*b^2-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e)
)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+
e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a*b^2-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)
^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a*b^2-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^2*a*b^2+3*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*a*b^2+3*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2
))*cos(f*x+e)*a*b^2-6*(a+b)^(5/2)*b)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2
)*cos(f*x+e)^3/sin(f*x+e)^2/(a+b)^(9/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.111 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{b(13a-2b)\sec(e+fx)}{8f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{7/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{5a\cot(e}{8f(a+b)^2}$$

```
[Out] -3/8*a*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+
b)^(7/2)/f-5/8*a*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1
/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-2*b)*
b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4134, 470, 527, 12, 377, 207}

$$-\frac{b(13a-2b)\sec(e+fx)}{8f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{7/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{5a\cot(e}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/(8*(a + b)^(7/2)*f) - (5*a*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*S
qrt[a + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*Sqr
t[a + b*Sec[e + f*x]^2]) - ((13*a - 2*b)*b*Sec[e + f*x])/(8*(a + b)^3*f*Sqr
t[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f} \ldots$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a - \ldots}{8(a+b)^3 \ldots}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a - \ldots}{8(a+b)^3 \ldots}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a - \ldots}{8(a+b)^3 \ldots}$$

$$= -\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{co}\ldots}{4(a+\ldots}$$

Mathematica [C] time = 0.47, size = 100, normalized size = 0.56

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)^2\csc^4(e+fx)-a(a-4b)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\right)}{8f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

[Out] -1/8*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)^2*Csc[e + f*x]^4 - a*(a - 4*b
)*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e +
f*x]^3)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))

fricas [B] time = 1.04, size = 873, normalized size = 4.93

$$\left[-\frac{3\left(\left(a^{3}-4\,a^{2}b\right)\cos\left(fx+e\right)^{6}-\left(2\,a^{3}-9\,a^{2}b+4\,ab^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b-4\,ab^{2}+\left(a^{3}-6\,a^{2}b+8\,ab^{2}\right)\cos\left(fx\right.}{16\left(\left(a^{5}+4\,a^{4}b+6\,a^{3}b^{2}+4\,a^{2}b^{3}+ab^{4}\right)f\cos\left(fx+e\right)^{6}-\left(2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^2)*cos
(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*s
qrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a
^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b
^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
+ a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*
b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*
b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 +
b^5)*f), 1/8*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^
2)*cos(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e
)^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)/(a + b)) + (3*(a^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5
- (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b
^2 - 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*
a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*
a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b
+ 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p

```
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)2/f*(2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-tan((f*x
+exp(1))/2)^2*(-536870912*a*b^8-3758096384*a^2*b^7-11274289152*a^3*b^6-1879
0481920*a^4*b^5-18790481920*a^5*b^4-11274289152*a^6*b^3-3758096384*a^7*b^2-
536870912*a^8*b)/(137438953472*a*b^9*sign(tan((f*x+exp(1))/2)^2-1)+10995116
27776*a^2*b^8*sign(tan((f*x+exp(1))/2)^2-1)+3848290697216*a^3*b^7*sign(tan(
(f*x+exp(1))/2)^2-1)+7696581394432*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)+96
20726743040*a^5*b^5*sign(tan((f*x+exp(1))/2)^2-1)+7696581394432*a^6*b^4*sig
n(tan((f*x+exp(1))/2)^2-1)+3848290697216*a^7*b^3*sign(tan((f*x+exp(1))/2)^2
-1)+1099511627776*a^8*b^2*sign(tan((f*x+exp(1))/2)^2-1)+137438953472*a^9*b*
sign(tan((f*x+exp(1))/2)^2-1))-(1610612736*a*b^8+5905580032*a^2*b^7+1610612
736*a^3*b^6-24159191040*a^4*b^5-51002736640*a^5*b^4-46707769344*a^6*b^3-209
37965568*a^7*b^2-3758096384*a^8*b)/(137438953472*a*b^9*sign(tan((f*x+exp(1)
)/2)^2-1)+1099511627776*a^2*b^8*sign(tan((f*x+exp(1))/2)^2-1)+3848290697216
*a^3*b^7*sign(tan((f*x+exp(1))/2)^2-1)+7696581394432*a^4*b^6*sign(tan((f*x+
exp(1))/2)^2-1)+9620726743040*a^5*b^5*sign(tan((f*x+exp(1))/2)^2-1)+7696581
394432*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-1)+3848290697216*a^7*b^3*sign(tan
((f*x+exp(1))/2)^2-1)+1099511627776*a^8*b^2*sign(tan((f*x+exp(1))/2)^2-1)+1
37438953472*a^9*b*sign(tan((f*x+exp(1))/2)^2-1)))-(4831838208*a*b^8-2630667
4688*a^2*b^7-194884141056*a^3*b^6-410706247680*a^4*b^5-389231411200*a^5*b^4
-156229435392*a^6*b^3-4831838208*a^7*b^2+9126805504*a^8*b)/(137438953472*a*
b^9*sign(tan((f*x+exp(1))/2)^2-1)+1099511627776*a^2*b^8*sign(tan((f*x+exp(1
))/2)^2-1)+3848290697216*a^3*b^7*sign(tan((f*x+exp(1))/2)^2-1)+769658139443
2*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)+9620726743040*a^5*b^5*sign(tan((f*x
+exp(1))/2)^2-1)+7696581394432*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-1)+384829
0697216*a^7*b^3*sign(tan((f*x+exp(1))/2)^2-1)+1099511627776*a^8*b^2*sign(ta
n((f*x+exp(1))/2)^2-1)+137438953472*a^9*b*sign(tan((f*x+exp(1))/2)^2-1)))-(
2684354560*a*b^8-23085449216*a^2*b^7-160524402688*a^3*b^6-362387865600*a^4*
b^5-399968829440*a^5*b^4-228170137600*a^6*b^3-60666413056*a^7*b^2-483183820
8*a^8*b)/(137438953472*a*b^9*sign(tan((f*x+exp(1))/2)^2-1)+1099511627776*a^
2*b^8*sign(tan((f*x+exp(1))/2)^2-1)+3848290697216*a^3*b^7*sign(tan((f*x+exp
(1))/2)^2-1)+7696581394432*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)+9620726743
040*a^5*b^5*sign(tan((f*x+exp(1))/2)^2-1)+7696581394432*a^6*b^4*sign(tan((f
*x+exp(1))/2)^2-1)+3848290697216*a^7*b^3*sign(tan((f*x+exp(1))/2)^2-1)+1099
511627776*a^8*b^2*sign(tan((f*x+exp(1))/2)^2-1)+137438953472*a^9*b*sign(tan
((f*x+exp(1))/2)^2-1)))/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^
4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+2*(-((-tan((f*x+
exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4
-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))*(-6*a^3-4*b^3+4*
a*b^2+2*a^2*b)+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)
^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2
)^2+a+b))^3*(4*a^2+2*b^2-8*a*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)
+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/
2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2*(3*a^2-b^2+2*a*b)+sqrt(a+b)*(-5*a^3+
3*b^3+a*b^2-7*a^2*b))/((-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+ex
p(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+e
```

```
xp(1))/2)^2+a+b))^2-a-b)^2/(64*a^3+64*b^3+192*a*b^2+192*a^2*b)/sign(tan((f*
x+exp(1))/2)^2-1)+1/2*(-3*a^2+12*a*b)*atan((-tan((f*x+exp(1))/2)^2*sqrt(a+b
)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))
/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a-b))/sqrt(-a-b)/(16*a^3+16*b^3
+48*a*b^2+48*a^2*b)/sign(tan((f*x+exp(1))/2)^2-1)+(-3*a^2+12*a*b)*ln(abs(-s
qrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*t
an((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+
b))-a+b))/sqrt(a+b)/(64*a^3+64*b^3+192*a*b^2+192*a^2*b)/sign(tan((f*x+exp(1
))/2)^2-1)))
```

maple [B] time = 2.07, size = 8268, normalized size = 46.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.112 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}-\frac{(a+b)(33a+35b)\sin(e+fx)\cos(e+fx)}{48a^3f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(9a+7b)\sin(e+fx)}{24a^2f\sqrt{a+b\tan^2(}}$$

```
[Out] 5/16*(a+b)^2*(a+7*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
a^(9/2)/f-1/48*(a+b)*(33*a+35*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x
+e)^2)^(1/2)+1/24*(9*a+7*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)
^2)^(1/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/48
*b*(81*a^2+190*a*b+105*b^2)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4132, 470, 578, 527, 12, 377, 203}

$$-\frac{b\left(81a^2+190ab+105b^2\right)\tan(e+fx)}{48a^4f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{(9a+7b)\sin(e+fx)\cos}{24a^2f\sqrt{a+b\tan^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (5*(a + b)^2*(a + 7*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*a^(9/2)*f) - ((a + b)*(33*a + 35*b)*Cos[e + f*x]*Sin[e + f*x
])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((9*a + 7*b)*Cos[e + f*x]^3*
Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*S
in[e + f*x]^3)/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (b*(81*a^2 + 190*a*
b + 105*b^2)*Tan[e + f*x])/(48*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+2(b-3(a+b))x^2)}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+f\right.}{6af}$$

$$= \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= \frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)\cos(e+fx)\,\text{si}}{48a^3f\sqrt{a+b+b\tan^2(e+}}$$

Mathematica [A] time = 8.17, size = 256, normalized size = 1.06

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(120(a+b)^2(a+7b)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(120*(a + b)^2*(a + 7*b)*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*
Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(3
7*a^3 + 439*a^2*b + 830*a*b^2 + 420*b^3 + a*(29*a^2 + 108*a*b + 70*b^2)*Cos
[2*(e + f*x)] - 7*a^2*(a + b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*Sin[
e + f*x]))/(1536*a^(9/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a
 + b - a*Sin[e + f*x]^2)/(a + b)])
```

fricas [A] time = 10.05, size = 813, normalized size = 3.36

$$\left[-\frac{15\left(a^{3}b+9\,a^{2}b^{2}+15\,ab^{3}+7\,b^{4}+\left(a^{4}+9\,a^{3}b+15\,a^{2}b^{2}+7\,ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\,\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2
*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^4*cos(f*x + e)^7 - 2*(13*a^4 + 7*a^
3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x + e)^3 + (81
*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), -1/192*(15
*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2*b^2 + 7*a*
b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*
b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^
3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^4*cos(f*x + e)^7 - 2*(
13*a^4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x
 + e)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.93, size = 2437, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/48/f*(b+a*cos(f*x+e)^2)*(-30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
```

```
/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)+68*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+35*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+81*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)*a^2*b+190*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)*a*b^2-14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+225*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-210*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*b^3*sin(f*x+e)+14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^4*a^2*b-68*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b-
35*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2-8*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3+26*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^4*a^3+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^7*a^3-26*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3+
33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^3-33*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^3+105*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)*b^3-81*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
*b-190*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+105*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-270*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-4
50*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+135*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a^2*b*sin(f*x+e)-105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(3/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^{6}}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{6}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.113 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5(a+b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+-$$

4

[Out] 3/8*(a+b)*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f-5/8*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+15*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)

Rubi [A] time = 0.22, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4132, 470, 527, 12, 377, 203}

$$\frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5(a+b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+-$$

4

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(8*a^(7/2)*f) - (5*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (b*(13*a + 15*b)*Tan[e + f*x])/(8*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b-4(a+b)x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a+}{(}\right.}{}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15}{8a^3f\sqrt{a+b}}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15}{8a^3f\sqrt{a+b}}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15}{8a^3f\sqrt{a+b}}$$

$$= \frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{}{4a}$$

Mathematica [A] time = 3.43, size = 229, normalized size = 1.31

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(24\left(a^2+6ab+5b^2\right)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\right.}{256a^{7/2}f\sqrt{a+b}\sqrt{\frac{-a\sin}{}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(24*(a^2 + 6*a*b + 5*b^2)*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2
*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(
7*a^2 + 62*a*b + 60*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
 f*x)])*Sin[e + f*x]))/(256*a^(7/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3
/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

fricas [A] time = 3.21, size = 703, normalized size = 4.02

$$\left[-\frac{3\left(a^2b+6\,ab^2+5\,b^3+\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2*b + 6*a*b^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)
^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
)) - 8*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b +
 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/32*(3*(a^2*b + 6*a*b^2 + 5*b^3
 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*
a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b + 15*a*b^2)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.30, size = 1714, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/8/f*(b+a*cos(f*x+e)^2)*(-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)^5*a^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e
)+18*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+15*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*a^2*sin(f*x+e)-36*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-30*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^
2*sin(f*x+e)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2+5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2+5*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^2*a^2-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e
)^2*a*b+13*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b+15*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-13*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a*b-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*sin(f*
x+e)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.114 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] 1/2*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan(f*x+e)/
a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4132, 471, 527, 12, 377, 203}

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - (3*b*Tan[e + f*x])/(2*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3b)\,\text{Sub}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3b)\,\text{Sub}}{}$$

$$= \frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b}{2a^2f\sqrt{a+}}$$

Mathematica [A] time = 1.17, size = 190, normalized size = 1.57

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(4(a+3b)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-2\sqrt{2}\sqrt{a}\sqrt{}\right.}{32a^{5/2}f\sqrt{a+b}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(4*(a + 3*b)*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*Sqrt[2]*Sqrt
```

```
[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(a + 6*b + a*C
os[2*(e + f*x)])*Sin[e + f*x]))/(32*a^(5/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*
x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

fricas [B] time = 1.09, size = 607, normalized size = 5.02

$$\left[-\frac{\left(\left(a^2+3\,ab\right)\cos\left(fx+e\right)^2+ab+3\,b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4\right.\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(a^2*cos(f*x + e)^3
 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), -1/8*(((a^2 + 3*a*b)*cos(f*x + e)^2
 + a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(a^2*cos(f*x + e)^3 + 3*a*b*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f*
cos(f*x + e)^2 + a^3*b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.10, size = 1069, normalized size = 8.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(b+a*cos(f*x+e)^2)*(-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
```

```
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)-6*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*b*sin(f*x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+
e)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*sin(f*x+e)/(-1+cos(f*x+e
))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.115 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4128, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+
$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 1.31, size = 168, normalized size = 2.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}\,b\sin
}{4a^{3/2}f(a+b)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

fricas [B] time = 0.91, size = 601, normalized size = 7.81

$$\left[-\frac{8\,ab\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+ab\right)\cos\left(fx+e\right)^2+ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx
}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
```

```
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

maple [C] time = 1.84, size = 1007, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*
sin(f*x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b)*sin(f*x+e)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/c
os(f*x+e)^3/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

maxima [B] time = 0.92, size = 2055, normalized size = 26.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

3.116 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2b\tan(e+fx)}{f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/(a+b)^2/f/(a+
b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4132, 271, 191}

$$-\frac{2b\tan(e+fx)}{f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 1.62, size = 76, normalized size = 1.12

$$-\frac{\csc(e+fx)\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)((a-b)\cos(2(e+fx))+a+3b)}{4f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 3*b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]*Sec[e + f*x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.66, size = 102, normalized size = 1.50

$$-\frac{\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2+\left(a^2b+2\,ab^2+b^3\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.38, size = 89, normalized size = 1.31

$$-\frac{\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+2b\right)\left(\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\left(\cos^3\left(fx+e\right)\right)}{f\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2\sin\left(fx+e\right)(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(b+a*cos(f*x+e)^2)^2*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b)*((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3/sin(f*x+e)/(a+b)^2
```

maxima [A] time = 0.37, size = 64, normalized size = 0.94

$$-\frac{\frac{2\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + 1/(sqrt(b*t
an(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)))/f
```

mupad [B] time = 12.12, size = 2151, normalized size = 31.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] -((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*
2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*4i + f*x*4i)*(((a + 4*b)*(((a
 + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a +
3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(
a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a
+ 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3*b)^3
- ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b +
a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a - b) - (
a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b
 + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)/(8*f*(a*b
 + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a
*b^2 + a^2*b))))/a + ((a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2)
)/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i
)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*
b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*
b))))/a + (((a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b)
 - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (((a^2*
(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3
*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3
*(a + 3*b)*1i)/(4*f*(a*b + a^2)*(a*b^2 + a^2*b))) + exp(e*2i + f*x*2i)*((((
a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b
)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - ((a + 4*b)*(((a + 4*
b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^
2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3
*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b
)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3*b)^3 - ((a
 + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))
*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a - b) - (a + 3
*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^
2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^
2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2
+ a^2*b))))/a + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (
a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(4*f*(a*b^2 + a
```

```
^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (
a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))) - ((a + 4*b)
*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*
(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b
)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*
(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a + (((a + 3*b)^3 - ((a +
 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1
i)/(4*f*(a*b^2 + a^2*b)*(a + 3*b)) + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b
)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2)
)*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)
*(a*b^2 + a^2*b)) - (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 +
a^2*b))))/((exp(e*2i + f*x*2i) + 1)*(a - a*exp(e*6i + f*x*6i) + exp(e*2i +
f*x*2i)*(a + 4*b) - exp(e*4i + f*x*4i)*(a + 4*b)))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.117 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b(3a-b)\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^3(e+fx)}{3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(3a-b)\cot(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -1/3*(3*a-b)*cot(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)
^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/3*(3*a-b)*b*tan(f*x+e)/(a+b)^3/f/(a
+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4132, 453, 271, 191}

$$-\frac{2b(3a-b)\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^3(e+fx)}{3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(3a-b)\cot(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a - b)*Cot[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) -
Cot[e + f*x]^3/(3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (2*(3*a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e\right.}{3(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2(3}{}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{}{3(a}$$

Mathematica [A] time = 0.62, size = 102, normalized size = 0.83

$$-\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\left(a^2-2ab-3b^2\right)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)-2a(\right.}{6f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(-2*a*(a - 3*b) + (a^2 - 2*a*b - 3*b^2
)*Csc[e + f*x]^2 + (a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x])/(
(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [A] time = 1.46, size = 189, normalized size = 1.54

$$-\frac{\left(2\left(a^2-3ab\right)\cos\left(fx+e\right)^5-\left(3a^2-10ab+3b^2\right)\cos\left(fx+e\right)^3-2\left(3ab-b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos}{\cos}}}{3\left(\left(a^4+3a^3b+3a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-\left(a^4+2a^3b-2ab^3-b^4\right)f\cos\left(fx+e\right)^2-\left(a^3b+3a^2b^2+3ab^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 - 3*a*b)*cos(f*x + e)^5 - (3*a^2 - 10*a*b + 3*b^2)*cos(f*x + e
)^3 - 2*(3*a*b - b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f
)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.70, size = 137, normalized size = 1.11

$$\frac{\left(2\left(\cos^4\left(fx+e\right)\right)a^2-6\left(\cos^4\left(fx+e\right)\right)ab-3a^2\left(\cos^2\left(fx+e\right)\right)+10\left(\cos^2\left(fx+e\right)\right)ab-3b^2\left(\cos^2\left(fx+e\right)\right.}{3f\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2\sin\left(fx+e\right)^3\left(a+b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/(b+a*cos(f*x+e)^2)^2*(2*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4*a*b-3*a^2*cos
(f*x+e)^2+10*cos(f*x+e)^2*a*b-3*b^2*cos(f*x+e)^2-6*a*b+2*b^2)*cos(f*x+e)^3*
((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^3/(a+b)^3
```

maxima [A] time = 0.43, size = 156, normalized size = 1.27

$$-\frac{\frac{6\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^2}-\frac{8\,b^2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^3}+\frac{3}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)\tan\left(fx+e\right)}-\frac{4\,b}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^2\tan\left(fx\right.}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 8*b^2*t
an(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 3/(sqrt(b*tan(f*x
+ e)^2 + a + b)*(a + b)*tan(f*x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a + b)
*(a + b)^2*tan(f*x + e)) + 1/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*
x + e)^3))/f
```

mupad [B] time = 27.53, size = 124682, normalized size = 1013.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] (a^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128i)
/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a
^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128
*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*
f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*
x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i))) - (5*a*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(
cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3
i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*
```

```
(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f
*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + si
n(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (5*b*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*
3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1
i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*
b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(c
os(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (a^8*(a + b/(((c
os(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(a^3*b^7*f + 6
*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9
*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*
a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 7
2*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5
*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i))) - ((a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 +
 ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*32
i)/(24*a^2*f + 24*b^2*f + 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 24*a^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 48*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*
b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i)) + (b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e)
 - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
)/4 + 1/2))^(1/2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3
*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 3
2*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x)
+ sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + si
n(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*
1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 1
28*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*
b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f
*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos
```

```
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a^7*(a + b/(((cos(2*
f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(3*a^2*b^7*f + 16*a
^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b
*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a
^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*
a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i))) - ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((co
s(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*64i)/(24*a^2*f + 24*b^
2*f + 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*
a^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 48*b^2*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*b^2*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))
 - ((cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f
*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*32i)/(24*a^2*f + 24*b^2*f +
 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*a^2*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 48*b^2*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a
^7*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*5i)/(
3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6
*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*
b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)
```

*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 5
4*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*
b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i))) - (a^2*b^7*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(
2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i))/4 + 1/2))^(1/2)*117i)/(4*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f +
 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^
10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a
^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^
7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a
^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^3*b^6
*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*515i)/(4*
(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f +
33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402
*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*
b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s

in(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a
^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^5*(a + b/(((cos(2*f*x) - sin(2*f
*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1357i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f +
51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a
^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 4
02*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^
6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
))) - (a^5*b^4*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1
i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(
1/2)*1207i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f +
65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102
*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b
^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1

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i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 7
5*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*
b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b^3*(a + b/(((cos(2
*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1135i)/(12*(3*a^3*b^8*f +
 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f +
 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) +
12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3
*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^
9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i))) - (a^7*b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2
*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i))/4 + 1/2))^(1/2)*265i)/(12*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f +
 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^
10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a
^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^
7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a
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^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^6*(
a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*45i)/(4*(3*
a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a
^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^
8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*
a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i))) + (a^6*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2
*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i))/4 + 1/2))^(1/2)*19i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 4
0*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230
*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*
b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40
*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b
^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^6*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*6i)/(a^3*b^7*f + 6*a
^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b
*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^
9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*
```

(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*
a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
) + (a^3*b^5*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i)
)/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/
2)*20i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3
*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) +
4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 1
6*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i)) + (a^4*b^4*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*
(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i))/4 + 1/2))^(1/2)*91i)/(4*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f
 + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54
*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b
^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)
1i)(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(

8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20
*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b
^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^3*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*25i)/(3*(a^3*b^7*f +
 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a
^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) -
2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a
^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i))) - (a^6*b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e
)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2)
)^(1/2)*11i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15
*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^
4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^
6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^5*(a + b/(((cos(2*f*x) - sin(2*
f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*81i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 3
5*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) +

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 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a
^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3
*b^4*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*231i)
/(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*
f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20
*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^
7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*125i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4
*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*
a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^
2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x)
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+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^
7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*
(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*193i)/(12*
(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f +
8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7
*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i))) - (a^8*b*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(co
s(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i))/4 + 1/2))^(1/2)*2i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75
*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 1
2*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*
b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
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33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*
b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a*b*(a + b/
(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*256i)/(3*(32*a^4*
f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin
(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*
e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 1
28*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(co
s(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x
) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)
1i)(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(
4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i))) + (a^2*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*256i)/(
3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a^4
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e
) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a
*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*f*
(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*x)
1i)(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i))) + (a^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) +
sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i)
)/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/
2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*
f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4
f(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i
)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 12
8*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) +
sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (b^2*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - si
n(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 +
1/2))^(1/2)*256i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f +
128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*

```
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin
(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e
)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) +
 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^
3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos
(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (b^2*(cos(4*f*x) + sin(4*f
*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*
a*b^3*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 32*a^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i
) + 32*a^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e)
 + sin(4*e)*1i) - 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(
4*e)*1i) + 128*a*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) + 128*a^3*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192
*a^2*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*
b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a^7*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x
)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b
^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^
6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*
b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^7*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2
*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*3i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 3
5*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) +
```

sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a
^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)
1i)(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^7
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x)
- sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(3*a^2*b^7*f + 16*a^3*b^6*
f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12
*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^
2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) +
(a^7*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*
f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(3*a^2*b^7*f + 16*a
^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b
*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a
^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*

a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i))) + (b^7*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(
((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*117i)/(2*(3*a^2*b^
7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2
*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12
*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*
a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^
3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i))) + (b^7*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i
)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*261i)/(2
*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f +
 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^
7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x

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)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i))) + (b^7*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))
/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2
)*171i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a
^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3
*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (b^7*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin
(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 +
1/2))^(1/2)*27i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4
*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 7
0*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3
*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^
4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (10*a*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(
2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i -
a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos
(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x)
+ sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*
f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (5*a*(cos(4*f*x) + sin(4*f*x)*1i)*
```

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(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) -
sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4
 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(
cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x)
 + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4
*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i
)*(cos(4*e) + sin(4*e)*1i)*3i)) - (10*b*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*
e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2
))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x) + sin
(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4*f*x)*
1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos
(4*e) + sin(4*e)*1i)*3i)) - (5*b*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + s
in(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))
/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2
))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) +
sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)
*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(co
s(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) +
 sin(4*e)*1i)*3i)) - (a^8*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(
a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^
8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a
^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i)) - (a^8*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1
i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*3i)/(2*
(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a
^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
```

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) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*
a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i))) - (a^8*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(
a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^
8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a
^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i)) - (a^8*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*
(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a
^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*
a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x)
+ sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)
```

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*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i))) + (a*b*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*512i)
/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a
^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128
*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*
f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*
x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i))) + (a*b*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e)
+ sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1
i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(
1/2)*256i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*
b*f - 32*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a
^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*
1i)*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) -
128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b
^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x)
+ sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) + (a*b^7*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e)
- sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))
/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*
f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a
^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^
4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*
a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^
5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(c
```

```
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e)
 - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
)/4 + 1/2))^(1/2)*49i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*
b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
10*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^
6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^
5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^7*(cos(4*f*x) + sin(4*f
*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^
6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*
a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^
3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*
a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^
4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(4*f*x) + sin(4*
f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(co
s(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i))/4 + 1/2))^(1/2)*27i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 2
0*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
```

```
s(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^
7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^
6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a*b^7*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i
)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i))/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f +
 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*
a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^
2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*
a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^
3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*55i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5
*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^
8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x)
```

+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^
7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a*b^7*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f
*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*24i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b
^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*
a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*
a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2
f(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^7*b*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*
f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*13i)/(6*(a^3*b^7*f + 6*a^4*b^6*f + 15*a
^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2
*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8

```
*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^2*b^7*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) -
 sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1339i)/(2*(3*a^3*b^8*f + 19*a^4*
b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b
^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b
^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262
*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*
b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i))) - (a^3*b^6*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*4310i)/(
3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f
+ 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 4
02*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^
6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19
*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^5*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) -
```

```
sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/
4 + 1/2))^(1/2)*4999i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a
^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*
a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^
8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33
*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^
2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2249i)/(2*(3*a^3*b^8*f + 19*a^4*b^7
*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*
f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a
^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 4
10*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^
7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i))) - (a^6*b^3*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(
a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2641i)/(6*(
3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 3
```

```
3*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*
a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b
^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^
4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^7*b^2*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - si
n(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 +
 1/2))^(1/2)*277i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b
^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
2*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*
b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8
*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^2*b^7*(cos(4*f
*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*
f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1030i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51
*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^1
0*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) -
2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f
```

```
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402
*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*
b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))
 - (a^3*b^6*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/((
(cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*13109i)/(6*(3*a^3*b
^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b
^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^
4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3
*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i))) - (a^4*b^5*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e)
 + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*
1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^
(1/2)*7504i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f +
 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
```

+ sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 10
2*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*
b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7
*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos(4*f*x) +
sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*10009i)/(6*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a
^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*
b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*
a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a
^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^
5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)))
- (a^6*b^3*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1946i)/(3*(3*a^3*b^8
*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3
*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82
*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos

```
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
2*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b
^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i))) - (a^7*b^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) +
 sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i
))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1
/2)*273i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65
*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a
^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*
a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^
4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^2*b^7*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*
(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i))/4 + 1/2))^(1/2)*337i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^
6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f +
 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*
b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
```

n(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a
^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^
6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^
3*b^6*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2
*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1724i)/(3*(3*a^3*b^8*f +
19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f +
9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 1
2*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*
b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 1
02*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9
*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(
8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)
1i)(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i))) - (a^4*b^5*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4
 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2
725i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7
*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*
a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b
^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) +

```
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 5
1*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*
b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*1535i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f
 + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*
a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82
*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*
b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9
*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b
^3*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*
x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2131i)/(6*(3*a^3*b^8*f + 19*
a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a
^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a
^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
```

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1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*
a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i))) - (a^7*b^2*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + (
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*253i
)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4
*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410
*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*
b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^
5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^7*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e
) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
))/4 + 1/2))^(1/2)*885i)/(4*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75
*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 1
2*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*
b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6
```

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*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*
b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3*b^6*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) -
sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1209i)/(4*(3*a^3*b^8*f + 19*a^4*b
^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^
2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^
9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*
a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b
^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i))) + (a^4*b^5*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*917i)/(6*
(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f +
33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402
*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*
b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a
```

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^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^4*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - s
in(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4
+ 1/2))^(1/2)*68i)/(3*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b
^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
2*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*
b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8
*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^6*b^3*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*
f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*625i)/(12*(3*a^3*b^8*f + 19*a^4*b^7*f +
 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f +
a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*
b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a
^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
```

s(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin (8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1 i))) - (a^7*b^2*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*217i)/(12*(3*a^ 3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^ 8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)* (cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8* e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242 *a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5* b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f *(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos (2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f* x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + s in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f *x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co s(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s in(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6* e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3* a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^ 7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(c os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f *x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8* f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^6*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e) *1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2)) ^(1/2)*261i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25 *a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2* e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin (6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b ^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f* (cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + si n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f* x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i) *(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos (6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si n(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e) *1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3 *b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f *(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^6*b*(cos(2*f*x) + sin(2*f*x)*1i)* (cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4

+ 1/2))^(1/2)*85i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*
b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
+ 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*
a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b
^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35
*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b
^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a*b^6*(cos(4*f*x) + sin(
4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(
cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i))/4 + 1/2))^(1/2)*1107i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^
5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
80*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6
*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 2
5*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b
^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^6*b*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin
(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*45i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f
+ 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a
*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2

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*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
0*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*
b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (
a*b^6*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2
*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*369i)/(3*a^2*b^7*f + 16*a
^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b
*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a
^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
- 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*
a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b
^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i)) + (a^6*b*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/
(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*91i)/(6*(3*a^2*b^
7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2
*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12
*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*
a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^
3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*
```

```
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i))) + (a*b^6*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*261i)/
(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f
 + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*
a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i))) + (a^6*b*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*
1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^
(1/2)*11i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 2
5*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^
3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^6*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e
) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
))/4 + 1/2))^(1/2)*98i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^
```

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4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110
*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*
b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*
b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^3*b^5*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(
2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i))/4 + 1/2))^(1/2)*299i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20
*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
00*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7
*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6
*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^4*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*286i)/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^
5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a
```

```
^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a
^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^3*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(
2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*8i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5
*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 1
6*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b
^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 1
5*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b
^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^6*b^2*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*19i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*
a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f
*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^
2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
```

```
in(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^
8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^2*b^6*(
cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) -
 sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*86i)/(a^3*b^7*f + 6*a^4*b^6*f +
15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4
*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b
^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6
*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^3*b^
5*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x
) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*203i)/(2*(a^3*b^7*f + 6*a^4*b
^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f +
 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
6*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*
b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) +
 (a^4*b^4*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((c
os(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*121i)/(6*(a^3*b^7*f +
 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a
^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) -
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2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a
^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i))) - (a^5*b^3*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a
+ b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*152i)/(3*(a^3
*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b
^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a
^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 10
0*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*
b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i))) - (a^6*b^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*130i)
/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f +
 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^
2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
```

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(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^
8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i))) - (a^2*b^6*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i)
)/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/
2)*98i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3
*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) +
4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*
b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 1
6*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i)) - (a^3*b^5*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*
1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^
(1/2)*331i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*
a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4
*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*
```

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x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^4*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - s
in(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4
+ 1/2))^(1/2)*464i)/(3*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4
*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*
a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b
^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6
*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b
^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b^3*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(
2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i))/4 + 1/2))^(1/2)*92i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6
*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a
^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a
^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^6*b^2*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(c
os(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i))/4 + 1/2))^(1/2)*37i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*
a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
```

8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^2*b^6*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*92i)/(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a^3*b^5*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*275i)/(2*(a^3*b^7*f + 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(cos(6*f*

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x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 15*
a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^8*b^2
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^4*b^4*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin
(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*409i)/(4*(a^3*b^7*f + 6*a^4*b^6*f +
15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f + 2*a^9*
b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + sin(2*f*x
)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 4
*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 26*a^3*b
^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4*b^6*f*(
cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6
*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^5*b
^3*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*
x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*125i)/(3*(a^3*b^7*f + 6*a^4*
b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9*b*f
+ 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^9*b
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 72*a^4
*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5*b^5*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
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*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)))
 - (a^6*b^2*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((
cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*65i)/(6*(a^3*b^7*f +
 6*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a
^9*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) -
2*a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 72*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a
^5*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^
4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(
cos(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i))) + (a^2*b^5*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a
+ b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*957i)/(2*(3*a
^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^
7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8
*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a
^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*
x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i))) + (a^3*b^4*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/
4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)
*1397i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a
^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
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a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7
f(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x)
+ sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3
*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) -
sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/
4 + 1/2))^(1/2)*519i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^
5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 17
2*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4
*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + si
n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
n(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5
*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1
i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i))/4 + 1/2))^(1/2)*83i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5
*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 18
0*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*
b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + si

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n(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8
*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i
) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25
*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^
2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^2*b^5*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin
(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1923i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*
f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12
*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^
2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) +
 (a^3*b^4*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((c
os(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*5303i)/(6*(3*a^2*b^7*
f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f
 + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1
i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a
*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2
30*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^
5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8
*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
```

```
in(8*e)*1i))) + (a^4*b^3*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*
1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*2785i)
/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*
f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
 - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*
e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20
*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^
7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*
e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2
))^(1/2)*839i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f
 + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*
a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b
^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 1
6*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*
b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8
*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^2*b^5*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos
(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i))/4 + 1/2))^(1/2)*1299i)/(2*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f
 + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*
f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
```

```
(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*
a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^
3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*
(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + s
in(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e
)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
- 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a
^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^3*b^4*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2*f*x) - sin(
2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1805i)/(3*(3*a^2*b^7*f + 16*a^3*b^6*f
 + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*
a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*
x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2
*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) +
(a^4*b^3*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((co
s(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*633i)/(2*(3*a^2*b^7*f
+ 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f +
 a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
 + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b
^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*
e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e
)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230
*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*
```

```
b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6
*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*
1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i))) + (a^5*b^2*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos
(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*95i)/(3*
a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a
^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^
8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*
a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^
2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*
x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + s
in(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i)) + (a^2*b^5*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/
4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)
*126i)/(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b
^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos
(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*
1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8
*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)
*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2
*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)
*1i)*(cos(8*e) + sin(8*e)*1i)) + (a^3*b^4*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
```

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(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(
2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1
/2))^(1/2)*503i)/(4*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*f + 40*a^5*b^4
*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 7
0*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 172*a^3
*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 230*a^4*b^5*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180*a^5*b^4*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b^3*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*f*x) + sin(6*f
*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x) + sin(6*f*x)*1
i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + s
in(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 35*a^
4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 40*a^5*b^4*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*a^6*b^3*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^4*b^3*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(c
os(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i))/4 + 1/2))^(1/2)*421i)/(6*(3*a^2*b^7*f + 16*a^3*b^6*f + 35*a^4*b^5*
f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12*a*b^8*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(co
s(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
+ 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 180
*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^6*b
^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 20*a^7*b^2*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^2*b^7*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*f*x) + sin(6*f*
x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 25*
a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 8*a^7*b^2
*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) + (a^5*b^2*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin
(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*265i)/(12*(3*a^2*b^7*f + 16*a^3*b^6*
f + 35*a^4*b^5*f + 40*a^5*b^4*f + 25*a^6*b^3*f + 8*a^7*b^2*f + a^8*b*f + 12
*a*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2*a^8*b*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a*b^8*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 2*a^8*b*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^8*b*f*(cos(8*f*x) + sin(8*f*x)*1i)
*(cos(8*e) + sin(8*e)*1i) + 70*a^2*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
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2*e) + sin(2*e)*1i) + 172*a^3*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 230*a^4*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 180*a^5*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^6*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 20*a^7*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 70*a^
2*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 172*a^3*b^6
*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 230*a^4*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 180*a^5*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^6*b^3*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 20*a^7*b^2*f*(cos(6*f*x) + sin(6
*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^2*b^7*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 16*a^3*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 35*a^4*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 40*a^5*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 25*a^6*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 8*a^7*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) -
 (a*b^8*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)*(a + b/(((cos
(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*261i)/(2*(3*a^3*b^8*f +
 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f +
 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) +
12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3
*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*
f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(
2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)
*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*
e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6
*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^
9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f
*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(
8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x)
 + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin
(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*
(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*
e) + sin(8*e)*1i))) - (a^8*b*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 +
 ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*8i
)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f
+ 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
 sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 4
02*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^
6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4
*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f
*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) +
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```
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(
cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6
*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) +
 sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19
*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b
^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(
cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (a*b^8*(cos(4*f*x) + sin(4*f*x)*1i)*(c
os(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - si
n(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 +
 1/2))^(1/2)*405i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b
^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(c
os(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1
i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + s
in(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*
e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 2
2*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*
b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*
(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*
f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*
1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(c
os(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*
1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8
*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*
(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^8*b*(cos(4*f*x
) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*
x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e
) + sin(2*e)*1i))/4 + 1/2))^(1/2)*12i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5
*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*
f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^
10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*
x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + s
in(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*
f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(
cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2
*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i
) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24
2*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5
```

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*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) +
sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f
*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i
)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos
(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(
8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)) - (
a*b^8*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)*(a + b/(((cos(2
*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*27i)/(2*(3*a^3*b^8*f + 19
*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*
a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*
a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^
8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2
*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f
*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin
(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102
*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b
^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(c
os(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f
*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) +
sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*
f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
+ sin(8*e)*1i))) - (a^8*b*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)
*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*8i)/(
3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 3
3*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 402*
a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 410*a^6*b
^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 102*a^8*b^3*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x)
 + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin
(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos
(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
 + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + si
```

n(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e
)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i)
- 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^
4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 51*a^5*b^6*
f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x
) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^3*f*(cos(8*f*x) + si
n(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i)) + (a*b^8*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2
*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/
2))^(1/2)*117i)/(2*(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6*f + 75*a^6*b^5*
f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f + 2*a^10*b*f*(cos(
2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*f*x) + sin(8*f*x
)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2
*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) +
 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 22*a
^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 12*a^2*b^9
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 82*a^3*b^8*f*(co
s(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^4*b^7*f*(cos(6*f
*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6*f*(cos(6*f*x) +
 sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*f*x) + sin(6*f*x
)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)
*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(
6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) +
sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*
e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)
 - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 65*
a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 33*a^8*b^
3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 9*a^9*b^2*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))) - (a^8*b*(cos(8*f*x) +
 sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i)*(a + b/(((cos(2*f*x) - sin(2*f*x)*
1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) +
 sin(2*e)*1i))/4 + 1/2))^(1/2)*2i)/(3*a^3*b^8*f + 19*a^4*b^7*f + 51*a^5*b^6
*f + 75*a^6*b^5*f + 65*a^7*b^4*f + 33*a^8*b^3*f + 9*a^9*b^2*f + a^10*b*f +
2*a^10*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*a^10*b
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^10*b*f*(cos(8*
f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 12*a^2*b^9*f*(cos(2*f*x) +
 sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 82*a^3*b^8*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 242*a^4*b^7*f*(cos(2*f*x) + sin(2*f*x)
1i)(cos(2*e) + sin(2*e)*1i) + 402*a^5*b^6*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 410*a^6*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(
2*e) + sin(2*e)*1i) + 262*a^7*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
+ sin(2*e)*1i) + 102*a^8*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) + 22*a^9*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*
1i) - 12*a^2*b^9*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) -
82*a^3*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 242*a^
4*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 402*a^5*b^6
f(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 410*a^6*b^5*f*(c
os(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 262*a^7*b^4*f*(cos(6*
f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 102*a^8*b^3*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 22*a^9*b^2*f*(cos(6*f*x) + sin(
6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 3*a^3*b^8*f*(cos(8*f*x) + sin(8*f*x)*

```
1i)*(cos(8*e) + sin(8*e)*1i) - 19*a^4*b^7*f*(cos(8*f*x) + sin(8*f*x)*1i)*(c
os(8*e) + sin(8*e)*1i) - 51*a^5*b^6*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e
) + sin(8*e)*1i) - 75*a^6*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + si
n(8*e)*1i) - 65*a^7*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 33*a^8*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) -
 9*a^9*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.118 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2b\left(15a^2-10ab-b^2\right)\tan(e+fx)}{15f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\left(15a^2-10ab-b^2\right)\cot(e+fx)}{15f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -1/15*(15*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2
/15*(5*a+2*b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/5*cot(f*x
+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/15*b*(15*a^2-10*a*b-b^2)*tan(f*x
+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4132, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2-10ab-b^2\right)\tan(e+fx)}{15f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\left(15a^2-10ab-b^2\right)\cot(e+fx)}{15f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(15*(a + b)^3*f*Sqrt[a + b + b*Tan[
e + f*x]^2]) - (2*(5*a + 2*b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*Sqrt[a + b +
b*Tan[e + f*x]^2]) - Cot[e + f*x]^5/(5*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x
]^2]) - (2*b*(15*a^2 - 10*a*b - b^2)*Tan[e + f*x])/(15*(a + b)^4*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a+2b)+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+\right.}{5(a+b)f}$$

$$= -\frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} +$$

$$= -\frac{\left(15a^2-10ab-b^2\right)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{\left(15a^2-10ab-b^2\right)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.92, size = 126, normalized size = 0.69

$$-\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(4a\left(a^2-4ab-5b^2\right)\csc^2(e+fx)-8a^2(a-5b)+3(a+\right.}{30f(a+b)^4\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*a^2*(a - 5*b) + 4*a*(a^2 - 4*a*b
- 5*b^2)*Csc[e + f*x]^2 + (a - 5*b)*(a + b)^2*Csc[e + f*x]^4 + 3*(a + b)^3*
Csc[e + f*x]^6)*Sec[e + f*x]^2*Tan[e + f*x])/((a + b)^4*f*(a + b*Sec[e + f*
x]^2)^(3/2))
```

fricas [A] time = 4.67, size = 314, normalized size = 1.72

$$-\frac{\left(8\left(a^3-5a^2b\right)\cos\left(fx+e\right)^7-4\left(5a^3-26a^2b+5ab^2\right)\cos\left(fx+e\right)^5+\left(15a^3\right.\right.}{15\left(\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)f\cos\left(fx+e\right)^6-\left(2a^5+7a^4b+8a^3b^2+2a^2b^3-2ab^4-b^5\right)f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^3 - 5*a^2*b)*cos(f*x + e)^7 - 4*(5*a^3 - 26*a^2*b + 5*a*b^2)*co
s(f*x + e)^5 + (15*a^3 - 85*a^2*b + 49*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 2*(1
5*a^2*b - 10*a*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)
^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x +
e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 2.11, size = 204, normalized size = 1.11

$$-\frac{\left(8\left(\cos^{6}\left(fx+e\right)\right)a^{3}-40\left(\cos^{6}\left(fx+e\right)\right)a^{2}b-20\left(\cos^{4}\left(fx+e\right)\right)a^{3}+104\left(\cos^{4}\left(fx+e\right)\right)a^{2}b-20\left(\cos^{4}\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/f/(b+a*cos(f*x+e)^2)^2*(8*cos(f*x+e)^6*a^3-40*cos(f*x+e)^6*a^2*b-20*c
os(f*x+e)^4*a^3+104*cos(f*x+e)^4*a^2*b-20*cos(f*x+e)^4*a*b^2+15*cos(f*x+e)^
2*a^3-85*a^2*cos(f*x+e)^2*b+49*cos(f*x+e)^2*a*b^2+5*cos(f*x+e)^2*b^3+30*a^2
*b-20*b^2*a-2*b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3/sin
(f*x+e)^5/(a+b)^4
```

maxima [A] time = 0.46, size = 282, normalized size = 1.54

$$-\frac{\frac{30\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}-\frac{80\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{3}}+\frac{48\,b^{3}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{4}}+\frac{15}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)\tan\left(fx+e\right)}-\frac{}{\sqrt{b\tan}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 80*b^
2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 48*b^3*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) + 15/(sqrt(b*tan(f*x + e)^2
 + a + b)*(a + b)*tan(f*x + e)) - 40*b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a +
 b)^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(
f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^3) - 6*
b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)^3) + 3/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^5))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] \text{Hanged}
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.119 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{8b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^4f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(5a^2+\cdots\right.}{5a^3f\cdots}$$

```
[Out] -1/5*(5*a^2+20*a*b+16*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/2)+2/15*(
5*a+4*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/5*cos(f*x+e)^5/a/f/(
a+b*sec(f*x+e)^2)^(3/2)-4/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^4/f/(a+b*
sec(f*x+e)^2)^(3/2)-8/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^5/f/(a+b*sec(
f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4134, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^4f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(\frac{4b(5a\cdots}{a^2}\cdots\right.}{5af\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((5 + (4*b*(5*a + 4*b))/a^2)*Cos[e + f*x])/(5*a*f*(a + b*Sec[e + f*x]^2)^(
3/2)) + (2*(5*a + 4*b)*Cos[e + f*x]^3)/(15*a^2*f*(a + b*Sec[e + f*x]^2)^(3/
2)) - Cos[e + f*x]^5/(5*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 + 2
0*a*b + 16*b^2)*Sec[e + f*x])/(15*a^4*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*
b*(5*a^2 + 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^5*f*Sqrt[a + b*Sec[e + f*x]
^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a+4b)+5ax^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\left(5a^2+20ab+1\right.}{}$$

$$= -\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{co}}{5af\left(a+\right.}$$

$$= -\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{co}}{5af\left(a+\right.}$$

$$= -\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{co}}{5af\left(a+\right.}$$

Mathematica [A] time = 3.21, size = 182, normalized size = 0.83

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(-16a^4\cos(6(e+fx))+3a^4\cos(8(e+fx))+425a^4-32a^3b\cos(6(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3840*((a + 2*b + a*Cos[2*(e + f*x)])*(425*a^4 + 6400*a^3*b + 22784*a^2*b
^2 + 32768*a*b^3 + 16384*b^4 + 48*a*(11*a^3 + 150*a^2*b + 384*a*b^2 + 256*b
^3)*Cos[2*(e + f*x)] + 12*a^2*(7*a^2 + 64*a*b + 64*b^2)*Cos[4*(e + f*x)] -
16*a^4*Cos[6*(e + f*x)] - 32*a^3*b*Cos[6*(e + f*x)] + 3*a^4*Cos[8*(e + f*x)
])*Sec[e + f*x]^5)/(a^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [A] time = 0.96, size = 189, normalized size = 0.86

$$-\frac{\left(3\,a^4\cos\left(fx+e\right)^9-2\left(5\,a^4+4\,a^3b\right)\cos\left(fx+e\right)^7+3\left(5\,a^4+20\,a^3b+16\,a^2b^2\right)\cos\left(fx+e\right)^5+12\left(5\,a^3b+20\,\right.}{15\left(a^7f\cos\left(fx+e\right)^4+2\,a^6bf\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^4*cos(f*x + e)^9 - 2*(5*a^4 + 4*a^3*b)*cos(f*x + e)^7 + 3*(5*a^4
 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^5 + 12*(5*a^3*b + 20*a^2*b^2 + 16*a*
b^3)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*co
s(f*x + e)^2 + a^5*b^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*
(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-1/2304*tan((f*x+exp(1))/2)^
2*(-2112*a^13*b^6*sign(tan((f*x+exp(1))/2)^2-1)-5376*a^14*b^5*sign(tan((f*x
+exp(1))/2)^2-1)-4416*a^15*b^4*sign(tan((f*x+exp(1))/2)^2-1)-1152*a^16*b^3*
sign(tan((f*x+exp(1))/2)^2-1))/a^18/b^2-1/2304*(-6336*a^13*b^6*sign(tan((f*
x+exp(1))/2)^2-1)-6912*a^14*b^5*sign(tan((f*x+exp(1))/2)^2-1)+576*a^15*b^4*
sign(tan((f*x+exp(1))/2)^2-1)+1152*a^16*b^3*sign(tan((f*x+exp(1))/2)^2-1))/
a^18/b^2)-1/2304*(-6336*a^13*b^6*sign(tan((f*x+exp(1))/2)^2-1)-6912*a^14*b^
5*sign(tan((f*x+exp(1))/2)^2-1)+576*a^15*b^4*sign(tan((f*x+exp(1))/2)^2-1)+
1152*a^16*b^3*sign(tan((f*x+exp(1))/2)^2-1))/a^18/b^2)-1/2304*(-2112*a^13*b
^6*sign(tan((f*x+exp(1))/2)^2-1)-5376*a^14*b^5*sign(tan((f*x+exp(1))/2)^2-1
)-4416*a^15*b^4*sign(tan((f*x+exp(1))/2)^2-1)-1152*a^16*b^3*sign(tan((f*x+e
xp(1))/2)^2-1))/a^18/b^2)/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2
)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan((f*x+ex
p(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+e
xp(1))/2)^2+a+b)+2/15*((45*b^2+30*a*b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sq
rt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+2*b*tan((f*x+exp(1))/2)^2+a+b))^9+sqrt(a+b)*(-315*b^2-330*a*b)*(-tan((f*x
+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^
4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^8+(-tan((f*x+ex
```

```
p(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2
*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^7*(320*a^3+900*b^3
+1260*a*b^2+760*a^2*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*t
an((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*
tan((f*x+exp(1))/2)^2+a+b))^6*(640*a^3-1260*b^3+1020*a*b^2+2360*a^2*b)+(-ta
n((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1
))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^5*(-832*a
^4+630*b^4-5280*a*b^3-9330*a^2*b^2-5180*a^3*b)+(-tan((f*x+exp(1))/2)^2*sqrt
(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp
(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))*(2880*a^6-315*b^6+2610*a*b^5-3610
*a^2*b^4-13480*a^3*b^3+14125*a^4*b^2+20110*a^5*b)+sqrt(a+b)*(-tan((f*x+exp(
1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a
*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^4*(-2560*a^4+630*b^4
+6480*a*b^3-4050*a^2*b^2-12620*a^3*b)+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqr
t(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+2*b*tan((f*x+exp(1))/2)^2+a+b))^3*(-320*a^5-1260*b^5-1500*a*b^4+16740*a^2*
b^3+18220*a^3*b^2+2200*a^4*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+s
qrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)
^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2*(3200*a^5+900*b^5-3660*a*b^4-7500*a^2*
b^3+20380*a^3*b^2+22680*a^4*b)+sqrt(a+b)*(768*a^6+45*b^6-630*a*b^5+3190*a^2
*b^4-4920*a^3*b^3-907*a^4*b^2+4806*a^5*b))/a^4/(2*sqrt(a+b)*(-tan((f*x+exp(
1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a
*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-tan((f*x+exp(1))/2
)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan(
(f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2+3*a-b)^5/sign(tan((f*x+
exp(1))/2)^2-1))
```

maple [A] time = 3.41, size = 229, normalized size = 1.05

$$\frac{(a+b)^7 \left(b+a\left(\cos^2\left(fx+e\right)\right)\right)\left(3\left(\cos^8\left(fx+e\right)\right)a^4-10\left(\cos^6\left(fx+e\right)\right)a^4-8\left(\cos^6\left(fx+e\right)\right)a^3b+15\left(\cos^4 \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/30/f*(a+b)^7*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^8*a^4-10*cos(f*x+e)^6*a^4-8
*cos(f*x+e)^6*a^3*b+15*cos(f*x+e)^4*a^4+60*cos(f*x+e)^4*a^3*b+48*cos(f*x+e)
^4*a^2*b^2+60*cos(f*x+e)^2*a^3*b+240*cos(f*x+e)^2*a^2*b^2+192*cos(f*x+e)^2*
a*b^3+40*a^2*b^2+160*a*b^3+128*b^4)*4^(1/2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^
2)^(5/2)/cos(f*x+e)^5*a^2/((-a*b)^(1/2)+a)^7/((-a*b)^(1/2)-a)^7
```

maxima [A] time = 0.42, size = 334, normalized size = 1.53

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3}-\frac{10\left(\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-9\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)\right)}{a^4}+\frac{3\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5-20\left(a+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^4 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a + b/cos(f*x +
```

```
 e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x +
 e))/a^5 + 5*(6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(
f*x + e)^2)^(3/2)*a^3*cos(f*x + e)^3) + 10*(9*(a + b/cos(f*x + e)^2)*b^2*co
s(f*x + e)^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3) + 5*(
12*(a + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a + b/cos(f*x + e)^2)
^(3/2)*a^5*cos(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.120 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -(a+2*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)+1/3*cos(f*x+e)^3/a/f/(a+
b*sec(f*x+e)^2)^(3/2)-4/3*b*(a+2*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/
2)-8/3*b*(a+2*b)*sec(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4134, 453, 271, 192, 191}

$$-\frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + 2*b)*Cos[e + f*x])/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))) + Cos[e +
f*x]^3/(3*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + 2*b)*Sec[e + f*x])/
(3*a^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a + 2*b)*Sec[e + f*x])/(3*a^
4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b(a+2b))\,\text{Subst}\big(}{}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e}{3a^3f\left(a+b\sec^2(e+\right.}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e}{3a^3f\left(a+b\sec^2(e+\right.}$$

Mathematica [A] time = 2.31, size = 129, normalized size = 0.88

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^3(-\cos(6(e+fx)))+26a^3+3a\left(11a^2+96ab+128b^2\right)\cos(2(e+fx)\right.}{192a^4f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/192*((a + 2*b + a*Cos[2*(e + f*x)])*(26*a^3 + 264*a^2*b + 640*a*b^2 + 51
2*b^3 + 3*a*(11*a^2 + 96*a*b + 128*b^2)*Cos[2*(e + f*x)] + 6*a^2*(a + 4*b)*
Cos[4*(e + f*x)] - a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^5)/(a^4*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

fricas [A] time = 0.71, size = 138, normalized size = 0.95

$$\frac{\left(a^3\cos\left(fx+e\right)^7-3\left(a^3+2a^2b\right)\cos\left(fx+e\right)^5-12\left(a^2b+2ab^2\right)\cos\left(fx+e\right)^3-8\left(ab^2+2b^3\right)\cos\left(fx+e\right)\right)\sqrt{}}{3\left(a^6f\cos\left(fx+e\right)^4+2a^5bf\cos\left(fx+e\right)^2+a^4b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(f*x + e)^7 - 3*(a^3 + 2*a^2*b)*cos(f*x + e)^5 - 12*(a^2*b + 2*
a*b^2)*cos(f*x + e)^3 - 8*(a*b^2 + 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e
```

```
)^2 + b)/cos(f*x + e)^2)/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 +
 a^4*b^2*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 2.10, size = 159, normalized size = 1.09

$$-\frac{(a+b)^5\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)\left(\left(\cos^6\left(fx+e\right)\right)a^3-3\left(\cos^4\left(fx+e\right)\right)a^3-6\left(\cos^4\left(fx+e\right)\right)a^2b-12a^2\left(\cos\right.\right.}{6f\left(\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5\left(\sqrt{-ab}-a\right)^5\left(\sqrt{-ab}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/6/f*(a+b)^5*(b+a*cos(f*x+e)^2)*(cos(f*x+e)^6*a^3-3*cos(f*x+e)^4*a^3-6*co
s(f*x+e)^4*a^2*b-12*a^2*cos(f*x+e)^2*b-24*cos(f*x+e)^2*a*b^2-8*b^2*a-16*b^3
)*4^(1/2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5*a/((-a*b)^(1
/2)-a)^5/((-a*b)^(1/2)+a)^5
```

maxima [A] time = 0.41, size = 195, normalized size = 1.34

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3}-\frac{\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-9\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)}{a^4}+\frac{6\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)b\cos\left(fx+e\right)^2-b^2}{\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}a^3\cos\left(fx+e\right)^3}+\frac{9\left(a+\frac{}{\cos}\right.}{\left(a+\frac{}{\cos}\right.}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^4
+ (6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)
^(3/2)*a^3*cos(f*x + e)^3) + (9*(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 -
 b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/a^2/f/(a+b*sec(f*
x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 271, 192, 191}

$$-\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e + f*x])/(3*
a^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x])/(3*a^3*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(8b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right.}\right.}{3a}$$

$$= -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+}}$$

Mathematica [A] time = 1.32, size = 88, normalized size = 0.91

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(3a^2\cos(4(e+fx))+12a(a+4b)\cos(2(e+fx))+(3a+8b)^2\right)}{48a^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 2*b + a*Cos[2*(e + f*x)])*((3*a + 8*b)^2 + 12*a*(a + 4*b)*Cos[2
*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^5)/(a^3*f*(a + b*Sec[e +
 f*x]^2)^(5/2))
```

fricas [A] time = 0.62, size = 101, normalized size = 1.04

$$-\frac{\left(3\,a^2\cos\left(fx+e\right)^5+12\,ab\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^5 + 12*a*b*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f
*cos(f*x + e)^2 + a^3*b^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
```

```
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)2/f*2*(2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-1/
36864*tan((f*x+exp(1))/2)^2*(-7680*a^7*b^4*sign(tan((f*x+exp(1))/2)^2-1)-92
16*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2-1/36864*(-23040*a^7*b^4*
sign(tan((f*x+exp(1))/2)^2-1)+9216*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1))/a
^10/b^2)-1/36864*(-23040*a^7*b^4*sign(tan((f*x+exp(1))/2)^2-1)+9216*a^8*b^3
*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2)-1/36864*(-7680*a^7*b^4*sign(tan((
f*x+exp(1))/2)^2-1)-9216*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2)/s
qrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)
^2+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1
))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+(-tan((f*x
+exp(1))/2)^2*sqrt(a+b)+sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+e
xp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/a^2/(
2*sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+
b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2
+a+b))-(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan
((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)
)^2+3*a-b)/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 0.30, size = 90, normalized size = 0.93

$$-\frac{\frac{1}{a\sec\left(fx+e\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec\left(fx+e\right)}{3a\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{2\sec\left(fx+e\right)}{3a^{2}\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2)-4*b/a*(1/3*sec(f*x+e)/a/(a+b*
sec(f*x+e)^2)^(3/2)+2/3/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)))
```

maxima [A] time = 0.39, size = 86, normalized size = 0.89

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a^{3}}+\frac{6\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)b\cos\left(fx+e\right)^{2}-b^{2}}{\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}a^{3}\cos\left(fx+e\right)^{3}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 + (6*(a + b/cos(f*x + e
)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x + e)
^3))/f
```

mupad [B] time = 17.23, size = 26927, normalized size = 277.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
+ f*x*3i)*(((2*a + 4*b)*(((2*a + 4*b)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(48*
a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a +
```

```
b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) - (16
*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a - (a + 6*b)/(24*a*b*f*(a + b)
) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a + 4*b)*(8*a*
b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))/a - (((32*a*b^2 + 30*a^2*b + 3
*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^
4*b*f*(a + b)))*(2*a + 4*b))/a - (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a +
 b)) + (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b)) + (32*a*b^2 + 40*a^2*b
+ 3*a^3)/(48*a^3*b*f*(a + b))) + exp(e*1i + f*x*1i)*(((2*a + 4*b)*((((32*a*
b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^
2*b + a^3))/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3
)/(48*a^3*b*f*(a + b)) - (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a +
 (48*a*b^2 + 18*a^2*b + a^3 + 32*b^3)/(48*a^3*b*f*(a + b)) - (a + 6*b)/(24*
a*b*f*(a + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a
 + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))*(2*exp(e*2i + f*x
*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)*(a + exp(e*2i + f
*x*2i)*(2*a + 4*b) + a*exp(e*4i + f*x*4i))) - ((a + b/(exp(- e*1i - f*x*1i)
/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*1i + f*x*1i)*(((a*2i + b*4i)*(((
a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a
*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^
2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))
/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b
^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*
b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((
8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*2i)/(3*f*(2*a*b + a^2)) + (
(a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^
2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^
2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*
b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a*2i + b*
4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2
*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*
a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*
a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a
*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)
/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2
*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(4
8*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(1
2*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)
^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (3*a*(a + 2*b)*(8*a*b
 + a^2 + 8*b^2))/(16*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a + 2*b)*(
a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b))
) + exp(e*3i + f*x*3i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a
^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a
^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2
*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((
a*2i + b*4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i
)/(3*f*(2*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*
1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a
^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2
*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i
)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b +
a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^
2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2 + 8
*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*2i + b*4i)*(((a*2i
 + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i
)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*
1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a
^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b
```

```
+ a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (a*(a
+ 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a -
 ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*3i)/(4*f*(2*a*b
 + a^2)) + ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b
+ a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b
+ a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a
+ 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a +
 ((a*2i + b*4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))
*1i)/(3*f*(2*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i
 + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - (
(a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b
+ a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 +
a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))
*1i)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b
 + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8
*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2
+ 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b +
a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (3*a*(a +
 2*b)*(8*a*b + a^2 + 8*b^2))/(16*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a -
((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)))*1i)/a + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(6*f*(2*a*b + a^2
)*(a*b^2 + a^2*b))))*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp
(e*2i + f*x*2i) + 1)*(a*1i + exp(e*2i + f*x*2i)*(a*2i + b*4i) + a*exp(e*4i
+ f*x*4i)*1i)^2) - exp(- e*1i - f*x*1i)*(a + b/(exp(- e*1i - f*x*1i)/2 + ex
p(e*1i + f*x*1i)/2)^2)^(1/2)*(1/(2*a^3*f) + exp(e*2i + f*x*2i)/(2*a^3*f)) -
 ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
 + f*x*3i)*(((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a
^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 +
10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
 b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4
i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
+ b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((
a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*
(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2
/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b
 + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8
*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*
b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^
2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2
*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
+ (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^
2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*
b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)
*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*
1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a
*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i
)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
 + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a
*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a
```

```
^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^
2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^
2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*
a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i +
b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*
1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) -
 ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^
2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2
 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(
a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2
)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^
2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
 b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a
^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 +
 a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a
*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a
^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(
2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
 + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b
^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)
*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8
*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2
i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3
)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
+ ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1
i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i
)/a - ((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a
 + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(
a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*
b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)
^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*
(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i
+ b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^
2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*
b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2
+ 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*
b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2
+ 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a
^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 +
10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
 b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)
) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(1
2*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)
^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
+ ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2
*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b
)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i +
 b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/
(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*
a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*
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a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1

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i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b +
 a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2
*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((
(8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i
))*(44*a*b + 5*a^2 + 44*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8
*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))
*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b +
 a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a
*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b
*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(2
4*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*
b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2
 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)
*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a
^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46
*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*
b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*((((8*a*b + a^2 + 8
*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2
+ 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4
i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i
+ b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*
a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*
((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b
*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a +
 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*
b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b
^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2
+ a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4
*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b +
a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*
b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)
*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b +
 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i
+ b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b
)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*
b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (
a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))
/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b
^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2
)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
- ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i
)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)
/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
 + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i))
- (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i +
b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i
 + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i
)/(a*1i + b*2i))*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3))/(12*a^2*b*f*(2*a*b +
 a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*
b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^4*(8*a*b + a^2 + 8*b
^2)*(16*a*b + a^2 + 16*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3
 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 +
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a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)
*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i +
b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a
+ 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a
*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*
b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2
 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(
4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b +
 a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a
*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i
)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b
+ 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i
 + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*
b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a*b*f*
(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)
) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b
 + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a
*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/
(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*((8*a*b + a^2 + 8*b
^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(16*a*b + a^2 +
 16*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b +
a^2 + 8*b^2)*(a*b - a^2 + 2*b^2)*1i)/(8*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(
a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b +
3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*
1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*
f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i)/(48*a*b*f*(2*a*
b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4
i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a
^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(
2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b
^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)
) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*
1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1
i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*
1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*
(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b +
 a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b +
a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(
2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b
 + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*
a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2
 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3
 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)
) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*
f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2
*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a
^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14
*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b
)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2
 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(a*b - a^2 + 2*b^2))/(2*a^2*f*
(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)
) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*
a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) +
 (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a
^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 +
 a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(46*a*b^2 + 10*a^2*b + a^3 + 36*

b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + b)*(a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*1i)/(16*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^4*(8*a*b + a^2 + 8*b^2)*(16*a*b + a^2 + 16*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))) - exp(e*1i + f*x*1i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b

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^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2
/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b
 + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8
*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*
1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*
b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*
1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^
2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2
*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
 + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^
2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*
b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)
*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*
1i)/a - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a
*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2))/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i
)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i
 + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a
*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a
^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^
2 + 8*b^2)*(44*a*b + 5*a^2 + 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^
2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*
a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i + b*4i)*(((a*2i +
b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*
1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) -
 ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^
2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2
 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(
a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2
)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^
2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
 b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a
^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 +
 a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44*b^2))/(48*a
*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a
^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(
2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b))
 + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b
^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 +
 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)
*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8
*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2
i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3
)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
+ ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1
i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i
)/a - ((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a
 + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(
a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*
b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)
^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*
(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((a*2i
+ b*4i)*(((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^
2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a*
```

```
b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2
+ 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*
b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i
)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b +
a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*
a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a
*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2*
b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a - (((
8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i)
)*(8*a*b + a^2 + 8*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b
+ a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*
a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i +
b*1i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(
24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2
*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^
2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2
)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 +
a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(
a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(13*a*b + 3*a^2 + 10*b^2)
)/(6*a^2*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b
^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 +
 4*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)
^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(46*a*b^2 + 10*a^2
*b + a^3 + 36*b^3))/(12*a^2*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a + 2*b)^2
*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i +
 b*1i)*(a*1i + b*2i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(44*a*b + 5*a^2
+ 44*b^2)*1i)/(192*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
b*2i)) + ((a + 2*b)^4*(8*a*b + a^2 + 8*b^2)*(16*a*b + a^2 + 16*b^2)*1i)/(19
2*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) - ((a
+ 2*b)^2*(8*a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*
1i)/(192*a^2*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))
)*1i)/a + ((a*2i + b*4i)*(((a*2i + b*4i)*((((8*a*b + a^2 + 8*b^2)^2/(4*(a*b
^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))/(48*a
*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2
 + 8*b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i +
 b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4
*b^3)*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2
i)))*1i)/a + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1
i)/(a*1i + b*2i))*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3))/(12*a^2*b*f*(2*a*b +
 a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)^2*1i)/(192*b*f*(2
*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*
a*b + a^2 + 8*b^2)*(64*a*b^3 + 68*a^3*b + 5*a^4 + 132*a^2*b^2)*1i)/(192*a^2
*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)))*1i)/a + ((
a + b)*((8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1
i + b*2i)))/(4*a*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2
/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(44*a*b + 5*a^2 + 44
*b^2))/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - (((8*a*b + a^2 + 8*b^2)^2/(
4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(8*a*b + a^2 + 8*b^2))
/(48*a*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + (((8*a*b + a^2 + 8*b^2)^2/(4*(a*b
^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i))*(64*a*b^3 + 68*a^3*b + 5*a^4
 + 132*a^2*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) - ((a + 2*b)^2*((
8*a*b + a^2 + 8*b^2)^2/(4*(a*b^2 + a^2*b)) + (a*(a + 2*b)*1i)/(a*1i + b*2i)
)*(16*a*b + a^2 + 16*b^2))/(48*a^3*b*f*(2*a*b + a^2)*(a*1i + b*1i)) + ((a +
 2*b)^2*(8*a*b + a^2 + 8*b^2)*(a*b - a^2 + 2*b^2)*1i)/(8*a*f*(2*a*b + a^2)*
(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(8*a*b + a^2 +
8*b^2)*(13*a*b + 3*a^2 + 10*b^2)*1i)/(24*a*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*
(a*1i + b*1i)*(a*1i + b*2i)) + ((a + 2*b)^2*(a*2i + b*4i)*(8*a*b + a^2 + 8*
```

```
b^2)^2)/(192*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i
)) - ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(14*a*b^2 + 10*a^2*b + a^3 + 4*b^3)
*1i)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i)) +
 ((a + 2*b)^2*(8*a*b + a^2 + 8*b^2)*(46*a*b^2 + 10*a^2*b + a^3 + 36*b^3)*1i
)/(48*a*b*f*(2*a*b + a^2)*(a*b^2 + a^2*b)*(a*1i + b*1i)*(a*1i + b*2i))))*(2
*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)*(a
*1i + exp(e*2i + f*x*2i)*(a*2i + b*4i) + a*exp(e*4i + f*x*4i)*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.122 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{b(5a+2b)\sec(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\sec(e+fx)}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f-1/3
*b*sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+2*b)*sec(f*x+e)
/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4134, 414, 527, 12, 377, 207}

$$-\frac{b(5a+2b)\sec(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\sec(e+fx)}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
5/2)*f)) - (b*Sec[e + f*x])/(3*a*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) -
(b*(5*a + 2*b)*Sec[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{3a+2b-2bx^2}{\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+
}{3a(a+b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2}{3a^2(a+b)^2f}$$

Mathematica [C] time = 5.11, size = 108, normalized size = 0.85

$$\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)\left(3a\sin^2(e+fx)-2(2a+
\right)\right)}{6a^2f(a+b)\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(a^2*Hypergeometric2F1[-3/2,
 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*(-2*(2*a + b) + 3*a*Sin
[e + f*x]^2)))/(6*a^2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 0.70, size = 592, normalized size = 4.66

$$\left[\frac{3\left(a^4\cos\left(fx+e\right)^4+2\,a^3b\cos\left(fx+e\right)^2+a^2b^2\right)\sqrt{a+b}\,\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\,\sqrt{a+b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)+a+2\,b\right)}{\cos\left(fx+e\right)^2-1}\right)-}{6\left(\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)f\cos\left(fx+e\right)^4+2\left(a^6b+3\,a^5b^2+3\,a^4b^3+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(a + b)
*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(2*a^3*b + 3
*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a^5*b^
2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)
*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f), 1/3*(3*
(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b
^3 + 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
+ a^2*b^5)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
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```
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>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
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step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
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p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, in
tegration of abs or sign assumes constant sign by intervals (correct if the
 argument is real):Check [abs(t_nostep^2-1)]Warning, replacing 0 by ` u`, a
 substitution variable should perhaps be purged.Warning, replacing 0 by ` u
`, a substitution variable should perhaps be purged.Warning, replacing 0 by
 ` u`, a substitution variable should perhaps be purged.Evaluation time: 1.
77Error: Bad Argument Type
```

maple [B] time = 2.14, size = 5056, normalized size = 39.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.123 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{b(13a-2b)\sec(e+fx)}{6af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{5b\sec(e+fx)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{7/2}}-\frac{\ldots}{2f(a\ldots}$$

```
[Out] -1/2*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)
^(7/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-5/6*b*s
ec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-2*b)*b*sec(f*x+e)/a/
(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4134, 471, 527, 12, 377, 207}

$$-\frac{b(13a-2b)\sec(e+fx)}{6af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{5b\sec(e+fx)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{7/2}}-\frac{\ldots}{2f(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])
/(2*(a + b)^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*(a + b*Sec[
e + f*x]^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^
2)^(3/2)) - ((13*a - 2*b)*b*Sec[e + f*x])/(6*a*(a + b)^3*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13}{6a(a+}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13}{6a(a+}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13}{6a(a+}$$

$$= -\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{6(a+b)^2}$$

Mathematica [C] time = 1.40, size = 151, normalized size = 0.88

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)\csc^2(e+fx)\left(\left(3a^2+2b^2\right)\cos(2(e+fx))+3a^2+6ab-2b^2\right)-\right.}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/24*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*(3*a^2 + 6*a*b - 2*b^2 + (3*
a^2 + 2*b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - 3*a*(a - 4*b)*(a + 2*b + a*
Cos[2*(e + f*x)])*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a
 + b)])*Sec[e + f*x]^5)/(a*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 0.77, size = 941, normalized size = 5.50

$$\left[-\frac{3\left(\left(a^{4}-4\,a^{3}b\right)\cos\left(fx+e\right)^{6}-\left(a^{4}-6\,a^{3}b+8\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}-a^{2}b^{2}+4\,ab^{3}-\left(2\,a^{3}b-9\,a^{2}b^{2}+4\,ab^{3}\right)c}{12\left(\left(a^{7}+4\,a^{6}b+6\,a^{5}b^{2}+4\,a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{6}-\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a^4 - 4*a^3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos
(f*x + e)^4 - a^2*b^2 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x +
 e)^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) -
 2*(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 -
 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^
2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*
a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 +
8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*
a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/6*(3*((a^4 - 4*a^3*b)*cos(f*
x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 4*a*b^3 -
 (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4
*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4
*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
```

```
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*(tan((f*x+exp(1))/2
)^2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-tan((f*x+exp(1))/2)^2*(
1207959552*a^2*b^12*sign(tan((f*x+exp(1))/2)^2-1)+12079595520*a^3*b^11*sign
(tan((f*x+exp(1))/2)^2-1)+54358179840*a^4*b^10*sign(tan((f*x+exp(1))/2)^2-1
)+144955146240*a^5*b^9*sign(tan((f*x+exp(1))/2)^2-1)+253671505920*a^6*b^8*s
ign(tan((f*x+exp(1))/2)^2-1)+304405807104*a^7*b^7*sign(tan((f*x+exp(1))/2)^
2-1)+253671505920*a^8*b^6*sign(tan((f*x+exp(1))/2)^2-1)+144955146240*a^9*b^
5*sign(tan((f*x+exp(1))/2)^2-1)+54358179840*a^10*b^4*sign(tan((f*x+exp(1))/
2)^2-1)+12079595520*a^11*b^3*sign(tan((f*x+exp(1))/2)^2-1)+1207959552*a^12*
b^2*sign(tan((f*x+exp(1))/2)^2-1))/(-38654705664*a^2*b^13-425201762304*a^3*
b^12-2126008811520*a^4*b^11-6378026434560*a^5*b^10-12756052869120*a^6*b^9-1
7858474016768*a^7*b^8-17858474016768*a^8*b^7-12756052869120*a^9*b^6-6378026
434560*a^10*b^5-2126008811520*a^11*b^4-425201762304*a^12*b^3-38654705664*a^
13*b^2)-(-3221225472*a*b^13*sign(tan((f*x+exp(1))/2)^2-1)-4831838208*a^2*b^
12*sign(tan((f*x+exp(1))/2)^2-1)+96636764160*a^3*b^11*sign(tan((f*x+exp(1))
/2)^2-1)+555661393920*a^4*b^10*sign(tan((f*x+exp(1))/2)^2-1)+1449551462400*
a^5*b^9*sign(tan((f*x+exp(1))/2)^2-1)+2232309252096*a^6*b^8*sign(tan((f*x+e
xp(1))/2)^2-1)+2164663517184*a^7*b^7*sign(tan((f*x+exp(1))/2)^2-1)+13045963
16160*a^8*b^6*sign(tan((f*x+exp(1))/2)^2-1)+434865438720*a^9*b^5*sign(tan((
f*x+exp(1))/2)^2-1)+40265318400*a^10*b^4*sign(tan((f*x+exp(1))/2)^2-1)-1932
7352832*a^11*b^3*sign(tan((f*x+exp(1))/2)^2-1)-4831838208*a^12*b^2*sign(tan
((f*x+exp(1))/2)^2-1))/(-38654705664*a^2*b^13-425201762304*a^3*b^12-2126008
811520*a^4*b^11-6378026434560*a^5*b^10-12756052869120*a^6*b^9-1785847401676
8*a^7*b^8-17858474016768*a^8*b^7-12756052869120*a^9*b^6-6378026434560*a^10*
b^5-2126008811520*a^11*b^4-425201762304*a^12*b^3-38654705664*a^13*b^2))-(-9
663676416*a*b^13*sign(tan((f*x+exp(1))/2)^2-1)-21743271936*a^2*b^12*sign(ta
n((f*x+exp(1))/2)^2-1)+149786984448*a^3*b^11*sign(tan((f*x+exp(1))/2)^2-1)+
828660252672*a^4*b^10*sign(tan((f*x+exp(1))/2)^2-1)+1816771166208*a^5*b^9*s
ign(tan((f*x+exp(1))/2)^2-1)+2198486384640*a^6*b^8*sign(tan((f*x+exp(1))/2)
^2-1)+1555851902976*a^7*b^7*sign(tan((f*x+exp(1))/2)^2-1)+632970805248*a^8*
```

```
b^6*sign(tan((f*x+exp(1))/2)^2-1)+164282499072*a^9*b^5*sign(tan((f*x+exp(1)
)/2)^2-1)+65229815808*a^10*b^4*sign(tan((f*x+exp(1))/2)^2-1)+33822867456*a^
11*b^3*sign(tan((f*x+exp(1))/2)^2-1)+7247757312*a^12*b^2*sign(tan((f*x+exp(
1))/2)^2-1))/(-38654705664*a^2*b^13-425201762304*a^3*b^12-2126008811520*a^4
*b^11-6378026434560*a^5*b^10-12756052869120*a^6*b^9-17858474016768*a^7*b^8-
17858474016768*a^8*b^7-12756052869120*a^9*b^6-6378026434560*a^10*b^5-212600
8811520*a^11*b^4-425201762304*a^12*b^3-38654705664*a^13*b^2))-(-9663676416*
a*b^13*sign(tan((f*x+exp(1))/2)^2-1)-24159191040*a^2*b^12*sign(tan((f*x+exp
(1))/2)^2-1)+135291469824*a^3*b^11*sign(tan((f*x+exp(1))/2)^2-1)+7875896279
04*a^4*b^10*sign(tan((f*x+exp(1))/2)^2-1)+1720134402048*a^5*b^9*sign(tan((f
*x+exp(1))/2)^2-1)+1961726312448*a^6*b^8*sign(tan((f*x+exp(1))/2)^2-1)+1082
331758592*a^7*b^7*sign(tan((f*x+exp(1))/2)^2-1)-9663676416*a^8*b^6*sign(tan
((f*x+exp(1))/2)^2-1)-396210733056*a^9*b^5*sign(tan((f*x+exp(1))/2)^2-1)-23
6760072192*a^10*b^4*sign(tan((f*x+exp(1))/2)^2-1)-57982058496*a^11*b^3*sign
(tan((f*x+exp(1))/2)^2-1)-4831838208*a^12*b^2*sign(tan((f*x+exp(1))/2)^2-1)
)/(-38654705664*a^2*b^13-425201762304*a^3*b^12-2126008811520*a^4*b^11-63780
26434560*a^5*b^10-12756052869120*a^6*b^9-17858474016768*a^7*b^8-17858474016
768*a^8*b^7-12756052869120*a^9*b^6-6378026434560*a^10*b^5-2126008811520*a^1
1*b^4-425201762304*a^12*b^3-38654705664*a^13*b^2))-(-3221225472*a*b^13*sign
(tan((f*x+exp(1))/2)^2-1)-8455716864*a^2*b^12*sign(tan((f*x+exp(1))/2)^2-1)
+70061654016*a^3*b^11*sign(tan((f*x+exp(1))/2)^2-1)+479559942144*a^4*b^10*s
ign(tan((f*x+exp(1))/2)^2-1)+1362578374656*a^5*b^9*sign(tan((f*x+exp(1))/2)
^2-1)+2283043553280*a^6*b^8*sign(tan((f*x+exp(1))/2)^2-1)+2469069324288*a^7
*b^7*sign(tan((f*x+exp(1))/2)^2-1)+1761205026816*a^8*b^6*sign(tan((f*x+exp(
1))/2)^2-1)+811748818944*a^9*b^5*sign(tan((f*x+exp(1))/2)^2-1)+225083129856
*a^10*b^4*sign(tan((f*x+exp(1))/2)^2-1)+31406948352*a^11*b^3*sign(tan((f*x+
exp(1))/2)^2-1)+1207959552*a^12*b^2*sign(tan((f*x+exp(1))/2)^2-1))/(-386547
05664*a^2*b^13-425201762304*a^3*b^12-2126008811520*a^4*b^11-6378026434560*a
^5*b^10-12756052869120*a^6*b^9-17858474016768*a^7*b^8-17858474016768*a^8*b^
7-12756052869120*a^9*b^6-6378026434560*a^10*b^5-2126008811520*a^11*b^4-4252
01762304*a^12*b^3-38654705664*a^13*b^2))/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan
((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)
/(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+2*b*tan((f*x+exp(1))/2)^2+a+b)+2*(((a-b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)
+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/
2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))+sqrt(a+b)*(a+b))/sqrt(a+b)/(sqrt(a+b)*
(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+e
xp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2+sqr
t(a+b)*(-a-b))/(-16*a^2-16*b^2-32*a*b)/sign(tan((f*x+exp(1))/2)^2-1)+1/2*(-
a+4*b)*atan((-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+
b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2
+a+b))/sqrt(-a-b))/sqrt(-a-b)/(4*a^3+4*b^3+12*a*b^2+12*a^2*b)/sign(tan((f*x
+exp(1))/2)^2-1)+(-a+4*b)*ln(abs(sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b
)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))
/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))+a-b))/sqrt(a+b)/(16*a^3+16*b^3+48*a*b
^2+48*a^2*b)/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [B] time = 2.60, size = 11110, normalized size = 64.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.124 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(3a^2-24ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{9/2}}-\frac{5b(11a-10b)\sec(e+fx)}{24f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}-\frac{b(23a-12b)\sec(e+fx}{24f(a+b)^3\left(a+b\sec^2(e+f\right.}$$

```
[Out] -1/8*(3*a^2-24*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))/(a+b)^(9/2)/f-1/8*(5*a-2*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*se
c(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3
/2)-1/24*(23*a-12*b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)-5/24*(
11*a-10*b)*b*sec(f*x+e)/(a+b)^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4134, 470, 527, 12, 377, 207}

$$-\frac{\left(3a^2-24ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{9/2}}-\frac{5b(11a-10b)\sec(e+fx)}{24f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}-\frac{b(23a-12b)\sec(e+fx}{24f(a+b)^3\left(a+b\sec^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Se
c[e + f*x]^2]])/(8*(a + b)^(9/2)*f) - ((5*a - 2*b)*Cot[e + f*x]*Csc[e + f*x
])/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f
*x])/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 12*b)*b*Sec[e +
f*x])/(24*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 10*b)*b*Se
c[e + f*x])/(24*(a + b)^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a-2(2a-b)x^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right.}{4(a+b)f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23}{24(a+}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23}{24(a+}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23}{24(a+}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23}{24(a+}$$

$$= -\frac{\left(3a^2-24ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Mathematica [C] time = 1.80, size = 129, normalized size = 0.55

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(3(a+b)\csc^4(e+fx)((a+8b)\cos(2(e+fx))+3a-4b)-2\left(3a^2-\right.\right.}{96f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/96*((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a + b)*(3*a - 4*b + (a + 8*b)*Cos
[2*(e + f*x)])*Csc[e + f*x]^4 - 2*(3*a^2 - 24*a*b + 8*b^2)*Hypergeometric2F
1[-3/2, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/((a + b)^
3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 1.00, size = 1307, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3
*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2
 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a
^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(a*
cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 21*a^3*b - 16
*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117*a^3*b + 4*a^2*b^2 + 104*
a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^2*b^2 - 61*a*b^3 + 88*b^4
)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 -
5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2
- 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(
a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2
 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/24
*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3*b +
32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2 - 56
*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a^3*b
- 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(3*a^4 - 21*a^3*b - 16*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117
*a^3*b + 4*a^2*b^2 + 104*a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^
2*b^2 - 61*a*b^3 + 88*b^4)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 +
(a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^
6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5
+ 5*a*b^6 + b^7)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
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: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
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)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-tan((f*x+exp(1))/2)^2*(-39582418599936*a^2*b^17-593736278999040*a^3*b^16-4156153952993280*a^4*b^15-18010000462970880*a^5*b^14-54030001388912640*a^6*b^13-118866003055607808*a^7*b^12-198110005092679680*a^8*b^11-254712863690588160*a^9*b^10-254712863690588160*a^10*b^9-198110005092679680*a^11*b^8-118866003055607808*a^12*b^7-54030001388912640*a^13*b^6-18010000462970880*a^14*b^5-4156153952993280*a^15*b^4-593736278999040*a^16*b^3-39582418599936*a^17*b^2)/(10133099161583616*a^2*b^18*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^3*b^17*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^4*b^16*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^5*b^15*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^6*b^14*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^7*b^13*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^8*b^12*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^9*b^11*sign(tan((f*x+exp(1))/2)^2-1)+130412986209581137920*a^10*b^10*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^11*b^9*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^12*b^8*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^13*b^7*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^14*b^6*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^15*b^5*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)+10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1))-(356241767399424*a^2*b^17+4789472650592256*a^3*b^16+29647231531352064*a^4*b^15+111662002870419456*a^5*b^14+284558007314939904*a^6*b^13+515086013240967168*a^7*b^12+673574017315110912*a^8*b^11+628291730436784128*a^9*b^10+390559724325568512*a^10*b^9+118866003055607808*a^11*b^8-39622001018535936*a^12*b^7-68438001759289344*a^13*b^6-39622001018535936*a^14*b^5-13022615719378944*a^15*b^4-2414527534596096*a^16*b^3-197912092999680*a^17*b^2)/(10133099161583616*a^2*b^18*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^3*b^17*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^4*b^16*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^5*b^15*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^6*b^14*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^7*b^13*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^8*b^12*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^9*b^11*sign(tan((f*x+exp(1))/2)^2-1)+130412986209581137920*a^10*b^10*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^11*b^9*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^12*b^8*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^13*b^7*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^14*b^6*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^15*b^5*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)+10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))-(5198490976124928*a^2*b^17+59188909946437632*a^3*b^16+297580623034318848*a^4*b^15+847670688457162752*a^5*b^14+1409582702901854208*a^6*b^13+1030172026481934336*a^7*b^12-1030172026481934336*a^8*b^11-3950879530134011904*a^9*b^10-5671606431510429696*a^10*b^9-5045201463026909184*a^11*b^8-2984857410063040512*a^12*b^7-1145436029444947968*a^13*b^6-247337339691466752*a^14*b^5-11267795161448448*a^15*b^4+7045670510788608*a^16*b^3+1187472557998080*a^17*b^2)/(10133099161583616*a^2*b^18*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^3*b^17*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^4*b^16*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^5*b^15*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^6*b^14*sign(tan((f*x+exp(1))/2)^2-1)+44261377

137797234688*a^7*b^13*sign(tan((f*x+exp(1))/2)^2-1)+8114585808596159692 8*a^8*b^12*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^9*b^11*sign(tan((f*x+exp(1))/2)^2-1)+130412986209581137920*a^10*b^10*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^11*b^9*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^12*b^8*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^13*b^7*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^14*b^6*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^15*b^5*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)+10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))-(13062198137978880*a^2*b^17+137192662867378176*a^3*b^16+637197774621769728*a^4*b^15+1698006593100054528*a^5*b^14+2782881522087100416*a^6*b^13+2677750618285670400*a^7*b^12+967473475419635712*a^8*b^11-990114618858799104*a^9*b^10-1590975733205827584*a^10*b^9-922191188541308928*a^11*b^8-187224839977697280*a^12*b^7+28736835903553536*a^13*b^6-14328835533176832*a^14*b^5-33328396461146112*a^15*b^4-14012176184377344*a^16*b^3-1979120929996800*a^17*b^2)/(10133099161583616*a^2*b^18*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^3*b^17*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^4*b^16*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^5*b^15*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^6*b^14*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^7*b^13*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^8*b^12*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^9*b^11*sign(tan((f*x+exp(1))/2)^2-1)+130412986209581137920*a^10*b^10*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^11*b^9*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^12*b^8*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^13*b^7*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^14*b^6*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^15*b^5*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)+10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))-(12151802510180352*a^2*b^17+129078267054391296*a^3*b^16+603988125416423424*a^4*b^15+1604394173111205888*a^5*b^14+2538935076255694848*a^6*b^13+2063391899196063744*a^7*b^12-297382710941319168*a^8*b^11-2872377370541555712*a^9*b^10-3478463364143775744*a^10*b^9-2045975635012091904*a^11*b^8-353985569539227648*a^12*b^7+362218712608014336*a^13*b^6+306724161730904064*a^14*b^5+109366222591623168*a^15*b^4+19514132369768448*a^16*b^3+1385384650997760*a^17*b^2)/(10133099161583616*a^2*b^18*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^3*b^17*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^4*b^16*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^5*b^15*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^6*b^14*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^7*b^13*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^8*b^12*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^9*b^11*sign(tan((f*x+exp(1))/2)^2-1)+130412986209581137920*a^10*b^10*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^11*b^9*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^12*b^8*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^13*b^7*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^14*b^6*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^15*b^5*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)+10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))-(3892271162327040*a^2*b^17+45691305203859456*a^3*b^16+239434050111012864*a^4*b^15+725717256750759936*a^5*b^14+1351436129978548224*a^6*b^13+1398090607368339456*a^7*b^12+107545431336026112*a^8*b^11-2201851199458639872*a^9*b^10-4024463246311292928*a^10*b^9-4145216011320164352*a^11*b^8-2854670835287851008*a^12*b^7-1354866606257209344*a^13*b^6-434126773064564736*a^14*b^5-87701445477924864*a^15*b^4-9539362882584576*a^16*b^3-356241767399424*a^17*b^2)/(10133099161583616*a^2*b^18*sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^3*b^17*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^4*b^16*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486824960*a^5*b^15*sign(tan((f*x+exp(1))/2)^2-1)+18442240474082181120*a^6*b^14*sign(tan((f*x+exp(1))/2)^2-1)+44261377137797234688*a^7*b^13*sign(tan((f*x+exp

```
(1))/2)^2-1)+811458580859615969 28*a^8*b^12*sign(tan((f*x+exp(1))/2)^2-1)+11
59226544085165670 40*a^9*b^11*sign(tan((f*x+exp(1))/2)^2-1)+1304129862095811
37920*a^10*b^10*sign(tan((f*x+exp(1))/2)^2-1)+115922654408516567040*a^11*b^
9*sign(tan((f*x+exp(1))/2)^2-1)+81145858085961596928*a^12*b^8*sign(tan((f*x
+exp(1))/2)^2-1)+44261377137797234688*a^13*b^7*sign(tan((f*x+exp(1))/2)^2-1
)+18442240474082181120*a^14*b^6*sign(tan((f*x+exp(1))/2)^2-1)+5674535530486
824960*a^15*b^5*sign(tan((f*x+exp(1))/2)^2-1)+1215971899390033920*a^16*b^4*
sign(tan((f*x+exp(1))/2)^2-1)+162129586585337856*a^17*b^3*sign(tan((f*x+exp
(1))/2)^2-1)+10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))/sqr
t(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))
/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+2*(-((-tan((
f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/
2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))*(-6*a^3-8*b^
3+4*a*b^2+6*a^2*b)+(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1)
)/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1
))/2)^2+a+b))^3*(4*a^2+6*b^2-12*a*b)+sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt
(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp
(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2*(3*a^2-5*b^2-2*a*b)+sqrt(a+b)*(
-5*a^3+7*b^3+9*a*b^2-3*a^2*b))/((-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*ta
n((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*t
an((f*x+exp(1))/2)^2+a+b))^2-a-b)^2/(64*a^4+64*b^4+256*a*b^3+384*a^2*b^2+25
6*a^3*b)/sign(tan((f*x+exp(1))/2)^2-1)-1/2*(3*a^2+8*b^2-24*a*b)*atan((-tan(
(f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))
/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a-b))
/sqrt(-a-b)/(16*a^4+16*b^4+64*a*b^3+96*a^2*b^2+64*a^3*b)/sign(tan((f*x+exp(
1))/2)^2-1)-sqrt(a+b)*(3*a^2+8*b^2-24*a*b)*ln(abs((a+b)*(-tan((f*x+exp(1))/
2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan
((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))+sqrt(a+b)*(a-b)))/(64*a^
5+64*b^5+320*a*b^4+640*a^2*b^3+640*a^3*b^2+320*a^4*b)/sign(tan((f*x+exp(1))
/2)^2-1)))
```

maple [B] time = 3.23, size = 15551, normalized size = 66.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.125 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(a+b)(11a+21b)\sin(e+fx)\cos(e+fx)}{16a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{3(a+b)\sin(e+fx)\cos^3(e+\ldots}{8a^2f\left(a+b\tan^2(e+fx)+b\ldots\right.}$$

```
[Out] 5/16*(a+b)*(a^2+14*a*b+21*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/a^(11/2)/f-1/48*b*(113*a^2+420*a*b+315*b^2)*tan(f*x+e)/a^5/f/(a+b
+b*tan(f*x+e)^2)^(1/2)-1/16*(a+b)*(11*a+21*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(
a+b+b*tan(f*x+e)^2)^(3/2)+3/8*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*ta
n(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(3
/2)-7/48*b*(a+b)*(7*a+15*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4132, 470, 578, 527, 12, 377, 203}

$$\frac{5(a+b)\left(a^2+14ab+21b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}-\frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{7b(a+b)(7a+\ldots}{48a^4f\left(a+b\,\mathrm{ta}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(16*a^(11/2)*f) - ((a + b)*(11*a + 21*b)*Cos[e + f*x
]*Sin[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (3*(a + b)*Co
s[e + f*x]^3*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Co
s[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (7*
b*(a + b)*(7*a + 15*b)*Tan[e + f*x])/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^(
3/2)) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(48*a^5*f*Sqrt[a + b
 + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-6(a+b)x^2)}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx\right)}{6af}$$

$$= \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+f}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+f}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+f}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+f}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+f}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

$$= \frac{5(a+b)\left(a^2+14ab+21b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} - \frac{(a+b)(11a+21b)\cos}{16a^3f\left(a+b+b\,\text{t}\right.}$$

Mathematica [B] time = 19.16, size = 1705, normalized size = 5.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3072*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*
e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*
b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2
*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 1
120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4
)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*S
in[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a
 + b*Sec[e + f*x]^2)^(5/2)) + (((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/
```

```
2)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(420*Sqrt[a + b]*(a^
4 + 9*a^3*b + 26*a^2*b^2 + 30*a*b^3 + 12*b^4)*ArcSin[(Sqrt[a]*Sin[e + f*x])
/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - Sqrt[a]*Sin[e + f*x]*Sqrt[
(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(561*a^6 + 6161*a^5*b + 25200*a^4*b^
2 + 50960*a^3*b^3 + 54880*a^2*b^4 + 30240*a*b^5 + 6720*b^6) - 2*a*(1151*a^5
 + 11230*a^4*b + 39200*a^3*b^2 + 62720*a^2*b^3 + 47040*a*b^4 + 13440*b^5)*S
in[e + f*x]^2 + 672*a^2*(a + b)^2*(a^2 + 3*a*b + 6*b^2)*Sin[e + f*x]^4 - 57
6*a^3*(a - 2*b)*(a + b)^2*Sin[e + f*x]^6 + 512*a^4*(a + b)^2*Sin[e + f*x]^8
)))/(3072*Sqrt[2]*a^(11/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a + b*Se
c[e + f*x]^2)^(5/2)) - (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]
*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin
[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2
)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*
Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sq
rt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(
12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/
2)) + (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
 + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b
- a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e +
 f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a +
 b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6
*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a
 + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(
a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e +
f*x]^2)/(a + b))^2))/a^4))/(12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a
 + b - a*Sin[e + f*x]^2)^(3/2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a +
2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^
2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5
*(b + (3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Se
c[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])
^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [A] time = 39.59, size = 1003, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b + 3
5*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*b^3
 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 9*a^4*b
)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3*b^2)*cos(f*x + e)^5 + 2*(8
1*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e)^3 + (113*a^3*b^2 + 420*a^
2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*
f), -1/192*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b
+ 35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*
b^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
```

```
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^5*cos(f*x +
 e)^9 - 2*(13*a^5 + 9*a^4*b)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3
*b^2)*cos(f*x + e)^5 + 2*(81*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e
)^3 + (113*a^3*b^2 + 420*a^2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*
f*cos(f*x + e)^2 + a^6*b^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.85, size = 4477, normalized size = 15.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/48/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(450*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*a^3*b+630*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(
1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^4+315*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*b^4-315*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*b^4+630*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3-225*sin(f
*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b-52
5*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
```

(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
^2*b^2-315*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a*b^3+113*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+33*cos(f*
x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+8*cos(f*x+e)^8*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+26*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a^4-26*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^4-315*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-33*cos(f*x+
e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+420*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a*b^3+30*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4-15*
sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4
+30*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b+450*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b
^2+1050*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2-8*cos(f*x+e)^9*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+1050*sin(f*x+e)*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
*b^3-15*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b-22
5*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2-525*si
n(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f

```
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3+18*cos(f*x+e
)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-18*cos(f*x+e)^6*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-96*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^3*b-63*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b^2+96*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+
63*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-162*cos(f*x
+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-574*cos(f*x+e)^3*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-420*cos(f*x+e)^3*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^3+162*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^3*b+574*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^2*b^2+420*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-113*
cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-420*cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3)/(-1+cos(f*x+e))/((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.126 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(5a+7b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{\left(3a^2+3\right.\cdots}{\cdots}$$

```
[Out] 1/8*(3*a^2+30*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(9/2)/f-5/24*b*(11*a+21*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/
2)-1/8*(5*a+7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/24*b*(23*a+35*b)*
tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.30, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4132, 470, 527, 12, 377, 203}

$$\frac{\left(3a^2+30ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}-\frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a + 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 35*b)*Tan[e + f*x])/(24*a
^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 21*b)*Tan[e + f*x])/(
24*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+3b)x^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24a^3f\Big(a}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24a^3f\Big(a}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24a^3f\Big(a}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a}{24a^3f\Big(a}$$

$$= \frac{\left(3a^2+30ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e+fx)\sin(e+fx}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

Mathematica [B] time = 13.95, size = 1315, normalized size = 5.79

$$-\frac{\left(\frac{\cos(2(e+fx))a+a+2b}{a+b}\right)^{3/2}(\cos(2e+2fx)a+a+2b)^{5/2}\left(\sqrt{a}\sin(e+fx)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(192a^3(a+b)^2\sin^6(e+fx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/768*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*b
^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*
(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b
)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 11
```

```
20*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4)
*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*Si
n[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a
+ b*Sec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e +
f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)
])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a
*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*
x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
- a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3
))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(
3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
 + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b
- a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e +
 f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a +
 b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6
*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a
 + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(
a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e +
f*x]^2)/(a + b))^2))/a^4))/(3072*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a
+ b - a*Sin[e + f*x]^2)^(3/2)) + ((2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b
 + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(256*(a + b)^2*f*
(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b +
(3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e +
f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*
(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [A] time = 12.09, size = 873, normalized size = 3.85

$$\left[-\frac{3\left(\left(3\,a^4+30\,a^3b+35\,a^2b^2\right)\cos\left(fx+e\right)^4+3\,a^2b^2+30\,ab^3+35\,b^4+2\left(3\,a^3b+30\,a^2b^2+35\,ab^3\right)\cos\left(fx\,-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((3*a^4 + 30*a^3*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30
*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70
*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2
*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/96*(3*((3*a^4 + 30*a^3*b + 35*a^2*
b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b
^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
```

```
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6*a^4*cos(f*x +
e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70*a^2*b^2)*cos(f
*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*
x + e)^2 + a^5*b^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.12, size = 3223, normalized size = 14.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/24/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-180*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2
)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos
(f*x+e)^2*a^2*b+78*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2
*b+140*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+55*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2+21*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+90*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a*b^2*sin(f*x+e)-210*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-21*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b-78*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b-140*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^2*a*b^2-210*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a
*b^2+90*cos(f*x+e)^2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
```

```
1/2)*a^2*b+6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3-15*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3-6*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^3+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^5*a^3+105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)*b^3-55*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+105*cos(f*x+e)^2*s
in(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2+105*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-18*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*a^2*b*sin(f*x+e)-180*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+9*2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-105*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*b^3-18*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3+9*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3)/(-
1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.127 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=167

$$\frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{6a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5b\tan(e+fx)}{6a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-2a$$

```
[Out] 1/2*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f
-1/6*b*(13*a+15*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/2*co
s(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e)/a^2/f/(
a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4132, 471, 527, 12, 377, 203}

$$\frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{6a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5b\tan(e+fx)}{6a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-2a$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) - (5*b*Tan[e + f*x])/(6*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) -
(b*(13*a + 15*b)*Tan[e + f*x])/(6*a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-4bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \text{(}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13}{6a^3(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13}{6a^3(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13}{6a^3(a+b)}$$

$$= \frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b}{6a^2f\left(a+b\right.}$$

Mathematica [B] time = 10.53, size = 983, normalized size = 5.89

$$-\frac{(\cos(2e+2fx)a+a+2b)^{5/2}\csc(e+fx)\left(-\frac{12\sin^4(e+fx)}{a+b}+\frac{(\cos(2(e+fx))a+a+2b)\sin^2(e+fx)}{(a+b)^2}+\frac{\sin^2(e+fx)}{a+b}+\frac{16\left(-a\sin^2(e+\right.}{}\right.}{256\sqrt{2}f\left(b\sec^2(e+fx)+a\right)^{5/2}\left(-a s\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(S
in[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
+ b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1 -
(a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos
[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2
+ (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
 f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(Sqrt[2]*f*(a + b*S
ec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[
2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + (
(a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]
^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (
a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[
2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 +
 (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*(a + b - a*Sin[
e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)^2*Sin[e + f*x]^
2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/2)*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)/(a + b))^2))/a^4))
/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/
2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5
/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f
*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b + (3*a + 2*b)*Cos[2*(e + f*
x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384
*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5
/2))
```

fricas [B] time = 3.66, size = 879, normalized size = 5.26

$$\left[-\frac{3\left(\left(a^4+6a^3b+5a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+6ab^3+5b^4+2\left(a^3b+6a^2b^2+5ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3 +
 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a
```

```
^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b
 + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(3*(a^4 + a^3*b
)*cos(f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 +
15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)
^2 + (a^5*b^2 + a^4*b^3)*f), -1/24*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^2*b^2 + 6*a*b^3 + 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*(a^4 + a^3*b)*cos(f*x + e)^5 + 2*(9
*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 + 15*a*b^3)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f
*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^2 + a^4*b^3
)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 1.71, size = 3158, normalized size = 18.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/6/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3+18*cos(f*x+e)^2*sin(f*x+e)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*b+15*cos(f*x+e)^2*sin(f*x+e)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2-6*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
```

```
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin
(f*x+e)*cos(f*x+e)^2*a^3-36*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b-
30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b^2+3*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^5*a^2*b+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b
*sin(f*x+e)+18*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*
x+e)+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-6*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*a^2*b*sin(f*x+e)-36*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-30*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*b^3*sin(f*x+e)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(
f*x+e)^4*a^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+1
8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+20*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2-18*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b-20*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*cos(f*x+e)^2*a*b^2+13*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)
*a*b^2+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-13*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b^3)/(a+b)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(5/2)/a^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.128 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a}{3a^2(a+b)^2f}$$

Mathematica [C] time = 16.95, size = 1927, normalized size = 15.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
```

```
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

fricas [B] time = 1.23, size = 881, normalized size = 7.05

$$\left[-\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

maple [C] time = 2.20, size = 3024, normalized size = 24.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3+6*cos(f*x+e)^2*sin(f*x+e)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*b+3*cos(f*x+e)^2*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2-6*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
```

```
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f
*x+e)*cos(f*x+e)^2*a^3-12*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b-6*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^3*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-12*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+6*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^3*a*b^2-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)^2*a^2*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2+3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/(a^2+2*a*b+b^2)/a^2/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

3.129 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8b\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\tan(e+fx)}{3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)\left(a+b\tan^2(e+fx)+b\right.}$$

```
[Out] -8/3*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-cot(f*x+e)/(a+b)/f/(
a+b+b*tan(f*x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(
3/2)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4132, 271, 192, 191}

$$-\frac{8b\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\tan(e+fx)}{3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)\left(a+b\tan^2(e+fx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e +
 f*x])/(3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x]
)/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+f\right.}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} -$$

Mathematica [A] time = 1.97, size = 108, normalized size = 1.02

$$-\frac{\tan^3(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-6\left(a^2-b^2\right)\csc^2(e+fx)+3a^2+3(a+b)^2\csc^4(e+fx)-\right.}{6f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(3*a^2 - 6*a*b - b^2 - 6*(a^2 - b^2)*C
sc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x]^3)/
((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [A] time = 1.46, size = 192, normalized size = 1.81

$$-\frac{\left(\left(3a^2-6ab-b^2\right)\cos\left(fx+e\right)^5+4\left(3ab-b^2\right)\cos\left(fx+e\right)^3+8b^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(f}{\cos(}}}{3\left(\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)f\cos\left(fx+e\right)^4+2\left(a^4b+3a^3b^2+3a^2b^3+ab^4\right)f\cos\left(fx+e\right)^2+\left(a^3b^2+3a^2b^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^5 + 4*(3*a*b - b^2)*cos(f*x + e)^3
 + 8*b^2*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3
*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*
f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.61, size = 146, normalized size = 1.38

$$-\frac{\left(3\left(\cos^4\left(fx+e\right)\right)a^2-6\left(\cos^4\left(fx+e\right)\right)ab-\left(\cos^4\left(fx+e\right)\right)b^2+12\left(\cos^2\left(fx+e\right)\right)ab-4b^2\left(\cos^2\left(fx+e\right)\right.}{3f\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^4\sin\left(fx+e\right)\left(a^2+2ab+b^2\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/(b+a*cos(f*x+e)^2)^4*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4*a*b-cos(f*x+
e)^4*b^2+12*cos(f*x+e)^2*a*b-4*b^2*cos(f*x+e)^2+8*b^2)*((b+a*cos(f*x+e)^2)/
cos(f*x+e)^2)^(5/2)*cos(f*x+e)^5/sin(f*x+e)/(a^2+2*a*b+b^2)/(a+b)
```

maxima [A] time = 0.36, size = 94, normalized size = 0.89

$$-\frac{\frac{8\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^3}+\frac{4\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}+\frac{3}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*tan
(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) + 3/((b*tan(f*x + e)
^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)))/f
```

mupad [B] time = 16.30, size = 336, normalized size = 3.17

$$-\frac{\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(-a\,b\,6i+a^2\,3i-b^2\,1i+a^2\,e^{e\,2i+f\,x\,2i}\,12i+a^2\,e^{e\,4i+f\,x\,4i}\,18i+a^2\,e^{\cdots}\right.}{3\,f\,(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*
1i)/2)^2)^(1/2)*(a^2*3i - a*b*6i - b^2*1i + a^2*exp(e*2i + f*x*2i)*12i + a^
2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(e*8i + f*x*
8i)*3i - b^2*exp(e*2i + f*x*2i)*20i + b^2*exp(e*4i + f*x*4i)*90i - b^2*exp(
e*6i + f*x*6i)*20i - b^2*exp(e*8i + f*x*8i)*1i + a*b*exp(e*2i + f*x*2i)*24i
 + a*b*exp(e*4i + f*x*4i)*60i + a*b*exp(e*6i + f*x*6i)*24i - a*b*exp(e*8i +
 f*x*8i)*6i))/(3*f*(a + b)^3*(exp(e*2i + f*x*2i) - 1)*(a + 2*a*exp(e*2i + f
*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.130 $\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{8b(a-b)\tan(e+fx)}{3f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(a-b)\tan(e+fx)}{3f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^3(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3}}$$

```
[Out] -8/3*(a-b)*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+
e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*cot(f*x+e)^3/(a+b)/f/(a+b+b*tan
(f*x+e)^2)^(3/2)-4/3*(a-b)*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2
)
```

Rubi [A] time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4132, 453, 271, 192, 191}

$$-\frac{8b(a-b)\tan(e+fx)}{3f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(a-b)\tan(e+fx)}{3f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^3(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a - b)*Cot[e + f*x])/((a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) -
Cot[e + f*x]^3/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*(a -
b)*b*Tan[e + f*x])/(3*(a + b)^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \text{ta}\right.}{(a+b)f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 4.59, size = 138, normalized size = 0.87

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{4b^2(a+b)}{(a\cos(2(e+fx))+a+2b)^2}+\frac{4b(b-3a)}{a\cos(2(e+fx))+a+2b}-\left((a+b)\csc^4(e\right.\right.}{24f(a+b)^4\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((4*b^2*(a + b))/(a + 2*b + a*Cos[2*(e +
f*x)])^2 + (4*b*(-3*a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a - 3*b)*Cs
c[e + f*x]^2 - (a + b)*Csc[e + f*x]^4)*Sec[e + f*x]^4*Tan[e + f*x])/(24*(a
+ b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 4.85, size = 320, normalized size = 2.03

$$-\frac{\left(2\left(a^3-6a^2b+ab^2\right)\cos\left(fx+e\right)^7-3\left(a^3-7a^2b+7ab^2-b^3\right)\cos\left(\right.\right.}{3\left(\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^6-\left(a^6+2a^5b-2a^4b^2-8a^3b^3-7a^2b^4-2ab^5\right)f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^7 - 3*(a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e)^5 - 12*(a^2*b - 2*a*b^2 + b^3)*cos(f*x + e)^3 - 8*(a*b
```

```
^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^6
+ 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^
5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^
5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 -
 (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.67, size = 225, normalized size = 1.42

$$\frac{\left(2\left(\cos^{6}\left(fx+e\right)\right)a^{3}-12\left(\cos^{6}\left(fx+e\right)\right)a^{2}b+2\left(\cos^{6}\left(fx+e\right)\right)a\,b^{2}-3\left(\cos^{4}\left(fx+e\right)\right)a^{3}+21\left(\cos^{4}\left(fx+e\right)\right)\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(b+a*cos(f*x+e)^2)^4*(2*cos(f*x+e)^6*a^3-12*cos(f*x+e)^6*a^2*b+2*cos(
f*x+e)^6*a*b^2-3*cos(f*x+e)^4*a^3+21*cos(f*x+e)^4*a^2*b-21*cos(f*x+e)^4*a*b
^2+3*cos(f*x+e)^4*b^3-12*a^2*cos(f*x+e)^2*b+24*cos(f*x+e)^2*a*b^2-12*cos(f*
x+e)^2*b^3-8*b^2*a+8*b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)*cos(f*x+e
)^5/sin(f*x+e)^3/(a^2+2*a*b+b^2)/(a+b)^2
```

maxima [A] time = 0.36, size = 216, normalized size = 1.37

$$-\frac{\frac{8\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{3}}+\frac{4\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}(a+b)^{2}}-\frac{16\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{4}}-\frac{8\,b^{2}\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}(a+b)^{3}}+\frac{}{\left(b\tan\left(fx+e\right)^{2}+\right.}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*tan
(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 16*b^2*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 8*b^2*tan(f*x + e)/((b*tan(f
*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 3/((b*tan(f*x + e)^2 + a + b)^(3/2)*(
a + b)*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(
f*x + e)) + 1/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.131 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{8b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^5\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx}{5f(a+b)^3\left(a+b\tan^2(e+fx)+\right.}$$

```
[Out] -8/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)^(1/2)-
1/5*(5*a^2-10*a*b+b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)-2/15
*(5*a+b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/5*cot(f*x+e)^5
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-4/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a
+b)^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4132, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^5\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx}{5f(a+b)^3\left(a+b\tan^2(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((5*a^2 - 10*a*b + b^2)*Cot[e + f*x])/(5*(a + b)^3*f*(a + b + b*Tan[e + f*
x]^2)^(3/2)) - (2*(5*a + b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*(a + b + b*Tan[
e + f*x]^2)^(3/2)) - Cot[e + f*x]^5/(5*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (4*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*(a + b)^4*f*(a + b
+ b*Tan[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*
(a + b)^5*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(5a+b)+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e\right.}{5(a+b)f}$$

$$= -\frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3}}$$

$$= -\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 7.41, size = 173, normalized size = 0.77

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\left(-8a^2+50ab-15b^2\right)\csc^2(e+fx)+\frac{20ab^2(a+b)}{(a\cos(2(e+fx))+a+2b)^2}\right.}{120f(a+b)^5\left(a+b\sec^2(e+fx)\right)^{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((20*a*b^2*(a + b))/(a + 2*b + a*Cos[2*(e
 + f*x)])^2 + (10*a*b*(-6*a + 5*b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (-8*a^
2 + 50*a*b - 15*b^2)*Csc[e + f*x]^2 + 2*(a + b)*(-2*a + 5*b)*Csc[e + f*x]^4
 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^4*Tan[e + f*x])/(120*(a + b)^5*
f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 15.77, size = 460, normalized size = 2.04

$$-\frac{\left(8\left(a^4-10a^3b+5a^2b^2\right)\cos\left(fx+e\right)^9-4\left(5a^4-53a^3b+55a^2b^2-15\right.\right.}{15\left(\left(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^8-2\left(a^7+4a^6b+5a^5b^2-5a^3b^4-4a^2b^5-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f*x + e)^9 - 4*(5*a^4 - 53*a^3*b
+ 55*a^2*b^2 - 15*a*b^3)*cos(f*x + e)^7 + 3*(5*a^4 - 60*a^3*b + 126*a^2*b^2
 - 60*a*b^3 + 5*b^4)*cos(f*x + e)^5 + 4*(15*a^3*b - 55*a^2*b^2 + 53*a*b^3 -
 5*b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 - 10*a*b^3 + b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b
^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5
*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4
+ 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x +
 e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*
sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.74, size = 324, normalized size = 1.43

$$-\frac{\left(8\left(\cos^8\left(fx+e\right)\right)a^4-80\left(\cos^8\left(fx+e\right)\right)a^3b+40\left(\cos^8\left(fx+e\right)\right)a^2b^2-20\left(\cos^6\left(fx+e\right)\right)a^4+212\left(\cos^6\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/15/f/(b+a*cos(f*x+e)^2)^4*(8*cos(f*x+e)^8*a^4-80*cos(f*x+e)^8*a^3*b+40*c
os(f*x+e)^8*a^2*b^2-20*cos(f*x+e)^6*a^4+212*cos(f*x+e)^6*a^3*b-220*cos(f*x+
e)^6*a^2*b^2+60*cos(f*x+e)^6*a*b^3+15*cos(f*x+e)^4*a^4-180*cos(f*x+e)^4*a^3
*b+378*cos(f*x+e)^4*a^2*b^2-180*cos(f*x+e)^4*a*b^3+15*cos(f*x+e)^4*b^4+60*c
os(f*x+e)^2*a^3*b-220*cos(f*x+e)^2*a^2*b^2+212*cos(f*x+e)^2*a*b^3-20*cos(f*
x+e)^2*b^4+40*a^2*b^2-80*a*b^3+8*b^4)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/cos(
f*x+e)^2)^(5/2)/sin(f*x+e)^5/(a^2+2*a*b+b^2)/(a+b)^3
```

maxima [A] time = 0.38, size = 373, normalized size = 1.65

$$-\frac{\frac{40\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^3}+\frac{20\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{160\,b^2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^4}-\frac{80\,b^2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{128\,b^3}{\sqrt{b\tan(fx}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 20*b*
tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 160*b^2*tan(f*x
 + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 80*b^2*tan(f*x + e)/((b*
tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 128*b^3*tan(f*x + e)/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)^5) + 64*b^3*tan(f*x + e)/((b*tan(f*x + e)^2 +
a + b)^(3/2)*(a + b)^4) + 15/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(
f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e))
+ 48*b^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3*tan(f*x + e)) + 10/((b
*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3) - 8*b/((b*tan(f*x +
e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)^3) + 3/((b*tan(f*x + e)^2 + a +
b)^(3/2)*(a + b)*tan(f*x + e)^5))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.132 $\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx$

Optimal. Leaf size=123

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}(d\sin(e+fx))^m\left(\frac{-a\sin^2(e+fx)+a+b}{a+b}\right)^{-p}\left(a+b\sec^2(e+fx)\right)^pF_1\left(\frac{m+1}{2};p+\frac{1}{2},-p;\frac{m+3}{2};\sin\right.}{f(m+1)}$$

```
[Out] AppellF1(1/2+1/2*m,1/2+p,-p,3/2+1/2*m,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(c
os(f*x+e)^2)^(1/2+p)*(a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m*tan(f*x+e)/f/(1+
m)/(((a+b-a*sin(f*x+e)^2)/(a+b))^p)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m, x]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx=\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx$$

Mathematica [B] time = 4.00, size = 286, normalized size = 2.33

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^m\left(a+b\sec^2(e+fx)\right)^pF_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\right.}{f(m+1)\left(F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)-\frac{\tan^2(e+fx)\left((m+2)(a+b)F_1\left(\frac{m+3}{2};\frac{m+4}{2},-p;\frac{m+5}{2};-\tan^2(e+fx),-\frac{b\,t}{}\right.\right.}{(m+}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*S
in[e + f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - ((-2*b*p*AppellF1[(3 + m)/
2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] + (a + b)*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)/((a + b)*(3 + m)
)))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

maple [F] time = 3.48, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
[Out] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*sin(f*x+e))**m,x)
[Out] Timed out
```

3.133 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx$

Optimal. Leaf size=182

$$-\frac{\left(15a^2+10ab(1-2p)+b^2\left(4p^2-8p+3\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\right.}{15a^2f}$$

```
[Out] 1/15*(10*a+b*(3-2*p))*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1+p)/a^2/f-1/5*cos(f
*x+e)^5*(a+b*sec(f*x+e)^2)^(1+p)/a/f-1/15*(15*a^2+10*a*b*(1-2*p)+b^2*(4*p^2
-8*p+3))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(
f*x+e)^2)^p/a^2/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.19, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4134, 462, 453, 365, 364}

$$-\frac{\left(15a^2+10ab(1-2p)+b^2\left(4p^2-8p+3\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\right.}{15a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] ((10*a + b*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(15*a^
2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/(5*a*f) - ((15*a^2 +
 10*a*b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Cos[e + f*x]*Hypergeometric2F1[-
1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(15*a^2*f*
(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin^5(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{(-1+x^2)^2(a+bx^2)^p}{x^6}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af}+\frac{\text{Subst}\left(\int\frac{(-10a-b(3-2p)+5ax^2}{x^4}\right.}{5}$$

$$=\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f}-\frac{\cos^5(e+f}{}$$

$$=\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f}-\frac{\cos^5(e+f}{}$$

$$=\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f}-\frac{\cos^5(e+f}{}$$

Mathematica [A] time = 7.77, size = 253, normalized size = 1.39

$$\frac{2\sin^4(e+fx)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(4\left(15a^2+10ab(1-2p)+b^2\left(4p^2-8p+3\right)\right){}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\right.\right.}{15a^2f\left(4\cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p-2^{-p}\left(2^p\cos(4(e+fx))\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] (2*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4*(4*(15*a^2 + 10*a*b
*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*S
ec[e + f*x]^2)/a)] + (a + 2*b + a*Cos[2*(e + f*x)])*(-17*a - 6*b + 4*b*p +
3*a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(15*a^2*f*(4*Cos[2
*(e + f*x)]*((a + b + b*Tan[e + f*x]^2)/a)^p - (3*(((a + 2*b + a*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2)/a)^p + 2^p*Cos[4*(e + f*x)]*((a + b + b*Tan[e + f*x
]^2)/a)^p)/2^p))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\left(b\sec\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

maple [F] time = 4.77, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(\sin^5\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**5,x)
[Out] Timed out
```

3.134 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx$

Optimal. Leaf size=117

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3af}-\frac{(3a-2bp+b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(\right.}{3af}$$

[Out] 1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1+p)/a/f-1/3*(-2*b*p+3*a+b)*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^p/a/f/((1+b*sec(f*x+e)^2/a)^p)

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 453, 365, 364}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3af}-\frac{(3a-2bp+b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(\right.}{3af}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]

[Out] (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a + b - 2*b*p)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(3*a*f*(1 + (b*Sec[e + f*x]^2)/a)^p)

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{(3a+b-2bp)\,\text{Subst}\left(\int \frac{(a+bx^2)}{x^2}\right.}{3af}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{\left((3a+b-2bp)\left(a+b\sec^2(e+\right.\right.}{}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a+b-2bp)\cos(e+fx)\,{}_2F_1}{}$$

Mathematica [A] time = 3.89, size = 178, normalized size = 1.52

$$-\frac{\sin^2(e+fx)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left((a\cos(2(e+fx))+a+2b)\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p - 2(3a-2bp+b)\right.}{3af\left(\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p - 2\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p + \cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*(-2*(3*a + b - 2
*b*p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)] + (a + 2*b
+ a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(a*f*(-2*(1 + (b*S
ec[e + f*x]^2)/a)^p + ((a + b + b*Tan[e + f*x]^2)/a)^p + Cos[2*(e + f*x)]*(
(a + b + b*Tan[e + f*x]^2)/a)^p))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(b\sec\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

maple [F] time = 4.52, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\sin^3\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \sin \left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{3}\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^{2}}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.135 $\int \left(a+b\sec^2(e+fx)\right)^p \sin(e+fx)\,dx$

Optimal. Leaf size=68

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2
)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4134, 365, 364}

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sec^2(e+fx)\right)^p \left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx, x, \text{se}\right.}{f}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^p \left(1+\right.}{f}$$

Mathematica [A] time = 1.65, size = 68, normalized size = 1.00

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)

mupad [B] time = 4.92, size = 79, normalized size = 1.16

$$\frac{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a\,{\cos\left(e+f\,x\right)}^2}{b}\right)}{f\,\left(2\,p-1\right)\,{\left(\frac{a\,{\cos\left(e+f\,x\right)}^2}{b}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)

[Out] (cos(e + f*x)*(a + b/cos(e + f*x)^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -(
a*cos(e + f*x)^2)/b))/(f*(2*p - 1)*((a*cos(e + f*x)^2)/b + 1)^p)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e),x)

[Out] Timed out

3.136 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=77

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)*(a+b*sec(
f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4134, 430, 429}

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
 + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{-1+x^2}\,dx,x,\sec(e\right.}{f}$$

$$=-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec^2(e+\right.}{f}$$

<u>Mathematica</u> [B] time = 16.88, size = 1532, normalized size = 19.90

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]*(Sec[e + f*x]^2)^p*(a + b*Se
c[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
-(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a +
b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b + b*T
an[e + f*x]^2)/(a + b))^p))/(2*f*(-(a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1
+ p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((2*AppellF1[-1/2 - p, -1/2, -p, 1
/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2
])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[
e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p)) + p*(a + 2*b + a*Cos[2
*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((2*AppellF1[-1/2 - p, -1/2,
 -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e +
f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]
) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p) + ((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2
- p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[
e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x
]^2]) + (4*(a + b)*p*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
 -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*(1 + ((a + b)*Cot[e + f*x]^2)/
b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*(
(-2*(a + b)*(-1/2 - p)*p*AppellF1[1/2 - p, -1/2, 1 - p, 3/2 - p, -Cot[e + f
*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(b*(1/2
- p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(
((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(1/2 - p))*Sqrt[S
ec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e +
f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a
+ b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 + 2*p)*(1 +
 ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[1, 1
/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*T
an[e + f*x]^3*((a + b + b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (2*A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/((a + b + b*Tan[e + f*x]^2)/(a + b))^p - (Tan[e +
f*x]^2*((b*p*AppellF1[2, 1/2, 1 - p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(a + b) - (AppellF1[2, 3/2, -p, 3
```

```
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/2))/((a + b + b*Tan[e + f*x]^2)/(a + b))^p))/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int\csc\left(fx+e\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.137 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=81

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)^3*(a+b
*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4134, 511, 510}

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e +
 f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-1+x^2)^2}\,dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec^2(e\right.}{3f}$$

Mathematica [B] time = 4.41, size = 266, normalized size = 3.28

$$\frac{\csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\text{co}\right.}{f(2p-1)\left(\sec(e+fx)F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)-\frac{\cot(e+fx)\csc(e+fx)\left(2p(a+b)F_1\left(\frac{3}{2}-p;-\right.\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f
*x]^2)/b)]*Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(f*(-1 + 2*p)*(-(((2*(a
 + b)*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p, -Cot[e + f*x]^2, -(((a + b)
*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5/2 - p, -Cot[e + f*x]^
2, -(((a + b)*Cot[e + f*x]^2)/b)])*Cot[e + f*x]*Csc[e + f*x])/(b*(-3 + 2*p)
)) + AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e
 + f*x]^2)/b)]*Sec[e + f*x]))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

maple [F] time = 1.56, size = 0, normalized size = 0.00

$$\int \left(\csc^3\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.138 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

```
[Out] 1/5*AppellF1(5/2,3,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4132, 511, 510}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^3}\,dx, \right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2}; 3, -p; \frac{7}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan\right.}{5f}$$

Mathematica [B] time = 25.72, size = 5878, normalized size = 66.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
[Out] Result too large to show
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 + 1\right)\left(b\sec\left(fx+e\right)^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2 + a\right)^p \sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

maple [F] time = 6.09, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(\sin^4\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2 + a\right)^p \sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**4,x)
[Out] Timed out
```

3.139 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4132, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^2}\, a\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\right.}{3f}$$

Mathematica [B] time = 21.13, size = 3781, normalized size = 42.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*(a +
b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*Appel
lF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) +
2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2
, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*
AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*
Tan[e + f*x]^2)))/(f*(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f
*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[
e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 6*a*(a + b)*p
*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*Sin[2*(e
 + f*x)]*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 6*(a +
b)*(-2 + p)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[
e + f*x]^2*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]
```

```
^2, -((b*Tan[e + f*x]^2)/(a + b))]]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e + f*x]*(((2*b*p*Ap
pellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3
)/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (2*AppellF1[1/2, -
p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*T
an[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Sec[e + f*x]
^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p
, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[
1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(4*(-(b*p*
AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2
, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[
e + f*x]^2*(-(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))
 - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)) + 2*(a + b)*((6*b*p*AppellF1[5/2,
 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x
]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*(
a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2 - (AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*(4*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e +
 f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[
5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2
, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*App
```

```
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*
(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(b\sec\left(fx+e\right)^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

maple [F] time = 4.44, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\sin^2\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**2,x)
[Out] Timed out
```

3.140 $\int \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+f\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{f}$$

Mathematica [B] time = 15.17, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
```

```
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

maple [F] time = 1.35, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos \left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec ^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=73

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*
x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4132, 365, 364}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2}\,dx\right.}{f}$$

$$= -\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p}{f}$$

Mathematica [A] time = 1.09, size = 72, normalized size = 0.99

$$-\frac{\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

maple [F] time = 1.83, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.142 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=128

$$-\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f(a+b)}-\frac{\cot^3(}{}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1+p)/(a+b)/f-1/3*(3*a+2*b*(1+p))*co
t(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*x+e
)^2)^p/(a+b)/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 453, 365, 364}

$$-\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f(a+b)}-\frac{\cot^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(3*(a + b)*f) - ((3*a
+ 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f
*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*f*(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^p}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{(3a+2b(1+p))\,\text{Subst}\left(\int\right.}{3(}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{\left((3a+2b(1+p))\left(a+b\right.\right.}{}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} - \frac{(3a+2b(1+p))\cot(e+}{}$$

Mathematica [A] time = 2.25, size = 132, normalized size = 1.03

$$-\frac{\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left((3a+2b(p+1))\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+\cot^2(e+fx\right.}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p*((3*a + 2*b*(1 + p))*Hypergeome
tric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + Cot[e + f*x]^2*(a +
 b + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/((a + b)*f*(1 +
 (b*Tan[e + f*x]^2)/(a + b))^p)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \left(\csc^4\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \csc \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}}{\sin\left(e+f\,x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.143 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=192

$$-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\right.}{15f(a+b)^2}$$

```
[Out] -1/15*(10*a+b*(7+2*p))*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1+p)/(a+b)^2/f-1/
5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1+p)/(a+b)/f-1/15*(15*a^2+20*a*b*(1+p)
+4*b^2*(p^2+3*p+2))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(
a+b))*(a+b+b*tan(f*x+e)^2)^p/(a+b)^2/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.17, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 462, 453, 365, 364}

$$-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\right.}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(
15*(a + b)^2*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(5*(a
 + b)*f) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x]*
Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b + b*
Tan[e + f*x]^2)^p)/(15*(a + b)^2*f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)^p}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p(10\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5\ldots}{\ldots}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5\ldots}{\ldots}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5\ldots}{\ldots}$$

Mathematica [A] time = 2.00, size = 149, normalized size = 0.78

$$-\frac{\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(15\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+3\cot^4(e+fx)\,{}_2F_1\left(-\frac{5}{2}\ldots\right.\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((
b*Tan[e + f*x]^2)/(a + b))] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p,
 -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + 15*Hypergeometric2F1[-1/2, -p, 1/2,
 -((b*Tan[e + f*x]^2)/(a + b))])*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e
 + f*x]^2)/(a + b))^p)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

maple [F] time = 1.75, size = 0, normalized size = 0.00

$$\int \left(\csc^{6}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}}{\sin\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.144 $\int \left(a-a\sec^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=74

$$\frac{a^4\tan^7(c+dx)}{7d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan(c+dx)}{d}+a^4x$$

```
[Out] a^4*x-a^4*tan(d*x+c)/d+1/3*a^4*tan(d*x+c)^3/d-1/5*a^4*tan(d*x+c)^5/d+1/7*a^
4*tan(d*x+c)^7/d
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 3473, 8}

$$\frac{a^4\tan^7(c+dx)}{7d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan(c+dx)}{d}+a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x - (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d) - (a^4*Tan[c + d*
x]^5)/(5*d) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sec^2(c+dx)\right)^4\,dx &= a^4\int \tan^8(c+dx)\,dx\\
&= \frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^6(c+dx)\,dx\\
&= -\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int \tan^4(c+dx)\,dx\\
&= \frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^2(c+dx)\,dx\\
&= -\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int 1
\\
&= a^4x-\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.97

$$a^4\left(\frac{\tan^{-1}(\tan(c+dx))}{d}+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c
 + d*x]^5/(5*d) + Tan[c + d*x]^7/(7*d))
```

fricas [A] time = 0.45, size = 82, normalized size = 1.11

$$\frac{105\,a^4dx\cos\left(dx+c\right)^7-\left(176\,a^4\cos\left(dx+c\right)^6-122\,a^4\cos\left(dx+c\right)^4+66\,a^4\cos\left(dx+c\right)^2-15\,a^4\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 - (176*a^4*cos(d*x + c)^6 - 122*a^4*cos(d
*x + c)^4 + 66*a^4*cos(d*x + c)^2 - 15*a^4)*sin(d*x + c))/(d*cos(d*x + c)^7
)
```

giac [A] time = 0.36, size = 66, normalized size = 0.89

$$\frac{15\,a^4\tan\left(dx+c\right)^7-21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+105\,(dx+c)a^4-105\,a^4\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*tan(d*x + c)^7 - 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^
3 + 105*(d*x + c)*a^4 - 105*a^4*tan(d*x + c))/d
```

maple [A] time = 1.49, size = 125, normalized size = 1.69

$$\frac{-a^4\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan\left(dx+c\right)+4a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^4,x)
```

```
[Out] 1/d*(-a^4*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+4*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-6*a^4*(
-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-4*a^4*tan(d*x+c)+a^4*(d*x+c))
```

maxima [A] time = 0.33, size = 129, normalized size = 1.74

$$a^4x+\frac{\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)a^4}{35\,d}-\frac{4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx\right.\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/35*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35
*tan(d*x + c))*a^4/d - 4/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(
d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - 4*a^4*tan(d*x
 + c)/d
```

mupad [B] time = 4.62, size = 61, normalized size = 0.82

$$\frac{\frac{a^4\tan(c+d\,x)^7}{7}-\frac{a^4\tan(c+d\,x)^5}{5}+\frac{a^4\tan(c+d\,x)^3}{3}-a^4\tan\left(c+d\,x\right)+d\,x\,a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^4,x)
```

```
[Out] ((a^4*tan(c + d*x)^3)/3 - a^4*tan(c + d*x) - (a^4*tan(c + d*x)^5)/5 + (a^4*
tan(c + d*x)^7)/7 + a^4*d*x)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 1\,dx+\int\left(-4\sec^2(c+dx)\right)dx+\int 6\sec^4(c+dx)\,dx+\int\left(-4\sec^6(c+dx)\right)dx+\int\sec^8(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] a**4*(Integral(1, x) + Integral(-4*sec(c + d*x)**2, x) + Integral(6*sec(c +
 d*x)**4, x) + Integral(-4*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**8,
x))
```

3.145 $\int \left(a - a\sec^2(c+dx)\right)^3 dx$

Optimal. Leaf size=56

$$-\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x$$

```
[Out] a^3*x-a^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d-1/5*a^3*tan(d*x+c)^5/d
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4120, 3473, 8}

$$-\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - (a^3*Tan[c + d*x])/d + (a^3*Tan[c + d*x]^3)/(3*d) - (a^3*Tan[c + d*
x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sec^2(c+dx)\right)^3 dx &= -\left(a^3\int \tan^6(c+dx)\,dx\right)\\
&= -\frac{a^3\tan^5(c+dx)}{5d}+a^3\int \tan^4(c+dx)\,dx\\
&= \frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}-a^3\int \tan^2(c+dx)\,dx\\
&= -\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}+a^3\int 1\,dx\\
&= a^3x-\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.04

$$-a^3\left(-\frac{\tan^{-1}(\tan(c+dx))}{d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] -(a^3*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) +
Tan[c + d*x]^5/(5*d)))
```

fricas [A] time = 0.44, size = 69, normalized size = 1.23

$$\frac{15\,a^3dx\cos\left(dx+c\right)^5-\left(23\,a^3\cos\left(dx+c\right)^4-11\,a^3\cos\left(dx+c\right)^2+3\,a^3\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 - (23*a^3*cos(d*x + c)^4 - 11*a^3*cos(d*x +
 c)^2 + 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 0.42, size = 53, normalized size = 0.95

$$-\frac{3\,a^3\tan\left(dx+c\right)^5-5\,a^3\tan\left(dx+c\right)^3-15\left(dx+c\right)a^3+15\,a^3\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*tan(d*x + c)^5 - 5*a^3*tan(d*x + c)^3 - 15*(d*x + c)*a^3 + 15*
a^3*tan(d*x + c))/d
```

maple [A] time = 1.26, size = 81, normalized size = 1.45

$$\frac{a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(dx+c\right)-3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)-3a^3\tan\left(dx+c\right)+a^3\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-3*a^3*(-2/3-
1/3*sec(d*x+c)^2)*tan(d*x+c)-3*a^3*tan(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.33, size = 81, normalized size = 1.45

$$a^3x-\frac{\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^3}{15\,d}+\frac{\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^3}{d}-\frac{3\,a^3\tan\left(d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3/d
 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3/d - 3*a^3*tan(d*x + c)/d
```

mupad [B] time = 4.47, size = 49, normalized size = 0.88

$$-\frac{\frac{a^3\tan(c+d\,x)^5}{5}-\frac{a^3\tan(c+d\,x)^3}{3}+a^3\tan\left(c+d\,x\right)-d\,x\,a^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^3,x)
```

```
[Out] -(a^3*tan(c + d*x) - (a^3*tan(c + d*x)^3)/3 + (a^3*tan(c + d*x)^5)/5 - a^3*
d*x)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^3\left(\int(-1)\,dx+\int 3\sec^2(c+dx)\,dx+\int\left(-3\sec^4(c+dx)\right)dx+\int\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -a**3*(Integral(-1, x) + Integral(3*sec(c + d*x)**2, x) + Integral(-3*sec(c
 + d*x)**4, x) + Integral(sec(c + d*x)**6, x))
```

3.146 $\int \left(a-a\sec^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=38

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x$$

```
[Out] a^2*x-a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 3473, 8}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)^2\,dx&=a^2\int \tan^4(c+dx)\,dx\\&=\frac{a^2\tan^3(c+dx)}{3d}-a^2\int \tan^2(c+dx)\,dx\\&=-\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}+a^2\int 1\,dx\\&=a^2x-\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.11

$$a^2\left(\frac{\tan^{-1}(\tan(c+dx))}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

fricas [A] time = 0.54, size = 56, normalized size = 1.47

$$\frac{3\,a^2 dx\cos\left(dx+c\right)^3-\left(4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^2*d*x*cos(d*x + c)^3 - (4*a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

giac [A] time = 0.22, size = 39, normalized size = 1.03

$$\frac{a^2\tan\left(dx+c\right)^3+3\left(dx+c\right)a^2-3\,a^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*(d*x + c)*a^2 - 3*a^2*tan(d*x + c))/d
```

maple [A] time = 1.02, size = 49, normalized size = 1.29

$$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)-2a^2\tan\left(dx+c\right)+a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^2,x)
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-2*a^2*tan(d*x+c)+a^2*(d*x+c))
```

maxima [A] time = 0.32, size = 45, normalized size = 1.18

$$a^2x+\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2}{3\,d}-\frac{2\,a^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2/d - 2*a^2*tan(d*x + c)/d
```

mupad [B] time = 4.40, size = 33, normalized size = 0.87

$$a^2\,x-\frac{a^2\,\left(3\tan\left(c+d\,x\right)-\tan\left(c+d\,x\right)^3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^2,x)
[Out] a^2*x - (a^2*(3*tan(c + d*x) - tan(c + d*x)^3))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 1\,dx+\int\left(-2\sec^2\left(c+dx\right)\right)\,dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**2,x)
```

```
[Out] a**2*(Integral(1, x) + Integral(-2*sec(c + d*x)**2, x) + Integral(sec(c + d
*x)**4, x))
```

3.147 $\int \left(a-a\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$ax-\frac{a\tan(c+dx)}{d}$$

```
[Out] a*x-a*tan(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = $\{3767, 8\}$

$$ax-\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a - a*Sec[c + d*x]^2,x]
```

```
[Out] a*x - (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)\,dx &= ax-a\int \sec^2(c+dx)\,dx\\ &= ax+\frac{a\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= ax-\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.62

$$-a\left(\frac{\tan(c+dx)}{d}-\frac{\tan^{-1}(\tan(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - a*Sec[c + d*x]^2,x]
```

```
[Out] -(a*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d))
```

fricas [A] time = 0.50, size = 32, normalized size = 2.00

$$\frac{adx\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*d*x*cos(d*x + c) - a*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.24, size = 16, normalized size = 1.00

$$ax - \frac{a \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="giac")
[Out] a*x - a*tan(d*x + c)/d
```

maple [A] time = 0.78, size = 17, normalized size = 1.06

$$ax - \frac{a \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a-a*sec(d*x+c)^2,x)
[Out] a*x-a*tan(d*x+c)/d
```

maxima [A] time = 0.34, size = 16, normalized size = 1.00

$$ax - \frac{a \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="maxima")
[Out] a*x - a*tan(d*x + c)/d
```

mupad [B] time = 4.44, size = 16, normalized size = 1.00

$$a\,x - \frac{a \tan(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a - a/cos(c + d*x)^2,x)
[Out] a*x - (a*tan(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\int(-1)\,dx + \int\sec^2(c + dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)**2,x)
[Out] -a*(Integral(-1, x) + Integral(sec(c + d*x)**2, x))
```

3.148 $\int \frac{1}{a-a\sec^2(c+dx)}\,dx$

Optimal. Leaf size=19

$$\frac{\cot(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a+cot(d*x+c)/a/d
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 3473, 8}

$$\frac{\cot(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] x/a + Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\sec^2(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cot(c+dx)}{ad}\end{aligned}$$

Mathematica [C] time = 0.03, size = 31, normalized size = 1.63

$$\frac{\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/(a*d)
```

fricas [A] time = 0.41, size = 31, normalized size = 1.63

$$\frac{dx\sin\left(dx+c\right)+\cos\left(dx+c\right)}{ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="fricas")
[Out] (d*x*sin(d*x + c) + cos(d*x + c))/(a*d*sin(d*x + c))
```

giac [B] time = 0.24, size = 45, normalized size = 2.37

$$\frac{\frac{2\,(dx+c)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/a + 1/(a*tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.52, size = 31, normalized size = 1.63

$$\frac{1}{ad\tan\left(dx+c\right)}+\frac{\arctan\left(\tan\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2),x)
[Out] 1/a/d/tan(d*x+c)+1/a/d*arctan(tan(d*x+c))
```

maxima [A] time = 0.42, size = 26, normalized size = 1.37

$$\frac{\frac{dx+c}{a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="maxima")
[Out] ((d*x + c)/a + 1/(a*tan(d*x + c)))/d
```

mupad [B] time = 4.70, size = 19, normalized size = 1.00

$$\frac{x}{a}+\frac{\cot\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2),x)
[Out] x/a + cot(c + d*x)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{\sec^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2),x)
[Out] -Integral(1/(sec(c + d*x)**2 - 1), x)/a
```

3.149 $\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 3473, 8}

$$-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-2),x]
```

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx &= \frac{\int \cot^4(c+dx)\,dx}{a^2}\\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{\int \cot^2(c+dx)\,dx}{a^2}\\
&= \frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\int 1\,dx}{a^2}\\
&= \frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 36, normalized size = 0.97

$$-\frac{\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-2),x]
```

```
[Out] -1/3*(Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(a^
2*d)
```

fricas [B] time = 0.45, size = 75, normalized size = 2.03

$$\frac{4 \cos(dx+c)^3 + 3\left(dx\cos(dx+c)^2 - dx\right)\sin(dx+c) - 3\cos(dx+c)}{3\left(a^2 d\cos(dx+c)^2 - a^2 d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(d*x + c)^3 + 3*(d*x*cos(d*x + c)^2 - d*x)*sin(d*x + c) - 3*cos(d
*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [B] time = 0.64, size = 80, normalized size = 2.16

$$\frac{\frac{24(dx+c)}{a^2} + \frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a^2 + (15*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

maple [A] time = 0.68, size = 47, normalized size = 1.27

$$-\frac{1}{3d\,a^2\tan(dx+c)^3} + \frac{1}{d\,a^2\tan(dx+c)} + \frac{\arctan(\tan(dx+c))}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d/a^2/tan(d*x+c)^3+1/d/a^2/tan(d*x+c)+1/d/a^2*arctan(tan(d*x+c))
```

maxima [A] time = 0.42, size = 40, normalized size = 1.08

$$\frac{\frac{3(dx+c)}{a^2} + \frac{3\tan(dx+c)^2 - 1}{a^2\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^2 - 1)/(a^2*tan(d*x + c)^3))/d
```

mupad [B] time = 4.41, size = 31, normalized size = 0.84

$$\frac{x}{a^2} + \frac{\tan(c+d\,x)^2 - \frac{1}{3}}{a^2\,d\tan(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^2,x)
```

```
[Out] x/a^2 + (tan(c + d*x)^2 - 1/3)/(a^2*d*tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^4(c+dx)-2\sec^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**2,x)
[Out] Integral(1/(sec(c + d*x)**4 - 2*sec(c + d*x)**2 + 1), x)/a**2
```

3.150 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 3473, 8}

$$\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx &= -\frac{\int \cot^6(c+dx)\,dx}{a^3}\\
&= \frac{\cot^5(c+dx)}{5a^3d}+\frac{\int \cot^4(c+dx)\,dx}{a^3}\\
&= -\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\int \cot^2(c+dx)\,dx}{a^3}\\
&= \frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}+\frac{\int 1\,dx}{a^3}\\
&= \frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 36, normalized size = 0.65

$$\frac{\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] (Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*a^3*d
)
```

fricas [B] time = 0.46, size = 109, normalized size = 1.98

$$\frac{23\cos(dx+c)^5-35\cos(dx+c)^3+15\left(dx\cos(dx+c)^4-2\,dx\cos(dx+c)^2+dx\right)\sin(dx+c)+15\cos(dx+c}{15\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*(d*x*cos(d*x + c)^4 - 2*d*
x*cos(d*x + c)^2 + d*x)*sin(d*x + c) + 15*cos(d*x + c))/((a^3*d*cos(d*x + c
)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

giac [B] time = 0.25, size = 111, normalized size = 2.02

$$\frac{\frac{480\,(dx+c)}{a^3}+\frac{330\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+330\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(480*(d*x + c)/a^3 + (330*tan(1/2*d*x + 1/2*c)^4 - 35*tan(1/2*d*x + 1
/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
- 35*a^12*tan(1/2*d*x + 1/2*c)^3 + 330*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.90, size = 63, normalized size = 1.15

$$-\frac{1}{3d\,a^3\tan(dx+c)^3}+\frac{1}{5d\,a^3\tan(dx+c)^5}+\frac{1}{d\,a^3\tan(dx+c)}+\frac{\arctan(\tan(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^3,x)
```

```
[Out] -1/3/d/a^3/tan(d*x+c)^3+1/5/d/a^3/tan(d*x+c)^5+1/d/a^3/tan(d*x+c)+1/d/a^3*a
rctan(tan(d*x+c))
```

maxima [A] time = 0.42, size = 50, normalized size = 0.91

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(a^3*ta
n(d*x + c)^5))/d
```

mupad [B] time = 4.54, size = 41, normalized size = 0.75

$$\frac{x}{a^3}+\frac{\tan(c+d\,x)^4-\frac{\tan(c+d\,x)^2}{3}+\frac{1}{5}}{a^3\,d\tan(c+d\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^3,x)
```

```
[Out] x/a^3 + (tan(c + d*x)^4 - tan(c + d*x)^2/3 + 1/5)/(a^3*d*tan(c + d*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sec^6(c+dx)-3\sec^4(c+dx)+3\sec^2(c+dx)-1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -Integral(1/(sec(c + d*x)**6 - 3*sec(c + d*x)**4 + 3*sec(c + d*x)**2 - 1),
x)/a**3
```

3.151 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=73

$$-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot(c+dx)}{a^4d}+\frac{x}{a^4}$$

```
[Out] x/a^4+cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^4/d+1/5*cot(d*x+c)^5/a^4/d-1/7*co
t(d*x+c)^7/a^4/d
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 3473, 8}

$$-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot(c+dx)}{a^4d}+\frac{x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 + Cot[c + d*x]/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + Cot[c + d*x]^5/(5
*a^4*d) - Cot[c + d*x]^7/(7*a^4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx &= \frac{\int \cot^8(c+dx)\,dx}{a^4}\\
&= -\frac{\cot^7(c+dx)}{7a^4d}-\frac{\int \cot^6(c+dx)\,dx}{a^4}\\
&= \frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\int \cot^4(c+dx)\,dx}{a^4}\\
&= -\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}-\frac{\int \cot^2(c+dx)\,dx}{a^4}\\
&= \frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\int 1\,dx}{a^4}\\
&= \frac{x}{a^4}+\frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 36, normalized size = 0.49

$$-\frac{\cot^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\tan^2(c+dx)\right)}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] -1/7*(Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(a^
4*d)
```

fricas [B] time = 0.46, size = 147, normalized size = 2.01

$$\frac{176\,\cos(dx+c)^7-406\,\cos(dx+c)^5+350\,\cos(dx+c)^3+105\left(dx\cos(dx+c)^6-3\,dx\cos(dx+c)^4+3\,dx\,c\right.}{105\left(a^4d\cos(dx+c)^6-3\,a^4d\cos(dx+c)^4+3\,a^4d\cos(dx+c)^2-a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(d*x + c)^7 - 406*cos(d*x + c)^5 + 350*cos(d*x + c)^3 + 105*(
d*x*cos(d*x + c)^6 - 3*d*x*cos(d*x + c)^4 + 3*d*x*cos(d*x + c)^2 - d*x)*sin
(d*x + c) - 105*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

giac [B] time = 0.38, size = 139, normalized size = 1.90

$$\frac{\frac{13440\,(dx+c)}{a^4}+\frac{9765\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1295\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1295}{a^{28}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^4 + (9765*tan(1/2*d*x + 1/2*c)^6 - 1295*tan(1/2*
d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^2 - 15)/(a^4*tan(1/2*d*x + 1/2*c)
^7) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*a^24*tan(1/2*d*x + 1/2*c)^5 + 1
295*a^24*tan(1/2*d*x + 1/2*c)^3 - 9765*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.79, size = 79, normalized size = 1.08

$$-\frac{1}{7d\,a^4\tan(dx+c)^7}-\frac{1}{3d\,a^4\tan(dx+c)^3}+\frac{1}{5d\,a^4\tan(dx+c)^5}+\frac{1}{d\,a^4\tan(dx+c)}+\frac{\arctan(\tan(dx+c))}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^4,x)
```

```
[Out] -1/7/d/a^4/tan(d*x+c)^7-1/3/d/a^4/tan(d*x+c)^3+1/5/d/a^4/tan(d*x+c)^5+1/d/a
^4/tan(d*x+c)+1/d/a^4*arctan(tan(d*x+c))
```

maxima [A] time = 0.43, size = 60, normalized size = 0.82

$$\frac{\frac{105\,(dx+c)}{a^4}+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{a^4\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(105*(d*x + c)/a^4 + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/(a^4*tan(d*x + c)^7))/d
```

mupad [B] time = 4.93, size = 51, normalized size = 0.70

$$\frac{x}{a^4}+\frac{\tan\left(c+d\,x\right)^6-\frac{\tan\left(c+d\,x\right)^4}{3}+\frac{\tan\left(c+d\,x\right)^2}{5}-\frac{1}{7}}{a^4\,d\,\tan\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^4,x)
```

```
[Out] x/a^4 + (tan(c + d*x)^2/5 - tan(c + d*x)^4/3 + tan(c + d*x)^6 - 1/7)/(a^4*d
*tan(c + d*x)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^8(c+dx)-4\sec^6(c+dx)+6\sec^4(c+dx)-4\sec^2(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**8 - 4*sec(c + d*x)**6 + 6*sec(c + d*x)**4 - 4*sec
(c + d*x)**2 + 1), x)/a**4
```

3.152 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=98

$$\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{(6a+5b)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b\tan(}{}$$

```
[Out] 1/16*(6*a+5*b)*arctanh(sin(f*x+e))/f+1/16*(6*a+5*b)*sec(f*x+e)*tan(f*x+e)/f
+1/24*(6*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3768, 3770}

$$\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{(6a+5b)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b\tan(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*a + 5*b)*ArcTanh[Sin[e + f*x]])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]*Tan[
e + f*x])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (b*Se
c[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{6}(6a+5b)\int \sec^5(e+fx)\,dx\\ &= \frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{8}\\ &= \frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}+\frac{(6a+5b)\sec^3(e+fx)\tan(e+}{24f}\\ &= \frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}\end{aligned}$$

Mathematica [A] time = 0.34, size = 75, normalized size = 0.77

$$\frac{3(6a+5b)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(2(6a+5b)\sec^2(e+fx)+3(6a+5b)+8b\sec^4(e+fx\right.}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (3*(6*a + 5*b)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(6*a + 5*b) + 2*(6*a
 + 5*b)*Sec[e + f*x]^2 + 8*b*Sec[e + f*x]^4)*Tan[e + f*x])/(48*f)
```

fricas [A] time = 0.49, size = 114, normalized size = 1.16

$$\frac{3\,(6\,a+5\,b)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)+2\left(3\,(6\,a+\right.}{96\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(6*a + 5*b)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*co
s(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(6*a + 5*b)*cos(f*x + e)^4 + 2*(
6*a + 5*b)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-(6*a+5*b)/64*ln(abs(sin(f*x+exp(1))-1))+(6*a+5*b)/64*ln(abs(s
in(f*x+exp(1))+1))+(-18*sin(f*x+exp(1))^5*a-15*sin(f*x+exp(1))^5*b+48*sin(f
*x+exp(1))^3*a+40*sin(f*x+exp(1))^3*b-30*sin(f*x+exp(1))*a-33*sin(f*x+exp(1
))*b)*1/96/(sin(f*x+exp(1))^2-1)^3)
```

maple [A] time = 0.80, size = 138, normalized size = 1.41

$$\frac{a\tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{4f}+\frac{3a\tan\left(fx+e\right)\sec\left(fx+e\right)}{8f}+\frac{3a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}+\frac{b\left(\sec^5\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/4/f*a*tan(f*x+e)*sec(f*x+e)^3+3/8/f*a*tan(f*x+e)*sec(f*x+e)+3/8/f*a*ln(se
c(f*x+e)+tan(f*x+e))+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f+5/24*b*sec(f*x+e)^3*ta
n(f*x+e)/f+5/16*b*sec(f*x+e)*tan(f*x+e)/f+5/16/f*b*ln(sec(f*x+e)+tan(f*x+e)
)
```

maxima [A] time = 0.34, size = 126, normalized size = 1.29

$$\frac{3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(6\,a+5\,b)\sin\left(fx+e\right)^5-8\,(6\,a+5\,b)\sin\left(fx+e\right)^3+3\,(\right.}{\sin\left(fx+e\right)^6-3\,\sin\left(fx+e\right)^4+3\,\sin\left(fx\right.}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(sin(f*x + e)
- 1) - 2*(3*(6*a + 5*b)*sin(f*x + e)^5 - 8*(6*a + 5*b)*sin(f*x + e)^3 + 3*(
10*a + 11*b)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x +
 e)^2 - 1))/f
```

mupad [B] time = 4.65, size = 102, normalized size = 1.04

$$\frac{\operatorname{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)}{f}-\frac{\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\,{\sin\left(e+f\,x\right)}^5+\left(-a-\frac{5\,b}{6}\right)\,{\sin\left(e+f\,x\right)}^3+\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\,\sin\left(e+f\,x\right)}{f\,\left({\sin\left(e+f\,x\right)}^6-3\,{\sin\left(e+f\,x\right)}^4+3\,{\sin\left(e+f\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^5,x)
```

```
[Out] (atanh(sin(e + f*x))*((3*a)/8 + (5*b)/16))/f - (sin(e + f*x)^5*((3*a)/8 + (
5*b)/16) + sin(e + f*x)*((5*a)/8 + (11*b)/16) - sin(e + f*x)^3*(a + (5*b)/6
))/(f*(3*sin(e + f*x)^2 - 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

3.153 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

```
[Out] 1/8*(4*a+3*b)*arctanh(sin(f*x+e))/f+1/8*(4*a+3*b)*sec(f*x+e)*tan(f*x+e)/f+1
/4*b*sec(f*x+e)^3*tan(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3768, 3770}

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]])/(8*f) + ((4*a + 3*b)*Sec[e + f*x]*Tan[e
 + f*x])/(8*f) + (b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{4}(4a+3b)\int \sec^3(e+fx)\,dx\\ &= \frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{8}(4a\\ &= \frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b}{}\end{aligned}$$

Mathematica [A] time = 0.13, size = 54, normalized size = 0.77

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(4a+2b\sec^2(e+fx)+3b\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(4*a + 3*b + 2*b*Sec[e +
f*x]^2)*Tan[e + f*x])/(8*f)
```

fricas [**A**] time = 0.51, size = 95, normalized size = 1.36

$$\frac{(4\,a+3\,b)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-(4\,a+3\,b)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left((4\,a+3\right.}{16\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a + 3*b)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (4*a + 3*b)*cos(f*
x + e)^4*log(-sin(f*x + e) + 1) + 2*((4*a + 3*b)*cos(f*x + e)^2 + 2*b)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-4*a-3*b)/32*ln(abs(sin(f*x+exp(1))-1))-(-4*a-3*b)/32*ln(abs(
sin(f*x+exp(1))+1))-(4*sin(f*x+exp(1))^3*a+3*sin(f*x+exp(1))^3*b-4*sin(f*x+
exp(1))*a-5*sin(f*x+exp(1))*b)*1/16/(sin(f*x+exp(1))^2-1)^2)
```

maple [**A**] time = 1.03, size = 98, normalized size = 1.40

$$\frac{a\tan\left(fx+e\right)\sec\left(fx+e\right)}{2f}+\frac{a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{b\left(\sec^3\left(fx+e\right)\right)\tan\left(fx+e\right)}{4f}+\frac{3b\sec\left(fx+e\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2/f*a*tan(f*x+e)*sec(f*x+e)+1/2/f*a*ln(sec(f*x+e)+tan(f*x+e))+1/4*b*sec(f
*x+e)^3*tan(f*x+e)/f+3/8*b*sec(f*x+e)*tan(f*x+e)/f+3/8/f*b*ln(sec(f*x+e)+ta
n(f*x+e))
```

maxima [**A**] time = 0.34, size = 97, normalized size = 1.39

$$\frac{(4\,a+3\,b)\log\left(\sin\left(fx+e\right)+1\right)-(4\,a+3\,b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left((4\,a+3\,b)\sin\left(fx+e\right)^3-(4\,a+5\,b)\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(sin(f*x + e) - 1)
 - 2*((4*a + 3*b)*sin(f*x + e)^3 - (4*a + 5*b)*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 0.14, size = 78, normalized size = 1.11

$$\frac{\operatorname{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(\frac{a}{2}+\frac{3\,b}{8}\right)}{f}-\frac{\sin\left(e+f\,x\right)^3\,\left(\frac{a}{2}+\frac{3\,b}{8}\right)-\sin\left(e+f\,x\right)\,\left(\frac{a}{2}+\frac{5\,b}{8}\right)}{f\,\left(\sin\left(e+f\,x\right)^4-2\sin\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^3,x)
```

```
[Out] (atanh(sin(e + f*x))*(a/2 + (3*b)/8))/f - (sin(e + f*x)^3*(a/2 + (3*b)/8) -
 sin(e + f*x)*(a/2 + (5*b)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f*x)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

3.154 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*(2*a+b)*arctanh(sin(f*x+e))/f+1/2*b*sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4046, 3770}

$$\frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2*
f)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}(2a+b)\int \sec(e+fx)\,dx\\ &= \frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.20

$$\frac{a\tanh^{-1}(\sin(e+fx))}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/f + (b*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e +
f*x]*Tan[e + f*x])/(2*f)
```

fricas [A] time = 0.47, size = 72, normalized size = 1.80

$$\frac{(2\,a+b)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,a+b)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\,b\sin\left(fx+\right.}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a + b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a + b)*cos(f*x + e
)^2*log(-sin(f*x + e) + 1) + 2*b*sin(f*x + e))/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-(2*a+b)/8*ln(abs(sin(f*x+exp(1))-1))+(2*a+b)/8*ln(abs(sin(f*x
+exp(1))+1))-sin(f*x+exp(1))*b*1/4/(sin(f*x+exp(1))^2-1))
```

maple [A] time = 0.79, size = 59, normalized size = 1.48

$$\frac{a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{b\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{b\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*a*ln(sec(f*x+e)+tan(f*x+e))+1/2*b*sec(f*x+e)*tan(f*x+e)/f+1/2/f*b*ln(se
c(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.34, size = 58, normalized size = 1.45

$$\frac{(2\,a+b)\log\left(\sin\left(fx+e\right)+1\right)-(2\,a+b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\,b\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(sin(f*x + e) - 1) - 2*
b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

mupad [B] time = 4.41, size = 41, normalized size = 1.02

$$\frac{\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(a+\frac{b}{2}\right)}{f}-\frac{b\,\sin\left(e+f\,x\right)}{2\,f\,\left({\sin\left(e+f\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x),x)
```

```
[Out] (atanh(sin(e + f*x))*(a + b/2))/f - (b*sin(e + f*x))/(2*f*(sin(e + f*x)^2 -
 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

3.155 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(e+fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

```
[Out] b*arctanh(sin(f*x+e))/f+a*sin(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4045, 3770}

$$\frac{a\sin(e+fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\sin(e+fx)}{f}+b\int \sec(e+fx)\,dx\\ &= \frac{b\tanh^{-1}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$\frac{a\sin(e)\cos(fx)}{f}+\frac{a\cos(e)\sin(fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Cos[f*x]*Sin[e])/f + (a*Cos[e]*Sin[f*x])/f
```

fricas [A] time = 0.50, size = 40, normalized size = 1.67

$$\frac{b\log\left(\sin\left(fx+e\right)+1\right)-b\log\left(-\sin\left(fx+e\right)+1\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e))
/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b/4*ln(abs(sin(f*x+exp(1))-1))+b/4*ln(abs(sin(f*x+exp(1))+1))
+sin(f*x+exp(1))*a/2)
```

maple [A] time = 0.67, size = 32, normalized size = 1.33

$$\frac{a \sin \left(f x+e\right)}{f}+\frac{b \ln \left(\sec \left(f x+e\right)+\tan \left(f x+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] a*sin(f*x+e)/f+1/f*b*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.34, size = 38, normalized size = 1.58

$$\frac{b\left(\log \left(\sin \left(f x+e\right)+1\right)-\log \left(\sin \left(f x+e\right)-1\right)\right)+2\, a \sin \left(f x+e\right)}{2\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)) + 2*a*sin(f*x + e))/
f
```

mupad [B] time = 0.06, size = 22, normalized size = 0.92

$$\frac{a\,\sin\left(e+f\,x\right)+b\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*sin(e + f*x) + b*atanh(sin(e + f*x)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b \sec ^{2}\left(e+f x\right)\right) \cos \left(e+f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

3.156 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

```
[Out] (a+b)*sin(f*x+e)/f-1/3*a*sin(f*x+e)^3/f
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4044, 3013}

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - (a*Sin[e + f*x]^3)/(3*f)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \int \cos(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx\\ &= -\frac{\text{Subst}\left(\int \left(a+b-ax^2\right)\,dx,x,-\sin(e+fx)\right)}{f}\\ &= \frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.67

$$-\frac{a\sin^3(e+fx)}{3f}+\frac{a\sin(e+fx)}{f}+\frac{b\sin(e)\cos(fx)}{f}+\frac{b\cos(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*Cos[f*x]*Sin[e])/f + (b*Cos[e]*Sin[f*x])/f + (a*Sin[e + f*x])/f - (a*Sin
[e + f*x]^3)/(3*f)
```

fricas [A] time = 0.49, size = 28, normalized size = 0.93

$$\frac{\left(a\cos\left(fx+e\right)^2+2\,a+3\,b\right)\sin\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a + 3*b)*sin(f*x + e)/f
```

giac [A] time = 0.17, size = 37, normalized size = 1.23

$$-\frac{a \sin\left(fx+e\right)^{3} - 3 \, a \sin\left(fx+e\right) - 3 \, b \sin\left(fx+e\right)}{3 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/3*(a*sin(f*x + e)^3 - 3*a*sin(f*x + e) - 3*b*sin(f*x + e))/f
```

maple [A] time = 1.48, size = 33, normalized size = 1.10

$$\frac{\frac{a\left(2+\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)}{3}+b \sin\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
[Out] 1/f*(1/3*a*(2+cos(f*x+e)^2)*sin(f*x+e)+b*sin(f*x+e))
```

maxima [A] time = 0.34, size = 27, normalized size = 0.90

$$-\frac{a \sin\left(fx+e\right)^{3} - 3 \, {\left(a+b\right)} \sin\left(fx+e\right)}{3 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/3*(a*sin(f*x + e)^3 - 3*(a + b)*sin(f*x + e))/f
```

mupad [B] time = 0.05, size = 28, normalized size = 0.93

$$-\frac{\frac{a\,{\sin\left(e+f\,x\right)}^3}{3}-\sin\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
[Out] -((a*sin(e + f*x)^3)/3 - sin(e + f*x)*(a + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + fx \right)}\right) \cos^{3}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**3, x)
```

3.157 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=50

$$-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)\sin(e+fx)}{f}+\frac{a\sin^5(e+fx)}{5f}$$

```
[Out] (a+b)*sin(f*x+e)/f-1/3*(2*a+b)*sin(f*x+e)^3/f+1/5*a*sin(f*x+e)^5/f
```

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4044, 3013, 373}

$$-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)\sin(e+fx)}{f}+\frac{a\sin^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - ((2*a + b)*Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*
x]^5)/(5*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \int \cos^3(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx\\
&= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-ax^2\right)\,dx, x, -\sin(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(2a+b)x^2+ax^4\right)\,dx, x, -\sin(e+fx)\right)}{f}\\
&= \frac{(a+b)\sin(e+fx)}{f}-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.42

$$\frac{a\sin^5(e+fx)}{5f}-\frac{2a\sin^3(e+fx)}{3f}+\frac{a\sin(e+fx)}{f}-\frac{b\sin^3(e+fx)}{3f}+\frac{b\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*Sin[e + f*x])/f + (b*Sin[e + f*x])/f - (2*a*Sin[e + f*x]^3)/(3*f) - (b*S
in[e + f*x]^3)/(3*f) + (a*Sin[e + f*x]^5)/(5*f)
```

fricas [A] time = 0.45, size = 45, normalized size = 0.90

$$\frac{\left(3\,a\cos\left(fx+e\right)^{4}+\left(4\,a+5\,b\right)\cos\left(fx+e\right)^{2}+8\,a+10\,b\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*cos(f*x + e)^4 + (4*a + 5*b)*cos(f*x + e)^2 + 8*a + 10*b)*sin(f*x
 + e)/f
```

giac [A] time = 0.27, size = 62, normalized size = 1.24

$$\frac{3\,a\sin\left(fx+e\right)^{5}-10\,a\sin\left(fx+e\right)^{3}-5\,b\sin\left(fx+e\right)^{3}+15\,a\sin\left(fx+e\right)+15\,b\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 10*a*sin(f*x + e)^3 - 5*b*sin(f*x + e)^3 + 15*a*
sin(f*x + e) + 15*b*sin(f*x + e))/f
```

maple [A] time = 1.88, size = 54, normalized size = 1.08

$$\frac{\frac{a\left(\frac{8}{3}+\cos^{4}\left(fx+e\right)+\frac{4\left(\cos^{2}\left(fx+e\right)\right)}{3}\right)\sin\left(fx+e\right)}{5}+\frac{b\left(2+\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/5*a*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+1/3*b*(2+cos(f*x+
e)^2)*sin(f*x+e))
```

maxima [A] time = 0.32, size = 43, normalized size = 0.86

$$\frac{3\,a\sin\left(fx+e\right)^{5}-5\,\left(2\,a+b\right)\sin\left(fx+e\right)^{3}+15\,\left(a+b\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 5*(2*a + b)*sin(f*x + e)^3 + 15*(a + b)*sin(f*x
+ e))/f
```

mupad [B] time = 4.32, size = 43, normalized size = 0.86

$$\frac{\frac{a\sin\left(e+f\,x\right)^{5}}{5}+\left(-\frac{2\,a}{3}-\frac{b}{3}\right)\,\sin\left(e+f\,x\right)^{3}+\left(a+b\right)\,\sin\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*sin(e + f*x)^5)/5 - sin(e + f*x)^3*((2*a)/3 + b/3) + sin(e + f*x)*(a +
b))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.158 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{(7a+6b)\tan^5(e+fx)}{35f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

```
[Out] 1/7*(7*a+6*b)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^6*tan(f*x+e)/f+2/21*(7*a+6*b)*t
an(f*x+e)^3/f+1/35*(7*a+6*b)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4046, 3767}

$$\frac{(7a+6b)\tan^5(e+fx)}{35f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((7*a + 6*b)*Tan[e + f*x])/(7*f) + (b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) +
(2*(7*a + 6*b)*Tan[e + f*x]^3)/(21*f) + ((7*a + 6*b)*Tan[e + f*x]^5)/(35*f)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= \frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{1}{7}(7a+6b)\int \sec^6(e+fx)\,dx\\ &= \frac{b\sec^6(e+fx)\tan(e+fx)}{7f}-\frac{(7a+6b)\operatorname{Subst}\left(\int\left(1+2x^2+x^4\right)dx,\ldots\right.}{7f}\\ &= \frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{2(7a+6b)\tan\ldots}{21f}\end{aligned}$$

Mathematica [A] time = 0.29, size = 81, normalized size = 0.93

$$\frac{a\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}+\frac{b\left(\frac{1}{7}\tan^7(e+fx)+\frac{3}{5}\tan^5(e+fx)+\tan^3(e+fx)+\tan(e\ldots\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f + (b*(Tan[e
+ f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*x]^7/7))/f
```

fricas [A] time = 0.44, size = 74, normalized size = 0.85

$$\frac{\left(8\,(7\,a+6\,b)\cos\left(fx+e\right)^{6}+4\,(7\,a+6\,b)\cos\left(fx+e\right)^{4}+3\,(7\,a+6\,b)\cos\left(fx+e\right)^{2}+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a + 6*b)*cos(f*x + e)^6 + 4*(7*a + 6*b)*cos(f*x + e)^4 + 3*(7*a
 + 6*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

giac [A] time = 0.23, size = 86, normalized size = 0.99

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,a\tan\left(fx+e\right)^{5}+63\,b\tan\left(fx+e\right)^{5}+70\,a\tan\left(fx+e\right)^{3}+105\,b\tan\left(fx+e\right)^{3}+105\,a\tan}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 + 63*b*tan(f*x + e)^5 + 70
*a*tan(f*x + e)^3 + 105*b*tan(f*x + e)^3 + 105*a*tan(f*x + e) + 105*b*tan(f
*x + e))/f
```

maple [A] time = 1.04, size = 78, normalized size = 0.90

$$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^{2}\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)-b\left(-\frac{16}{35}-\frac{\left(\sec^{6}\left(fx+e\right)\right)}{7}-\frac{6\left(\sec^{4}\left(fx+e\right)\right)}{35}-\frac{8\left(\sec^{2}\left(fx+e\right)\right)}{35}\right)\tan\left(fx+e\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b*(-16/35-1/7
*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.35, size = 60, normalized size = 0.69

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,(a+3\,b)\tan\left(fx+e\right)^{5}+35\,(2\,a+3\,b)\tan\left(fx+e\right)^{3}+105\,(a+b)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a + 3*b)*tan(f*x + e)^5 + 35*(2*a + 3*b)*t
an(f*x + e)^3 + 105*(a + b)*tan(f*x + e))/f
```

mupad [B] time = 4.31, size = 56, normalized size = 0.64

$$\frac{\frac{b\tan\left(e+f\,x\right)^{7}}{7}+\left(\frac{a}{5}+\frac{3\,b}{5}\right)\,\tan\left(e+f\,x\right)^{5}+\left(\frac{2\,a}{3}+b\right)\,\tan\left(e+f\,x\right)^{3}+(a+b)\,\tan\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)^5*(a/5 + (3*b)/5) + (b*tan(e + f*x)^7)/7 + tan(e + f*x)^3*((2
*a)/3 + b) + tan(e + f*x)*(a + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\sec^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

3.159 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=65

$$\frac{(5a+4b)\tan^3(e+fx)}{15f}+\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

```
[Out] 1/5*(5*a+4*b)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^4*tan(f*x+e)/f+1/15*(5*a+4*b)*t
an(f*x+e)^3/f
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4046, 3767}

$$\frac{(5a+4b)\tan^3(e+fx)}{15f}+\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tan[e + f*x])/(5*f) + (b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) +
((5*a + 4*b)*Tan[e + f*x]^3)/(15*f)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{1}{5}(5a+4b)\int \sec^4(e+fx)\,dx\\ &= \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}-\frac{(5a+4b)\,\text{Subst}\left(\int\left(1+x^2\right)dx,x,-\tan(e\right.}{5f}\\ &= \frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{(5a+4b)\tan^3(e}{15f}\end{aligned}$$

Mathematica [A] time = 0.20, size = 61, normalized size = 0.94

$$\frac{a\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}+\frac{b\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f + (b*(Tan[e + f*x] + (2*Tan[e + f*x
]^3)/3 + Tan[e + f*x]^5/5))/f
```

fricas [A] time = 0.44, size = 56, normalized size = 0.86

$$\frac{\left(2\,(5\,a+4\,b)\cos\left(fx+e\right)^{4}+(5\,a+4\,b)\cos\left(fx+e\right)^{2}+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a + 4*b)*cos(f*x + e)^4 + (5*a + 4*b)*cos(f*x + e)^2 + 3*b)*sin(
f*x + e)/(f*cos(f*x + e)^5)
```

giac [A] time = 0.82, size = 62, normalized size = 0.95

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,a\tan\left(fx+e\right)^{3}+10\,b\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)+15\,b\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 10*b*tan(f*x + e)^3 + 15*a*
tan(f*x + e) + 15*b*tan(f*x + e))/f
```

maple [A] time = 1.06, size = 58, normalized size = 0.89

$$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)-b\left(-\frac{8}{15}-\frac{\left(\sec^{4}\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^{2}\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b*(-8/15-1/5*sec(f*x+e)^4-4/15*s
ec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.34, size = 43, normalized size = 0.66

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,(a+2\,b)\tan\left(fx+e\right)^{3}+15\,(a+b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a + 2*b)*tan(f*x + e)^3 + 15*(a + b)*tan(f*x
+ e))/f
```

mupad [B] time = 4.50, size = 42, normalized size = 0.65

$$\frac{\frac{b\tan\left(e+f\,x\right)^5}{5}+\left(\frac{a}{3}+\frac{2\,b}{3}\right)\,\tan\left(e+f\,x\right)^3+\left(a+b\right)\,\tan\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^4,x)
```

```
[Out] (tan(e + f*x)^3*(a/3 + (2*b)/3) + (b*tan(e + f*x)^5)/5 + tan(e + f*x)*(a +
b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\sec^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.160 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=43

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

```
[Out] 1/3*(3*a+2*b)*tan(f*x+e)/f+1/3*b*sec(f*x+e)^2*tan(f*x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3767, 8}

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}+\frac{1}{3}(3a+2b)\int \sec^2(e+fx)\,dx\\
&= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}-\frac{(3a+2b)\operatorname{Subst}\left(\int 1\,dx,x,-\tan(e+fx\right.}{3f}\\
&= \frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\sec^2(e+fx)\tan(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.84

$$\frac{a\tan(e+fx)}{f}+\frac{b\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + (b*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

fricas [A] time = 0.52, size = 37, normalized size = 0.86

$$\frac{\left((3\,a+2\,b)\cos\left(fx+e\right)^{2}+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/3*((3*a + 2*b)*cos(f*x + e)^2 + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

giac [A] time = 0.36, size = 37, normalized size = 0.86

$$\frac{b\tan\left(fx+e\right)^{3}+3\,a\tan\left(fx+e\right)+3\,b\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/3*(b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 3*b*tan(f*x + e))/f
```

maple [A] time = 0.96, size = 35, normalized size = 0.81

$$\frac{a\tan\left(fx+e\right)-b\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
[Out] 1/f*(a*tan(f*x+e)-b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.34, size = 34, normalized size = 0.79

$$\frac{\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)b+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*b + 3*a*tan(f*x + e))/f
```

mupad [B] time = 4.46, size = 28, normalized size = 0.65

$$\frac{b\tan\left(e+f\,x\right)^{3}}{3\,f}+\frac{\tan\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^2,x)
[Out] (b*tan(e + f*x)^3)/(3*f) + (tan(e + f*x)*(a + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.161 $\int \left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\,\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

fricas [B] time = 0.48, size = 31, normalized size = 2.07

$$\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

giac [A] time = 0.20, size = 16, normalized size = 1.07

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

maple [A] time = 0.75, size = 16, normalized size = 1.07

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

maxima [A] time = 0.34, size = 15, normalized size = 1.00

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

mupad [B] time = 4.42, size = 17, normalized size = 1.13

$$\frac{b\tan\left(e + f\,x\right) + a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + a*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

3.162 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] 1/2*(a+2*b)*x+1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4045, 8}

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}(a+2b)\int 1\,dx\\ &= \frac{1}{2}(a+2b)x+\frac{a\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.06

$$\frac{a(e+fx)}{2f}+\frac{a\sin(2(e+fx))}{4f}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] b*x + (a*(e + f*x))/(2*f) + (a*Sin[2*(e + f*x)])/(4*f)
```

fricas [A] time = 0.50, size = 28, normalized size = 0.90

$$\frac{(a+2\,b)fx+a\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*((a + 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 40, normalized size = 1.29

$$\frac{\left(fx+e\right)(a+2\,b)+\frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{2}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

maple [A] time = 0.69, size = 37, normalized size = 1.19

$$\frac{a\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+\left(fx+e\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
[Out] 1/f*(a*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+(f*x+e)*b)
```

maxima [A] time = 0.44, size = 37, normalized size = 1.19

$$\frac{\left(fx+e\right)(a+2\,b)+\frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{2}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.31, size = 25, normalized size = 0.81

$$\frac{\frac{a\,\sin\left(2\,e+2\,f\,x\right)}{4}+f\,x\,\left(\frac{a}{2}+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
[Out] ((a*sin(2*e + 2*f*x))/4 + f*x*(a/2 + b))/f
```

sympy [A] time = 5.79, size = 51, normalized size = 1.65

$$a\left(\begin{cases}\frac{x\sin^{2}\left(e+fx\right)}{2}+\frac{x\cos^{2}\left(e+fx\right)}{2}+\frac{\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\ f\neq 0\\ x\cos^{2}\left(e\right) & \text{otherwise}\end{cases}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
[Out] a*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*cos(e
 + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True)) + b*x
```

3.163 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(3a+4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(3a+4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] 1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*si
n(f*x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4045, 2635, 8}

$$\frac{(3a+4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(3a+4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}(3a+4b)\int \cos^2(e+fx)\,dx\\ &= \frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{8}(3a\\ &= \frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e}{4f}\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.74

$$\frac{4(3a+4b)(e+fx)+8(a+b)\sin(2(e+fx))+a\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a + 4*b)*(e + f*x) + 8*(a + b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)])
/(32*f)
```

fricas [A] time = 0.72, size = 49, normalized size = 0.80

$$\frac{(3\,a+4\,b)fx+\left(2\,a\cos\left(fx+e\right)^{3}+(3\,a+4\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + 4*b)*f*x + (2*a*cos(f*x + e)^3 + (3*a + 4*b)*cos(f*x + e))*sin(
f*x + e))/f
```

giac [A] time = 1.15, size = 79, normalized size = 1.30

$$\frac{\left(fx+e\right)(3\,a+4\,b)+\frac{3\,a\tan\left(fx+e\right)^{3}+4\,b\tan\left(fx+e\right)^{3}+5\,a\tan\left(fx+e\right)+4\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((f*x + e)*(3*a + 4*b) + (3*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 5*a
*tan(f*x + e) + 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f
```

maple [A] time = 1.29, size = 65, normalized size = 1.07

$$\frac{a\left(\frac{\left(\cos^{3}\left(fx+e\right)+\frac{3\cos\left(fx+e\right)}{2}\right)\sin\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+b*(1/2*
sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.44, size = 73, normalized size = 1.20

$$\frac{\left(fx+e\right)(3\,a+4\,b)+\frac{(3\,a+4\,b)\tan\left(fx+e\right)^{3}+(5\,a+4\,b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((f*x + e)*(3*a + 4*b) + ((3*a + 4*b)*tan(f*x + e)^3 + (5*a + 4*b)*tan(
f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.48, size = 67, normalized size = 1.10

$$x\left(\frac{3\,a}{8}+\frac{b}{2}\right)+\frac{\left(\frac{3\,a}{8}+\frac{b}{2}\right)\tan\left(e+f\,x\right)^{3}+\left(\frac{5\,a}{8}+\frac{b}{2}\right)\tan\left(e+f\,x\right)}{f\left(\tan\left(e+f\,x\right)^{4}+2\tan\left(e+f\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 + b/2) + (tan(e + f*x)^3*((3*a)/8 + b/2) + tan(e + f*x)*((5*a)/8
 + b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\cos^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

3.164 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{(5a+6b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(5a+6b)+\frac{a\sin(e+fx)\cos^5(e+f
$$

```
[Out] 1/16*(5*a+6*b)*x+1/16*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(5*a+6*b)*cos(
f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4045, 2635, 8}

$$\frac{(5a+6b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(5a+6b)+\frac{a\sin(e+fx)\cos^5(e+f
$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 6*b)*x)/16 + ((5*a + 6*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((5*a
 + 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{6}(5a+6b)\int \cos^4(e+fx)\,dx\\
&= \frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{8}\\
&= \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(e+fx)\sin(e+
\\
&= \frac{1}{16}(5a+6b)x+\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(
\end{aligned}$$

Mathematica [A] time = 0.11, size = 68, normalized size = 0.76

$$\frac{(45a+48b)\sin(2(e+fx))+(9a+6b)\sin(4(e+fx))+a\sin(6(e+fx))+60ae+60afx+72be+72bfx}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (60*a*e + 72*b*e + 60*a*f*x + 72*b*f*x + (45*a + 48*b)*Sin[2*(e + f*x)] + (
9*a + 6*b)*Sin[4*(e + f*x)] + a*Sin[6*(e + f*x)])/(192*f)
```

fricas [A] time = 0.57, size = 68, normalized size = 0.76

$$\frac{3\,(5\,a+6\,b)fx+\left(8\,a\cos\left(fx+e\right)^{5}+2\,(5\,a+6\,b)\cos\left(fx+e\right)^{3}+3\,(5\,a+6\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a + 6*b)*f*x + (8*a*cos(f*x + e)^5 + 2*(5*a + 6*b)*cos(f*x + e)^
3 + 3*(5*a + 6*b)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.23, size = 104, normalized size = 1.17

$$\frac{3\left(fx+e\right)(5\,a+6\,b)+\frac{15\,a\tan\left(fx+e\right)^{5}+18\,b\tan\left(fx+e\right)^{5}+40\,a\tan\left(fx+e\right)^{3}+48\,b\tan\left(fx+e\right)^{3}+33\,a\tan\left(fx+e\right)+30\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{3}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (15*a*tan(f*x + e)^5 + 18*b*tan(f*x + e)^5
+ 40*a*tan(f*x + e)^3 + 48*b*tan(f*x + e)^3 + 33*a*tan(f*x + e) + 30*b*tan(
f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

maple [A] time = 1.68, size = 86, normalized size = 0.97

$$\frac{a\left(\frac{\left(\cos^{5}\left(fx+e\right)+\frac{5\left(\cos^{3}\left(fx+e\right)\right)}{4}+\frac{15\cos\left(fx+e\right)}{8}\right)\sin\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos^{3}\left(fx+e\right)+\frac{3\cos\left(fx+e\right)}{2}\right)\sin\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16
*f*x+5/16*e)+b*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)
)
```

maxima [A] time = 0.44, size = 103, normalized size = 1.16

$$\frac{3\left(fx+e\right)(5\,a+6\,b)+\frac{3\,(5\,a+6\,b)\tan\left(fx+e\right)^{5}+8\,(5\,a+6\,b)\tan\left(fx+e\right)^{3}+3\,(11\,a+10\,b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{6}+3\,\tan\left(fx+e\right)^{4}+3\,\tan\left(fx+e\right)^{2}+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (3*(5*a + 6*b)*tan(f*x + e)^5 + 8*(5*a + 6*
b)*tan(f*x + e)^3 + 3*(11*a + 10*b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f
*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.94, size = 91, normalized size = 1.02

$$x\left(\frac{5a}{16}+\frac{3b}{8}\right)+\frac{\left(\frac{5a}{16}+\frac{3b}{8}\right)\tan\left(e+fx\right)^5+\left(\frac{5a}{6}+b\right)\tan\left(e+fx\right)^3+\left(\frac{11a}{16}+\frac{5b}{8}\right)\tan\left(e+fx\right)}{f\left(\tan\left(e+fx\right)^6+3\tan\left(e+fx\right)^4+3\tan\left(e+fx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 + (3*b)/8) + (tan(e + f*x)^5*((5*a)/16 + (3*b)/8) + tan(e + f*x
)*((11*a)/16 + (5*b)/8) + tan(e + f*x)^3*((5*a)/6 + b))/(f*(3*tan(e + f*x)^
2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.165 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=165

$$\frac{\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))}{128f}+\frac{\left(48a^2+80ab+35b^2\right)\tan(e+fx)\sec^3(e+fx)}{192f}+\frac{\left(48a^2+80ab+35b\right.}{}$$

```
[Out] 1/128*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e))/f+1/128*(48*a^2+80*a*b+35*
b^2)*sec(f*x+e)*tan(f*x+e)/f+1/192*(48*a^2+80*a*b+35*b^2)*sec(f*x+e)^3*tan(
f*x+e)/f+1/48*b*(10*a+7*b)*sec(f*x+e)^5*tan(f*x+e)/f+1/8*b*sec(f*x+e)^7*(a+
b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{4147, 413, 385, 199, 206\}$

$$\frac{\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))}{128f}+\frac{\left(48a^2+80ab+35b^2\right)\tan(e+fx)\sec^3(e+fx)}{192f}+\frac{\left(48a^2+80ab+35b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]])/(128*f) + ((48*a^2 + 80*
a*b + 35*b^2)*Sec[e + f*x]*Tan[e + f*x])/(128*f) + ((48*a^2 + 80*a*b + 35*b
^2)*Sec[e + f*x]^3*Tan[e + f*x])/(192*f) + (b*(10*a + 7*b)*Sec[e + f*x]^5*T
an[e + f*x])/(48*f) + (b*Sec[e + f*x]^7*(a + b - a*Sin[e + f*x]^2)*Tan[e +
f*x])/(8*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^5}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} - \frac{\text{Subst}\left(\int \frac{-(a+}{}\right.}{}$$

$$= \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} + \frac{b\sec^7(e+fx)\left(a+b-a\,\text{si}\right.}{8f}$$

$$= \frac{\left(48a^2+80ab+35b^2\right)\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{b(10a+7b)\sec^5}{}$$

$$= \frac{\left(48a^2+80ab+35b^2\right)\sec(e+fx)\tan(e+fx)}{128f} + \frac{\left(48a^2+80ab+3\right.}{}$$

$$= \frac{\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))}{128f} + \frac{\left(48a^2+80ab+35b^2\right.}{1}$$

Mathematica [A] time = 0.53, size = 119, normalized size = 0.72

$$\frac{3\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(2\left(48a^2+80ab+35b^2\right)\sec^2(e+fx)+\right.}{384f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(48*a
^2 + 80*a*b + 35*b^2) + 2*(48*a^2 + 80*a*b + 35*b^2)*Sec[e + f*x]^2 + 8*b*(
16*a + 7*b)*Sec[e + f*x]^4 + 48*b^2*Sec[e + f*x]^6)*Tan[e + f*x])/(384*f)
```

fricas [A] time = 0.48, size = 168, normalized size = 1.02

$$\frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(-\text{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(sin(f*x + e) + 1) -
3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(-sin(f*x + e) + 1) + 2*(3*(
48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^6 + 2*(48*a^2 + 80*a*b + 35*b^2)*cos
```

```
(f*x + e)^4 + 8*(16*a*b + 7*b^2)*cos(f*x + e)^2 + 48*b^2)*sin(f*x + e))/(f*
cos(f*x + e)^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-48*a^2-80*a*b-35*b^2)/512*ln(abs(sin(f*x+exp(1))-1))-(-48*a^
2-80*a*b-35*b^2)/512*ln(abs(sin(f*x+exp(1))+1))+(-144*sin(f*x+exp(1))^7*a^2
-240*sin(f*x+exp(1))^7*a*b-105*sin(f*x+exp(1))^7*b^2+528*sin(f*x+exp(1))^5*
a^2+880*sin(f*x+exp(1))^5*a*b+385*sin(f*x+exp(1))^5*b^2-624*sin(f*x+exp(1))
^3*a^2-1168*sin(f*x+exp(1))^3*a*b-511*sin(f*x+exp(1))^3*b^2+240*sin(f*x+exp
(1))*a^2+528*sin(f*x+exp(1))*a*b+279*sin(f*x+exp(1))*b^2)*1/768/(sin(f*x+ex
p(1))^2-1)^4)
```

maple [A] time = 1.29, size = 256, normalized size = 1.55

$$\frac{a^2\tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{4f}+\frac{3a^2\tan\left(fx+e\right)\sec\left(fx+e\right)}{8f}+\frac{3a^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}+\frac{ab\tan\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/4/f*a^2*tan(f*x+e)*sec(f*x+e)^3+3/8/f*a^2*tan(f*x+e)*sec(f*x+e)+3/8/f*a^2
*ln(sec(f*x+e)+tan(f*x+e))+1/3/f*a*b*tan(f*x+e)*sec(f*x+e)^5+5/12/f*a*b*tan
(f*x+e)*sec(f*x+e)^3+5/8/f*a*b*tan(f*x+e)*sec(f*x+e)+5/8/f*a*b*ln(sec(f*x+e
)+tan(f*x+e))+1/8/f*b^2*tan(f*x+e)*sec(f*x+e)^7+7/48/f*b^2*tan(f*x+e)*sec(f
*x+e)^5+35/192/f*b^2*tan(f*x+e)*sec(f*x+e)^3+35/128*b^2*sec(f*x+e)*tan(f*x+
e)/f+35/128/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.34, size = 200, normalized size = 1.21

$$\frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(48\,a^2+80\,ab+\right.\right.}{}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80*
a*b + 35*b^2)*log(sin(f*x + e) - 1) - 2*(3*(48*a^2 + 80*a*b + 35*b^2)*sin(f
*x + e)^7 - 11*(48*a^2 + 80*a*b + 35*b^2)*sin(f*x + e)^5 + (624*a^2 + 1168*
a*b + 511*b^2)*sin(f*x + e)^3 - 3*(80*a^2 + 176*a*b + 93*b^2)*sin(f*x + e))
/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 +
 1))/f
```

mupad [B] time = 4.68, size = 170, normalized size = 1.03

$$\frac{\left(-\frac{3\,a^2}{8}-\frac{5\,a\,b}{8}-\frac{35\,b^2}{128}\right)\,\sin\left(e+f\,x\right)^7+\left(\frac{11\,a^2}{8}+\frac{55\,a\,b}{24}+\frac{385\,b^2}{384}\right)\,\sin\left(e+f\,x\right)^5+\left(-\frac{13\,a^2}{8}-\frac{73\,a\,b}{24}-\frac{511\,b^2}{384}\right)\,\sin\left(e+f\,x\right.}{f\,\left(\sin\left(e+f\,x\right)^8-4\,\sin\left(e+f\,x\right)^6+6\,\sin\left(e+f\,x\right)^4-4\,\sin\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^5,x)
```

```
[Out] (sin(e + f*x)*((11*a*b)/8 + (5*a^2)/8 + (93*b^2)/128) - sin(e + f*x)^7*((5*
a*b)/8 + (3*a^2)/8 + (35*b^2)/128) + sin(e + f*x)^5*((55*a*b)/24 + (11*a^2)
/8 + (385*b^2)/384) - sin(e + f*x)^3*((73*a*b)/24 + (13*a^2)/8 + (511*b^2)/
384))/(f*(6*sin(e + f*x)^4 - 4*sin(e + f*x)^2 - 4*sin(e + f*x)^6 + sin(e +
f*x)^8 + 1)) + (atanh(sin(e + f*x))*((5*a*b)/8 + (3*a^2)/8 + (35*b^2)/128))
/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2 \sec^5\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**5, x)
```

3.166 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=129

$$\frac{\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{\left(8a^2+12ab+5b^2\right)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b(8a+5b)\tan(e+fx)\sec}{24f}$$

```
[Out] 1/16*(8*a^2+12*a*b+5*b^2)*arctanh(sin(f*x+e))/f+1/16*(8*a^2+12*a*b+5*b^2)*s
ec(f*x+e)*tan(f*x+e)/f+1/24*b*(8*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec
(f*x+e)^5*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 413, 385, 199, 206}

$$\frac{\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{\left(8a^2+12ab+5b^2\right)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b(8a+5b)\tan(e+fx)\sec}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]])/(16*f) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (b*(8*a + 5*b)*Sec[e + f*x]^3*
Tan[e + f*x])/(24*f) + (b*Sec[e + f*x]^5*(a + b - a*Sin[e + f*x]^2)*Tan[e +
 f*x])/(6*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-(a+\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\left(a+b-a\sin\cdots\right.}{6f}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\sec(e+fx)\tan(e+fx)}{16f} + \frac{b(8a+5b)\sec^3(e+\cdots}{24f}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))}{16f} + \frac{\left(8a^2+12ab+5b^2\right)\sec(\cdots}{16f}$$

Mathematica [A] time = 0.39, size = 94, normalized size = 0.73

$$\frac{3\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(3\left(8a^2+12ab+5b^2\right)+2b(12a+5b)\sec^{2}\cdots\right.}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(8*a^2
+ 12*a*b + 5*b^2) + 2*b*(12*a + 5*b)*Sec[e + f*x]^2 + 8*b^2*Sec[e + f*x]^4)
*Tan[e + f*x])/(48*f)
```

fricas [A] time = 0.49, size = 143, normalized size = 1.11

$$\frac{3\left(8\,a^2+12\,ab+5\,b^2\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,a^2+12\,ab+5\,b^2\right)\cos\left(fx+e\right)^6\log\left(-\sin\left(f\cdots\right.\right.}{96\,f\cos\left(f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(
8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(8*a^2
 + 12*a*b + 5*b^2)*cos(f*x + e)^4 + 2*(12*a*b + 5*b^2)*cos(f*x + e)^2 + 8*b
^2)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-(8*a^2+12*a*b+5*b^2)/64*ln(abs(sin(f*x+exp(1))-1))+(8*a^2+12*
a*b+5*b^2)/64*ln(abs(sin(f*x+exp(1))+1))+(-24*sin(f*x+exp(1))^5*a^2-36*sin(
f*x+exp(1))^5*a*b-15*sin(f*x+exp(1))^5*b^2+48*sin(f*x+exp(1))^3*a^2+96*sin(
f*x+exp(1))^3*a*b+40*sin(f*x+exp(1))^3*b^2-24*sin(f*x+exp(1))*a^2-60*sin(f*
x+exp(1))*a*b-33*sin(f*x+exp(1))*b^2)*1/96/(sin(f*x+exp(1))^2-1)^3)
```

maple [A] time = 1.31, size = 191, normalized size = 1.48

$$\frac{a^2 \tan\left(fx+e\right) \sec\left(fx+e\right)}{2f}+\frac{a^2 \ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{ab \tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{2f}+\frac{3ab \tan\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a^2*tan(f*x+e)*sec(f*x+e)+1/2/f*a^2*ln(sec(f*x+e)+tan(f*x+e))+1/2/f*a
*b*tan(f*x+e)*sec(f*x+e)^3+3/4/f*a*b*tan(f*x+e)*sec(f*x+e)+3/4/f*a*b*ln(sec
(f*x+e)+tan(f*x+e))+1/6/f*b^2*tan(f*x+e)*sec(f*x+e)^5+5/24/f*b^2*tan(f*x+e)
*sec(f*x+e)^3+5/16*b^2*sec(f*x+e)*tan(f*x+e)/f+5/16/f*b^2*ln(sec(f*x+e)+tan
(f*x+e))
```

maxima [A] time = 0.33, size = 166, normalized size = 1.29

$$\frac{3\left(8\,a^2+12\,ab+5\,b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,a^2+12\,ab+5\,b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(8\,a^2+12\,ab+5\,b^2\right)\sin\right.}{}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b
+ 5*b^2)*log(sin(f*x + e) - 1) - 2*(3*(8*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)
^5 - 8*(6*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^3 + 3*(8*a^2 + 20*a*b + 11*b^2
)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))
/f
```

mupad [B] time = 4.55, size = 134, normalized size = 1.04

$$\frac{\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)\left(\frac{a^2}{2}+\frac{3\,a\,b}{4}+\frac{5\,b^2}{16}\right)}{f}-\frac{\left(\frac{a^2}{2}+\frac{3\,a\,b}{4}+\frac{5\,b^2}{16}\right)\sin\left(e+f\,x\right)^5+\left(-a^2-2\,a\,b-\frac{5\,b^2}{6}\right)\sin\left(e+f\,x\right)^3+\left(\right.}{f\left(\sin\left(e+f\,x\right)^6-3\sin\left(e+f\,x\right)^4+3\sin\left(e+f\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^3,x)
```

```
[Out] (atanh(sin(e + f*x))*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/f - (sin(e + f*x)*((
5*a*b)/4 + a^2/2 + (11*b^2)/16) - sin(e + f*x)^3*(2*a*b + a^2 + (5*b^2)/6)
+ sin(e + f*x)^5*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/(f*(3*sin(e + f*x)^2 - 3
*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**3, x)
```

3.167 $\int \sec(e + fx) \left(a + b\sec^2(e + fx)\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{\left(8a^2 + 8ab + 3b^2\right)\tanh^{-1}(\sin(e + fx))}{8f} + \frac{3b(2a + b)\tan(e + fx)\sec(e + fx)}{8f} + \frac{b\tan(e + fx)\sec^3(e + fx)\left(-a\sin^2(\right.}{4f}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e))/f+3/8*b*(2*a+b)*sec(f*x+e)*tan(
f*x+e)/f+1/4*b*sec(f*x+e)^3*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 413, 385, 206}

$$\frac{\left(8a^2 + 8ab + 3b^2\right)\tanh^{-1}(\sin(e + fx))}{8f} + \frac{3b(2a + b)\tan(e + fx)\sec(e + fx)}{8f} + \frac{b\tan(e + fx)\sec^3(e + fx)\left(-a\sin^2(\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/(8*f) + (3*b*(2*a + b)*Sec[
e + f*x]*Tan[e + f*x])/(8*f) + (b*Sec[e + f*x]^3*(a + b - a*Sin[e + f*x]^2)
*Tan[e + f*x])/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^3}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{-(a+b}{}\right.}{}$$

$$= \frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(\right.}{4f}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}(\sin(e+fx))}{8f} + \frac{3b(2a+b)\sec(e+fx)\tan}{8f}$$

Mathematica [A] time = 0.14, size = 63, normalized size = 0.69

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}(\sin(e+fx))+b\tan(e+fx)\sec(e+fx)\left(8a+2b\sec^2(e+fx)+3b\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]] + b*Sec[e + f*x]*(8*a + 3*b
+ 2*b*Sec[e + f*x]^2)*Tan[e + f*x])/(8*f)
```

fricas [A] time = 0.62, size = 116, normalized size = 1.27

$$\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-\left(8\,a^2+8\,ab+3\,b^2\right)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right.\right.}{16\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (8*a^2
 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*((8*a*b + 3*b^2
)*cos(f*x + e)^2 + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-8*a^2-8*a*b-3*b^2)/32*ln(abs(sin(f*x+exp(1))-1))-(-8*a^2-8*a
*b-3*b^2)/32*ln(abs(sin(f*x+exp(1))+1))-(8*sin(f*x+exp(1))^3*a*b+3*sin(f*x+
exp(1))^3*b^2-8*sin(f*x+exp(1))*a*b-5*sin(f*x+exp(1))*b^2)*1/16/(sin(f*x+ex
p(1))^2-1)^2)
```

maple [A] time = 1.15, size = 125, normalized size = 1.37

$$\frac{a^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{ab\tan\left(fx+e\right)\sec\left(fx+e\right)}{f}+\frac{ab\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{b^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*a^2*ln(sec(f*x+e)+tan(f*x+e))+1/f*a*b*tan(f*x+e)*sec(f*x+e)+1/f*a*b*ln(
sec(f*x+e)+tan(f*x+e))+1/4/f*b^2*tan(f*x+e)*sec(f*x+e)^3+3/8*b^2*sec(f*x+e)
*tan(f*x+e)/f+3/8/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.33, size = 119, normalized size = 1.31

$$\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(8\,a^2+8\,ab+3\,b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(\left(8\,ab+3\,b^2\right)\sin\left(fx+e\right)^3-\left(8\,ab\right.\right.}{\sin\left(fx+e\right)^4-2\,\sin\left(fx\right.}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b^
2)*log(sin(f*x + e) - 1) - 2*((8*a*b + 3*b^2)*sin(f*x + e)^3 - (8*a*b + 5*b
^2)*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.46, size = 86, normalized size = 0.95

$$\frac{\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(a^2+a\,b+\frac{3\,b^2}{8}\right)}{f}+\frac{\sin\left(e+f\,x\right)\,\left(\frac{5\,b^2}{8}+a\,b\right)-{\sin\left(e+f\,x\right)}^3\,\left(\frac{3\,b^2}{8}+a\,b\right)}{f\,\left({\sin\left(e+f\,x\right)}^4-2\,{\sin\left(e+f\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x),x)
```

```
[Out] (atanh(sin(e + f*x))*(a*b + a^2 + (3*b^2)/8))/f + (sin(e + f*x)*(a*b + (5*b
^2)/8) - sin(e + f*x)^3*(a*b + (3*b^2)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f
*x)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x), x)
```

3.168 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sin(e+fx)}{f}+\frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*b*(4*a+b)*arctanh(sin(f*x+e))/f+a^2*sin(f*x+e)/f+1/2*b^2*sec(f*x+e)*tan
(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 390, 385, 206}

$$\frac{a^2\sin(e+fx)}{f}+\frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*Sin[e + f*x])/f + (b^2*Sec
[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{b(2a+b)-2abx^2}{\left(1-x^2\right)^2}\right)dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{a^2\sin(e+fx)}{f}+\frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{(b(4a+b))\,\text{Subst}\left(\int \frac{1}{1-x}\right.}{2f}$$

$$= \frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e}{2f}$$

Mathematica [A] time = 0.03, size = 80, normalized size = 1.43

$$\frac{a^2\sin(e)\cos(fx)}{f}+\frac{a^2\cos(e)\sin(fx)}{f}+\frac{2ab\tanh^{-1}(\sin(e+fx))}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+f}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[e + f*x]])/f + (b^2*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*
Cos[f*x]*Sin[e])/f + (a^2*Cos[e]*Sin[f*x])/f + (b^2*Sec[e + f*x]*Tan[e + f*
x])/(2*f)
```

fricas [A] time = 0.67, size = 94, normalized size = 1.68

$$\frac{\left(4\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-\left(4\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,a^2\cos\right.}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b + b^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (4*a*b + b^2)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(2*a^2*cos(f*x + e)^2 + b^2)*sin(f*x
 + e))/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(sin(f*x+exp(1))*a^2/2-sin(f*x+exp(1))*b^2*1/4/(sin(f*x+exp(1))
^2-1)-(4*a*b+b^2)/8*ln(abs(sin(f*x+exp(1))-1))+(4*a*b+b^2)/8*ln(abs(sin(f*x
+exp(1))+1)))
```

maple [A] time = 1.06, size = 78, normalized size = 1.39

$$\frac{a^2\sin\left(fx+e\right)}{f}+\frac{2ab\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{b^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{b^2\ln\left(\sec\left(fx+e\right)+\tan\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] a^2*sin(f*x+e)/f+2/f*a*b*ln(sec(f*x+e)+tan(f*x+e))+1/2*b^2*sec(f*x+e)*tan(f
*x+e)/f+1/2/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.32, size = 87, normalized size = 1.55

$$-\frac{b^2\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ab\left(\log\left(\sin\left(fx+e\right)+1\right)-\log\left(\sin\left(fx\right.\right.\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)
) - 4*a^2*sin(f*x + e))/f
```

mupad [B] time = 0.11, size = 55, normalized size = 0.98

$$\frac{a^2\,\sin\left(e+f\,x\right)+\frac{b\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(4\,a+b\right)}{2}-\frac{b^2\,\sin\left(e+f\,x\right)}{2\,\left(\sin\left(e+f\,x\right)^2-1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*sin(e + f*x) + (b*atanh(sin(e + f*x))*(4*a + b))/2 - (b^2*sin(e + f*x)
)/(2*(sin(e + f*x)^2 - 1)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x), x)
```

3.169 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=49

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a(a+2b)\sin(e+fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

```
[Out] b^2*arctanh(sin(f*x+e))/f+a*(a+2*b)*sin(f*x+e)/f-1/3*a^2*sin(f*x+e)^3/f
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 390, 206}

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a(a+2b)\sin(e+fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (a*(a + 2*b)*Sin[e + f*x])/f - (a^2*Sin[e + 
 f*x]^3)/(3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a(a+2b)-a^2x^2+\frac{b^2}{1-x^2}\right)\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{b^2\tanh^{-1}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.47

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a^2\sin(e+fx)}{f}+\frac{2ab\sin(e)\cos(fx)}{f}+\frac{2ab\cos(e)\sin(fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (2*a*b*Cos[f*x]*Sin[e])/f + (2*a*b*Cos[e]*S
in[f*x])/f + (a^2*Sin[e + f*x])/f - (a^2*Sin[e + f*x]^3)/(3*f)
```

fricas [A] time = 0.82, size = 66, normalized size = 1.35

$$\frac{3\,b^2\log\left(\sin\left(fx+e\right)+1\right)-3\,b^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(a^2\cos\left(fx+e\right)^2+2\,a^2+6\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*log(sin(f*x + e) + 1) - 3*b^2*log(-sin(f*x + e) + 1) + 2*(a^2*co
s(f*x + e)^2 + 2*a^2 + 6*a*b)*sin(f*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b^2/4*ln(abs(sin(f*x+exp(1))-1))+b^2/4*ln(abs(sin(f*x+exp(1))
+1))+(-4/3*sin(f*x+exp(1))^3*a^2+4*sin(f*x+exp(1))*a^2+8*sin(f*x+exp(1))*a*
b)/8)
```

maple [A] time = 1.04, size = 72, normalized size = 1.47

$$\frac{\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)a^2}{3f}+\frac{2a^2\sin\left(fx+e\right)}{3f}+\frac{2\sin\left(fx+e\right)ab}{f}+\frac{b^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f*sin(f*x+e)*cos(f*x+e)^2*a^2+2/3*a^2*sin(f*x+e)/f+2/f*sin(f*x+e)*a*b+1
/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.32, size = 63, normalized size = 1.29

$$-\frac{2\,a^2\sin\left(fx+e\right)^3-3\,b^2\log\left(\sin\left(fx+e\right)+1\right)+3\,b^2\log\left(\sin\left(fx+e\right)-1\right)-6\left(a^2+2\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(sin(f*
x + e) - 1) - 6*(a^2 + 2*a*b)*sin(f*x + e))/f
```

mupad [B] time = 4.53, size = 48, normalized size = 0.98

$$-\frac{\sin\left(e+f\,x\right)\,\left(a^2-2\,a\,\left(a+b\right)\right)+\frac{a^2\,{\sin\left(e+f\,x\right)}^3}{3}-b^2\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] -(sin(e + f*x)*(a^2 - 2*a*(a + b)) + (a^2*sin(e + f*x)^3)/3 - b^2*atanh(sin
(e + f*x)))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.170 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)^2\sin(e+fx)}{f}$$

```
[Out] (a+b)^2*sin(f*x+e)/f-2/3*a*(a+b)*sin(f*x+e)^3/f+1/5*a^2*sin(f*x+e)^5/f
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4147, 194}

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)^2\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Sin[e + f*x])/f - (2*a*(a + b)*Sin[e + f*x]^3)/(3*f) + (a^2*Sin[
e + f*x]^5)/(5*f)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx&=\frac{\operatorname{Subst}\left(\int\left(a+b-ax^2\right)^2\,dx,x,\sin(e+fx)\right)}{f}\\&=\frac{\operatorname{Subst}\left(\int\left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2a^2\left(1+\frac{b}{a}\right)x^2+a^2x^4\right)dx,x,\sin(e+f\right.}{f}\\&=\frac{(a+b)^2\sin(e+fx)}{f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{a^2\sin^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.02, size = 106, normalized size = 2.00

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a^2\sin^3(e+fx)}{3f}+\frac{a^2\sin(e+fx)}{f}-\frac{2ab\sin^3(e+fx)}{3f}+\frac{2ab\sin(e+fx)}{f}+\frac{b^2\sin(e)\cos(fx)}{f}+\frac{b^2\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*Cos[f*x]*Sin[e])/f + (b^2*Cos[e]*Sin[f*x])/f + (a^2*Sin[e + f*x])/f +
(2*a*b*Sin[e + f*x])/f - (2*a^2*Sin[e + f*x]^3)/(3*f) - (2*a*b*Sin[e + f*x]
^3)/(3*f) + (a^2*Sin[e + f*x]^5)/(5*f)
```

fricas [A] time = 0.53, size = 59, normalized size = 1.11

$$\frac{\left(3\,a^2\cos\left(fx+e\right)^4+2\left(2\,a^2+5\,ab\right)\cos\left(fx+e\right)^2+8\,a^2+20\,ab+15\,b^2\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*cos(f*x + e)^4 + 2*(2*a^2 + 5*a*b)*cos(f*x + e)^2 + 8*a^2 + 20*
a*b + 15*b^2)*sin(f*x + e)/f
```

giac [A] time = 0.21, size = 82, normalized size = 1.55

$$\frac{3\,a^2\sin\left(fx+e\right)^5-10\,a^2\sin\left(fx+e\right)^3-10\,ab\sin\left(fx+e\right)^3+15\,a^2\sin\left(fx+e\right)+30\,ab\sin\left(fx+e\right)+15\,b^2\sin(}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*a^2*sin(f*x + e)^3 - 10*a*b*sin(f*x + e)^3
+ 15*a^2*sin(f*x + e) + 30*a*b*sin(f*x + e) + 15*b^2*sin(f*x + e))/f
```

maple [A] time = 1.60, size = 67, normalized size = 1.26

$$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4\left(fx+e\right)+\frac{4\left(\cos^2\left(fx+e\right)\right)}{3}\right)\sin\left(fx+e\right)}{5}+\frac{2ab\left(2+\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)}{3}+b^2\sin\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/5*a^2*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+2/3*a*b*(2+cos(
f*x+e)^2)*sin(f*x+e)+b^2*sin(f*x+e))
```

maxima [A] time = 0.32, size = 55, normalized size = 1.04

$$\frac{3\,a^2\sin\left(fx+e\right)^5-10\left(a^2+ab\right)\sin\left(fx+e\right)^3+15\left(a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*(a^2 + a*b)*sin(f*x + e)^3 + 15*(a^2 + 2*a*
b + b^2)*sin(f*x + e))/f
```

mupad [B] time = 4.44, size = 44, normalized size = 0.83

$$\frac{\sin\left(e+f\,x\right)\,{\left(a+b\right)}^2+\frac{a^2\sin\left(e+f\,x\right)^5}{5}-\frac{2\,a\sin\left(e+f\,x\right)^3\,\left(a+b\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (sin(e + f*x)*(a + b)^2 + (a^2*sin(e + f*x)^5)/5 - (2*a*sin(e + f*x)^3*(a +
 b))/3)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.171 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=106

$$\frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2}{}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+2/3*(a+b)*(a+2*b)*tan(f*x+e)^3/f+1/5*(a^2+6*a*b+6*b^2)
*tan(f*x+e)^5/f+2/7*b*(a+2*b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$\frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/(3*f) + (
(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x]^7
)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+b+bx^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((a+b)^2+2(a+b)(a+2b)x^2+\left(a^2+6ab+6b^2\right)x^4+2b(a\right.\right.}{f}\\ &= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{\left(a^2+6ab+6b\right.}{5}\end{aligned}$$

Mathematica [A] time = 0.40, size = 96, normalized size = 0.91

$$\frac{63\left(a^2+6ab+6b^2\right)\tan^5(e+fx)+210\left(a^2+3ab+2b^2\right)\tan^3(e+fx)+90b(a+2b)\tan^7(e+fx)+315(a+b)^2\tan}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (315*(a + b)^2*Tan[e + f*x] + 210*(a^2 + 3*a*b + 2*b^2)*Tan[e + f*x]^3 + 63
*(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5 + 90*b*(a + 2*b)*Tan[e + f*x]^7 + 35*
b^2*Tan[e + f*x]^9)/(315*f)
```

fricas [A] time = 0.76, size = 120, normalized size = 1.13

$$\frac{\left(8\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^8+4\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+3\left(21\,a^2+36\,ab+16\,b^2\right)\text{co}\right.}{315\,f\cos\left(fx+e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^8 + 4*(21*a^2 + 36*a*b + 1
6*b^2)*cos(f*x + e)^6 + 3*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^4 + 10*(9
*a*b + 4*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*x + e)/(f*cos(f*x + e)^9)
```

giac [A] time = 0.29, size = 164, normalized size = 1.55

$$35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+180\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5+378\,ab\tan\left(fx+e\right)^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 180*b^2*tan(f*x + e)
^7 + 63*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 378*b^2*tan(f*x + e)^
5 + 210*a^2*tan(f*x + e)^3 + 630*a*b*tan(f*x + e)^3 + 420*b^2*tan(f*x + e)^
3 + 315*a^2*tan(f*x + e) + 630*a*b*tan(f*x + e) + 315*b^2*tan(f*x + e))/f
```

maple [A] time = 1.26, size = 134, normalized size = 1.26

$$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^2\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)-2ab\left(-\frac{16}{35}-\frac{\left(\sec^6\left(fx+e\right)\right)}{7}-\frac{6\left(\sec^4\left(fx+e\right)\right)}{35}-\frac{8\left(\sec^2\left(fx+e\right)\right)}{35}\right)\tan\left(\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-16/
35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-1
28/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-64/315*sec(f*
x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.34, size = 103, normalized size = 0.97

$$\frac{35\,b^2\tan\left(fx+e\right)^9+90\left(ab+2\,b^2\right)\tan\left(fx+e\right)^7+63\left(a^2+6\,ab+6\,b^2\right)\tan\left(fx+e\right)^5+210\left(a^2+3\,ab+2\,b^2\right.}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*(a*b + 2*b^2)*tan(f*x + e)^7 + 63*(a^2 +
6*a*b + 6*b^2)*tan(f*x + e)^5 + 210*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^3 +
315*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

mupad [B] time = 4.54, size = 94, normalized size = 0.89

$$\frac{\tan\left(e+f\,x\right)\,{\left(a+b\right)}^2+\frac{b^2\,{\tan\left(e+f\,x\right)}^9}{9}+{\tan\left(e+f\,x\right)}^3\,\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)+{\tan\left(e+f\,x\right)}^5\,\left(\frac{a^2}{5}+\frac{6\,a\,b}{5}+\frac{6\,b^2}{5}\right)+\frac{2\,b\,\mathrm{ta}\cdots}{}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^9)/9 + tan(e + f*x)^3*(2*a*b +
(2*a^2)/3 + (4*b^2)/3) + tan(e + f*x)^5*((6*a*b)/5 + a^2/5 + (6*b^2)/5) + (
2*b*tan(e + f*x)^7*(a + 2*b))/7)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**6, x)
```

3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+1/3*(a+b)*(a+3*b)*tan(f*x+e)^3/f+1/5*b*(2*a+3*b)*tan(f
*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/(3*f) + (b*
(2*a + 3*b)*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+b+bx^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((a+b)^2+(a+b)(a+3b)x^2+b(2a+3b)x^4+b^2x^6\right)\,dx,\right.}{f}\\ &= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{b(2a+3b)\,\mathrm{ta}}{5}\end{aligned}$$

Mathematica [A] time = 0.37, size = 75, normalized size = 0.94

$$\frac{35\left(a^2+4ab+3b^2\right)\tan^3(e+fx)+21b(2a+3b)\tan^5(e+fx)+105(a+b)^2\tan(e+fx)+15b^2\tan^7(e+fx)}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

[Out] (105*(a + b)^2*Tan[e + f*x] + 35*(a^2 + 4*a*b + 3*b^2)*Tan[e + f*x]^3 + 21*b*(2*a + 3*b)*Tan[e + f*x]^5 + 15*b^2*Tan[e + f*x]^7)/(105*f)

fricas [A] time = 0.59, size = 94, normalized size = 1.18

$$\frac{\left(2\left(35a^2+56ab+24b^2\right)\cos\left(fx+e\right)^6+\left(35a^2+56ab+24b^2\right)\cos\left(fx+e\right)^4+6\left(7ab+3b^2\right)\cos\left(fx+e\right)^2+\right.}{105f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/105*(2*(35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^6 + (35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^4 + 6*(7*a*b + 3*b^2)*cos(f*x + e)^2 + 15*b^2)*sin(f*x + e)/(f*cos(f*x + e)^7)

giac [A] time = 0.28, size = 123, normalized size = 1.54

$$\frac{15b^2\tan\left(fx+e\right)^7+42ab\tan\left(fx+e\right)^5+63b^2\tan\left(fx+e\right)^5+35a^2\tan\left(fx+e\right)^3+140ab\tan\left(fx+e\right)^3+105}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^5 + 35*a^2*tan(f*x + e)^3 + 140*a*b*tan(f*x + e)^3 + 105*b^2*tan(f*x + e)^3 + 105*a^2*tan(f*x + e) + 210*a*b*tan(f*x + e) + 105*b^2*tan(f*x + e))/f

maple [A] time = 1.25, size = 104, normalized size = 1.30

$$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)-2ab\left(-\frac{8}{15}-\frac{\left(\sec^4\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^2\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)-b^2\left(-\frac{16}{35}-\frac{\left(\sec^6\left(fx+e\right)\right)}{7}-\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)

[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))

maxima [A] time = 0.34, size = 81, normalized size = 1.01

$$\frac{15b^2\tan\left(fx+e\right)^7+21\left(2ab+3b^2\right)\tan\left(fx+e\right)^5+35\left(a^2+4ab+3b^2\right)\tan\left(fx+e\right)^3+105\left(a^2+2ab+b^2\right)\text{ta}}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + 3*b^2)*tan(f*x + e)^5 + 35*(a^2 + 4*a*b + 3*b^2)*tan(f*x + e)^3 + 105*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f

mupad [B] time = 4.67, size = 70, normalized size = 0.88

$$\frac{\tan\left(e+fx\right)\,\left(a+b\right)^2+\tan\left(e+fx\right)^3\,\left(\frac{a^2}{3}+\frac{4\,a\,b}{3}+b^2\right)+\frac{b^2\tan\left(e+fx\right)^7}{7}+\frac{b\tan\left(e+fx\right)^5\,\left(2\,a+3\,b\right)}{5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + tan(e + f*x)^3*((4*a*b)/3 + a^2/3 + b^2) + (b^2*t
an(e + f*x)^7)/7 + (b*tan(e + f*x)^5*(2*a + 3*b))/5)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**4, x)
```

3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+2/3*b*(a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 194}

$$\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*b*(a + b)*Tan[e + f*x]^3)/(3*f) + (b^2*Tan[
e + f*x]^5)/(5*f)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)+2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.26, size = 48, normalized size = 0.91

$$\frac{10b(a+b)\tan^3(e+fx)+15(a+b)^2\tan(e+fx)+3b^2\tan^5(e+fx)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (15*(a + b)^2*Tan[e + f*x] + 10*b*(a + b)*Tan[e + f*x]^3 + 3*b^2*Tan[e + f*
x]^5)/(15*f)
```

fricas [A] time = 0.90, size = 69, normalized size = 1.30

$$\frac{\left(\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+2\left(5\,ab+2\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2 + 20*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(5*a*b + 2*b^2)*cos(f*x
+ e)^2 + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

giac [A] time = 0.90, size = 82, normalized size = 1.55

$$\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+10\,b^2\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)+30\,ab\tan\left(fx+e\right)+15\,b^2}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 10*b^2*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e) + 30*a*b*tan(f*x + e) + 15*b^2*tan(f*x + e))/f
```

maple [A] time = 1.52, size = 71, normalized size = 1.34

$$\frac{a^2\tan\left(fx+e\right)-2ab\left(-\frac{2}{3}-\frac{\left(\sec^2\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)-b^2\left(-\frac{8}{15}-\frac{\left(\sec^4\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^2\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*tan(f*x+e)-2*a*b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-8/15-1/5
*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.34, size = 71, normalized size = 1.34

$$\frac{10\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ab+\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)b^2+15\,a^2\tan}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(10*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*b + (3*tan(f*x + e)^5 + 10*tan
(f*x + e)^3 + 15*tan(f*x + e))*b^2 + 15*a^2*tan(f*x + e))/f
```

mupad [B] time = 4.54, size = 44, normalized size = 0.83

$$\frac{\tan\left(e+f\,x\right)\,{\left(a+b\right)}^2+\frac{b^2\,{\tan\left(e+f\,x\right)}^5}{5}+\frac{2\,b\,{\tan\left(e+f\,x\right)}^3\,\left(a+b\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^5)/5 + (2*b*tan(e + f*x)^3*(a +
 b))/3)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

3.174 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] time = 0.38, size = 106, normalized size = 2.65

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a^2fx\cos^3(e+fx)+2b(3a+b)\sec(e)\sin(fx)\cos^2(e+fx)+b^2\tan(e)\cos\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 1.11, size = 58, normalized size = 1.45

$$\frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

giac [A] time = 0.23, size = 53, normalized size = 1.32

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

maple [A] time = 1.28, size = 48, normalized size = 1.20

$$\frac{a^2\left(fx+e\right)+2ab\tan\left(fx+e\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.35, size = 44, normalized size = 1.10

$$a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

mupad [B] time = 4.55, size = 42, normalized size = 1.05

$$\frac{\frac{b^2\tan\left(e+f\,x\right)^3}{3}-\tan\left(e+f\,x\right)\,\left(b^2-2\,b\,\left(a+b\right)\right)+a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

3.175 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ax(a+4b)+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] 1/2*a*(a+4*b)*x+1/2*a^2*cos(f*x+e)*sin(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 203}

$$\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ax(a+4b)+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*Tan[e + f*
x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b^2\tan(e+fx)}{f} + \frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{b^2\tan(e+fx)}{f} + \frac{(a(a+4b))\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{1}{2}a(a+4b)x + \frac{a^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{b^2\tan(e+fx)}{f}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 1.11

$$\frac{a^2(e+fx)}{2f} + \frac{a^2\sin(2(e+fx))}{4f} + 2abx + \frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(e + f*x))/(2*f) + (a^2*Sin[2*(e + f*x)])/(4*f) + (b^2*Tan[e
 + f*x])/f
```

fricas [A] time = 1.21, size = 56, normalized size = 1.19

$$\frac{\left(a^2+4\,ab\right)fx\cos\left(fx+e\right)+\left(a^2\cos\left(fx+e\right)^2+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 4*a*b)*f*x*cos(f*x + e) + (a^2*cos(f*x + e)^2 + 2*b^2)*sin(f*x
+ e))/(f*cos(f*x + e))
```

giac [A] time = 0.74, size = 57, normalized size = 1.21

$$\frac{2\,b^2\tan\left(fx+e\right)+\left(a^2+4\,ab\right)\left(fx+e\right)+\frac{a^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

maple [A] time = 1.22, size = 51, normalized size = 1.09

$$\frac{a^2\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(fx+e\right)+b^2\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(f*x+e)+b^2*tan(f*
x+e))
```

maxima [A] time = 0.43, size = 53, normalized size = 1.13

$$\frac{2\,b^2\tan\left(fx+e\right)+\left(a^2+4\,ab\right)\left(fx+e\right)+\frac{a^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

mupad [B] time = 4.52, size = 66, normalized size = 1.40

$$\frac{b^2\tan\left(e+f\,x\right)}{f}+\frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(e+f\,x\right)\,\left(a+4\,b\right)}{2\,\left(\frac{a^2}{2}+2\,b\,a\right)}\right)\,\left(a+4\,b\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x))/f + (a^2*sin(2*e + 2*f*x))/(4*f) + (a*atan((a*tan(e + f*
x)*(a + 4*b))/(2*(2*a*b + a^2/2)))*(a + 4*b))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**2, x)
```

3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=81

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{a\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4f}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f
*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)/f
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 203}

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{a\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) + (a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^3}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f} + \frac{\text{Subst}\left(\int \frac{(a+b)(3}{}\right.}{}$$

$$= \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+\right.}{4f}$$

$$= \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(}{}$$

Mathematica [A] time = 0.11, size = 58, normalized size = 0.72

$$\frac{4\left(3a^2+8ab+8b^2\right)(e+fx)+a^2\sin(4(e+fx))+8a(a+2b)\sin(2(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(e + f*x) + 8*a*(a + 2*b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)])/(32*f)
```

fricas [A] time = 1.39, size = 62, normalized size = 0.77

$$\frac{\left(3a^2+8ab+8b^2\right)fx+\left(2a^2\cos\left(fx+e\right)^3+\left(3a^2+8ab\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 + 8*a*b)*
cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.59, size = 93, normalized size = 1.15

$$\frac{\left(3a^2+8ab+8b^2\right)\left(fx+e\right)+\frac{3a^2\tan\left(fx+e\right)^3+8ab\tan\left(fx+e\right)^3+5a^2\tan\left(fx+e\right)+8ab\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + (3*a^2*tan(f*x + e)^3 + 8*a*b*tan(
f*x + e)^3 + 5*a^2*tan(f*x + e) + 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^
2)/f
```

maple [A] time = 1.23, size = 78, normalized size = 0.96

$$\frac{a^2\left(\frac{\left(\cos^3\left(fx+e\right)+\frac{3\cos\left(fx+e\right)}{2}\right)\sin\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+2*a*b
*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b^2*(f*x+e))
```

maxima [A] time = 0.46, size = 87, normalized size = 1.07

$$\frac{\left(3\,a^2+8\,ab+8\,b^2\right)\left(fx+e\right)+\frac{\left(3\,a^2+8\,ab\right)\tan\left(fx+e\right)^3+\left(5\,a^2+8\,ab\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + ((3*a^2 + 8*a*b)*tan(f*x + e)^3 +
(5*a^2 + 8*a*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.57, size = 76, normalized size = 0.94

$$x\,\left(\frac{3\,a^2}{8}+a\,b+b^2\right)+\frac{\left(\frac{3\,a^2}{8}+b\,a\right)\,{\mathrm{tan}\left(e+f\,x\right)}^3+\left(\frac{5\,a^2}{8}+b\,a\right)\,\mathrm{tan}\left(e+f\,x\right)}{f\,\left({\mathrm{tan}\left(e+f\,x\right)}^4+2\,{\mathrm{tan}\left(e+f\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*(a*b + (3*a^2)/8 + b^2) + (tan(e + f*x)*(a*b + (5*a^2)/8) + tan(e + f*x)^
3*(a*b + (3*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.177 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=119

$$\frac{\left(5a^2+12ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(5a^2+12ab+8b^2\right)+\frac{a(5a+8b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a\sin}{}$$

```
[Out] 1/16*(5*a^2+12*a*b+8*b^2)*x+1/16*(5*a^2+12*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)
/f+1/24*a*(5*a+8*b)*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)
*(a+b+b*tan(f*x+e)^2)/f
```

Rubi [A] time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 413, 385, 199, 203}

$$\frac{\left(5a^2+12ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(5a^2+12ab+8b^2\right)+\frac{a(5a+8b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 12*a*b + 8*b^2)*x)/16 + ((5*a^2 + 12*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(5*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (
a*Cos[e + f*x]^5*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f}+\frac{\text{Subst}\left(\int\frac{(a+}{}\right.}{}$$

$$=\frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos^5(e+fx)\sin(e+fx)\left(\right.}{6f}$$

$$=\frac{\left(5a^2+12ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f}+\frac{a(5a+8b)\cos^3(e+}{24f}$$

$$=\frac{1}{16}\left(5a^2+12ab+8b^2\right)x+\frac{\left(5a^2+12ab+8b^2\right)\cos(e+fx)\sin(e+}{16f}$$

Mathematica [A] time = 0.19, size = 99, normalized size = 0.83

$$\frac{\left(45a^2+96ab+48b^2\right)\sin(2(e+fx))+a^2\sin(6(e+fx))+60a^2e+60a^2fx+3a(3a+4b)\sin(4(e+fx))+144}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (60*a^2*e + 144*a*b*e + 96*b^2*e + 60*a^2*f*x + 144*a*b*f*x + 96*b^2*f*x +
(45*a^2 + 96*a*b + 48*b^2)*Sin[2*(e + f*x)] + 3*a*(3*a + 4*b)*Sin[4*(e + f*
x)] + a^2*Sin[6*(e + f*x)])/(192*f)
```

fricas [A] time = 1.73, size = 89, normalized size = 0.75

$$\frac{3\left(5a^2+12ab+8b^2\right)fx+\left(8a^2\cos\left(fx+e\right)^5+2\left(5a^2+12ab\right)\cos\left(fx+e\right)^3+3\left(5a^2+12ab+8b^2\right)\cos\left(f\right.\right.}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*f*x + (8*a^2*cos(f*x + e)^5 + 2*(5*a^2 + 1
2*a*b)*cos(f*x + e)^3 + 3*(5*a^2 + 12*a*b + 8*b^2)*cos(f*x + e))*sin(f*x +
e))/f
```

giac [A] time = 0.25, size = 161, normalized size = 1.35

$$\frac{3\left(5a^2+12ab+8b^2\right)\left(fx+e\right)+\frac{15a^2\tan\left(fx+e\right)^5+36ab\tan\left(fx+e\right)^5+24b^2\tan\left(fx+e\right)^5+40a^2\tan\left(fx+e\right)^3+96ab\tan\left(fx+e\right)^3+48b^2\tan}{\left(\tan\left(fx+e\right)^2+1\right)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (15*a^2*tan(f*x + e)^5 + 36*a*
b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 96*a*b*t
an(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) + 60*a*b*tan(f*
x + e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

maple [A] time = 1.62, size = 116, normalized size = 0.97

$$\frac{a^2\left(\frac{\left(\cos^5(fx+e)+\frac{5(\cos^3(fx+e))}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b^2\left(\frac{\sin}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/
16*f*x+5/16*e)+2*a*b*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+
3/8*e)+b^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.43, size = 135, normalized size = 1.13

$$\frac{3\left(5a^2+12ab+8b^2\right)\left(fx+e\right)+\frac{3\left(5a^2+12ab+8b^2\right)\tan\left(fx+e\right)^5+8\left(5a^2+12ab+6b^2\right)\tan\left(fx+e\right)^3+3\left(11a^2+20ab+8b^2\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^6+3\tan\left(fx+e\right)^4+3\tan\left(fx+e\right)^2+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (3*(5*a^2 + 12*a*b + 8*b^2)*ta
n(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(11*a^2 + 20*a
*b + 8*b^2)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x +
e)^2 + 1))/f
```

mupad [B] time = 5.32, size = 123, normalized size = 1.03

$$x\left(\frac{5a^2}{16}+\frac{3ab}{4}+\frac{b^2}{2}\right)+\frac{\left(\frac{5a^2}{16}+\frac{3ab}{4}+\frac{b^2}{2}\right)\tan\left(e+fx\right)^5+\left(\frac{5a^2}{6}+2ab+b^2\right)\tan\left(e+fx\right)^3+\left(\frac{11a^2}{16}+\frac{5ab}{4}+\frac{b^2}{2}\right)\mathrm{t}}{f\left(\tan\left(e+fx\right)^6+3\tan\left(e+fx\right)^4+3\tan\left(e+fx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a*b)/4 + (5*a^2)/16 + b^2/2) + (tan(e + f*x)*((5*a*b)/4 + (11*a^2)/16
 + b^2/2) + tan(e + f*x)^3*(2*a*b + (5*a^2)/6 + b^2) + tan(e + f*x)^5*((3*a
*b)/4 + (5*a^2)/16 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(
e + f*x)^6 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.178 $\int \left(a+b\sec^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x+b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(3*a+2*b)*tan(d*x+c)^3/d+1/5
*b^3*tan(d*x+c)^5/d
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+a^3x+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b^2*(3*a + 2*b)*Tan[c +
 d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^3}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(b\left(3a^2+3ab+b^2\right)+b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1+x^2}\right)\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}+\\
&= a^3x+\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [B] time = 1.02, size = 268, normalized size = 3.67

$$\frac{\sec(c)\sec^5(c+dx)\left(150a^3dx\cos(2c+dx)+75a^3dx\cos(2c+3dx)+75a^3dx\cos(4c+3dx)+15a^3dx\cos(4c+5d\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*(150*a^3*d*x*Cos[d*x] + 150*a^3*d*x*Cos[2*c + d*x] +
 75*a^3*d*x*Cos[2*c + 3*d*x] + 75*a^3*d*x*Cos[4*c + 3*d*x] + 15*a^3*d*x*Cos
[4*c + 5*d*x] + 15*a^3*d*x*Cos[6*c + 5*d*x] + 540*a^2*b*Sin[d*x] + 420*a*b^
2*Sin[d*x] + 160*b^3*Sin[d*x] - 360*a^2*b*Sin[2*c + d*x] - 180*a*b^2*Sin[2*
c + d*x] + 360*a^2*b*Sin[2*c + 3*d*x] + 300*a*b^2*Sin[2*c + 3*d*x] + 80*b^3
*Sin[2*c + 3*d*x] - 90*a^2*b*Sin[4*c + 3*d*x] + 90*a^2*b*Sin[4*c + 5*d*x] +
 60*a*b^2*Sin[4*c + 5*d*x] + 16*b^3*Sin[4*c + 5*d*x]))/(480*d)
```

fricas [A] time = 0.75, size = 90, normalized size = 1.23

$$\frac{15\,a^3dx\cos\left(dx+c\right)^5+\left(\left(45\,a^2b+30\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4+3\,b^3+\left(15\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 + ((45*a^2*b + 30*a*b^2 + 8*b^3)*cos(d*x +
c)^4 + 3*b^3 + (15*a*b^2 + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
+ c)^5)
```

giac [A] time = 0.33, size = 91, normalized size = 1.25

$$\frac{3\,b^3\tan\left(dx+c\right)^5+15\,ab^2\tan\left(dx+c\right)^3+10\,b^3\tan\left(dx+c\right)^3+15\,(dx+c)a^3+45\,a^2b\tan\left(dx+c\right)+45\,ab^2\tan\left(d\ldots\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 15*a*b^2*tan(d*x + c)^3 + 10*b^3*tan(d*x + c)^
3 + 15*(d*x + c)*a^3 + 45*a^2*b*tan(d*x + c) + 45*a*b^2*tan(d*x + c) + 15*b
^3*tan(d*x + c))/d
```

maple [A] time = 1.25, size = 84, normalized size = 1.15

$$\frac{a^3\left(dx+c\right)+3a^2b\tan\left(dx+c\right)-3b^2a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)-b^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(d\ldots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*tan(d*x+c)-3*b^2*a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x
+c)-b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.35, size = 83, normalized size = 1.14

$$a^3x+\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab^2}{d}+\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)b^3}{15\,d}+\frac{3\,a^2b\tan\left(dx\ldots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + (tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3/d + 3*a^2*b*tan(d*x + c)/d
```

mupad [B] time = 4.51, size = 73, normalized size = 1.00

$$\frac{\tan(c+d\,x)\,\left(3\,b\,(a+b)^2-3\,b^2\,(a+b)+b^3\right)+\frac{b^3\tan(c+d\,x)^5}{5}+\tan(c+d\,x)^3\,\left(b^2\,(a+b)-\frac{b^3}{3}\right)+a^3\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x)^2)^3,x)
```

```
[Out] (tan(c + d*x)*(3*b*(a + b)^2 - 3*b^2*(a + b) + b^3) + (b^3*tan(c + d*x)^5)/
5 + tan(c + d*x)^3*(b^2*(a + b) - b^3/3) + a^3*d*x)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**3, x)
```

3.179 $\int \left(a+b\sec^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=111

$$a^4x+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan\ldots}{}$$

```
[Out] a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2+8*a*b+3*b^2)*
tan(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+a^4x+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*a^2 +
8*a*b + 3*b^2)*Tan[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tan[c + d*x]^5)/(5*
d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(c+dx)\right)^4\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^4}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)+b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)x^4+\ldots\right.\right.}{d}\\ &= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b^3(4\ldots}{}\\ &= a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\ldots\end{aligned}$$

Mathematica [B] time = 1.57, size = 455, normalized size = 4.10

$$\frac{\sec(c)\sec^7(c+dx)\left(3675a^4dx\cos(2c+dx)+2205a^4dx\cos(2c+3dx)+2205a^4dx\cos(4c+3dx)+735a^4dx\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*(3675*a^4*d*x*Cos[d*x] + 3675*a^4*d*x*Cos[2*c + d*x]
 + 2205*a^4*d*x*Cos[2*c + 3*d*x] + 2205*a^4*d*x*Cos[4*c + 3*d*x] + 735*a^4*
d*x*Cos[4*c + 5*d*x] + 735*a^4*d*x*Cos[6*c + 5*d*x] + 105*a^4*d*x*Cos[6*c +
 7*d*x] + 105*a^4*d*x*Cos[8*c + 7*d*x] + 16800*a^3*b*Sin[d*x] + 18480*a^2*b
^2*Sin[d*x] + 11200*a*b^3*Sin[d*x] + 3360*b^4*Sin[d*x] - 12600*a^3*b*Sin[2*
c + d*x] - 10920*a^2*b^2*Sin[2*c + d*x] - 4480*a*b^3*Sin[2*c + d*x] + 12600
*a^3*b*Sin[2*c + 3*d*x] + 15120*a^2*b^2*Sin[2*c + 3*d*x] + 9408*a*b^3*Sin[2
*c + 3*d*x] + 2016*b^4*Sin[2*c + 3*d*x] - 5040*a^3*b*Sin[4*c + 3*d*x] - 252
0*a^2*b^2*Sin[4*c + 3*d*x] + 5040*a^3*b*Sin[4*c + 5*d*x] + 5880*a^2*b^2*Sin
[4*c + 5*d*x] + 3136*a*b^3*Sin[4*c + 5*d*x] + 672*b^4*Sin[4*c + 5*d*x] - 84
0*a^3*b*Sin[6*c + 5*d*x] + 840*a^3*b*Sin[6*c + 7*d*x] + 840*a^2*b^2*Sin[6*c
 + 7*d*x] + 448*a*b^3*Sin[6*c + 7*d*x] + 96*b^4*Sin[6*c + 7*d*x]))/(13440*d
)
```

fricas [A] time = 0.63, size = 130, normalized size = 1.17

$$\frac{105\,a^4dx\cos(dx+c)^7+\left(4\left(105\,a^3b+105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos(dx+c)^6+2\left(105\,a^2b^2+56\,ab^3+12\,b^4\right)\right.}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 + (4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3
+ 12*b^4)*cos(d*x + c)^6 + 2*(105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cos(d*x + c)
^4 + 15*b^4 + 6*(14*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
 + c)^7)
```

giac [A] time = 0.22, size = 148, normalized size = 1.33

$$\frac{15\,b^4\tan(dx+c)^7+84\,ab^3\tan(dx+c)^5+63\,b^4\tan(dx+c)^5+210\,a^2b^2\tan(dx+c)^3+280\,ab^3\tan(dx+c)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 84*a*b^3*tan(d*x + c)^5 + 63*b^4*tan(d*x + c
)^5 + 210*a^2*b^2*tan(d*x + c)^3 + 280*a*b^3*tan(d*x + c)^3 + 105*b^4*tan(d
*x + c)^3 + 105*(d*x + c)*a^4 + 420*a^3*b*tan(d*x + c) + 630*a^2*b^2*tan(d*
x + c) + 420*a*b^3*tan(d*x + c) + 105*b^4*tan(d*x + c))/d
```

maple [A] time = 1.48, size = 130, normalized size = 1.17

$$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*tan(d*x+c)-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d
*x+c)-4*a*b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-b^4*(-1
6/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.36, size = 134, normalized size = 1.21

$$a^4x + \frac{2\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^2b^2}{d} + \frac{4\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)ab^3}{15d} + \frac{(5\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c
)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3/d + 1/35*(5*tan(d*x + c)^7
 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*b^4/d + 4*a^3*b
*tan(d*x + c)/d
```

mupad [B] time = 4.58, size = 119, normalized size = 1.07

$$\frac{\tan(c+dx)\left(4b(a+b)^3 + 4b^3(a+b) - 6b^2(a+b)^2 - b^4\right) + \tan(c+dx)^3\left(2b^2(a+b)^2 - \frac{4b^3(a+b)}{3} + \frac{b^4}{3}\right) + \frac{b^4\tan}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x)^2)^4,x)
```

```
[Out] (tan(c + d*x)*(4*b*(a + b)^3 + 4*b^3*(a + b) - 6*b^2*(a + b)^2 - b^4) + tan
(c + d*x)^3*(2*b^2*(a + b)^2 - (4*b^3*(a + b))/3 + b^4/3) + (b^4*tan(c + d*
x)^7)/7 + tan(c + d*x)^5*((4*b^3*(a + b))/5 - b^4/5) + a^4*d*x)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2(c+dx)\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**4, x)
```

3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a+b}}-\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{\tan(e+fx)\sec(e+fx)}{2bf}$$

```
[Out] -1/2*(2*a-b)*arctanh(sin(f*x+e))/b^2/f+a^(3/2)*arctanh(sin(f*x+e)*a^(1/2)/(
a+b)^(1/2))/b^2/f/(a+b)^(1/2)+1/2*sec(f*x+e)*tan(f*x+e)/b/f
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 206, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a+b}}-\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{\tan(e+fx)\sec(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((2*a - b)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (a^(3/2)*ArcTanh[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*f) + (Sec[e + f*x]*Tan[e + f*x])
/(2*b*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{\text{Subst}\left(\int \frac{-a+b-ax^2}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{2bf}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{b^2 f} - \frac{(2a-b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2 f} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,f} + \frac{\sec(e+fx)\tan(e+fx)}{2bf}$$

Mathematica [C] time = 6.12, size = 1195, normalized size = 13.90

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^2(e+fx)\left(\frac{2i\tan^{-1}\left(\frac{2\sin(e)\left(\sin(2e)a+ia-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\sin(fx)\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\sin(2e+fx)\sqrt{a}\cdots\right.}{i(a+3b)\cos(e)+i(a+b)\cos(3e)+ia\cos(e+2fx)+ia\cos(3e+2fx)\cdots}\right)}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(4*a*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] - 2*b*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*a*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*b*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]] + (a^(3/2)*Cos[e]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)]
- (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[2*e + f*x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) - (a^(3/2)*Cos[
e]*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2
*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x]
 + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]])/(Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((2*I)*a^(3/2)*ArcTan[(2*Sin[e]*(I*a
 + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I
*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2
] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e +
 f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a
*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin
[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[e] + I*
Sin[e]))/Sqrt[a + b] - (I*a^(3/2)*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e +
 f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2]*Sin[2*e + f*x]]*Sin[e])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
 (I*a^(3/2)*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[
2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
```

```
*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x
]]*Sin[e])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + (2*a^(3/2)*ArcTan[((
a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*Sqrt[(Cos[e] - I*Sin[e])^2]*((
-I)*Cos[e] + Sin[e]))/Sqrt[a + b] + b/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2 - b/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(8*b^2*f*(a + b*Sec[e + f*
x]^2))
```

fricas [A] time = 1.99, size = 272, normalized size = 3.16

$$\left[\frac{2\,a\sqrt{\frac{a}{a+b}}\cos\left(fx+e\right)^{2}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\,(a+b)\sqrt{\frac{a}{a+b}}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^{2}+b}\right)-(2\,a-b)\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)\right.}{4\,b^{2}f\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(a/(a + b))*cos(f*x + e)^2*log(-(a*cos(f*x + e)^2 - 2*(a + b)
*sqrt(a/(a + b))*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - (2*a - b
)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*a - b)*cos(f*x + e)^2*log(-sin(
f*x + e) + 1) + 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2), -1/4*(4*a*sqrt(-a
/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e))*cos(f*x + e)^2 + (2*a - b)*
cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a - b)*cos(f*x + e)^2*log(-sin(f*
x + e) + 1) - 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-a^2*1/2/b^2/sqrt(-a^2-a*b)*atan(a*sin(f*x+exp(1))/sqrt(-a^2-a
*b))-(-2*a+b)*1/8/b^2*ln(abs(sin(f*x+exp(1))-1))+(-2*a+b)*1/8/b^2*ln(abs(si
n(f*x+exp(1))+1))-sin(f*x+exp(1))*1/4/b/(sin(f*x+exp(1))^2-1))
```

maple [A] time = 0.64, size = 141, normalized size = 1.64

$$\frac{a^{2}\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{f\,b^{2}\sqrt{(a+b)\,a}}-\frac{1}{4fb\left(-1+\sin\left(fx+e\right)\right)}+\frac{\ln\left(-1+\sin\left(fx+e\right)\right)a}{2f\,b^{2}}-\frac{\ln\left(-1+\sin\left(fx+e\right)\right)}{4fb}-\frac{}{4fb\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/b^2*a^2/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/4/f/b/(
-1+sin(f*x+e))+1/2/f/b^2*ln(-1+sin(f*x+e))*a-1/4/f/b*ln(-1+sin(f*x+e))-1/4/
f/b/(1+sin(f*x+e))-1/2/f/b^2*ln(1+sin(f*x+e))*a+1/4/f/b*ln(1+sin(f*x+e))
```

maxima [A] time = 0.45, size = 124, normalized size = 1.44

$$-\frac{\frac{2\,a^{2}\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^{2}}+\frac{(2\,a-b)\log\left(\sin\left(fx+e\right)+1\right)}{b^{2}}-\frac{(2\,a-b)\log\left(\sin\left(fx+e\right)-1\right)}{b^{2}}+\frac{2\,\sin\left(fx+e\right)}{b\sin\left(fx+e\right)^{2}-b}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((
a + b)*a)))/(sqrt((a + b)*a)*b^2) + (2*a - b)*log(sin(f*x + e) + 1)/b^2 - (
2*a - b)*log(sin(f*x + e) - 1)/b^2 + 2*sin(f*x + e)/(b*sin(f*x + e)^2 - b))
/f
```

mupad [B] time = 4.95, size = 591, normalized size = 6.87

$$\frac{b\left(a\sin\left(e+fx\right)-a\operatorname{atanh}\left(\sin\left(e+fx\right)\right)+a\sin\left(e+fx\right)^2\operatorname{atanh}\left(\sin\left(e+fx\right)\right)\right)+b^2\left(\sin\left(e+fx\right)+\operatorname{atanh}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] (b*(a*sin(e + f*x) - a*atanh(sin(e + f*x)) + a*sin(e + f*x)^2*atanh(sin(e +
 f*x))) + atan((a^5*sin(e + f*x)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*(a
^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(
e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*
1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b
+ a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*(a^3*b + a^4
)^(1/2)*2i + b^2*(sin(e + f*x) + atanh(sin(e + f*x)) - sin(e + f*x)^2*atanh
(sin(e + f*x))) - 2*a^2*atanh(sin(e + f*x)) - atan((a^5*sin(e + f*x)*(a^3*b
 + a^4)^(1/2)*8i - b*sin(e + f*x)*(a^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(
a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b
^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(
1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b + a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 +
5*a^4*b^3 + 3*a^5*b^2))*sin(e + f*x)^2*(a^3*b + a^4)^(1/2)*2i + 2*a^2*sin(e
 + f*x)^2*atanh(sin(e + f*x)))/(f*(2*a*b^2 + 2*b^3 - 2*b^3*sin(e + f*x)^2 -
 2*a*b^2*sin(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{bf\sqrt{a+b}}$$

```
[Out] arctanh(sin(f*x+e))/b/f-arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/f
/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 391, 206, 208}

$$\frac{\tanh^{-1}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{bf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b*f) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{bf} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{bf}$$
$$= \frac{\tanh^{-1}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}\,f}$$

Mathematica [C] time = 1.35, size = 1022, normalized size = 18.58

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^2(e+fx)\left(-4\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-(Sqrt[a]*Cos[e]*Log[a + 2*
(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e]
 + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]) + Sqrt[a]*Cos[e]*Lo
g[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b
*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]] - (2*I)*Sqr
t[a]*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]
*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*
Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a +
b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e]
+ I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin
[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(Cos[e] - I*Si
n[e]) - 4*Sqrt[a + b]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sqrt[(Cos[e]
 - I*Sin[e])^2] + 4*Sqrt[a + b]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sq
rt[(Cos[e] - I*Sin[e])^2] + I*Sqrt[a]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*
(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sq
rt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2]*Sin[2*e + f*x]]*Sin[e] - I*Sqrt[a]*Log[-a - 2*(a + b)*Cos[2*e]
+ a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sin[e] + 2*Sqrt[a]*ArcTan[((a + b)*Si
n[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Co
s[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(I*Cos[e] + Sin[e])))/(8*b*Sqrt[a + b]*
f*(a + b*Sec[e + f*x]^2)*Sqrt[(Cos[e] - I*Sin[e])^2])
```

fricas [A] time = 1.96, size = 157, normalized size = 2.85

$$\left[\frac{\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf},\ \frac{2\sqrt{-\frac{a}{a+b}}\ a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + log(sin(f*x + e) + 1) - log
(-sin(f*x + e) + 1))/(b*f), 1/2*(2*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))
*sin(f*x + e)) + log(sin(f*x + e) + 1) - log(-sin(f*x + e) + 1))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-1/4/b*ln(abs(sin(f*x+exp(1))-1))+1/4/b*ln(abs(sin(f*x+exp(1))
+1))+a/b*1/2/sqrt(-a^2-a*b)*atan(a*sin(f*x+exp(1))/sqrt(-a^2-a*b)))
```

maple [A] time = 0.62, size = 68, normalized size = 1.24

$$-\frac{a\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{fb\sqrt{(a+b)\,a}}-\frac{\ln\left(-1+\sin\left(fx+e\right)\right)}{2fb}+\frac{\ln\left(1+\sin\left(fx+e\right)\right)}{2fb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/b*a/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/2/f/b*ln(-
1+sin(f*x+e))+1/2/f/b*ln(1+sin(f*x+e))
```

maxima [A] time = 0.44, size = 83, normalized size = 1.51

$$\frac{\frac{a\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}+\frac{\log\left(\sin\left(fx+e\right)+1\right)}{b}-\frac{\log\left(\sin\left(fx+e\right)-1\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b
)*a)))/(sqrt((a + b)*a)*b) + log(sin(f*x + e) + 1)/b - log(sin(f*x + e) - 1
)/b)/f
```

mupad [B] time = 4.58, size = 456, normalized size = 8.29

$$\frac{\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)}{b\,f}+\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2-\frac{\sin\left(e+f\,x\right)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}\,1{}\mathrm{i}}{b^2+a\,b}+\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2+\frac{\sin\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2-\frac{\sin\left(e+f\,x\right)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2+\frac{\sin\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{f\,\left(b^2+a\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atanh(sin(e + f*x))/(b*f) + (atan((((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (si
n(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*
(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2*a^
3*sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a
 + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a +
b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (sin(e + f
*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b)
)^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))/(a*b + b^2) - ((2*a^3*sin(e +
f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/
2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))
/(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(f*(a*b + b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\sqrt{a+b}}$$

```
[Out] arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/f/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4147, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

fricas [A] time = 1.16, size = 117, normalized size = 3.25

$$\left[\frac{\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{a^{2}+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^{2}+b}\right)}{2\sqrt{a^{2}+ab}f}, -\frac{\sqrt{-a^{2}-ab}\arctan\left(\frac{\sqrt{-a^{2}-ab}\sin\left(fx+e\right)}{a+b}\right)}{\left(a^{2}+ab\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*
cos(f*x + e)^2 + b))/(sqrt(a^2 + a*b)*f), -sqrt(-a^2 - a*b)*arctan(sqrt(-a^
2 - a*b)*sin(f*x + e)/(a + b))/((a^2 + a*b)*f)]
```

giac [A] time = 0.26, size = 39, normalized size = 1.08

$$-\frac{\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^{2}-ab}}\right)}{\sqrt{-a^{2}-ab}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*f)
```

maple [A] time = 0.89, size = 28, normalized size = 0.78

$$\frac{\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{f\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))
```

maxima [A] time = 0.45, size = 50, normalized size = 1.39

$$-\frac{\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*
a)))/(sqrt((a + b)*a)*f)
```

mupad [B] time = 0.11, size = 28, normalized size = 0.78

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\sin\left(e+fx\right)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))/(a^(1/2)*f*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec \left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sin(e+fx)}{af}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

```
[Out] sin(f*x+e)/a/f-b*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/f/(a+b)^(1
/2)
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 388, 208}

$$\frac{\sin(e+fx)}{af}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*f))
+ Sin[e + f*x]/(a*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-ax^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\sin(e+fx)}{af}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx,x,\sin(e+fx)\right)}{af}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}\,f}+\frac{\sin(e+fx)}{af}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 1.00

$$\frac{\sqrt{a}\sin(e+fx)-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
in[e + f*x])/(a^(3/2)*f)
```

fricas [A] time = 0.46, size = 164, normalized size = 3.15

$$\left[\frac{\sqrt{a^2+ab}\,b\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{2\left(a^3+a^2b\right)f},\ \frac{\sqrt{-a^2-ab}\,b\arctan\left(\frac{\sqrt{-a^2-ab}\,\mathrm{s}}{a+}\right.}{\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + a*b)*b*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
+ e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b)*sin(f*x + e))/((a^3
 + a^2*b)*f), (sqrt(-a^2 - a*b)*b*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a +
 b)) + (a^2 + a*b)*sin(f*x + e))/((a^3 + a^2*b)*f)]
```

giac [A] time = 0.23, size = 55, normalized size = 1.06

$$\frac{\frac{b\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a}+\frac{\sin\left(fx+e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a) + sin(f*x +
 e)/a)/f
```

maple [A] time = 1.40, size = 45, normalized size = 0.87

$$\frac{\frac{\sin\left(fx+e\right)}{a}-\frac{b\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{a\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/a*sin(f*x+e)-1/a*b/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1
/2)))
```

maxima [A] time = 0.44, size = 67, normalized size = 1.29

$$\frac{\frac{b\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{2\sin\left(fx+e\right)}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b
)*a)))/(sqrt((a + b)*a)*a) + 2*sin(f*x + e)/a)/f
```

mupad [B] time = 4.40, size = 44, normalized size = 0.85

$$\frac{\sin\left(e+f\,x\right)}{a\,f}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{a^{3/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] sin(e + f*x)/(a*f) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(3/
2)*f*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}f\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

```
[Out] (a-b)*sin(f*x+e)/a^2/f-1/3*sin(f*x+e)^3/a/f+b^2*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/a^(5/2)/f/(a+b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}f\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*f) +
 ((a - b)*Sin[e + f*x])/(a^2*f) - Sin[e + f*x]^3/(3*a*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-b}{a^2}-\frac{x^2}{a}+\frac{b^2}{a^2(a+b-ax^2)}\right)dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{(a-b)\sin(e+fx)}{a^2f} - \frac{\sin^3(e+fx)}{3af} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{a^2f}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}\,f} + \frac{(a-b)\sin(e+fx)}{a^2f} - \frac{\sin^3(e+fx)}{3af}$$

Mathematica [**A**] time = 0.28, size = 105, normalized size = 1.38

$$\frac{a^{3/2}\sin(3(e+fx)) + \frac{6b^2\left(\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}} + 3\sqrt{a}\,(3a-4b)\sin(e+fx)}{12a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] + Log[Sqrt[a + b] + Sqrt[
a]*Sin[e + f*x]]))/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sin[e + f*x] + a^(3/
2)*Sin[3*(e + f*x)])/(12*a^(5/2)*f)
```

fricas [**A**] time = 0.70, size = 230, normalized size = 3.03

$$\left[\frac{3\sqrt{a^2+ab}\,b^2\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(2a^3-a^2b-3ab^2+\left(a^3+a^2b\right)\cos(fx+e)^2\right)\sin(f\right.}{6\left(a^4+a^3b\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 + a*b)*b^2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3 - a^2*b - 3*a*b^2 +
(a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)*f), -1/3*(3*sqrt
(-a^2 - a*b)*b^2*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (2*a^3 - a
^2*b - 3*a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)
*f)]
```

giac [**A**] time = 0.23, size = 89, normalized size = 1.17

$$-\frac{\frac{3b^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a^2} + \frac{a^2\sin(fx+e)^3-3a^2\sin(fx+e)+3ab\sin(fx+e)}{a^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^2)
+ (a^2*sin(f*x + e)^3 - 3*a^2*sin(f*x + e) + 3*a*b*sin(f*x + e))/a^3)/f
```

maple [A] time = 1.48, size = 70, normalized size = 0.92

$$-\frac{\frac{\frac{a\left(\sin^3\left(fx+e\right)\right)}{3}-a\sin\left(fx+e\right)+b\sin\left(fx+e\right)}{a^2}+\frac{b^2\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{a^2\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-1/a^2*(1/3*a*sin(f*x+e)^3-a*sin(f*x+e)+b*sin(f*x+e))+b^2/a^2/((a+b)*a
)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

maxima [A] time = 0.44, size = 88, normalized size = 1.16

$$-\frac{\frac{3\,b^2\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2}+\frac{2\left(a\sin\left(fx+e\right)^3-3\,(a-b)\sin\left(fx+e\right)\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((
a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2*(a*sin(f*x + e)^3 - 3*(a - b)*sin(f*x
 + e))/a^2)/f
```

mupad [B] time = 4.45, size = 72, normalized size = 0.95

$$\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{a^{5/2}\,f\,\sqrt{a+b}}-\frac{\sin\left(e+f\,x\right)^3}{3\,a\,f}-\frac{\sin\left(e+f\,x\right)\,\left(\frac{a+b}{a^2}-\frac{2}{a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(5/2)*f*(a + b)^(1/2))
 - sin(e + f*x)^3/(3*a*f) - (sin(e + f*x)*((a + b)/a^2 - 2/a))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=108

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f}+\frac{\sin^5(e+fx)}{5af}$$

```
[Out] (a^2-a*b+b^2)*sin(f*x+e)/a^3/f-1/3*(2*a-b)*sin(f*x+e)^3/a^2/f+1/5*sin(f*x+e
)^5/a/f-b^3*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/f/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{4147, 390, 208\}$

$$\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{\sin^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b]*f)
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/(a^3*f) - ((2*a - b)*Sin[e + f*x]^3)/(
3*a^2*f) + Sin[e + f*x]^5/(5*a*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{a^3} - \frac{(2a-b)x^2}{a^2} + \frac{x^4}{a} - \frac{b^3}{a^3(a+b-ax^2)}\right) dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3 f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2 f} + \frac{\sin^5(e+fx)}{5af} - \frac{b^3\,\text{Subst}\left(\int \frac{}{a\cdots}\right.}{}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}\,f} + \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3 f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2 f} + \frac{\text{si}\cdots}{}$$

Mathematica [A] time = 0.69, size = 136, normalized size = 1.26

$$\frac{5a^{3/2}(5a-4b)\sin(3(e+fx)) + 3a^{5/2}\sin(5(e+fx)) + 30\sqrt{a}\left(5a^2-6ab+8b^2\right)\sin(e+fx) + \frac{120b^3\left(\log\left(\sqrt{a+b}-\sqrt{a}\cdots\right.\right.}{}}{240a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((120*b^3*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqrt
[a]*Sin[e + f*x]]))/Sqrt[a + b] + 30*Sqrt[a]*(5*a^2 - 6*a*b + 8*b^2)*Sin[e
+ f*x] + 5*a^(3/2)*(5*a - 4*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)
])/(240*a^(7/2)*f)
```

fricas [A] time = 0.47, size = 305, normalized size = 2.82

$$\left[\frac{15\sqrt{a^2+ab}\,b^3\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right) + 2\left(3\left(a^4+a^3b\right)\cos(fx+e)^4 + 8a^4 - 2a^3b + 5a^2b^{2}\cdots\right.}{30\left(a^5+a^4b\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a^2 + a*b)*b^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(3*(a^4 + a^3*b)*cos(f*x
+ e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a^2*b^
2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f), 1/15*(15*sqrt(-a^2 - a*
b)*b^3*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (3*(a^4 + a^3*b)*cos
(f*x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f)]
```

giac [A] time = 0.24, size = 136, normalized size = 1.26

$$\frac{\frac{15\,b^3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a^3} + \frac{3\,a^4\sin(fx+e)^5 - 10\,a^4\sin(fx+e)^3 + 5\,a^3b\sin(fx+e)^3 + 15\,a^4\sin(fx+e) - 15\,a^3b\sin(fx+e) + 15\,a^2b^2\sin(fx+e)}{a^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b^3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^3)
 + (3*a^4*sin(f*x + e)^5 - 10*a^4*sin(f*x + e)^3 + 5*a^3*b*sin(f*x + e)^3 +
 15*a^4*sin(f*x + e) - 15*a^3*b*sin(f*x + e) + 15*a^2*b^2*sin(f*x + e))/a^5
)/f
```

maple [A] time = 1.39, size = 110, normalized size = 1.02

$$\frac{\frac{\frac{(\sin^5(fx+e))a^2}{5}-\frac{2(\sin^3(fx+e))a^2}{3}+\frac{(\sin^3(fx+e))ab}{3}+a^2\sin(fx+e)-\sin(fx+e)ab+b^2\sin(fx+e)}{a^3}-\frac{b^3\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a^3\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/a^3*(1/5*sin(f*x+e)^5*a^2-2/3*sin(f*x+e)^3*a^2+1/3*sin(f*x+e)^3*a*b+
a^2*sin(f*x+e)-sin(f*x+e)*a*b+b^2*sin(f*x+e))-b^3/a^3/((a+b)*a)^(1/2)*arcta
nh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

maxima [A] time = 0.46, size = 117, normalized size = 1.08

$$\frac{\frac{15\,b^3\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3}+\frac{2\left(3\,a^2\sin(fx+e)^5-5\left(2\,a^2-ab\right)\sin(fx+e)^3+15\left(a^2-ab+b^2\right)\sin(fx+e)\right)}{a^3}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/30*(15*b^3*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*a^3) + 2*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 - a
*b)*sin(f*x + e)^3 + 15*(a^2 - a*b + b^2)*sin(f*x + e))/a^3)/f
```

mupad [B] time = 0.14, size = 111, normalized size = 1.03

$$\frac{\sin\left(e+f\,x\right)\left(\frac{3}{a}+\frac{(a+b)\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right)}{f}+\frac{\sin\left(e+f\,x\right)^5}{5\,a\,f}+\frac{\sin\left(e+f\,x\right)^3\,\left(\frac{a+b}{3\,a^2}-\frac{1}{a}\right)}{f}-\frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{7/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (sin(e + f*x)*(3/a + ((a + b)*((a + b)/a^2 - 3/a))/a))/f + sin(e + f*x)^5/(
5*a*f) + (sin(e + f*x)^3*((a + b)/(3*a^2) - 1/a))/f - (b^3*atanh((a^(1/2)*s
in(e + f*x))/(a + b)^(1/2)))/(a^(7/2)*f*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}f\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/f/(a+b)^(1/2)-(a-b)*tan(
f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 390, 205}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}f\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*f) -
((a - b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a-b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{b^2f}$$
$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,f}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

Mathematica [C] time = 2.26, size = 224, normalized size = 2.91

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sqrt{b(\sin(e)+i\cos(e))^4}\sec(e+fx)\left(\sec(e)\sin(fx)\left(-3a+b\sec^2(\right.\right.\right.}{6b^2f\sqrt{a+b}\sqrt{b(\cos(e)-i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-3*a^2*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*
Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(Sec[e]*(-3*a + 2*b + b*Sec[e +
f*x]^2)*Sin[f*x] + b*Sec[e + f*x]*Tan[e])))/(6*b^2*Sqrt[a + b]*f*(a + b*Sec
[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [B] time = 0.86, size = 354, normalized size = 4.60

$$\left[-\frac{3\sqrt{-ab-b^2}\,a^2\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+2b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}}{a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b^2}\right.}{12\left(ab^3+b^4\right)f\cos\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a*b - b^2)*a^2*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)
^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)
^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b^2 + b^3 - (3*a^2*b + a*b^2 - 2*b
^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3), -1/6*(3
*sqrt(a*b + b^2)*a^2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b +
b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(a*b^2 + b^3 - (3*a^2*b
 + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x +
e)^3)]
```

giac [A] time = 0.24, size = 101, normalized size = 1.31

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2}{\sqrt{ab+b^2}\,b^2}+\frac{b^2\tan\left(fx+e\right)^3-3ab\tan\left(fx+e\right)+3b^2\tan\left(fx+e\right)}{b^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*a^2/(sqrt(a*b + b^2)*b^2) + (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x
+ e) + 3*b^2*tan(f*x + e))/b^3)/f
```

maple [A] time = 0.58, size = 79, normalized size = 1.03

$$\frac{\tan^3\left(fx+e\right)}{3bf}-\frac{\tan\left(fx+e\right)a}{f\,b^2}+\frac{\tan\left(fx+e\right)}{bf}+\frac{a^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\,b^2\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/3*tan(f*x+e)^3/b/f-1/f/b^2*tan(f*x+e)*a+tan(f*x+e)/b/f+1/f/b^2*a^2/((a+b)
*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.45, size = 65, normalized size = 0.84

$$\frac{\frac{3\,a^2\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b^2}+\frac{b\tan\left(fx+e\right)^3-3\,(a-b)\tan\left(fx+e\right)}{b^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b^2) + (
b*tan(f*x + e)^3 - 3*(a - b)*tan(f*x + e))/b^2)/f
```

mupad [B] time = 4.37, size = 72, normalized size = 0.94

$$\frac{\tan\left(e+f\,x\right)^3}{3\,b\,f}-\frac{\tan\left(e+f\,x\right)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{f}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{b^{5/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) - (tan(e + f*x)*((a + b)/b^2 - 2/b))/f + (a^2*atan((
b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(5/2)*f*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan(e+fx)}{bf}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}f\sqrt{a+b}}$$

```
[Out] -a*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/f/(a+b)^(1/2)+tan(f*x+e)/
b/f
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 388, 205}

$$\frac{\tan(e+fx)}{bf}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*f)) +
 Tan[e + f*x]/(b*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)}{bf}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{bf}\\ &= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,f}+\frac{\tan(e+fx)}{bf}\end{aligned}$$

Mathematica [C] time = 0.63, size = 192, normalized size = 3.69

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sec(e)\sin(fx)\sqrt{b(\sin(e)+i\cos(e))^4}\sec(e+fx)+a(\cos(2e)\right.}{2bf\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(a*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Sec[e
]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*Sin[f*x]))/(2*b*Sqrt[a + b]*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [B] time = 0.54, size = 286, normalized size = 5.50

$$\left[-\frac{\sqrt{-ab-b^2}\,a\cos\left(fx+e\right)\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\ldots}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right.}{4\left(ab^2+b^3\right)f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b - b^2)*a*cos(f*x + e)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*
cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b + b^2)*sin(f*x + e))/((a*b^2 + b^3)*f*c
os(f*x + e)), 1/2*(sqrt(a*b + b^2)*a*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*(a*b + b^
2)*sin(f*x + e))/((a*b^2 + b^3)*f*cos(f*x + e))]
```

giac [A] time = 0.35, size = 69, normalized size = 1.33

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)a}{\sqrt{ab+b^2}\,b}-\frac{\tan\left(fx+e\right)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*a/(sqrt(a*b + b^2)*b) - tan(f*x + e)/b)/f
```

maple [A] time = 0.43, size = 47, normalized size = 0.90

$$\frac{\tan\left(fx+e\right)}{bf}-\frac{a\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{fb\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] tan(f*x+e)/b/f-1/f/b*a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.45, size = 45, normalized size = 0.87

$$-\frac{\frac{a\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b}-\frac{\tan\left(fx+e\right)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(a*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b) - tan(f*x +
e)/b)/f
```

mupad [B] time = 4.42, size = 44, normalized size = 0.85

$$\frac{\tan\left(e+f\,x\right)}{b\,f}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{b^{3/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] tan(e + f*x)/(b*f) - (a*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(3/2
)*f*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,f\sqrt{a+b}}$$

```
[Out] arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/f/b^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

fricas [B] time = 0.56, size = 209, normalized size = 5.81

$$\left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4((a+2b)\cos(fx+e)^3-b\cos(fx+e))\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)}{4(ab+b^2)f}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sq
rt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2))/((a*b + b^2)*f), -1/2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))/(sqrt(a*b + b^2)*f)]
```

giac [A] time = 0.22, size = 50, normalized size = 1.39

$$\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2
)))/(sqrt(a*b + b^2)*f)
```

maple [A] time = 0.55, size = 28, normalized size = 0.78

$$\frac{\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.43, size = 27, normalized size = 0.75

$$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*f)
```

mupad [B] time = 4.51, size = 31, normalized size = 0.86

$$\frac{\mathrm{atan}\left(\frac{b\tan(e+fx)}{\sqrt{b^2+ab}}\right)}{f\sqrt{b^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atan((b*tan(e + f*x))/(a*b + b^2)^(1/2))/(f*(a*b + b^2)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sec^2(e+fx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\ &= \frac{x}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx,x,\cot(e+fx)\right)}{af}\\ &= \frac{x}{a}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [C] time = 0.29, size = 182, normalized size = 4.04

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+b(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\ldots}{\ldots}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.63, size = 231, normalized size = 5.13

$$\left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\,\sin}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right.}{4\,af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

giac [A] time = 0.21, size = 68, normalized size = 1.51

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

maple [A] time = 0.78, size = 48, normalized size = 1.07

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{fa\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/a*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a*arctan(
tan(f*x+e))
```

maxima [A] time = 0.43, size = 44, normalized size = 0.98

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\sqrt{\left(a+b\right)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

mupad [B] time = 4.71, size = 460, normalized size = 10.22

$$\frac{x}{a}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,b^3\,\tan\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2-\frac{\tan\left(e+f\,x\right)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,\left(a+b\right)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,\left(a+b\right)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,\left(a+b\right)}\,1{}\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\tan\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2+\frac{\tan\left(e+f\,x\right)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a}{4\,\left(a^2+b\,a\right)}}{2\,\left(a^2+b\,a\right)}}{a^2+b\,a}}{\frac{\left(2\,b^3\,\tan\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2-\frac{\tan\left(e+f\,x\right)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,\left(a+b\right)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,\left(a+b\right)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-b\,\left(a+b\right)}}{a^2+b\,a}-\frac{\left(2\,b^3\,\tan\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2+\frac{\tan\left(e+f\,x\right)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{-b\,(a}{4\,\left(a^2+b\,a\right)}}{2\,\left(a^2+b\,a\right)}}{a^2+b\,a}}\right)}{f\,\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sec^2\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

```
[Out] 1/2*(a-2*b)*x/a^2+1/2*cos(f*x+e)*sin(f*x+e)/a/f+b^(3/2)*arctan(b^(1/2)*tan(
f*x+e)/(a+b)^(1/2))/a^2/f/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a^2*Sqrt[a + b]*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{-a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^2f} + \frac{b^2\,\text{Subst}\left(\int \frac{}{a+}\right.}{}$$

$$= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,f} + \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

Mathematica [A] time = 0.23, size = 67, normalized size = 0.89

$$\frac{\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2(a-2b)(e+fx) + a\sin(2(e+fx))}{4a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(e + f*x) + (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + a*Sin[2*(e + f*x)])/(4*a^2*f)
```

fricas [A] time = 0.62, size = 272, normalized size = 3.63

$$\left[\frac{2(a-2b)fx + 2a\cos(fx+e)\sin(fx+e) + b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4 - 2(3ab+4b^2)\cos(fx+e)^2 - 4((a^2+3ab+}{a^2\cos(fx+e)^4 + 2ab\,c}\right.}{4a^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 2*b)*f*x + 2*a*cos(f*x + e)*sin(f*x + e) + b*sqrt(-b/(a + b))*
log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqr
t(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^
2 + b^2)))/(a^2*f), 1/2*((a - 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e) - b*sq
rt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*
cos(f*x + e)*sin(f*x + e))))/(a^2*f)]
```

giac [A] time = 0.23, size = 99, normalized size = 1.32

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{\sqrt{ab+b^2}\,a^2} + \frac{(fx+e)(a-2b)}{a^2} + \frac{\tan(fx+e)}{(\tan(fx+e)^2+1)a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*b^2/(sqrt(a*b + b^2)*a^2) + (f*x + e)*(a - 2*b)/a^2 + tan(f*x +
e)/((tan(f*x + e)^2 + 1)*a))/f
```

maple [A] time = 1.56, size = 92, normalized size = 1.23

$$\frac{b^{2}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^{2}\sqrt{\left(a+b\right)b}}+\frac{\tan\left(fx+e\right)}{2fa\left(1+\tan^{2}\left(fx+e\right)\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{2fa}-\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{f\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*b^2/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a*ta
n(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a*arctan(tan(f*x+e))-1/f/a^2*arctan(tan(f*x
+e))*b
```

maxima [A] time = 0.44, size = 72, normalized size = 0.96

$$\frac{\frac{2\,b^{2}\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^{2}}+\frac{\left(fx+e\right)(a-2\,b)}{a^{2}}+\frac{\tan\left(fx+e\right)}{a\tan\left(fx+e\right)^{2}+a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) + (
f*x + e)*(a - 2*b)/a^2 + tan(f*x + e)/(a*tan(f*x + e)^2 + a))/f
```

mupad [B] time = 5.24, size = 373, normalized size = 4.97

$$-\frac{2\,b^{2}\,\mathrm{atan}\left(\frac{\sin\left(e+f\,x\right)}{\cos\left(e+f\,x\right)}\right)-a\,\left(\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2}-b\,\mathrm{atan}\left(\frac{\sin\left(e+f\,x\right)}{\cos\left(e+f\,x\right)}\right)\right)-a^{2}\,\left(\frac{\sin\left(2\,e+2\,f\,x\right)}{2}+\mathrm{atan}\left(\frac{\sin\left(e+f\,x\right)}{\cos\left(e+f\,x\right)}\right)\right)+\mathrm{atan}\left(\frac{a}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] -(atan((a*sin(e + f*x)*(- a*b^3 - b^4)^(3/2)*4i + b*sin(e + f*x)*(- a*b^3 -
 b^4)^(3/2)*8i + b^5*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*8i + a*b^4*sin(e +
f*x)*(- a*b^3 - b^4)^(1/2)*12i + a^4*b*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1
i + a^2*b^3*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1i - a^3*b^2*sin(e + f*x)*(-
 a*b^3 - b^4)^(1/2)*2i)/(3*a^2*b^5*cos(e + f*x) + 5*a^3*b^4*cos(e + f*x) +
a^4*b^3*cos(e + f*x) - a^5*b^2*cos(e + f*x)))*(- a*b^3 - b^4)^(1/2)*2i + 2*
b^2*atan(sin(e + f*x)/cos(e + f*x)) - a*((b*sin(2*e + 2*f*x))/2 - b*atan(si
n(e + f*x)/cos(e + f*x))) - a^2*(sin(2*e + 2*f*x)/2 + atan(sin(e + f*x)/cos
(e + f*x))))/(f*(2*a^2*b + 2*a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a+b}}+\frac{(3a-4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))
/a^3/f/(a+b)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a+b}}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{(3a-4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a^3*Sqrt[a + b]*f) + ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x
])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-3bx^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2+(3}{(1+x^2)(a+b}\right.}{}$$

$$= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,d\right.}{a^3f}$$

$$= \frac{(3a^2-4ab+8b^2)x}{8a^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,f} + \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} +$$

Mathematica [A] time = 0.40, size = 95, normalized size = 0.81

$$\frac{4\left(3a^2-4ab+8b^2\right)(e+fx)+a^2\sin(4(e+fx))-\frac{32b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+8a(a-b)\sin(2(e+fx))}{32a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(e + f*x) - (32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sin[2*(e + f*x)] + a^2*Sin[4*
(e + f*x)])/(32*a^3*f)
```

fricas [A] time = 0.87, size = 343, normalized size = 2.93

$$\left[\frac{2\,b^2\sqrt{-\frac{b}{a+b}}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\,\sin\left(fx+e\right)+}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right.}{8\,a^3f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b^2*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*
(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
(a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
 + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a
^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/
8*(4*b^2*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(
a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a
^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

giac [A] time = 1.23, size = 149, normalized size = 1.27

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{\sqrt{ab+b^2}\,a^3}-\frac{(3a^2-4ab+8b^2)(fx+e)}{a^3}-\frac{3a\tan(fx+e)^3-4b\tan(fx+e)^3+5a\tan(fx+e)-4b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/(sqrt(a*b + b^2)*a^3) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a
^3 - (3*a*tan(f*x + e)^3 - 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) - 4*b*tan(
f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

maple [A] time = 1.75, size = 194, normalized size = 1.66

$$-\frac{b^3\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{fa^3\sqrt{(a+b)b}}+\frac{3\left(\tan^3(fx+e)\right)}{8fa\left(1+\tan^2(fx+e)\right)^2}-\frac{\left(\tan^3(fx+e)\right)b}{2fa^2\left(1+\tan^2(fx+e)\right)^2}-\frac{\tan(fx+e)b}{2fa^2\left(1+\tan^2(fx+e)\right)^2}+\frac{\cdots}{8f\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b^3/a^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/8/f/a/(
tan(f*x+e)^2+1)^2*tan(f*x+e)^3-1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-
1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)*b+5/8/f/a/(tan(f*x+e)^2+1)^2*tan(f*
x+e)+1/f/a^3*arctan(tan(f*x+e))*b^2+3/8/f/a*arctan(tan(f*x+e))-1/2/f/a^2*ar
ctan(tan(f*x+e))*b
```

maxima [A] time = 0.43, size = 126, normalized size = 1.08

$$-\frac{\frac{8b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3}-\frac{(3a-4b)\tan(fx+e)^3+(5a-4b)\tan(fx+e)}{a^2\tan(fx+e)^4+2a^2\tan(fx+e)^2+a^2}-\frac{(3a^2-4ab+8b^2)(fx+e)}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3) -
((3*a - 4*b)*tan(f*x + e)^3 + (5*a - 4*b)*tan(f*x + e))/(a^2*tan(f*x + e)^4
 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a^3)/f
```

mupad [B] time = 5.26, size = 1114, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((tan(e + f*x)*(5*a - 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a - 4*b))/(8*a^2))
/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((((-b^5*(a + b))^(1/2)
*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))
/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2
)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b))^(1/2))
/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4) + ((-b^5*(a
 + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 +
 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (
3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a
+ b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4)
)/(((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3*b^5)/32)/a^6 + ((-b^5*(a + b
))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a
^4*b^3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^
8*b^2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b)
)^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*b + a^4) - ((-b
^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*
b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/
2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^
5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*b + a^
4)))*(-b^5*(a + b))^(1/2)*1i)/(f*(a^3*b + a^4)) - (atan((63*b^4*tan(e + f*x
))/(64*((63*b^4)/64 - (81*a*b^3)/256 + (27*a^2*b^2)/256 - (35*b^5)/(32*a) +
 (5*b^6)/(4*a^2))) - (81*b^3*tan(e + f*x))/(256*((27*a*b^2)/256 - (81*b^3)/
256 + (63*b^4)/(64*a) - (35*b^5)/(32*a^2) + (5*b^6)/(4*a^3))) - (35*b^5*tan
(e + f*x))/(32*((63*a*b^4)/64 - (35*b^5)/32 - (81*a^2*b^3)/256 + (27*a^3*b^
2)/256 + (5*b^6)/(4*a))) + (5*b^6*tan(e + f*x))/(4*((5*b^6)/4 - (35*a*b^5)/
32 + (63*a^2*b^4)/64 - (81*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(
e + f*x))/(256*((27*b^2)/256 - (81*b^3)/(256*a) + (63*b^4)/(64*a^2) - (35*b
^5)/(32*a^3) + (5*b^6)/(4*a^4))))*(a^2*3i - a*b*4i + b^2*8i)*1i)/(8*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=163

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f\sqrt{a+b}}+\frac{(5a-6b)\sin(e+fx)\cos^3(e+fx)}{24a^2f}+\frac{\left(5a^2-6ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x\left(5a\right.}{}$$

```
[Out] 1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*x/a^4+1/16*(5*a^2-6*a*b+8*b^2)*cos(f*x+
e)*sin(f*x+e)/a^3/f+1/24*(5*a-6*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos(f*
x+e)^5*sin(f*x+e)/a/f+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^4/f/
(a+b)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f\sqrt{a+b}}+\frac{\left(5a^2-6ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x\left(-6a^2b+5a^3+8ab^2-16b^3\right)}{16a^4}+\frac{(5a-6b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*x)/(16*a^4) + (b^(7/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*Sqrt[a + b]*f) + ((5*a^2 - 6*a*b + 8*b^2
)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((5*a - 6*b)*Cos[e + f*x]^3*Sin[e
 + f*x])/(24*a^2*f) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-5bx^2}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af} + \frac{\text{Subst}\left(\int \frac{3(5a^2-ab+2b^2 \ldots}{(1+x^2)^2(\ldots}\right.}{\ldots}$$

$$= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos \ldots}{\ldots}$$

$$= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos \ldots}{\ldots}$$

$$= \frac{(5a^3-6a^2b+8ab^2-16b^3)x}{16a^4} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}\,f} + \frac{(5a^2-6ab+8b^2)\cos(e+ \ldots}{16a^3f}$$

Mathematica [A] time = 0.85, size = 133, normalized size = 0.82

$$\frac{a^3\sin(6(e+fx)) + 3a(15a^2-16ab+16b^2)\sin(2(e+fx)) + 3a^2(3a-2b)\sin(4(e+fx)) + 12(5a^3-6a^2b+8ab^2 \ldots}{192a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (12*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*(e + f*x) + (192*b^(7/2)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + 3*a*(15*a^2 - 16*a*b + 16*
b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a - 2*b)*Sin[4*(e + f*x)] + a^3*Sin[6*(e +
 f*x)])/(192*a^4*f)
```

fricas [A] time = 0.78, size = 424, normalized size = 2.60

$$\left[\frac{12\,b^{3}\sqrt{-\frac{b}{a+b}}\,\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\,\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\,\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{3}-\left(ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\,\sin\left(f\ldots\right.}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*b^3*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 1
6*b^3)*f*x + (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), -1/48*(24
*b^3*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))) - 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)
*f*x - (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3*(5*a^
3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)]
```

giac [A] time = 0.24, size = 229, normalized size = 1.40

$$\frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)b^{4}}{\sqrt{ab+b^{2}}\,a^{4}}+\frac{3\left(5\,a^{3}-6\,a^{2}b+8\,ab^{2}-16\,b^{3}\right)\left(fx+e\right)}{a^{4}}+\frac{15\,a^{2}\tan\left(fx+e\right)^{5}-18\,ab\tan\left(fx+e\right)^{5}+24\,b^{2}\tan\left(fx+e\right)^{5}+\ldots}{\ldots}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/(sqrt(a*b + b^2)*a^4) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*
b^3)*(f*x + e)/a^4 + (15*a^2*tan(f*x + e)^5 - 18*a*b*tan(f*x + e)^5 + 24*b^
2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 48*a*b*tan(f*x + e)^3 + 48*b^2*t
an(f*x + e)^3 + 33*a^2*tan(f*x + e) - 30*a*b*tan(f*x + e) + 24*b^2*tan(f*x
+ e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

maple [B] time = 1.65, size = 359, normalized size = 2.20

$$\frac{b^{4}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^{4}\sqrt{\left(a+b\right)b}}+\frac{5\left(\tan^{5}\left(fx+e\right)\right)}{16fa\left(1+\tan^{2}\left(fx+e\right)\right)^{3}}-\frac{3\left(\tan^{5}\left(fx+e\right)\right)b}{8f\,a^{2}\left(1+\tan^{2}\left(fx+e\right)\right)^{3}}+\frac{\left(\tan^{5}\left(fx+e\right)\right)b^{2}}{2f\,a^{3}\left(1+\tan^{2}\left(fx+e\right)\right)^{3}}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/a^4*b^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+5/16/f/a/(
tan(f*x+e)^2+1)^3*tan(f*x+e)^5-3/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b+
1/2/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b^2+1/f/a^3/(tan(f*x+e)^2+1)^3*ta
n(f*x+e)^3*b^2+5/6/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-1/f/a^2/(tan(f*x+e)^
2+1)^3*tan(f*x+e)^3*b-5/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b+1/2/f/a^3/(
tan(f*x+e)^2+1)^3*tan(f*x+e)*b^2+11/16/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)-1/
f/a^4*arctan(tan(f*x+e))*b^3+5/16/f/a*arctan(tan(f*x+e))-3/8/f/a^2*arctan(t
an(f*x+e))*b+1/2/f/a^3*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.45, size = 189, normalized size = 1.16

$$\frac{\frac{48\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^4}+\frac{3\,(5\,a^2-6\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2-6\,ab+6\,b^2)\tan(fx+e)^3+3\,(11\,a^2-10\,ab+8\,b^2)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3}+\frac{3\,(5\,a^3-6\,a^2b+8\,ab^2-}{a^4}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4) +
 (3*(5*a^2 - 6*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 - 6*a*b + 6*b^2)*tan(
f*x + e)^3 + 3*(11*a^2 - 10*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x + e)^6
+ 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) + 3*(5*a^3 - 6*a^2*b +
 8*a*b^2 - 16*b^3)*(f*x + e)/a^4)/f
```

mupad [B] time = 6.15, size = 1979, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((tan(e + f*x)*(11*a^2 - 10*a*b + 8*b^2))/(16*a^3) + (tan(e + f*x)^3*(5*a^2
 - 6*a*b + 6*b^2))/(6*a^3) + (tan(e + f*x)^5*(5*a^2 - 6*a*b + 8*b^2))/(16*a
^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (atan
((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(
e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*
16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - (ta
n(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 -
 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*1
6i)*1i)/(32*a^4) - (((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^
2)/4)/a^9 + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6
i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i
))/(32*a^4) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^
6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i
 + a^3*5i - b^3*16i)*1i)/(32*a^4))/((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^
3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a
*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i +
 a^3*5i - b^3*16i))/(32*a^4) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2
*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*
b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - ((5*a*b^10)/4 - b^11 - (1
1*a^2*b^9)/8 + (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (25*a^5*b^6)/128)/a^9 +
(((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 + (tan(e
+ f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16
i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) + (tan(
e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 6
0*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i
))/(32*a^4)))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(16*a^4*f) + (at
an(((((-b^7*(a + b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a
^11*b^2)/4)/(2*a^9) - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a
+ b))^(1/2))/(512*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) - (tan(e + f*x)*(5
12*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 +
 25*a^6*b^3))/(256*a^6))*(-b^7*(a + b))^(1/2)*1i)/(a^4*b + a^5) - ((((-b^7*
(a + b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(
2*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/
(512*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) + (tan(e + f*x)*(512*b^9 - 256*
a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))
/(256*a^6))*(-b^7*(a + b))^(1/2)*1i)/(a^4*b + a^5))/(((((-b^7*(a + b))^(1/2
```

```
)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^9) - (tan
(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512*a^6*(a^4
*b + a^5))))/(2*(a^4*b + a^5)) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a
^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(256*a^6))*(
-b^7*(a + b))^(1/2))/(a^4*b + a^5) - ((5*a*b^10)/4 - b^11 - (11*a^2*b^9)/8
+ (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (25*a^5*b^6)/128)/a^9 + ((((-b^7*(a +
 b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^
9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512
*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) + (tan(e + f*x)*(512*b^9 - 256*a*b^
8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(25
6*a^6))*(-b^7*(a + b))^(1/2))/(a^4*b + a^5)))*(-b^7*(a + b))^(1/2)*1i)/(f*(
a^4*b + a^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
[Out] Timed out
```

3.193 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2f(a+b)^{3/2}}-\frac{a\sin(e+fx)}{2bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}(\sin(e+fx))}{b^2f}$$

```
[Out] arctanh(sin(f*x+e))/b^2/f-1/2*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)-1
/2*(2*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3/2
)/f
```

Rubi [A] time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 206, 208}

$$-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2f(a+b)^{3/2}}-\frac{a\sin(e+fx)}{2bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}(\sin(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b^2*f) - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*f) - (a*Sin[e + f*x])/(2*b*(a +
 b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-ax^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-2b-ax^2}{(1-x^2)\left(a+b-ax^2\right)}\,dx,x,\sin(e+fx\right.}{2b(a+b)f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(e+fx)\right)}{b^2f} - \frac{(a(2\ldots}{}$$

$$= \frac{\tanh^{-1}(\sin(e+fx))}{b^2f} - \frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a\sin(e+\ldots}{2b(a+b)f\left(a+b-\ldots\right.}$$

Mathematica [C] time = 3.89, size = 980, normalized size = 9.61

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^3(e+fx)\left(-8b\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan(e+fx)a^{3/2}-2i(2a+3b)\tan^{-1}\left(\ldots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*a*(2*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) - 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*
(e + f*x)])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos
[e] - I*Sin[e])^2] + 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*(e + f*x)])
*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin
[e])^2] + 2*a*(2*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*
x])]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) - 8*a^
(3/2)*b*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x]))/(32*Sqrt[a]*
b^2*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(Cos[e] - I*Sin[e])^2])
```

fricas [A] time = 2.07, size = 392, normalized size = 3.84

$$\left[-\frac{2\,ab\sin\left(fx+e\right)-\left(\left(2\,a^2+3\,ab\right)\cos\left(fx+e\right)^2+2\,ab+3\,b^2\right)\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)}{4\left(\left(a^2b^2+ab^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b + 3*b^2
)*sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*sin(f*
x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*((a^2 + a*b)*cos(f*x + e)^2 +
 a*b + b^2)*log(sin(f*x + e) + 1) + 2*((a^2 + a*b)*cos(f*x + e)^2 + a*b + b
^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 +
b^4)*f), -1/2*(a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b +
 3*b^2)*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e)) - ((a^2 + a*
b)*cos(f*x + e)^2 + a*b + b^2)*log(sin(f*x + e) + 1) + ((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x +
 e)^2 + (a*b^3 + b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-1/4/b^2*ln(abs(sin(f*x+exp(1))-1))+1/4/b^2*ln(abs(sin(f*x+exp
(1))+1))+(-2*a^2-3*a*b)*1/2/(-2*a*b^2-2*b^3)/sqrt(-a^2-a*b)*atan(a*sin(f*x+
exp(1))/sqrt(-a^2-a*b))-sin(f*x+exp(1))*a/(-4*a*b-4*b^2)/(sin(f*x+exp(1))^2
*a-a-b))
```

maple [A] time = 0.56, size = 151, normalized size = 1.48

$$\frac{a\sin\left(fx+e\right)}{2fb\,(a+b)\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)}-\frac{a^2\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{f\,b^2\,(a+b)\sqrt{(a+b)\,a}}-\frac{3a\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{2fb\,(a+b)\sqrt{(a+b)\,a}}-\frac{\ln\left(-1+\sin\left(fx+e\right)\right.}{2f\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a/b/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/f*a^2/b^2/(a+b)/((a+b)*a
)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-3/2/f*a/b/(a+b)/((a+b)*a)^(1/
2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/2/f/b^2*ln(-1+sin(f*x+e))+1/2/f/
b^2*ln(1+sin(f*x+e))
```

maxima [A] time = 0.45, size = 146, normalized size = 1.43

$$\frac{\frac{(2\,a+3\,b)a\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(ab^2+b^3\right)\sqrt{(a+b)a}}-\frac{2\,a\sin\left(fx+e\right)}{a^2b+2\,ab^2+b^3-\left(a^2b+ab^2\right)\sin\left(fx+e\right)^2}+\frac{2\,\log\left(\sin\left(fx+e\right)+1\right)}{b^2}-\frac{2\,\log\left(\sin\left(fx+e\right)-1\right)}{b^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a + 3*b)*a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) +
 sqrt((a + b)*a)))/((a*b^2 + b^3)*sqrt((a + b)*a)) - 2*a*sin(f*x + e)/(a^2*
b + 2*a*b^2 + b^3 - (a^2*b + a*b^2)*sin(f*x + e)^2) + 2*log(sin(f*x + e) +
1)/b^2 - 2*log(sin(f*x + e) - 1)/b^2)/f
```

mupad [B] time = 5.87, size = 2039, normalized size = 19.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3))
 - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^
2*(2*a*b^3 + b^4 + a^2*b^2)))*1i)/(2*b^2) + (sin(e + f*x)*(20*a^4*b + 8*a^5
 + 13*a^3*b^2)*1i)/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((((4*a^2*b^6 + 6*a
^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e + f*x)*(16*a^2*b
^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2
)))*1i)/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2)*1i)/(4*(2*a
*b^3 + b^4 + a^2*b^2)))/b^2)/((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a
*b^4 + b^5 + a^2*b^3)) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^
5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) + (sin(e + f*x)
*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 + (((4
*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e +
f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 +
 b^4 + a^2*b^2)))/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/
(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((3*a^3*b)/2 + a^4)/(2*a*b^4 + b^5 + a
^2*b^3)))*1i)/(b^2*f) - (atan(((((sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b
^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*((4*a^2*b^6 + 6*a
^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e + f*x)*(a*(a + b)^3)
^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*
(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*
b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a + b)^3)^(1/2)*(2*a + 3
*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)) + (((sin(e + f*x)*(20*a^4
*b + 8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) - ((a*(a + b)^3)^(1
/2)*((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) + (sin(e
 + f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b
^5 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 +
 a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a +
 b)^3)^(1/2)*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))/(((
3*a^3*b)/2 + a^4)/(2*a*b^4 + b^5 + a^2*b^3) - (((sin(e + f*x)*(20*a^4*b + 8
*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*((
4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e + f*x
)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 3
2*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a + b)^3)
^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)) + (((sin(e +
f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) - ((a*(
a + b)^3)^(1/2)*((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b
^3) + (sin(e + f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^
6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 +
 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^
2)))*(a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2))))*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*1i)/(2*f*(3*a*b^4 + b^5 + 3*a^2*b^3
+ a^3*b^2)) - (a*sin(e + f*x))/(2*b*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

3.194 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,f(a+b)^{3/2}}$$

```
[Out] 1/2*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)+1/2*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/(a+b)^(3/2)/f/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 199, 208}

$$\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*f) + S
in[e + f*x]/(2*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,(a+b)^{3/2}f} + \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A] time = 0.27, size = 88, normalized size = 1.19

$$\frac{\sqrt{a}\sqrt{a+b}\,\sin(e+fx) + \left(-a\sin^2(e+fx)+a+b\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f(a+b)^{3/2}(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b]*Sin[e + f*x] + ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]]*(a + b - a*Sin[e + f*x]^2))/(Sqrt[a]*(a + b)^(3/2)*f*(a + 2*b + a*Cos[
2*(e + f*x)]))
```

fricas [A] time = 1.34, size = 262, normalized size = 3.54

$$\left[\frac{\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\,\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{4\left(\left(a^4+2a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b+2a^2b^2+ab^3\right)f\right)}, -\frac{\left(a\cos\left(fx\right.\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*cos(f*x + e)^2 + b)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqr
t(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b
)*sin(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^
2*b^2 + a*b^3)*f), -1/2*((a*cos(f*x + e)^2 + b)*sqrt(-a^2 - a*b)*arctan(sqr
t(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (a^2 + a*b)*sin(f*x + e))/((a^4 + 2*a
^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f)]
```

giac [A] time = 0.30, size = 79, normalized size = 1.07

$$-\frac{\frac{\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,(a+b)} + \frac{\sin\left(fx+e\right)}{\left(a\sin\left(fx+e\right)^2-a-b\right)(a+b)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*(a + b)) +
sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a + b)))/f
```

maple [A] time = 0.64, size = 68, normalized size = 0.92

$$\frac{-\frac{\sin(fx+e)}{2(a+b)(-a-b+a(\sin^2(fx+e)))}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{2(a+b)\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/2*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/((a+b)*a)^(1/2)*
arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

maxima [A] time = 0.44, size = 98, normalized size = 1.32

$$-\frac{\frac{2\sin(fx+e)}{(a^2+ab)\sin(fx+e)^2-a^2-2ab-b^2}+\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(f*x + e)/((a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*b - b^2) + log
((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sq
rt((a + b)*a)*(a + b)))/f
```

mupad [B] time = 0.14, size = 62, normalized size = 0.84

$$\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin(e+fx)^2+a+b\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}f(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] sin(e + f*x)/(2*f*(a + b)*(a + b - a*sin(e + f*x)^2)) + atanh((a^(1/2)*sin(
e + f*x))/(a + b)^(1/2))/(2*a^(1/2)*f*(a + b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

3.195 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}f(a+b)^{3/2}}-\frac{b\sin(e+fx)}{2af(a+b)\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] 1/2*(2*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/f-1
/2*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 385, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}f(a+b)^{3/2}}-\frac{b\sin(e+fx)}{2af(a+b)\left(-a\sin^2(e+fx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*f) - (b*Sin[e + f*x])/(2*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+\ldots\right.}{2a(a+b)f}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A] time = 0.38, size = 82, normalized size = 0.99

$$\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{2\sqrt{a}\,b\sin(e+fx)}{(a+b)(a\cos(2(e+fx))+a+2b)}}{2a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (2
*Sqrt[a]*b*Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*a^(3/
2)*f)
```

fricas [A] time = 1.08, size = 301, normalized size = 3.63

$$\left[\frac{\left(\left(2a^2+ab\right)\cos\left(fx+e\right)^2 + 2ab + b^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2 - 2\sqrt{a^2+ab}\,\sin\left(fx+e\right) - 2a - b}{a\cos\left(fx+e\right)^2 + b}\right) - 2\left(a^2b+ab^2\right)\sin\ldots}{4\left(\left(a^5+2a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2 + \left(a^4b+2a^3b^2+a^2b^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^
2 + b)) - 2*(a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(
f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/2*(((2*a^2 + a*b)*cos(f*x
 + e)^2 + 2*a*b + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e
)/(a + b)) + (a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos
(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

giac [A] time = 0.66, size = 94, normalized size = 1.13

$$-\frac{\frac{(2a+b)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^2+ab\right)\sqrt{-a^2-ab}} - \frac{b\sin\left(fx+e\right)}{\left(a\sin\left(fx+e\right)^2-a-b\right)\left(a^2+ab\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + a*b)*sqrt(-
a^2 - a*b)) - b*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a^2 + a*b)))/f
```

maple [A] time = 0.84, size = 80, normalized size = 0.96

$$\frac{\frac{b\sin\left(fx+e\right)}{2(a+b)a\left(-a-b+a\left(\sin^{2}\left(fx+e\right)\right)\right)}+\frac{(2a+b)\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{2(a+b)a\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/2*b/(a+b)/a*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)+1/2*(2*a+b)/(a+b)/a/((a
+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

maxima [A] time = 0.43, size = 111, normalized size = 1.34

$$-\frac{\frac{2\,b\sin\left(fx+e\right)}{a^{3}+2\,a^{2}b+ab^{2}-\left(a^{3}+a^{2}b\right)\sin\left(fx+e\right)^{2}}+\frac{(2\,a+b)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\left(a^{2}+ab\right)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*sin(f*x + e)/(a^3 + 2*a^2*b + a*b^2 - (a^3 + a^2*b)*sin(f*x + e)^
2) + (2*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqr
t((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)))/f
```

mupad [B] time = 4.43, size = 71, normalized size = 0.86

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)\,\left(2\,a+b\right)}{2\,a^{3/2}\,f\,{\left(a+b\right)}^{3/2}}-\frac{b\,\sin\left(e+f\,x\right)}{2\,a\,f\,\left(a+b\right)\,\left(-a\,{\sin\left(e+f\,x\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*f*(a + b
)^(3/2)) - (b*sin(e + f*x))/(2*a*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}f(a+b)^{3/2}}+\frac{b^2\sin(e+fx)}{2a^2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{a^2f}$$

```
[Out] -1/2*b*(4*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/2
)/f+sin(f*x+e)/a^2/f+1/2*b^2*sin(f*x+e)/a^2/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 390, 385, 208}

$$\frac{b^2\sin(e+fx)}{2a^2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}f(a+b)^{3/2}}+\frac{\sin(e+fx)}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(5/2)*(a
+ b)^(3/2)*f) + Sin[e + f*x]/(a^2*f) + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*f*
(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2} - \frac{b(2a+b)-2abx^2}{a^2\left(a+b-ax^2\right)^2}\right)dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{a^2 f} - \frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{a^2 f}$$

$$= \frac{\sin(e+fx)}{a^2 f} + \frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)} - \frac{(b(4a+3b))\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\right.}{2a^2(a+b)}$$

$$= -\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{a^2 f} + \frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+\right.}$$

Mathematica [C] time = 3.01, size = 945, normalized size = 9.36

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^3(e+fx)\left(8\sqrt{a}\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan(e+fx)b^2 - 2i(4a+3b)\tan^{-1}\left(\frac{2\sin}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*b*(4*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - b*(4*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + b*(4*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) + 8*Sqrt[a]*(a + b)^(3/2)*Cos[f*x]*(a + 2*b +
 a*Cos[2*(e + f*x)])*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[e] + 2*b*
(4*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b
+ a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) + 8*Sqrt[a]*(a + b)^
(3/2)*Cos[e]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 8*Sqrt[a]*b^2*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*
Tan[e + f*x]))/(32*a^(5/2)*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(C
os[e] - I*Sin[e])^2])
```

fricas [A] time = 0.69, size = 391, normalized size = 3.87

$$\left[\frac{\left(4\,ab^2+3\,b^3+\left(4\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,\sqrt{a^2+ab}\,\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(2\,a^3b+\ldots\right.}{4\left(\left(a^6+2\,a^5b+a^4b^2\right)f\cos\left(fx+e\right)^2+\left(a^5b+2\,a^4b^2\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((4*a*b^2 + 3*b^3 + (4*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(a^2 + a*b
)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos
(f*x + e)^2 + b)) + 2*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x
+ e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f), 1/2*((4*a*b^2 + 3*b^3 + (4*a^2*b
 + 3*a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*
x + e)/(a + b)) + (2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a^2*b
^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x + e)
^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f)]
```

giac [A] time = 0.33, size = 117, normalized size = 1.16

$$-\frac{\frac{b^2\sin\left(fx+e\right)}{\left(a^3+a^2b\right)\left(a\sin\left(fx+e\right)^2-a-b\right)}-\frac{\left(4\,ab+3\,b^2\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^3+a^2b\right)\sqrt{-a^2-ab}}-\frac{2\,\sin\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(f*x + e)/((a^3 + a^2*b)*(a*sin(f*x + e)^2 - a - b)) - (4*a*b
+ 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + a^2*b)*sqrt(-a^2 -
 a*b)) - 2*sin(f*x + e)/a^2)/f
```

maple [A] time = 1.16, size = 92, normalized size = 0.91

$$\frac{\frac{\sin\left(fx+e\right)}{a^2}+\frac{b\left(-\frac{b\sin\left(fx+e\right)}{2(a+b)\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)}-\frac{(4a+3b)\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{2(a+b)\sqrt{(a+b)a}}\right)}{a^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/a^2*sin(f*x+e)+b/a^2*(-1/2*b/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-
1/2*(4*a+3*b)/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

maxima [A] time = 0.43, size = 133, normalized size = 1.32

$$\frac{\frac{2\,b^2\sin\left(fx+e\right)}{a^4+2\,a^3b+a^2b^2-\left(a^4+a^3b\right)\sin\left(fx+e\right)^2}+\frac{\left(4\,ab+3\,b^2\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^3+a^2b\right)\sqrt{(a+b)a}}+\frac{4\,\sin\left(fx+e\right)}{a^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*sin(f*x + e)/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x +
e)^2) + (4*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x +
 e) + sqrt((a + b)*a)))/((a^3 + a^2*b)*sqrt((a + b)*a)) + 4*sin(f*x + e)/a^
2)/f
```

mupad [B] time = 0.18, size = 94, normalized size = 0.93

$$\frac{\sin\left(e+f\,x\right)}{a^2\,f}+\frac{b^2\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^3\,\sin\left(e+f\,x\right)^2+a^3+b\,a^2\right)}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)\,\left(4\,a+3\,b\right)}{2\,a^{5/2}\,f\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] sin(e + f*x)/(a^2*f) + (b^2*sin(e + f*x))/(2*f*(a + b)*(a^2*b + a^3 - a^3*s
in(e + f*x)^2)) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + 3*b
))/(2*a^(5/2)*f*(a + b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=126

$$\frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}f(a+b)^{3/2}}-\frac{b^3\sin(e+fx)}{2a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3/
2)/f+(a-2*b)*sin(f*x+e)/a^3/f-1/3*sin(f*x+e)^3/a^2/f-1/2*b^3*sin(f*x+e)/a^3
/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4147, 390, 385, 208}

$$-\frac{b^3\sin(e+fx)}{2a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}f(a+b)^{3/2}}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*f) + ((a - 2*b)*Sin[e + f*x])/(a^3*f) - Sin[e + f*x]^3/(3*a^2*f
) - (b^3*Sin[e + f*x])/(2*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{a^3} - \frac{x^2}{a^2} + \frac{b^2(3a+2b)-3ab^2x^2}{a^3\left(a+b-ax^2\right)^2}\right) dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a-2b)\sin(e+fx)}{a^3 f} - \frac{\sin^3(e+fx)}{3a^2 f} + \frac{\text{Subst}\left(\int \frac{b^2(3a+2b)-3ab^2x^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{a^3 f}$$

$$= \frac{(a-2b)\sin(e+fx)}{a^3 f} - \frac{\sin^3(e+fx)}{3a^2 f} - \frac{b^3\sin(e+fx)}{2a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\left(b^2(6\right.}{}$$

$$= \frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2} f} + \frac{(a-2b)\sin(e+fx)}{a^3 f} - \frac{\sin^3(e+fx)}{3a^2 f} - \frac{}{2a^3(a+}$$

Mathematica [A] time = 1.11, size = 139, normalized size = 1.10

$$\frac{a^{3/2}\sin(3(e+fx)) + 3\sqrt{a}\sin(e+fx)\left(-\frac{4b^3}{(a+b)(a\cos(2(e+fx))+a+2b)} + 3a - 8b\right) - \frac{3b^2(6a+5b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a}\right.\right.}{(a+b)^{3/2}}}{12a^{7/2} f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-3*b^2*(6*a + 5*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^
3)/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x] + a^(3/2)*Sin[3*(
e + f*x)])/(12*a^(7/2)*f)
```

fricas [B] time = 0.74, size = 490, normalized size = 3.89

$$\left[\frac{3\left(6ab^3 + 5b^4 + \left(6a^2b^2 + 5ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2 - 2\sqrt{a^2+ab}\sin\left(fx+e\right) - 2a - b}{a\cos\left(fx+e\right)^2 + b}\right) + 2\left(4a^4b - 4\right.}{12\left(\left(a^7 + 2a^6b + a^5b^2\right)f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 + 5*b^4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a^2
+ a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(
a*cos(f*x + e)^2 + b)) + 2*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2
*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 -
5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f), -1/6*(3*(6*a*b^3 + 5*b^4 + (6
*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b
)*sin(f*x + e)/(a + b)) - (4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2*
(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 - 5
*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f)]
```

giac [A] time = 0.29, size = 152, normalized size = 1.21

$$\frac{\frac{3\,b^3\sin\left(fx+e\right)}{\left(a^4+a^3b\right)\left(a\sin\left(fx+e\right)^2-a-b\right)}-\frac{3\left(6\,ab^2+5\,b^3\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^4+a^3b\right)\sqrt{-a^2-ab}}-\frac{2\left(a^4\sin\left(fx+e\right)^3-3\,a^4\sin\left(fx+e\right)+6\,a^3b\sin\left(fx+e\right)\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(f*x + e)/((a^4 + a^3*b)*(a*sin(f*x + e)^2 - a - b)) - 3*(6*a
*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4 + a^3*b)*sqrt(-
a^2 - a*b)) - 2*(a^4*sin(f*x + e)^3 - 3*a^4*sin(f*x + e) + 6*a^3*b*sin(f*x
+ e))/a^6)/f
```

maple [A] time = 1.40, size = 120, normalized size = 0.95

$$-\frac{\frac{\frac{a\left(\sin^3\left(fx+e\right)\right)}{3}-a\sin\left(fx+e\right)+2b\sin\left(fx+e\right)}{a^3}-\frac{b^2\left(-\frac{b\sin\left(fx+e\right)}{2(a+b)\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)}-\frac{(6a+5b)\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{2(a+b)\sqrt{(a+b)a}}\right)}{a^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/a^3*(1/3*a*sin(f*x+e)^3-a*sin(f*x+e)+2*b*sin(f*x+e))-b^2/a^3*(-1/2*
b/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(6*a+5*b)/(a+b)/((a+b)*a)^(1/2
)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

maxima [A] time = 0.44, size = 154, normalized size = 1.22

$$-\frac{\frac{6\,b^3\sin\left(fx+e\right)}{a^5+2\,a^4b+a^3b^2-\left(a^5+a^4b\right)\sin\left(fx+e\right)^2}+\frac{3\left(6\,ab^2+5\,b^3\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^4+a^3b\right)\sqrt{(a+b)a}}+\frac{4\left(a\sin\left(fx+e\right)^3-3\,(a-2\,b)\sin\left(fx+e\right)\right)}{a^3}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^3*sin(f*x + e)/(a^5 + 2*a^4*b + a^3*b^2 - (a^5 + a^4*b)*sin(f*x
+ e)^2) + 3*(6*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin
(f*x + e) + sqrt((a + b)*a)))/((a^4 + a^3*b)*sqrt((a + b)*a)) + 4*(a*sin(f*
x + e)^3 - 3*(a - 2*b)*sin(f*x + e))/a^3)/f
```

mupad [B] time = 4.62, size = 124, normalized size = 0.98

$$\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)\,\left(6\,a+5\,b\right)}{2\,a^{7/2}\,f\,{\left(a+b\right)}^{3/2}}-\frac{{\sin\left(e+f\,x\right)}^3}{3\,a^2\,f}-\frac{b^3\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^4\,{\sin\left(e+f\,x\right)}^2+a^4+b\,a^3\right)}-\frac{\sin\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(6*a + 5*b))/(2*a^(7/2)*f*
(a + b)^(3/2)) - sin(e + f*x)^3/(3*a^2*f) - (b^3*sin(e + f*x))/(2*f*(a + b)
```

```
*(a^3*b + a^4 - a^4*sin(e + f*x)^2)) - (sin(e + f*x)*((2*(a + b))/a^3 - 3/a
^2))/f
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.198 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=157

$$-\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}f(a+b)^{3/2}}+\frac{b^4\sin(e+fx)}{2a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{2(a-b)\sin^3(e+fx)}{3a^3f}+\frac{\sin^5(e+fx)}{5a^2f}$$

```
[Out] -1/2*b^3*(8*a+7*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9/2)/(a+b)^(3
/2)/f+(a^2-2*a*b+3*b^2)*sin(f*x+e)/a^4/f-2/3*(a-b)*sin(f*x+e)^3/a^3/f+1/5*s
in(f*x+e)^5/a^2/f+1/2*b^4*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 208}

$$\frac{b^4\sin(e+fx)}{2a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f}-\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}f(a+b)^{3/2}}-\frac{2(a-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(9/2)*(
a + b)^(3/2)*f) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/(a^4*f) - (2*(a - b)
*Sin[e + f*x]^3)/(3*a^3*f) + Sin[e + f*x]^5/(5*a^2*f) + (b^4*Sin[e + f*x])/
(2*a^4*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-2ab+3b^2}{a^4} - \frac{2(a-b)x^2}{a^3} + \frac{x^4}{a^2} - \frac{b^3(4a+3b)-4ab^3x^2}{a^4\left(a+b-ax^2\right)^2}\right)dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} - \frac{\text{Subst}\left(\int \frac{b^3}{}\right.}{}$$

$$= \frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{b}{2a^4(a+b)f}$$

$$= -\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(}{3a^3f}$$

Mathematica [A] time = 2.03, size = 171, normalized size = 1.09

$$\frac{5a^{3/2}(5a-8b)\sin(3(e+fx)) + 3a^{5/2}\sin(5(e+fx)) + 30\sqrt{a}\sin(e+fx)\left(5a^2+8b^2\left(\frac{b^2}{(a+b)(a\cos(2(e+fx))+a+2b)}+3\right)-\right.}{240a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((60*b^3*(8*a + 7*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 30*Sqrt[a]*(5*a^2 - 12*a*b +
 8*b^2*(3 + b^2/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))*Sin[e + f*x] + 5
*a^(3/2)*(5*a - 8*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)])/(240*a^
(9/2)*f)
```

fricas [A] time = 1.97, size = 583, normalized size = 3.71

$$\left[\frac{15\left(8\,ab^4+7\,b^5+\left(8\,a^2b^3+7\,ab^4\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{a^2+ab}\,\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(6\left(a^6+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(8*a*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(a^2
 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/
(a*cos(f*x + e)^2 + b)) + 2*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^6 + 1
6*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a^6 + a^5
*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2 + 54*a^
3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2
)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f), 1/30*(15*(8*a*b^4 +
7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x +
e)^6 + 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a
```

```
^6 + a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2
 + 54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b +
 a^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f)]
```

giac [A] time = 0.27, size = 197, normalized size = 1.25

$$-\frac{\frac{15\,b^4 \sin\left(fx+e\right)}{\left(a^5+a^4 b\right)\left(a \sin\left(fx+e\right)^2-a-b\right)}-\frac{15\left(8\,ab^3+7\,b^4\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^5+a^4 b\right)\sqrt{-a^2-ab}}-\frac{2\left(3\,a^8 \sin\left(fx+e\right)^5-10\,a^8 \sin\left(fx+e\right)^3+10\,a^7 b \sin\left(fx+e\right)^3+15\,a^8 \sin\left(fx+\right.\right.}{a^{10}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*sin(f*x + e)/((a^5 + a^4*b)*(a*sin(f*x + e)^2 - a - b)) - 15*
(8*a*b^3 + 7*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^5 + a^4*b)*sq
rt(-a^2 - a*b)) - 2*(3*a^8*sin(f*x + e)^5 - 10*a^8*sin(f*x + e)^3 + 10*a^7*
b*sin(f*x + e)^3 + 15*a^8*sin(f*x + e) - 30*a^7*b*sin(f*x + e) + 45*a^6*b^2
*sin(f*x + e))/a^10)/f
```

maple [A] time = 1.85, size = 158, normalized size = 1.01

$$\frac{\frac{\frac{\left(\sin^5\left(fx+e\right)\right)a^2}{5}-\frac{2\left(\sin^3\left(fx+e\right)\right)a^2}{3}+\frac{2\left(\sin^3\left(fx+e\right)\right)ab}{3}+a^2\sin\left(fx+e\right)-2\sin\left(fx+e\right)ab+3b^2\sin\left(fx+e\right)}{a^4}+\frac{b^3\left(-\frac{b\sin\left(fx+e\right)}{2(a+b)\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)}-\frac{(8a+7b)\operatorname{arctanh}\left(\frac{a\,\mathrm{s}}{\right.}}{2(a+b)\sqrt{(a+}}\right.}{a^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/a^4*(1/5*sin(f*x+e)^5*a^2-2/3*sin(f*x+e)^3*a^2+2/3*sin(f*x+e)^3*a*b+
a^2*sin(f*x+e)-2*sin(f*x+e)*a*b+3*b^2*sin(f*x+e))+b^3/a^4*(-1/2*b/(a+b)*sin
(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(8*a+7*b)/(a+b)/((a+b)*a)^(1/2)*arctanh(a
*sin(f*x+e)/((a+b)*a)^(1/2))))
```

maxima [A] time = 0.45, size = 183, normalized size = 1.17

$$\frac{\frac{30\,b^4\sin\left(fx+e\right)}{a^6+2\,a^5b+a^4b^2-\left(a^6+a^5b\right)\sin\left(fx+e\right)^2}+\frac{15\left(8\,ab^3+7\,b^4\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^5+a^4b\right)\sqrt{(a+b)a}}+\frac{4\left(3\,a^2\sin\left(fx+e\right)^5-10\left(a^2-ab\right)\sin\left(fx+e\right)^3+15\left(a^2-2\,ab+3\,b\right.\right.}{a^4}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(30*b^4*sin(f*x + e)/(a^6 + 2*a^5*b + a^4*b^2 - (a^6 + a^5*b)*sin(f*x
+ e)^2) + 15*(8*a*b^3 + 7*b^4)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*si
n(f*x + e) + sqrt((a + b)*a)))/((a^5 + a^4*b)*sqrt((a + b)*a)) + 4*(3*a^2*s
in(f*x + e)^5 - 10*(a^2 - a*b)*sin(f*x + e)^3 + 15*(a^2 - 2*a*b + 3*b^2)*si
n(f*x + e))/a^4)/f
```

mupad [B] time = 0.18, size = 173, normalized size = 1.10

$$\frac{\sin\left(e+f\,x\right)^5}{5\,a^2\,f}+\frac{\sin\left(e+f\,x\right)^3\,\left(\frac{2\,(a+b)}{3\,a^3}-\frac{4}{3\,a^2}\right)}{f}+\frac{\sin\left(e+f\,x\right)\,\left(\frac{6}{a^2}-\frac{(a+b)^2}{a^4}+\frac{2\,(a+b)\,\left(\frac{2\,(a+b)}{a^3}-\frac{4}{a^2}\right)}{a}\right)}{f}+\frac{b^4\,\mathrm{s}}{2\,f\,(a+b)\,\left(-a^5\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] sin(e + f*x)^5/(5*a^2*f) + (sin(e + f*x)^3*((2*(a + b))/(3*a^3) - 4/(3*a^2)
))/f + (sin(e + f*x)*(6/a^2 - (a + b)^2/a^4 + (2*(a + b)*((2*(a + b))/a^3 -
 4/a^2))/a))/f + (b^4*sin(e + f*x))/(2*f*(a + b)*(a^4*b + a^5 - a^5*sin(e +
 f*x)^2)) - (b^3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(8*a + 7*b))/(
2*a^(9/2)*f*(a + b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.199 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{a^2\tan(e+fx)}{2b^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2f}$$

```
[Out] -1/2*a*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2)
/f+tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 205}

$$\frac{a^2\tan(e+fx)}{2b^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*f) + Tan[e + f*x]/(b^2*f) + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a(a+2b)+2abx^2}{b^2\left(a+b+bx^2\right)^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{b^2f} - \frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{b^2f}$$

$$= \frac{\tan(e+fx)}{b^2f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(a(3a+4b))\,\text{Subst}\left(\int \frac{1}{a+b+bx}\right.}{2b^2(a+b)}$$

$$= -\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{b^2f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+f\right.}$$

Mathematica [C] time = 2.37, size = 248, normalized size = 2.48

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{a(a\sin(2fx)-(a+2b)\sin(2e))}{(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + 2\sec(e)\sin(fx)\sec(e+fx)(a\cos(2(e+f\right.}{8b^2f\left(a+b\sec^2(e+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((a*(3*a + 4*b)*ArcTan[(Sec[
f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])
*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2
*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x] + (a*(-((a + 2
*b)*Sin[2*e]) + a*Sin[2*f*x]))/((a + b)*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(8*b^2*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.70, size = 516, normalized size = 5.16

$$\left[-\frac{\left(\left(3a^3+4a^2b\right)\cos\left(fx+e\right)^3 + \left(3a^2b+4ab^2\right)\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\,\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4 - 2\left(3ab+4b^2\right)\cos}{a^2\cos}\right.}{8\left(\left(a^3b^3+2a^2b^4+ab^5\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e)
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - 4*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*c
```

```
os(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^
3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e)), 1/4*(((3*a^3 + 4*a^2*b)*cos(
f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e))*sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))
+ 2*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cos(f*x
+ e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^3 + (a^
2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e))]
```

giac [A] time = 1.42, size = 125, normalized size = 1.25

$$\frac{\frac{a^2\tan\left(fx+e\right)}{\left(ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,a^2+4\,ab\right)}{\left(ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{2\,\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)) - (pi*floo
r((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a
^2 + 4*a*b)/((a*b^2 + b^3)*sqrt(a*b + b^2)) + 2*tan(f*x + e)/b^2)/f
```

maple [A] time = 0.70, size = 128, normalized size = 1.28

$$\frac{\tan\left(fx+e\right)}{b^2f}+\frac{a^2\tan\left(fx+e\right)}{2b^2\left(a+b\right)f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3a^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2f\,b^2\left(a+b\right)\sqrt{\left(a+b\right)b}}-\frac{2a\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{fb\left(a+b\right)\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)-3/2/f*
a^2/b^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*a/b/
(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.45, size = 110, normalized size = 1.10

$$\frac{\frac{a^2\tan\left(fx+e\right)}{a^2b^2+2\,ab^3+b^4+\left(ab^3+b^4\right)\tan\left(fx+e\right)^2}-\frac{\left(3\,a^2+4\,ab\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{2\,\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/(a^2*b^2 + 2*a*b^3 + b^4 + (a*b^3 + b^4)*tan(f*x + e)
^2) - (3*a^2 + 4*a*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a*b^2 + b^3)
*sqrt((a + b)*b)) + 2*tan(f*x + e)/b^2)/f
```

mupad [B] time = 4.79, size = 113, normalized size = 1.13

$$\frac{\tan\left(e+f\,x\right)}{b^2\,f}+\frac{a^2\,\tan\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(b^3\,{\tan\left(e+f\,x\right)}^2+b^3+a\,b^2\right)}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\tan\left(e+f\,x\right)\,\left(3\,a+4\,b\right)}{\sqrt{a+b}\,\left(3\,a^2+4\,b\,a\right)}\right)\,\left(3\,a+4\,b\right)}{2\,b^{5/2}\,f\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*f*(a + b)*(a*b^2 + b^3 + b^3*t
an(e + f*x)^2)) - (a*atan((a*b^(1/2)*tan(e + f*x)*(3*a + 4*b))/((a + b)^(1/
2)*(4*a*b + 3*a^2)))*(3*a + 4*b))/(2*b^(5/2)*f*(a + b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

3.200 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}f(a+b)^{3/2}}-\frac{a\tan(e+fx)}{2bf(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] 1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/f-1/
2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 385, 205}

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}f(a+b)^{3/2}}-\frac{a\tan(e+fx)}{2bf(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(
3/2)*f) - (a*Tan[e + f*x])/(2*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx\right.}{2b(a+b)f}$$

$$= \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.28, size = 84, normalized size = 1.02

$$\frac{\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a\cos(2(e+fx))+a+2b)}}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*
Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*b^(3
/2)*f)
```

fricas [B] time = 0.77, size = 406, normalized size = 4.95

$$\left[-\frac{4\left(a^2b+ab^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right) + \left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2 + ab + 2\,b^2\right)\sqrt{-ab-b^2}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos}{\ }\right.}{8\left(\left(a^3b^2+2\,a^2b^3+ab^4\right)f\cos\left(fx+e\right)^2 + \left(a^2b^3+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x
 + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b
*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^
2 + (a^2*b^3 + 2*a*b^4 + b^5)*f), -1/4*(2*(a^2*b + a*b^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arc
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^2*b^3 + 2*a*
b^4 + b^5)*f)]
```

giac [A] time = 0.27, size = 93, normalized size = 1.13

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)(a+2\,b)}{\left(ab+b^2\right)^{\frac{3}{2}}} - \frac{a\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)\left(ab+b^2\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(a*b + b^2)^(3/2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 +
a + b)*(a*b + b^2)))/f
```

maple [A] time = 0.69, size = 106, normalized size = 1.29

$$-\frac{a\tan\left(fx+e\right)}{2b\left(a+b\right)f\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)}+\frac{a\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2fb\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\left(a+b\right)\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)+1/2/f*a/b/(a+b)/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/(a+b)/((a+b)*b)^(1/2)*arctan
(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.46, size = 88, normalized size = 1.07

$$-\frac{\frac{a\tan\left(fx+e\right)}{a^{2}b+2\,ab^{2}+b^{3}+\left(ab^{2}+b^{3}\right)\tan\left(fx+e\right)^{2}}-\frac{\left(a+2\,b\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\sqrt{\left(a+b\right)b}\left(ab+b^{2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*tan(f*x + e)/(a^2*b + 2*a*b^2 + b^3 + (a*b^2 + b^3)*tan(f*x + e)^2)
 - (a + 2*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a*b +
 b^2)))/f
```

mupad [B] time = 4.41, size = 70, normalized size = 0.85

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a+b}}\right)\,\left(a+2\,b\right)}{2\,b^{3/2}\,f\,{\left(a+b\right)}^{3/2}}-\frac{a\,\tan\left(e+f\,x\right)}{2\,b\,f\,\left(a+b\right)\,\left(b\,{\tan\left(e+f\,x\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*f*(a + b)
^(3/2)) - (a*tan(e + f*x))/(2*b*f*(a + b)*(a + b + b*tan(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.201 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,f(a+b)^{3/2}}+\frac{\tan(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] 1/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(3/2)/f/b^(1/2)+1/2*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,f(a+b)^{3/2}}+\frac{\tan(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*f) + Ta
n[e + f*x]/(2*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}f} + \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 0.89, size = 211, normalized size = 2.89

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{a\sin(2fx)-(a+2b)\sin(2e)}{a(\cos(e)-\sin(e))(\sin(e)+\cos(e))} - \frac{(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\frac{(\cos(2e)-}{}\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin}}\right.}{8f(a+b)\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-((ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]
*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] -
 I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) + (-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x])/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a +
b)*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.57, size = 368, normalized size = 5.04

$$\left[\frac{4\left(ab+b^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right) - \left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)c}{a^2}\right.}{8\left(\left(a^3b+2a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(a^2b^2+2ab^3+b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*sqrt
(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b -
b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))
)/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^4)
*f), 1/4*(2*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*
sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*
cos(f*x + e)*sin(f*x + e))))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2
+ (a^2*b^2 + 2*a*b^3 + b^4)*f)]
```

giac [A] time = 0.67, size = 87, normalized size = 1.19

$$\frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}\,(a+b)} + \frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)(a+b)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))/(sqrt(a*b + b^2)*(a + b)) + tan(f*x + e)/((b*tan(f*x + e)^2 + a +
b)*(a + b)))/f
```

maple [A] time = 0.57, size = 66, normalized size = 0.90

$$\frac{\tan\left(fx+e\right)}{2\left(a+b\right)f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2f\left(a+b\right)\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)+1/2/f/(a+b)/((a+b)*b)^(1/2)*arc
tan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.43, size = 71, normalized size = 0.97

$$\frac{\frac{\tan\left(fx+e\right)}{\left(ab+b^2\right)\tan\left(fx+e\right)^2+a^2+2\,ab+b^2}+\frac{\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\sqrt{\left(a+b\right)b}\,\left(a+b\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)/((a*b + b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2) + arctan
(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)))/f
```

mupad [B] time = 4.47, size = 69, normalized size = 0.95

$$\frac{\tan\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(b\,\tan\left(e+f\,x\right)^2+a+b\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\,\left(2\,a+2\,b\right)}{2\,{\left(a+b\right)}^{3/2}}\right)}{2\,\sqrt{b}\,f\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] tan(e + f*x)/(2*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + atan((b^(1/2)*tan(e
 + f*x)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(2*b^(1/2)*f*(a + b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.202 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} - \frac{(b(3a \cdots}{\cdots}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 1.91, size = 240, normalized size = 2.61

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(2x(a\cos(2(e+fx))+a+2b)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{f(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b(3a+2b)(c\cdots}{\cdots}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

fricas [B] time = 0.77, size = 435, normalized size = 4.73

$$\left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2-4ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\left(ab+b^2\right)fx+\left(\left(3a^2+2ab\right)\cos\left(fx+e\right)^2+3a\cdots\right.}{8\left(\left(a^4+a^3b\right)f\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
```

```
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

giac [A] time = 0.97, size = 119, normalized size = 1.29

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab+2\,b^2\right)}{\left(a^3+a^2b\right)\sqrt{ab+b^2}}+\frac{b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)\left(a^2+ab\right)}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

maple [A] time = 0.87, size = 127, normalized size = 1.38

$$-\frac{b\tan\left(fx+e\right)}{2a\left(a+b\right)f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2fa\left(a+b\right)\sqrt{\left(a+b\right)b}}-\frac{b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^2\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\tan\left(fx+\right.\right.}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a/(a+b)/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^2/a^2/(a+b)/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 106, normalized size = 1.15

$$-\frac{\frac{b\tan\left(fx+e\right)}{a^3+2\,a^2b+ab^2+\left(a^2b+ab^2\right)\tan\left(fx+e\right)^2}+\frac{\left(3\,ab+2\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\left(a^3+a^2b\right)\sqrt{\left(a+b\right)b}}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

mupad [B] time = 6.67, size = 2056, normalized size = 22.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
```

```
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

3.203 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f(a+b)^{3/2}}+\frac{x(a-4b)}{2a^3}+\frac{b(a+2b)\tan(e+fx)}{2a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sin(e+fx)\cos(e+fx}{2af\left(a+b\tan^2(e+fx)+\right.}$$

```
[Out] 1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))/a^3/(a+b)^(3/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1/
2*b*(a+2*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f(a+b)^{3/2}}+\frac{b(a+2b)\tan(e+fx)}{2a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(a-4b)}{2a^3}+\frac{\sin(e+fx)\cos(e+fx}{2af\left(a+b\tan^2(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*
(a + b + b*Tan[e + f*x]^2)) + (b*(a + 2*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b-3bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-2}{}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-4b)\,\text{Su}}{}$$

$$= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{}{2a}$$

Mathematica [A] time = 1.33, size = 103, normalized size = 0.73

$$\frac{\frac{2b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \sin(2(e+fx))\left(\frac{2ab^2}{(a+b)(a\cos(2(e+fx))+a+2b)}+a\right) + 2(a-4b)(e+fx)}{4a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(e + f*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])))*Sin[2*(e + f*x)])/(4*a^3*f)
```

fricas [A] time = 0.79, size = 544, normalized size = 3.83

$$\left[\frac{4\left(a^3-3a^2b-4ab^2\right)fx\cos\left(fx+e\right)^2 + 4\left(a^2b-3ab^2-4b^3\right)fx + \left(5ab^2+4b^3+\left(5a^2b+4ab^2\right)\cos\left(fx+e\right)^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 4*(a^2*b - 3*a*b^2 -
 4*b^3)*f*x + (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)) + 4*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2)*
cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a^3*
b^2)*f), 1/4*(2*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 2*(a^2*b - 3
*a*b^2 - 4*b^3)*f*x - (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2
)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))
/(b*cos(f*x + e)*sin(f*x + e))) + 2*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b
+ 2*a*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a
^4*b + a^3*b^2)*f)]
```

giac [A] time = 0.25, size = 203, normalized size = 1.43

$$\frac{\frac{\left(5\,ab^{2}+4\,b^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{4}+a^{3}b\right)\sqrt{ab+b^{2}}}+\frac{ab\tan\left(fx+e\right)^{3}+2\,b^{2}\tan\left(fx+e\right)^{3}+a^{2}\tan\left(fx+e\right)+2\,ab\tan\left(fx+e\right)+2\,b^{2}\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{4}+a\tan\left(fx+e\right)^{2}+2\,b\tan\left(fx+e\right)^{2}+a+b\right)\left(a^{3}+a^{2}b\right)}+\frac{(fx}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 4*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (a*b*tan(f*x +
 e)^3 + 2*b^2*tan(f*x + e)^3 + a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + 2*b^
2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2
+ a + b)*(a^3 + a^2*b)) + (f*x + e)*(a - 4*b)/a^3)/f
```

maple [A] time = 1.24, size = 174, normalized size = 1.23

$$\frac{b^{2}\tan\left(fx+e\right)}{2f\,a^{2}\left(a+b\right)\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)}+\frac{5b^{2}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2f\,a^{2}\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{2b^{3}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^{3}\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{\tan\left(fx+e\right.}{2f\,a^{2}\left(1+\tan^{2}\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^2/a^2/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+5/2/f*b^2/a^2/(a+b)/((a
+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+2/f*b^3/a^3/(a+b)/((a+b)*
b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a^2*tan(f*x+e)/(tan(f*x
+e)^2+1)+1/2/f/a^2*arctan(tan(f*x+e))-2/f/a^3*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.45, size = 175, normalized size = 1.23

$$\frac{\frac{\left(5\,ab^{2}+4\,b^{3}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^{4}+a^{3}b\right)\sqrt{(a+b)b}}+\frac{\left(ab+2\,b^{2}\right)\tan\left(fx+e\right)^{3}+\left(a^{2}+2\,ab+2\,b^{2}\right)\tan\left(fx+e\right)}{\left(a^{3}b+a^{2}b^{2}\right)\tan\left(fx+e\right)^{4}+a^{4}+2\,a^{3}b+a^{2}b^{2}+\left(a^{4}+3\,a^{3}b+2\,a^{2}b^{2}\right)\tan\left(fx+e\right)^{2}}+\frac{\left(fx+e\right)(a-4\,b)}{a^{3}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 + 4*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + a^3*b
)*sqrt((a + b)*b)) + ((a*b + 2*b^2)*tan(f*x + e)^3 + (a^2 + 2*a*b + 2*b^2)*
tan(f*x + e))/((a^3*b + a^2*b^2)*tan(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 +
 (a^4 + 3*a^3*b + 2*a^2*b^2)*tan(f*x + e)^2) + (f*x + e)*(a - 4*b)/a^3)/f
```

mupad [B] time = 7.69, size = 2401, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) + (b*tan(e + f*x)^3*(
a + 2*b))/(2*a^2*(a + b)))/(f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*(a
 + 2*b))) - (atan(((((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*
b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 +
2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i - b*
4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b +
a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3) + (((tan
(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5
*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2
*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*
b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i -
b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)
/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - (((tan(e + f*x)*(64*a*b^6 +
 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2))
- (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^
2) - (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16
*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i
 - b*4i))/(4*a^3) + (((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3
*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 +
 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b
*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b +
 a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i))/(4*a^3)))*(a*1i -
b*4i)*1i)/(2*a^3*f) - (atan(((((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e
+ f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b
+ a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^
7*b + a^8 + a^6*b^2) + (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(
32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4
*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3
)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^
3 + 3*a^4*b^2) + ((((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a
*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*
b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 +
a^6*b^2) - (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 +
 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5
*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*
a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^
2))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^
6*b^2) + ((((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 3
2*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) +
(((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2)
 + (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*
b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6
 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b +
a^6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) - ((((5*a
)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*
b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 +
8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x
)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^
```

```
3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*
a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 +
 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(
a + b)^3)^(1/2)*2i)/(f*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.204 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f(a+b)^{3/2}}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3(a-2b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(\cdots}{\cdots}$$

```
[Out] 1/8*(3*a^2-8*a*b+24*b^2)*x/a^4-1/2*b^(5/2)*(7*a+6*b)*arctan(b^(1/2)*tan(f*x
+e)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/f+3/8*(a-2*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/
(a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1
/8*(a-3*b)*b*(3*a+4*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.29, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4146, 414, 527, 522, 203, 205}

$$-\frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f(a+b)^{3/2}}+\frac{x\left(3a^2-8ab+24b^2\right)}{8a^4}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3(a-2b)\sin(e\cdots}{8a^2f\left(a+b\,\mathrm{ta}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 8*a*b + 24*b^2)*x)/(8*a^4) - (b^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*(a + b)^(3/2)*f) + (3*(a - 2*b)*Cos[e +
 f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*
Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) + ((a - 3*b)*b*(3*a + 4*b)
*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-3a+b-5bx^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3a}{}\right.}{}$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b}{8a^3(a+b)f}$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b}{8a^3(a+b)f}$$

$$= \frac{\left(3a^2-8ab+24b^2\right)x}{8a^4} - \frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} + \frac{3(a-2b)\cos(e+f}{8a^2f\left(a+b+b\tan\right.}$$

Mathematica [A] time = 1.62, size = 138, normalized size = 0.68

$$\frac{4\left(3a^2-8ab+24b^2\right)(e+fx)+a^2\sin(4(e+fx)) - \frac{16b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{16ab^3\sin(2(e+fx))}{(a+b)(a\cos(2(e+fx))+a+2b)} + 8a(a}{32a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 8*a*b + 24*b^2)*(e + f*x) - (16*b^(5/2)*(7*a + 6*b)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) + 8*a*(a - 2*b)*Sin[2*(e + f
*x)] - (16*a*b^3*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))
 + a^2*Sin[4*(e + f*x)])/(32*a^4*f)
```

fricas [A] time = 0.77, size = 656, normalized size = 3.23

$$\left[\frac{\left(3\,a^4-5\,a^3b+16\,a^2b^2+24\,ab^3\right)fx\cos\left(fx+e\right)^2+\left(3\,a^3b-5\,a^2b^2+16\,ab^3+24\,b^4\right)fx+\left(7\,ab^3+6\,b^4+\left(7\,a^2b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*cos(f*x + e)^2 + (3*a^3
*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + (7*a*b^3 + 6*b^4 + (7*a^2*b^2 + 6
*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*(a^4 + a^3*b)*cos(f
*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 5*a^2*b
^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2
+ (a^5*b + a^4*b^2)*f), 1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*
cos(f*x + e)^2 + (3*a^3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + 2*(7*a*b^3
 + 6*b^4 + (7*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e
))) + (2*(a^4 + a^3*b)*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x
 + e)^3 + (3*a^3*b - 5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

giac [A] time = 0.78, size = 205, normalized size = 1.01

$$-\frac{\frac{4\,b^3\tan\left(fx+e\right)}{\left(a^4+a^3b\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{4\left(7\,ab^3+6\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+a^4b\right)\sqrt{ab+b^2}}-\frac{\left(3\,a^2-8\,ab+24\,b^2\right)\left(fx+e\right)}{a^4}-\frac{3\,a\tan\left(fx+e\right)^3-8\,b\tan\left(f\right.}{\left(\tan\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(f*x + e)/((a^4 + a^3*b)*(b*tan(f*x + e)^2 + a + b)) + 4*(7*
a*b^3 + 6*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) - (3*a^2 - 8*a*b + 24*b^
2)*(f*x + e)/a^4 - (3*a*tan(f*x + e)^3 - 8*b*tan(f*x + e)^3 + 5*a*tan(f*x +
 e) - 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

maple [A] time = 1.32, size = 276, normalized size = 1.36

$$-\frac{b^3\tan\left(fx+e\right)}{2f\,a^3\left(a+b\right)\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{7b^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2f\,a^3\left(a+b\right)\sqrt{\left(a+b\right)b}}-\frac{3b^4\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^4\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{3\left(\tan^3\left(fx+e\right.\right.}{8f\,a^2\left(1+\tan^2\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b^3/a^3/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-7/2/f*b^3/a^3/(a+b)/((
a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f*b^4/a^4/(a+b)/((a+b)
*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/8/f/a^2/(tan(f*x+e)^2+1)^2
*tan(f*x+e)^3-1/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-1/f/a^3/(tan(f*x+e)
```

```
^2+1)^2*tan(f*x+e)*b+5/8/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)+3/f/a^4*arctan
(tan(f*x+e))*b^2+3/8/f/a^2*arctan(tan(f*x+e))-1/f/a^3*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.46, size = 268, normalized size = 1.32

$$-\frac{\frac{4\left(7\,ab^{3}+6\,b^{4}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^{5}+a^{4}b\right)\sqrt{(a+b)b}}-\frac{\left(3\,a^{2}b-5\,ab^{2}-12\,b^{3}\right)\tan(fx+e)^{5}+\left(3\,a^{3}+3\,a^{2}b-16\,ab^{2}-24\,b^{3}\right)\tan(fx+e)^{3}+\left(5\,a^{3}+2\,a^{2}b-11\,ab^{2}-12\,b^{3}\right)\tan(}{\left(a^{4}b+a^{3}b^{2}\right)\tan(fx+e)^{6}+a^{5}+2\,a^{4}b+a^{3}b^{2}+\left(a^{5}+4\,a^{4}b+3\,a^{3}b^{2}\right)\tan(fx+e)^{4}+\left(2\,a^{5}+5\,a^{4}b+3\,a^{3}b^{2}\right)\tan(fx+}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(7*a*b^3 + 6*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + a^
4*b)*sqrt((a + b)*b)) - ((3*a^2*b - 5*a*b^2 - 12*b^3)*tan(f*x + e)^5 + (3*a
^3 + 3*a^2*b - 16*a*b^2 - 24*b^3)*tan(f*x + e)^3 + (5*a^3 + 2*a^2*b - 11*a*
b^2 - 12*b^3)*tan(f*x + e))/((a^4*b + a^3*b^2)*tan(f*x + e)^6 + a^5 + 2*a^4
*b + a^3*b^2 + (a^5 + 4*a^4*b + 3*a^3*b^2)*tan(f*x + e)^4 + (2*a^5 + 5*a^4*
b + 3*a^3*b^2)*tan(f*x + e)^2) - (3*a^2 - 8*a*b + 24*b^2)*(f*x + e)/a^4)/f
```

mupad [B] time = 11.91, size = 2880, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 +
121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((
6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2
)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(
512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7
*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*
8i + b^2*24i)*1i)/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a
^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b +
 a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b
^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(a^2*3i
 - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11
*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(1
6*a^4))*(a^2*3i - a*b*8i + b^2*24i)*1i)/(16*a^4))/(((135*a*b^9)/4 + 27*b^10
 - (9*a^2*b^8)/2 - (149*a^3*b^7)/32 + (219*a^4*b^6)/64 - (63*a^5*b^5)/64)/(
2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a
^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b +
 a^8 + a^6*b^2)) - (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b
^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i
 - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11
*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(1
6*a^4))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8
 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b
^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^
10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (t
an(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*
a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*
b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4)))*(a^2*3i
- a*b*8i + b^2*24i)*1i)/(8*a^4*f) - ((tan(e + f*x)^3*(16*a*b^2 - 3*a^2*b -
3*a^3 + 24*b^3))/(8*a^3*(a + b)) + (tan(e + f*x)*(11*a*b^2 - 2*a^2*b - 5*a^
3 + 12*b^3))/(8*a^3*(a + b)) + (tan(e + f*x)^5*(5*a*b^2 - 3*a^2*b + 12*b^3)
)/(8*a^3*(a + b)))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan(e + f*x)^
6 + tan(e + f*x)^4*(a + 3*b))) + (atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*
```

```
b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32
*(2*a^7*b + a^8 + a^6*b^2)) - ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9
*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11
+ a^9*b^2) - (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5
+ 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b
^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7
+ a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*1i)/(4*(3*a^6*b
 + a^7 + a^4*b^3 + 3*a^5*b^2)) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 8
00*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7
*b + a^8 + a^6*b^2)) + ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2
 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b
^2) + (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*
a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*
a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b
^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*1i)/(4*(3*a^6*b + a^7
+ a^4*b^3 + 3*a^5*b^2)))/(((135*a*b^9)/4 + 27*b^10 - (9*a^2*b^8)/2 - (149*a
^3*b^7)/32 + (219*a^4*b^6)/64 - (63*a^5*b^5)/64)/(2*a^10*b + a^11 + a^9*b^2
) - (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121
*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b^5
*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3
)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(-b^5*(a
+ b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256
*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5
*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a +
b)^3)^(1/2)*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + (((tan
(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 -
 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b^5*(a + b)^3
)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a
^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(-b^5*(a + b)^3)^(1
/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2)
)/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7
*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2
)*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))))*(-b^5*(a + b)^3)
^(1/2)*(7*a + 6*b)*1i)/(2*f*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.205 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=278

$$\frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f(a+b)^{3/2}}+\frac{(5a-8b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{\left(15a^2-26ab+48b^2\right)\sin(e+fx)\cos(e$$

```
[Out] 1/16*(5*a^3-12*a^2*b+24*a*b^2-64*b^3)*x/a^5+1/2*b^(7/2)*(9*a+8*b)*arctan(b^
(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(3/2)/f+1/48*(15*a^2-26*a*b+48*b^2)
*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(5*a-8*b)*cos(f*x+e)
^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)+1/16*b*(5*a^3-7*a^2*b+12*a*b^2+32*b^3)*tan(f*x+e)/a^4/(a
+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.35, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f(a+b)^{3/2}}+\frac{b\left(-7a^2b+5a^3+12ab^2+32b^3\right)\tan(e+fx)}{16a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{\left(15a^2-26ab+48b^2\right)\sin(e+}{48a^3f\left(a+b\tan^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 - 12*a^2*b + 24*a*b^2 - 64*b^3)*x)/(16*a^5) + (b^(7/2)*(9*a + 8*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*(a + b)^(3/2)*f) + ((15*
a^2 - 26*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan[
e + f*x]^2)) + ((5*a - 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b +
 b*Tan[e + f*x]^2)) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e
 + f*x]^2)) + (b*(5*a^3 - 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(16*a^
4*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-5a+b-7bx^2}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^2}\,dx,x,\tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{15a^2}{(}\right.}{}$$

$$= \frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(5a^3-12a^2b+24ab^2-64b^3\right)x}{16a^5} + \frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f} + \frac{\left(15a^2-26a\right.}{48a^3}$$

Mathematica [C] time = 4.42, size = 499, normalized size = 1.79

$$\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{a^3\sin(6e)\cos(6fx)(a\cos(2(e+fx))+a+2b)}{f}+\frac{a^3\cos(6e)\sin(6fx)(a\cos(2(e+fx))+a+2b)}{f}+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(12*(5*a^3 - 12*a^2*b + 24*a
*b^2 - 64*b^3)*x*(a + 2*b + a*Cos[2*(e + f*x)]) - (96*b^4*(9*a + 8*b)*ArcTa
n[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*
x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
 f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e]
)^4]) + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[2*e])/f + (3*a^2*(3*a - 4*b)*Cos[4*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[4*e])/f + (a^3*Cos[6*f*x]*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[6*e
])/f + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*e]*(a + 2*b + a*Cos[2*(e + f*x
)])*Sin[2*f*x])/f - (96*b^4*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f
*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) + (3*a^2*(3*a - 4*b)*Cos[4*e]*(a + 2*
b + a*Cos[2*(e + f*x)])*Sin[4*f*x])/f + (a^3*Cos[6*e]*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sin[6*f*x])/f))/(768*a^5*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.95, size = 789, normalized size = 2.84

$$\left[\frac{3\left(5a^5-7a^4b+12a^3b^2-40a^2b^3-64ab^4\right)fx\cos\left(fx+e\right)^2+3\left(5a^4b-7a^3b^2+12a^2b^3-40ab^4-64b^5\right)f\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x
 + e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x + 6*
(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt
(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2
)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x +
e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*
x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f), 1/48*(3*(5*
a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x + e)^2 + 3*
(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x - 12*(9*a*b^4 +
8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))
 + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2)*cos(f*
x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x + e)^3 + 3
*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*x + e))/
((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

giac [A] time = 0.52, size = 285, normalized size = 1.03

$$\frac{\frac{24 b^4 \tan(fx+e)}{(a^5+a^4b)(b\tan(fx+e)^2+a+b)} + \frac{24(9ab^4+8b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+a^5b)\sqrt{ab+b^2}} + \frac{3(5a^3-12a^2b+24ab^2-64b^3)(fx+e)}{a^5} + \frac{15a^2\tan(fx+\cdots}{\cdots}}{48\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(24*b^4*tan(f*x + e)/((a^5 + a^4*b)*(b*tan(f*x + e)^2 + a + b)) + 24*(
9*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))/((a^6 + a^5*b)*sqrt(a*b + b^2)) + 3*(5*a^3 - 12*a^2*b
+ 24*a*b^2 - 64*b^3)*(f*x + e)/a^5 + (15*a^2*tan(f*x + e)^5 - 36*a*b*tan(f*
x + e)^5 + 72*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 96*a*b*tan(f*x +
 e)^3 + 144*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 60*a*b*tan(f*x + e)
+ 72*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

maple [A] time = 1.53, size = 442, normalized size = 1.59

$$\frac{b^4\tan(fx+e)}{2f a^4(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)} + \frac{9b^4\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{2f a^4(a+b)\sqrt{(a+b)b}} + \frac{4b^5\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f a^5(a+b)\sqrt{(a+b)b}} + \frac{5\left(\tan^5(fx+e\cdots\right.}{16f a^2\left(1+\tan^2(fx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/a^4*b^4/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+9/2/f*b^4/a^4/(a+b)/((a
+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+4/f/a^5*b^5/(a+b)/((a+b)*
b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+5/16/f/a^2/(tan(f*x+e)^2+1)^3
*tan(f*x+e)^5-3/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b+3/2/f/a^4/(tan(f*
x+e)^2+1)^3*tan(f*x+e)^5*b^2+3/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b^2+5/
6/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-2/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+
e)^3*b-5/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b+3/2/f/a^4/(tan(f*x+e)^2+1)
^3*tan(f*x+e)*b^2+11/16/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)-4/f/a^5*arctan(
tan(f*x+e))*b^3+5/16/f/a^2*arctan(tan(f*x+e))-3/4/f/a^3*arctan(tan(f*x+e))*
b+3/2/f/a^4*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.46, size = 369, normalized size = 1.33

$$\frac{\frac{24(9ab^4+8b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+a^5b)\sqrt{(a+b)b}} + \frac{3(5a^3b-7a^2b^2+12ab^3+32b^4)\tan(fx+e)^7+(15a^4+34a^3b-41a^2b^2+156ab^3+288b^4)\tan(fx+e)^5+(40a^4+17a\cdots}{(a^5b+a^4b^2)\tan(fx+e)^8+(a^6+5a^5b+4a^4b^2)\tan(fx+e)^6+a^6+2a^5b+a^4b^2+3(a^6+\cdots}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(24*(9*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 + a
^5*b)*sqrt((a + b)*b)) + (3*(5*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 32*b^4)*tan(f
*x + e)^7 + (15*a^4 + 34*a^3*b - 41*a^2*b^2 + 156*a*b^3 + 288*b^4)*tan(f*x
+ e)^5 + (40*a^4 + 17*a^3*b - 35*a^2*b^2 + 204*a*b^3 + 288*b^4)*tan(f*x + e
)^3 + 3*(11*a^4 + 2*a^3*b - 5*a^2*b^2 + 28*a*b^3 + 32*b^4)*tan(f*x + e))/((
a^5*b + a^4*b^2)*tan(f*x + e)^8 + (a^6 + 5*a^5*b + 4*a^4*b^2)*tan(f*x + e)^
6 + a^6 + 2*a^5*b + a^4*b^2 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x + e)^4
+ (3*a^6 + 7*a^5*b + 4*a^4*b^2)*tan(f*x + e)^2) + 3*(5*a^3 - 12*a^2*b + 24*
a*b^2 - 64*b^3)*(f*x + e)/a^5)/f
```

mupad [B] time = 8.56, size = 3310, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)*(28*a*b^3 + 2*a^3*b + 11*a^4 + 32*b^4 - 5*a^2*b^2))/(16*a^4*
(a + b)) + (tan(e + f*x)^5*(156*a*b^3 + 34*a^3*b + 15*a^4 + 288*b^4 - 41*a^
2*b^2))/(48*a^4*(a + b)) + (tan(e + f*x)^3*(204*a*b^3 + 17*a^3*b + 40*a^4 +
 288*b^4 - 35*a^2*b^2))/(48*a^4*(a + b)) + (b*tan(e + f*x)^7*(12*a*b^2 - 7*
a^2*b + 5*a^3 + 32*b^3))/(16*a^4*(a + b)))/(f*(a + b + tan(e + f*x)^2*(3*a
+ 4*b) + tan(e + f*x)^4*(3*a + 6*b) + b*tan(e + f*x)^8 + tan(e + f*x)^6*(a
+ 4*b))) - (atan(-((((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^1
3*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) -
(tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5
120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 +
a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) - (tan(e +
f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 5
68*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 +
 a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(32*a^5) - (((((
8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/
4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e + f*x)*(a*b^2*24i
 - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12
*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i -
a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) + (tan(e + f*x)*(14336*a*b^10 + 819
2*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5
 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i -
 a^2*b*12i + a^3*5i - b^3*64i)*1i)/(32*a^5))/((72*a*b^12 + 64*b^13 - 11*a^2
*b^11 + (19*a^3*b^10)/8 + (267*a^4*b^9)/32 - (101*a^5*b^8)/16 + (655*a^6*b^
7)/256 - (225*a^7*b^6)/256)/(2*a^13*b + a^14 + a^12*b^2) + (((((8*a^10*b^7
+ 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15
*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) - (tan(e + f*x)*(a*b^2*24i - a^2*b*12
i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024
*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i +
 a^3*5i - b^3*64i))/(32*a^5) - (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 52
48*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b
^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i
+ a^3*5i - b^3*64i))/(32*a^5) + (((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^
5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 +
 a^12*b^2) + (tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048
*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^
9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5
) + (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 6
4*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a
^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^
5)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(16*a^5*f) - (atan((((-
b^7*(a + b)^3)^(1/2)*((tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^
9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a
^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)) - (((8*a^10*b^7 + 15*a^11*b^6 + (
23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*
b + a^14 + a^12*b^2) - (tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(20
48*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b
 + a^10 + a^8*b^2)*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)
^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b)*
1i)/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-b^7*(a + b)^3)^(1/2)*((t
an(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*
b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b +
 a^10 + a^8*b^2)) + (((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13
```

```
*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (
tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*
b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^
7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(
3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b)*1i)/(4*(3*a^7*b + a^8 +
a^5*b^3 + 3*a^6*b^2)))/((72*a*b^12 + 64*b^13 - 11*a^2*b^11 + (19*a^3*b^10)/
8 + (267*a^4*b^9)/32 - (101*a^5*b^8)/16 + (655*a^6*b^7)/256 - (225*a^7*b^6)
/256)/(2*a^13*b + a^14 + a^12*b^2) - ((-b^7*(a + b)^3)^(1/2)*((tan(e + f*x)
*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a
^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8
*b^2)) - (((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (
3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) - (tan(e + f*x
)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*
a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^7*b + a^8 +
 a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a
^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^
6*b^2)) + ((-b^7*(a + b)^3)^(1/2)*((tan(e + f*x)*(14336*a*b^10 + 8192*b^11
+ 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a
^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)) + (((8*a^10*b^7 + 15
*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2
)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9
*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/
(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b
^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))
*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))))*(-b^7*(a + b)^3)^
(1/2)*(9*a + 8*b)*1i)/(2*f*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.206 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\sin(e+fx)}{8f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}f(a+b)^{5/2}}$$

```
[Out] 1/4*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+3/8*sin(f*x+e)/(a+b)^2/f/(a+b
-a*sin(f*x+e)^2)+3/8*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f/
a^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 199, 208}

$$\frac{3\sin(e+fx)}{8f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*f)
 + Sin[e + f*x]/(4*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*Sin[e + f*x
])/(8*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^3}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^2}\,dx,x,\sin(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{3\,\text{Subs}}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,(a+b)^{5/2}f} + \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+}{8(a+b)^2f\left(a+b-\right.}$$

Mathematica [A] time = 0.48, size = 128, normalized size = 1.19

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{4\sin(e+fx)\left(5(a+b)-3a\sin^2(e+fx)\right)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2}\right)}{64f\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((3*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b)
 - 3*a*Sin[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(64*
f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 1.68, size = 472, normalized size = 4.37

$$\left[\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\,\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(2\,a^3+7\,a^2b\right.}{16\left(\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)f\cos\left(fx+e\right)^4+2\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a^2 + a*b)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f
*x + e)^2 + b)) + 2*(2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2
+ 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f), -1/8*(3*(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(
a + b)) - (2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x + e)^2)*sin(
f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 + 2*(a^5*
b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2 + 3*a^3*b^
3 + 3*a^2*b^4 + a*b^5)*f)]
```

giac [A] time = 0.36, size = 122, normalized size = 1.13

$$-\frac{\frac{3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+2ab+b^2)\sqrt{-a^2-ab}}+\frac{3a\sin(fx+e)^3-5a\sin(fx+e)-5b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)^2(a^2+2ab+b^2)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)*sqrt(-
a^2 - a*b)) + (3*a*sin(f*x + e)^3 - 5*a*sin(f*x + e) - 5*b*sin(f*x + e))/((
a*sin(f*x + e)^2 - a - b)^2*(a^2 + 2*a*b + b^2)))/f
```

maple [A] time = 0.90, size = 108, normalized size = 1.00

$$\frac{\frac{\sin(fx+e)}{4(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{8(a+b)\sqrt{(a+b)a}}}{a+b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(1/4*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)^2+3/4/(a+b)*(-1/2*sin(f*x+e
)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e
)/((a+b)*a)^(1/2))))
```

maxima [A] time = 0.43, size = 179, normalized size = 1.66

$$-\frac{\frac{2\left(3a\sin(fx+e)^3-5(a+b)\sin(fx+e)\right)}{(a^4+2a^3b+a^2b^2)\sin(fx+e)^4+a^4+4a^3b+6a^2b^2+4ab^3+b^4-2(a^4+3a^3b+3a^2b^2+ab^3)\sin(fx+e)^2}+\frac{3\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2ab+b^2)}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a*sin(f*x + e)^3 - 5*(a + b)*sin(f*x + e))/((a^4 + 2*a^3*b + a^
2*b^2)*sin(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sin(f*x + e)^2) + 3*log((a*sin(f*x + e) - sq
rt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 +
2*a*b + b^2)))/f
```

mupad [B] time = 0.22, size = 113, normalized size = 1.05

$$\frac{\frac{5\sin(e+fx)}{8(a+b)}-\frac{3a\sin(e+fx)^3}{8(a+b)^2}}{f\left(2ab+a^2+b^2-\sin\left(e+fx\right)^2\left(2a^2+2ba\right)+a^2\sin\left(e+fx\right)^4\right)}+\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((5*sin(e + f*x))/(8*(a + b)) - (3*a*sin(e + f*x)^3)/(8*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2) + a^2*sin(e + f*x)^4)) + (3
*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(8*a^(1/2)*f*(a + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**3, x)
```

3.207 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}f(a+b)^{5/2}}+\frac{(4a+b)\sin(e+fx)}{8af(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[Out] 1/8*(4*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/f-1
/4*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+1/8*(4*a+b)*sin(f*x+e)/a/(
a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 385, 199, 208}

$$\frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}f(a+b)^{5/2}}+\frac{(4a+b)\sin(e+fx)}{8af(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*f) - (b*Sin[e + f*x])/(4*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) +
((4*a + b)*Sin[e + f*x])/(8*a*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\,\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+ \ldots\right.}{4a(a+b)f}$$

$$= -\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(4a \ldots}{\ldots}$$

$$= \frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a \ldots}{8a(a+b)^2 f \ldots}$$

Mathematica [A] time = 0.61, size = 163, normalized size = 1.30

$$-\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{8\sin(e+fx)}{\left(-a\sin^2(e+fx)+a+b\right)^2}-(4a+b)\left(\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{4\sin(e+fx)\left(5(a+b)-3a\sin \ldots\right.}{(a+b)^2(a\cos(2(e+fx)) \ldots}\right.\right.}{192af\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/192*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((8*Sin[e + f*x])/(
a + b - a*Sin[e + f*x]^2)^2 - (4*a + b)*((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b) - 3*a*Si
n[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))))/(a*f*(a + b*
Sec[e + f*x]^2)^3)
```

fricas [B] time = 1.28, size = 544, normalized size = 4.35

$$\left[\frac{\left(\left(4a^3+a^2b\right)\cos\left(fx+e\right)^4+4ab^2+b^3+2\left(4a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\,\sin(f \ldots}{a\cos\left(fx+e\right)^2+b}\right.}{16\left(\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)f\cos\left(fx+e\right)^4+2\left(a^6b+3a^5b^2+3a^4b^3+a^3b^4 \ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*b + a*b^2
)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b
)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3*b + a^2*b^2 -
a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 3
*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a
^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2
*b^5)*f), -1/8*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*
b + a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x
 + e)/(a + b)) - (2*a^3*b + a^2*b^2 - a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f
*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 +
(a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

giac [A] time = 0.74, size = 162, normalized size = 1.30

$$-\frac{\frac{(4\,a+b)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^3+2\,a^2b+ab^2\right)\sqrt{-a^2-ab}}+\frac{4\,a^2\sin\left(fx+e\right)^3+ab\sin\left(fx+e\right)^3-4\,a^2\sin\left(fx+e\right)-3\,ab\sin\left(fx+e\right)+b^2\sin\left(fx+e\right)}{\left(a^3+2\,a^2b+ab^2\right)\left(a\sin\left(fx+e\right)^2-a-b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + 2*a^2*b + a
*b^2)*sqrt(-a^2 - a*b)) + (4*a^2*sin(f*x + e)^3 + a*b*sin(f*x + e)^3 - 4*a^
2*sin(f*x + e) - 3*a*b*sin(f*x + e) + b^2*sin(f*x + e))/((a^3 + 2*a^2*b + a
*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

maple [A] time = 0.93, size = 124, normalized size = 0.99

$$\frac{\frac{-\frac{(4a+b)\left(\sin^3\left(fx+e\right)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin\left(fx+e\right)}{8(a+b)a}}{\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)^2}+\frac{(4a+b)\,\mathrm{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*((-1/8*(4*a+b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(4*a-b)/(a+b)/a*sin(f*x
+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/((a+b)*a)^(1/2)*
arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

maxima [A] time = 0.45, size = 212, normalized size = 1.70

$$-\frac{\frac{(4\,a+b)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^3+2\,a^2b+ab^2\right)\sqrt{(a+b)a}}+\frac{2\left(\left(4\,a^2+ab\right)\sin\left(fx+e\right)^3-\left(4\,a^2+3\,ab-b^2\right)\sin\left(fx+e\right)\right)}{a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+\left(a^5+2\,a^4b+a^3b^2\right)\sin\left(fx+e\right)^4-2\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sin\left(fx+e\right)^2}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((4*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + s
qrt((a + b)*a)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + 2*((4*a^2 + a*
b)*sin(f*x + e)^3 - (4*a^2 + 3*a*b - b^2)*sin(f*x + e))/(a^5 + 4*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 2*a^4*b + a^3*b^2)*sin(f*x + e)^4 - 2*
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

mupad [B] time = 4.68, size = 129, normalized size = 1.03

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{a}\ \sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)(4\,a+b)}{8\,a^{3/2}\,f\,(a+b)^{5/2}}-\frac{\frac{\sin\left(e+f\,x\right)^3\,(4\,a+b)}{8\,(a+b)^2}-\frac{\sin\left(e+f\,x\right)(4\,a-b)}{8\,a\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\sin\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + b))/(8*a^(3/2)*f*(a + b
)^(5/2)) - ((sin(e + f*x)^3*(4*a + b))/(8*(a + b)^2) - (sin(e + f*x)*(4*a -
```

```
 b))/(8*a*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2)
+ a^2*sin(e + f*x)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**3, x)
```

3.208 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{3b(2a+b)\sin(e+fx)}{8a^2f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}f(a+b)^{5/2}}-\frac{b\sin(e+fx)\cos^2(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+
b)^(5/2)/f-1/4*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-3
/8*b*(2*a+b)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 413, 385, 208}

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}f(a+b)^{5/2}}-\frac{3b(2a+b)\sin(e+fx)}{8a^2f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)\cos^2(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*f) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(4*a*(a + b)*f*(a
+ b - a*Sin[e + f*x]^2)^2) - (3*b*(2*a + b)*Sin[e + f*x])/(8*a^2*(a + b)^2*
f*(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-b+(4a+3b)x^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right.}{4a(a+b)f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(8}{}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} - \frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{}{8}$$

Mathematica [C] time = 6.82, size = 927, normalized size = 6.44

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^5(e+fx)\left(32\sqrt{a}\,(a+b)^{3/2}\sqrt{(\cos(e)-i\sin(e))^2}\,\tan(e+fx)b^2-8\sqrt{a}\,\sqrt{a+b}\,(8a+5\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((-2*I)*(8*a^2 + 8*a*b + 3*b
^2)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos
[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b + a*Cos[
2*(e + f*x)])^2*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + (8*a^2 + 8*a*b + 3*b^2)*
(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e +
f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - (8*a^2 + 8*a*b +
3*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos
[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]
*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + 2*(8*a^2
+ 8*a*b + 3*b^2)*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]
*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos
[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[
a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]
*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a +
b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Si
n[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a
 + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]*(I*Cos[e] + Sin[e]) + 32*Sqrt[a
]*b^2*(a + b)^(3/2)*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x] - 8*Sqrt[a]*b*
Sqrt[a + b]*(8*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Sqrt[(Cos[e] - I*Sin
[e])^2]*Tan[e + f*x]))/(256*a^(5/2)*(a + b)^(5/2)*f*(a + b*Sec[e + f*x]^2)^
3*Sqrt[(Cos[e] - I*Sin[e])^2])
```

fricas [B] time = 1.20, size = 613, normalized size = 4.26

$$\left[\frac{\left(\left(8\,a^4+8\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+8\,a^2b^2+8\,ab^3+3\,b^4+2\left(8\,a^3b+8\,a^2b^2+3\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2}\cdots}{16\left(\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3\right)f\cos\left(fx+e\right)^4+2\left(a^7b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3
+ 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(a^2 + a*b)
*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(
f*x + e)^2 + b)) - 2*(6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a^3*b
^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*b^2 +
 a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*
cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f), -1/8*(((8*
a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3*b^4 + 2
*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sq
rt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (
8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a
^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5
*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b
^5)*f)]
```

giac [A] time = 1.10, size = 185, normalized size = 1.28

$$-\frac{\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{-a^2-ab}}-\frac{8\,a^2b\sin\left(fx+e\right)^3+5\,ab^2\sin\left(fx+e\right)^3-8\,a^2b\sin\left(fx+e\right)-11\,ab^2\sin\left(fx+e\right)-3\,b^3\sin\left(fx+e\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\left(a\sin\left(fx+e\right)^2-a-b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4
 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) - (8*a^2*b*sin(f*x + e)^3 + 5*a*b^2
*sin(f*x + e)^3 - 8*a^2*b*sin(f*x + e) - 11*a*b^2*sin(f*x + e) - 3*b^3*sin(
f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

maple [A] time = 0.96, size = 142, normalized size = 0.99

$$\frac{-\frac{-\frac{b(8a+5b)\left(\sin^3\left(fx+e\right)\right)}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin\left(fx+e\right)}{8a^2(a+b)}}{\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{(a+b)a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(-(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(8*a+3*b)/a^2*b/
(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*
b+b^2)/a^2/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

maxima [A] time = 0.45, size = 233, normalized size = 1.62

$$-\frac{\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{(a+b)a}}-\frac{2\left(\left(8\,a^2b+5\,ab^2\right)\sin\left(fx+e\right)^3-\left(8\,a^2b+11\,ab^2+3\,b^3\right)\sin\left(fx+e\right)\right)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+\left(a^6+2\,a^5b+a^4b^2\right)\sin\left(fx+e\right)^4-2\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sin\left(fx+\cdots\right.}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((8*a^2 + 8*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*si
n(f*x + e) + sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a))
- 2*((8*a^2*b + 5*a*b^2)*sin(f*x + e)^3 - (8*a^2*b + 11*a*b^2 + 3*b^3)*sin(
f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6 + 2*a^5*b
 + a^4*b^2)*sin(f*x + e)^4 - 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*
x + e)^2))/f
```

mupad [B] time = 0.25, size = 149, normalized size = 1.03

$$\frac{\frac{\sin\left(e+f\,x\right)^3\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,{\left(a+b\right)}^2}-\frac{\sin\left(e+f\,x\right)\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,\left(a+b\right)}}{f\,\left(2\,a\,b+a^2+b^2-{\sin\left(e+f\,x\right)}^2\,\left(2\,a^2+2\,b\,a\right)+a^2\,{\sin\left(e+f\,x\right)}^4\right)}+\frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)\,\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,f\,{\left(a+b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((sin(e + f*x)^3*(8*a*b + 5*b^2))/(8*a*(a + b)^2) - (sin(e + f*x)*(8*a*b +
3*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*
a^2) + a^2*sin(e + f*x)^4)) + (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*
(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*f*(a + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**3, x)
```

3.209 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}f(a+b)^{5/2}}-\frac{b^3\sin(e+fx)}{4a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3b^2(4a+3b}{8a^3f(a+b)^2\left(-a\sin\right.}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2
)/(a+b)^(5/2)/f+sin(f*x+e)/a^3/f-1/4*b^3*sin(f*x+e)/a^3/(a+b)/f/(a+b-a*sin(
f*x+e)^2)^2+3/8*b^2*(4*a+3*b)*sin(f*x+e)/a^3/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 390, 1157, 385, 208}

$$-\frac{b^3\sin(e+fx)}{4a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sin(e+fx)}{8a^3f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\text{t}}{8a^{7/2}f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*f) + Sin[e + f*x]/(a^3*f) - (b^3*Sin[e + f*x])
/(4*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e
+ f*x])/(8*a^3*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3} - \frac{b\left(3a^2+3ab+b^2\right)-3ab(2a+b)x^2+3a^2bx^4}{a^3\left(a+b-ax^2\right)^3}\right) dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{a^3f} - \frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)-3ab(2a+b)x^2+3a^2bx^4}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{a^3f}$$

$$= \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2+12ab(a+b)\cdots}{\left(a+b-ax^2\right)^2} \cdots\right)}{4a^3(a+b\cdots}$$

$$= \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3b^2(4a+3b)\sin(e+\cdots}{8a^3(a+b)^2f\left(a+b-a\sin^2\cdots\right.}$$

$$= -\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin\cdots}{4a^3(a+b)f\left(a+\cdots\right.}$$

Mathematica [C] time = 7.51, size = 2382, normalized size = 15.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[e])/(8*a^3*f*
(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[
2*e + 2*f*x])^3*Sec[e + f*x]^6*((((-3*I)/128)*ArcTan[((-I)*a*Cos[e] - I*b*C
os[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a
+ b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e
 + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*S
qrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a
+ b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[
Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e]
+ b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b
*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e +
2*f*x])]*Cos[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (3*A
rcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e]
+ b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
 Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e
```

```
] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] +
 I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e
] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2
*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin
[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(128*a^(7/2)*Sqrt[a + b]*f*Sqr
t[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*
b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*A
rcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[
a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3
*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e]
 - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2
*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*
Sin[2*e]]) - (((3*I)/128)*ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*
I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]]
+ Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a
]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a +
 b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Sin[e])/(a^(7/2)*Sqrt[a +
b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) +
((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]
^6*((-3*Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (
2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] -
I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Si
n[2*e + f*x]])/(256*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + ((
(3*I)/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (2*I)
*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Si
n[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*
e + f*x]]*Sin[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a
 + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b
+ a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*Cos[e]*Log[-a - 2*a*Cos[2*e] - 2
*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a
 + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(7/2)*Sqrt[a + b]
*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (((3*I)/256)*Log[-a - 2*a*Cos[2*e] - 2*b*
Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqr
t[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a +
b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(7/2)*Sqrt[a + b]
*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + (C
os[e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(8*a^3*f*(a
 + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^
6*(4*a*b^2*Sin[e + f*x] + 3*b^3*Sin[e + f*x]))/(32*a^3*(a + b)^2*f*(a + b*S
ec[e + f*x]^2)^3) - (b^3*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[
e + f*x])/(8*a^3*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 1.64, size = 727, normalized size = 4.66

$$\left[\frac{3\left(8\,a^2b^3+12\,ab^4+5\,b^5+\left(8\,a^4b+12\,a^3b^2+5\,a^2b^3\right)\cos\left(fx+e\right)^4+2\left(8\,a^3b^2+12\,a^2b^3+5\,ab^4\right)\cos\left(fx+e\right.\right.}{16\left(\left(a^9+3\,a^8b+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)
*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x + e)^2)*sqrt
(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a -
 b)/(a*cos(f*x + e)^2 + b)) + 2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a
*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b +
```

```
60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 +
3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 +
a^4*b^5)*f), 1/8*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 +
5*a^2*b^3)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x +
e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (8*
a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2
+ a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b + 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b
^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 + 3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)
^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)*f)]
```

giac [A] time = 0.48, size = 205, normalized size = 1.31

$$\frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{-a^2-ab}}-\frac{12a^2b^2\sin\left(fx+e\right)^3+9ab^3\sin\left(fx+e\right)^3-12a^2b^2\sin\left(fx+e\right)-19ab^3\sin\left(fx+e\right)-7b^4\sin\left(fx+e\right)}{\left(a^5+2a^4b+a^3b^2\right)\left(a\sin\left(fx+e\right)^2-a-b\right)^2}+\frac{8\sin\left(fx\right.}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a^2 - a*b)) - (12*a^2*b^2*sin(f*x + e)^3
+ 9*a*b^3*sin(f*x + e)^3 - 12*a^2*b^2*sin(f*x + e) - 19*a*b^3*sin(f*x + e)
- 7*b^4*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - a - b)
^2) + 8*sin(f*x + e)/a^3)/f
```

maple [A] time = 1.40, size = 149, normalized size = 0.96

$$\frac{\frac{\sin\left(fx+e\right)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\left(\sin^3\left(fx+e\right)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(12a+7b)b\sin\left(fx+e\right)}{8a+8b}}{\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)^2}-\frac{3\left(8a^2+12ab+5b^2\right)\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)a}}\right)}{a^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(1/a^3*sin(f*x+e)+1/a^3*b*((-3/8*a*b*(4*a+3*b)/(a^2+2*a*b+b^2)*sin(f*x+
e)^3+1/8*(12*a+7*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-3/8*(8*a^2+
12*a*b+5*b^2)/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a
)^(1/2))))
```

maxima [A] time = 0.44, size = 253, normalized size = 1.62

$$\frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{(a+b)a}}-\frac{2\left(3\left(4a^2b^2+3ab^3\right)\sin\left(fx+e\right)^3-\left(12a^2b^2+19ab^3+7b^4\right)\sin\left(fx+e\right)\right)}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+\left(a^7+2a^6b+a^5b^2\right)\sin\left(fx+e\right)^4-2\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\sin\left(fx+e\right.}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))
/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b
)*a)) - 2*(3*(4*a^2*b^2 + 3*a*b^3)*sin(f*x + e)^3 - (12*a^2*b^2 + 19*a*b^3
+ 7*b^4)*sin(f*x + e))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (
```

```
a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*sin(f*x + e)^2) + 16*sin(f*x + e)/a^3)/f
```

mupad [B] time = 4.74, size = 175, normalized size = 1.12

$$\frac{\sin\left(e+f\,x\right)}{a^3\,f}+\frac{\frac{\sin\left(e+f\,x\right)\,\left(7\,b^3+12\,a\,b^2\right)}{8\,(a+b)}-\frac{3\,\sin\left(e+f\,x\right)^3\,\left(4\,a^2\,b^2+3\,a\,b^3\right)}{8\,(a+b)^2}}{f\,\left(2\,a^4\,b-\sin\left(e+f\,x\right)^2\,\left(2\,a^5+2\,b\,a^4\right)+a^5+a^3\,b^2+a^5\,\sin\left(e+f\,x\right)^4\right)}-\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\right.}{\sqrt{a+b}}\right.}{8\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] sin(e + f*x)/(a^3*f) + ((sin(e + f*x)*(12*a*b^2 + 7*b^3))/(8*(a + b)) - (3*
sin(e + f*x)^3*(3*a*b^3 + 4*a^2*b^2))/(8*(a + b)^2))/(f*(2*a^4*b - sin(e +
f*x)^2*(2*a^4*b + 2*a^5) + a^5 + a^3*b^2 + a^5*sin(e + f*x)^4)) - (3*b*atan
h((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^(7/2
)*f*(a + b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.210 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{b^4\sin(e+fx)}{4a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+f}{3a^3f}$$

```
[Out] 1/8*b^2*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9
/2)/(a+b)^(5/2)/f+(a-3*b)*sin(f*x+e)/a^4/f-1/3*sin(f*x+e)^3/a^3/f+1/4*b^4*s
in(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-1/8*b^3*(16*a+13*b)*sin(f*x+e)
/a^4/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.24, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 390, 1157, 385, 208}

$$\frac{b^4\sin(e+fx)}{4a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b^2\left(48a^2+80ab+35b^2\right)\tanh^{-1}}{8a^{9/2}f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
)/(8*a^(9/2)*(a + b)^(5/2)*f) + ((a - 3*b)*Sin[e + f*x])/(a^4*f) - Sin[e +
f*x]^3/(3*a^3*f) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*f*(a + b - a*Sin[e + f
*x]^2)^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*f*(a + b - a*
Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a+b-ax^2\right)^3}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{a^4}-\frac{x^2}{a^3}+\frac{b^2\left(6a^2+8ab+3b^2\right)-4ab^2(3a+2b)x^2+6a^2b^2x^4}{a^4\left(a+b-ax^2\right)^3}\right)dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{\text{Subst}\left(\int \frac{b^2\left(6a^2+8ab+3b^2\right)-4ab^2(3a+2b)x^2+6a^2b^2}{\left(a+b-ax^2\right)^3}\right.}{a^4f}$$

$$= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}-$$

$$= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}-$$

$$= \frac{b^2\left(48a^2+80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+}{3a^3}$$

Mathematica [A] time = 4.39, size = 194, normalized size = 1.07

$$\frac{4a^{3/2}\sin(3(e+fx))-\frac{3b^2\left(48a^2+80ab+35b^2\right)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{5/2}}+12\sqrt{a}\sin(e+fx)\left(-\frac{b^4(1}{(a+}\right.}{48a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^2*(48*a^2 + 80*a*b + 35*b^2)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]
] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(5/2) + 12*Sqrt[a]*(-
12*b - (b^4*(9*a + 22*b + 13*a*Cos[2*(e + f*x)]))/((a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2) + a*(3 - (16*b^3)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)]))))*Sin[e + f*x] + 4*a^(3/2)*Sin[3*(e + f*x)])/(48*a^(9/2)*f)
```

fricas [B] time = 0.90, size = 856, normalized size = 4.73

$$\left[\frac{3\left(48\,a^2b^4+80\,ab^5+35\,b^6+\left(48\,a^4b^2+80\,a^3b^3+35\,a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48\,a^3b^3+80\,a^2b^4+35\,ab^5\right)\text{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^
2*b^4)*cos(f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)
^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e)
 - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(16*a^5*b^2 - 24*a^4*b^3 - 210*a^3*
b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos
(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4)*cos(f
*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175*a^2*b^
5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*f*
cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f*x + e)
^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f), -1/24*(3*(48*a^2*b^4 +
 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4
+ 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*a
rctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (16*a^5*b^2 - 24*a^4*b^3 - 2
10*a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4
)*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175
*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*
b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f
*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f)]
```

giac [A] time = 0.44, size = 239, normalized size = 1.32

$$-\frac{\frac{3\left(48\,a^2b^2+80\,ab^3+35\,b^4\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{-a^2-ab}}-\frac{3\left(16\,a^2b^3\sin\left(fx+e\right)^3+13\,ab^4\sin\left(fx+e\right)^3-16\,a^2b^3\sin\left(fx+e\right)-27\,ab^4\sin\left(fx+e\right)-11\,b^5\sin\left(fx+e\right)\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\left(a\sin\left(fx+e\right)^2-a-b\right)^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 -
 a*b))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a^2 - a*b)) - 3*(16*a^2*b^3*sin(f*x
 + e)^3 + 13*a*b^4*sin(f*x + e)^3 - 16*a^2*b^3*sin(f*x + e) - 27*a*b^4*sin(
f*x + e) - 11*b^5*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(a*sin(f*x + e)^
2 - a - b)^2) + 8*(a^6*sin(f*x + e)^3 - 3*a^6*sin(f*x + e) + 9*a^5*b*sin(f*
x + e))/a^9)/f
```

maple [A] time = 1.24, size = 177, normalized size = 0.98

$$\frac{-\frac{\frac{a\left(\sin^3\left(fx+e\right)\right)}{3}-a\sin\left(fx+e\right)+3b\sin\left(fx+e\right)}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)\left(\sin^3\left(fx+e\right)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(16a+11b)b\sin\left(fx+e\right)}{8a+8b}}{\left(-a-b+a\left(\sin^2\left(fx+e\right)\right)\right)^2}-\frac{\left(48a^2+80ab+35b^2\right)\operatorname{arctanh}\left(\frac{a\sin\left(fx+e\right)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)a}}\right)}{a^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(-1/a^4*(1/3*a*sin(f*x+e)^3-a*sin(f*x+e)+3*b*sin(f*x+e))-b^2/a^4*((-1/8
*a*b*(16*a+13*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(16*a+11*b)*b/(a+b)*sin(f
*x+e))/(-a-b+a*sin(f*x+e)^2)^2-1/8*(48*a^2+80*a*b+35*b^2)/(a^2+2*a*b+b^2)/(
(a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

maxima [A]   time = 0.44, size = 272, normalized size = 1.50

$$-\frac{\frac{3\left(48\,a^{2}b^{2}+80\,ab^{3}+35\,b^{4}\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^{6}+2\,a^{5}b+a^{4}b^{2}\right)\sqrt{(a+b)a}}-\frac{6\left(\left(16\,a^{2}b^{3}+13\,ab^{4}\right)\sin\left(fx+e\right)^{3}-\left(16\,a^{2}b^{3}+27\,ab^{4}+11\,b^{5}\right)\sin\left(fx+e\right)\right)}{a^{8}+4\,a^{7}b+6\,a^{6}b^{2}+4\,a^{5}b^{3}+a^{4}b^{4}+\left(a^{8}+2\,a^{7}b+a^{6}b^{2}\right)\sin\left(fx+e\right)^{4}-2\left(a^{8}+3\,a^{7}b+3\,a^{6}b^{2}+a^{5}b^{3}\right)}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*log((a*sin(f*x + e) - sqrt((a + b
)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((
a + b)*a)) - 6*((16*a^2*b^3 + 13*a*b^4)*sin(f*x + e)^3 - (16*a^2*b^3 + 27*a
*b^4 + 11*b^5)*sin(f*x + e))/(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b
^4 + (a^8 + 2*a^7*b + a^6*b^2)*sin(f*x + e)^4 - 2*(a^8 + 3*a^7*b + 3*a^6*b^
2 + a^5*b^3)*sin(f*x + e)^2) + 16*(a*sin(f*x + e)^3 - 3*(a - 3*b)*sin(f*x +
 e))/a^4)/f
```

mupad [B]   time = 0.39, size = 256, normalized size = 1.41

$$\frac{b^2\,\ln\left(\sqrt{a+b}+\sqrt{a}\,\sin\left(e+f\,x\right)\right)\,\left(3\,a^2+5\,a\,b+\frac{35\,b^2}{16}\right)}{a^{9/2}\,f\,{\left(a+b\right)}^{5/2}}-\frac{\frac{\sin\left(e+f\,x\right)\,\left(11\,b^4+16\,a\,b^3\right)}{8\,\left(a+b\right)}-\frac{{\sin\left(e+f\,x\right)}^3\,\left(16\,a^2\,b^3\right.}{8\,{\left(a+b\right)}^2}}{f\,\left(2\,a^5\,b-{\sin\left(e+f\,x\right)}^2\,\left(2\,a^6+2\,b\,a^5\right)+a^6+a^4\,b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (b^2*log((a + b)^(1/2) + a^(1/2)*sin(e + f*x))*(5*a*b + 3*a^2 + (35*b^2)/16
))/(a^(9/2)*f*(a + b)^(5/2)) - ((sin(e + f*x)*(16*a*b^3 + 11*b^4))/(8*(a +
b)) - (sin(e + f*x)^3*(13*a*b^4 + 16*a^2*b^3))/(8*(a + b)^2))/(f*(2*a^5*b -
 sin(e + f*x)^2*(2*a^5*b + 2*a^6) + a^6 + a^4*b^2 + a^6*sin(e + f*x)^4)) -
sin(e + f*x)^3/(3*a^3*f) - (b^2*log(a^(1/2)*sin(e + f*x) - (a + b)^(1/2))*(
80*a*b + 48*a^2 + 35*b^2))/(16*a^(9/2)*f*(a + b)^(5/2)) - (sin(e + f*x)*((3
*(a + b))/a^4 - 4/a^3))/f
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.211 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=214

$$-\frac{b^5\sin(e+fx)}{4a^5f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{(2a-3b)\sin^3(e+fx)}{3a^4f}+\frac{\sin^5(e}{5a}$$

```
[Out] -1/8*b^3*(80*a^2+140*a*b+63*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^
(11/2)/(a+b)^(5/2)/f+(a^2-3*a*b+6*b^2)*sin(f*x+e)/a^5/f-1/3*(2*a-3*b)*sin(f
*x+e)^3/a^4/f+1/5*sin(f*x+e)^5/a^3/f-1/4*b^5*sin(f*x+e)/a^5/(a+b)/f/(a+b-a*
sin(f*x+e)^2)^2+1/8*b^4*(20*a+17*b)*sin(f*x+e)/a^5/(a+b)^2/f/(a+b-a*sin(f*x
+e)^2)
```

Rubi [A] time = 0.26, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{4147, 390, 1157, 385, 208\}$

$$-\frac{b^5\sin(e+fx)}{4a^5f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(11/2)*(a + b)^(5/2)*f) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/(a^
5*f) - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4*f) + Sin[e + f*x]^5/(5*a^3*f) -
(b^5*Sin[e + f*x])/(4*a^5*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*(2
0*a + 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
```

```
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{a^5} - \frac{(2a-3b)x^2}{a^4} + \frac{x^4}{a^3} - \frac{b^3\left(10a^2+15ab+6b^2\right)-5ab^3(4a+3b)x^2+10a^2b^3x^4}{a^5\left(a+b-ax^2\right)^3}\right) dx$$

$$= \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{}{4a^5(a+}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{}{4a^5(a+}$$

$$= -\frac{b^3\left(80a^2+140ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f} + \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f}$$

Mathematica [C] time = 7.60, size = 2670, normalized size = 12.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((5*a^2 - 18*a*b + 48*b^2)*Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*Sin[e])/(64*a^5*f*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*
a*b^4 - 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((I/128)*A
rcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e]
+ b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e
] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] +
I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e
] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2
*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin
[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[C
```

```
os[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e]
 + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sq
rt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e
] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[
2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e
]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos
[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*
e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(12
8*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*S
ec[e + f*x]^2)^3) + ((80*a^2*b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] -
 (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*
e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sq
rt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt
[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(11/2)*
Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/128)*ArcTanh[(2*(a + b)*Si
n[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*
Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2
*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*a*b^4 - 63*b^5)*(a + 2*b +
 a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*C
os[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b
]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e]
 - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqr
t[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[
Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt
[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((80*a^2*
b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(
(Cos[e]*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a
*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[
2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e
+ f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256
)*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2
*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*
Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((5*a - 12*b)*Cos[3*f*x]*(a + 2*b + a*Cos[2*e
+ 2*f*x])^3*Sec[e + f*x]^6*Sin[3*e])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) +
 (Cos[5*f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*e])/(640
*a^3*f*(a + b*Sec[e + f*x]^2)^3) + ((5*a^2 - 18*a*b + 48*b^2)*Cos[e]*(a + 2
*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(64*a^5*f*(a + b*Sec[e
+ f*x]^2)^3) + ((5*a - 12*b)*Cos[3*e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[
e + f*x]^6*Sin[3*f*x])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) + (Cos[5*e]*(a
+ 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*f*x])/(640*a^3*f*(a + b*
Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^6*(20*a
*b^4*Sin[e + f*x] + 17*b^5*Sin[e + f*x]))/(32*a^5*(a + b)^2*f*(a + b*Sec[e
+ f*x]^2)^3) - (b^5*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[e + f
*x])/(8*a^5*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.93, size = 995, normalized size = 4.65

$$\left[\frac{15\left(80\,a^{2}b^{5}+140\,ab^{6}+63\,b^{7}+\left(80\,a^{4}b^{3}+140\,a^{3}b^{4}+63\,a^{2}b^{5}\right)\cos\left(fx+e\right)^{4}+2\left(80\,a^{3}b^{4}+140\,a^{2}b^{5}+63\,ab^{6}\right)\mathrm{c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(80*a^2*b^5 + 140*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 6
3*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x
 + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
 + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(24*(a^8 + 3*a^7*b + 3*a^6*b^2
 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a
^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^
5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a
^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2
 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2
)*sin(f*x + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 +
 2*(a^10*b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 +
 3*a^8*b^3 + 3*a^7*b^4 + a^6*b^5)*f), 1/120*(15*(80*a^2*b^5 + 140*a*b^6 + 6
3*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*
b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (24*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a^3*b^5 +
2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^5*b^3 - 9
*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a^5*b^3 +
167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2 + 360*a^
5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2)*sin(f*x
 + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 + 2*(a^10*
b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 + 3*a^8*b^
3 + 3*a^7*b^4 + a^6*b^5)*f)]
```

giac [A] time = 0.32, size = 284, normalized size = 1.33

$$\frac{\frac{15\left(80\,a^{2}b^{3}+140\,ab^{4}+63\,b^{5}\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^{2}-ab}}\right)}{\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)\sqrt{-a^{2}-ab}}-\frac{15\left(20\,a^{2}b^{4}\sin\left(fx+e\right)^{3}+17\,ab^{5}\sin\left(fx+e\right)^{3}-20\,a^{2}b^{4}\sin\left(fx+e\right)-35\,ab^{5}\sin\left(fx+e\right)-15\,b^{6}\sin\left(fx \cdots\right)\right)}{\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)\left(a\sin\left(fx+e\right)^{2}-a-b\right)^{2}}\cdots}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*arctan(a*sin(f*x + e)/sqrt(-a^2
 - a*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt(-a^2 - a*b)) - 15*(20*a^2*b^4*sin(
f*x + e)^3 + 17*a*b^5*sin(f*x + e)^3 - 20*a^2*b^4*sin(f*x + e) - 35*a*b^5*s
in(f*x + e) - 15*b^6*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*(a*sin(f*x +
e)^2 - a - b)^2) + 8*(3*a^12*sin(f*x + e)^5 - 10*a^12*sin(f*x + e)^3 + 15*a
^11*b*sin(f*x + e)^3 + 15*a^12*sin(f*x + e) - 45*a^11*b*sin(f*x + e) + 90*a
^10*b^2*sin(f*x + e))/a^15)/f
```

maple [A] time = 1.32, size = 214, normalized size = 1.00

$$\frac{\frac{\frac{\left(\sin^{5}\left(fx+e\right)\right)a^{2}}{5}-\frac{2\left(\sin^{3}\left(fx+e\right)\right)a^{2}}{3}+\left(\sin^{3}\left(fx+e\right)\right)ab+a^{2}\sin\left(fx+e\right)-3\sin\left(fx+e\right)ab+6b^{2}\sin\left(fx+e\right)}{a^{5}}+\frac{b^{3}\left(\frac{-\frac{ab(20a+17b)\left(\sin^{3}\left(fx+e\right)\right)}{8\left(a^{2}+2ab+b^{2}\right)}+\frac{5(4a+3b)b\sin\left(fx+e\right)}{8(a+b)}}{\left(-a-b+a\left(\sin^{2}\left(fx+e\right)\right)\right)^{2}}-\cdots\right)}{a^{5}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(1/a^5*(1/5*sin(f*x+e)^5*a^2-2/3*sin(f*x+e)^3*a^2+sin(f*x+e)^3*a*b+a^2*
sin(f*x+e)-3*sin(f*x+e)*a*b+6*b^2*sin(f*x+e))+b^3/a^5*((-1/8*a*b*(20*a+17*b
```

```
)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+5/8*(4*a+3*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*si
n(f*x+e)^2)^2-1/8*(80*a^2+140*a*b+63*b^2)/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)*a
rctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

maxima [A] time = 0.46, size = 303, normalized size = 1.42

$$\frac{\frac{15\left(80a^2b^3+140ab^4+63b^5\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^7+2a^6b+a^5b^2\right)\sqrt{(a+b)a}}-\frac{30\left(\left(20a^2b^4+17ab^5\right)\sin\left(fx+e\right)^3-5\left(4a^2b^4+7ab^5+3b^6\right)\sin\left(fx+e\right)\right)}{a^9+4a^8b+6a^7b^2+4a^6b^3+a^5b^4+\left(a^9+2a^8b+a^7b^2\right)\sin\left(fx+e\right)^4-2\left(a^9+3a^8b+3a^7b^2+a^6b^3\right)\sin\left(\cdots\right.}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*log((a*sin(f*x + e) - sqrt((a +
 b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt
((a + b)*a)) - 30*((20*a^2*b^4 + 17*a*b^5)*sin(f*x + e)^3 - 5*(4*a^2*b^4 +
7*a*b^5 + 3*b^6)*sin(f*x + e))/(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5
*b^4 + (a^9 + 2*a^8*b + a^7*b^2)*sin(f*x + e)^4 - 2*(a^9 + 3*a^8*b + 3*a^7*
b^2 + a^6*b^3)*sin(f*x + e)^2) + 16*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 - 3*a*
b)*sin(f*x + e)^3 + 15*(a^2 - 3*a*b + 6*b^2)*sin(f*x + e))/a^5)/f
```

mupad [B] time = 0.33, size = 257, normalized size = 1.20

$$\frac{\frac{5\sin\left(e+fx\right)\left(3b^5+4ab^4\right)}{8(a+b)}-\frac{\sin\left(e+fx\right)^3\left(20a^2b^4+17ab^5\right)}{8(a+b)^2}}{f\left(2a^6b-\sin\left(e+fx\right)^2\left(2a^7+2ba^6\right)+a^7+a^5b^2+a^7\sin\left(e+fx\right)^4\right)}+\frac{\sin\left(e+fx\right)^5}{5a^3f}+\frac{\sin\left(e+fx\right)^3\left(\frac{a+b}{a^4}-\cdots\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((5*sin(e + f*x)*(4*a*b^4 + 3*b^5))/(8*(a + b)) - (sin(e + f*x)^3*(17*a*b^5
 + 20*a^2*b^4))/(8*(a + b)^2))/(f*(2*a^6*b - sin(e + f*x)^2*(2*a^6*b + 2*a^
7) + a^7 + a^5*b^2 + a^7*sin(e + f*x)^4)) + sin(e + f*x)^5/(5*a^3*f) + (sin
(e + f*x)^3*((a + b)/a^4 - 5/(3*a^3)))/f + (sin(e + f*x)*(10/a^3 - (3*(a +
b)^2)/a^5 + (3*(a + b)*((3*(a + b))/a^4 - 5/a^3))/a))/f + (b^3*atan((a^(1/2
)*sin(e + f*x)*1i)/(a + b)^(1/2))*(140*a*b + 80*a^2 + 63*b^2)*1i)/(8*a^(11/
2)*f*(a + b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.212 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=142

$$\frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}f(a+b)^{5/2}}-\frac{3a(a+2b)\tan(e+fx)}{8b^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)\sec^2(e+fx}{4bf(a+b)\left(a+b\tan^2(e+fx)\right.}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b
)^(5/2)/f-1/4*a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-3/
8*a*(a+2*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 413, 385, 205}

$$\frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}f(a+b)^{5/2}}-\frac{3a(a+2b)\tan(e+fx)}{8b^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)\sec^2(e+fx}{4bf(a+b)\left(a+b\tan^2(e+fx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(
5/2)*(a + b)^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(4*b*(a + b)*f*(a +
 b + b*Tan[e + f*x]^2)^2) - (3*a*(a + 2*b)*Tan[e + f*x])/(8*b^2*(a + b)^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+4b+(3a+4b)x^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \cdots$$

$$= \frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \cdots$$

Mathematica [A] time = 0.90, size = 125, normalized size = 0.88

$$\frac{\frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\sin(2(e+fx))\left(3a^2+3a(a+2b)\cos(2(e+fx))+16ab+16b^2\right)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2}}{8b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a +
b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f
*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8*b^
(5/2)*f)
```

fricas [B] time = 1.45, size = 722, normalized size = 5.08

$$\left[-\frac{\left(\left(3a^4+8a^3b+8a^2b^2\right)\cos\left(fx+e\right)^4+3a^2b^2+8ab^3+8b^4+2\left(3a^3b+8a^2b^2+8ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-ab} \cdots}{32\left(\left(a^5b^3+3a^4b^4+3 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3
 + 8*b^4 + 2*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(3*(a
^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8*
a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*
b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(f
*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f), -1/16*(((3*a^4 + 8*a^
3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 2*(3*a^3*b
+ 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b
```

```
)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(
a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8
*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2
*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(
f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f)]
```

giac [A] time = 0.36, size = 193, normalized size = 1.36

$$\frac{\frac{\left(\pi\left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,a^2+8\,ab+8\,b^2\right)}{\left(a^2b^2+2\,ab^3+b^4\right)\sqrt{ab+b^2}}-\frac{5\,a^2b\tan\left(fx+e\right)^3+8\,ab^2\tan\left(fx+e\right)^3+3\,a^3\tan\left(fx+e\right)+11\,a^2b\tan\left(fx+e\right)+8\,ab^2\tan\left(f\right.}{\left(a^2b^2+2\,ab^3+b^4\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 8*a*b + 8*b^2)/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(a*b + b^2)
) - (5*a^2*b*tan(f*x + e)^3 + 8*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) +
 11*a^2*b*tan(f*x + e) + 8*a*b^2*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)^2))/f
```

maple [B] time = 0.64, size = 294, normalized size = 2.07

$$-\frac{5a^2\left(\tan^3\left(fx+e\right)\right)}{8f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2 b\left(a^2+2ab+b^2\right)}-\frac{a\left(\tan^3\left(fx+e\right)\right)}{f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{}{8f\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -5/8/f/(a+b+b*tan(f*x+e)^2)^2*a^2/b/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/f/(a+b+b
*tan(f*x+e)^2)^2*a/(a^2+2*a*b+b^2)*tan(f*x+e)^3-3/8/f/(a+b+b*tan(f*x+e)^2)^
2*a^2/b^2/(a+b)*tan(f*x+e)-a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/
8/f/(a^2+2*a*b+b^2)/b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2)
)*a^2+1/f/(a^2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(
1/2))*a+1/f/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(
1/2))
```

maxima [A] time = 0.45, size = 210, normalized size = 1.48

$$\frac{\frac{\left(3\,a^2+8\,ab+8\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^2b^2+2\,ab^3+b^4\right)\sqrt{(a+b)b}}-\frac{\left(5\,a^2b+8\,ab^2\right)\tan\left(fx+e\right)^3+\left(3\,a^3+11\,a^2b+8\,ab^2\right)\tan\left(fx+e\right)}{a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6+\left(a^2b^4+2\,ab^5+b^6\right)\tan\left(fx+e\right)^4+2\left(a^3b^3+3\,a^2b^4+3\,ab^5+b^6\right)\tan\left(fx+e\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b
^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)) - ((5*a^2*b + 8*a*b^2)*tan(f*x + e)^3
+ (3*a^3 + 11*a^2*b + 8*a*b^2)*tan(f*x + e))/(a^4*b^2 + 4*a^3*b^3 + 6*a^2*b
^4 + 4*a*b^5 + b^6 + (a^2*b^4 + 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*b^3
+ 3*a^2*b^4 + 3*a*b^5 + b^6)*tan(f*x + e)^2))/f
```

mupad [B] time = 5.21, size = 149, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a+b}}\right)\left(3\,a^2+8\,a\,b+8\,b^2\right)}{8\,b^{5/2}\,f\,{\left(a+b\right)}^{5/2}}-\frac{\frac{{\tan\left(e+f\,x\right)}^3\,\left(5\,a^2+8\,b\,a\right)}{8\,b\,{\left(a+b\right)}^2}+\frac{\tan\left(e+f\,x\right)\,\left(3\,a^2+8\,b\,a\right)}{8\,b^2\,\left(a+b\right)}}{f\,\left(2\,a\,b+a^2+b^2+{\tan\left(e+f\,x\right)}^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,{\tan\left(e+f\,x\right)}^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(8*a*b + 3*a^2 + 8*b^2))/(8*b^(
5/2)*f*(a + b)^(5/2)) - ((tan(e + f*x)^3*(8*a*b + 5*a^2))/(8*b*(a + b)^2) +
 (tan(e + f*x)*(8*a*b + 3*a^2))/(8*b^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + ta
n(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

3.213 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=123

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}f(a+b)^{5/2}}+\frac{(a+4b)\tan(e+fx)}{8bf(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] 1/8*(a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/f-1/
4*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)*tan(f*x+e)/b/(a
+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 385, 199, 205}

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}f(a+b)^{5/2}}+\frac{(a+4b)\tan(e+fx)}{8bf(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(
5/2)*f) - (a*Tan[e + f*x])/(4*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (
(a + 4*b)*Tan[e + f*x])/(8*b*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+\right.}{4b(a+b)f}$$

$$= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a}{}$$

$$= \frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f} - \frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+}{8b(a+b)^2f}$$

Mathematica [C] time = 3.50, size = 283, normalized size = 2.30

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{((a+4b)\sin(2e)-(a-2b)\sin(2fx))(a\cos(2(e+fx))+a+2b)}{b(\cos(e)-\sin(e))(\sin(e)+\cos(e))} - \frac{4(a+b)((a+2b)\sin(2e)-a\sin(2fx))}{a(\cos(e)-\sin(e))(\sin(e)+\cos(e))}\right.}{64f(a+b)^2\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-(((a + 4*b)*ArcTan[(Sec[f*
x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2
*(Cos[2*e] - I*Sin[2*e]))/(b*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) -
(4*(a + b)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e
] + Sin[e])) + ((a + 2*b + a*Cos[2*(e + f*x)])*((a + 4*b)*Sin[2*e] - (a - 2
*b)*Sin[2*f*x]))/(b*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(64*(a + b)^2*f*
(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.88, size = 654, normalized size = 5.32

$$\left[-\frac{\left(\left(a^3+4a^2b\right)\cos\left(fx+e\right)^4+ab^2+4b^3+2\left(a^2b+4ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-}{}\right.}{32\left(\left(a^5b^2+3a^4b^3+3a^3b^4+a^2b^5\right)f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^
2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos
(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b
*cos(f*x + e)^2 + b^2)) + 4*((a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^3 - (
a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^
3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3 + 3*a^3*b^4 + 3*a^2*
```

```
b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f), -
1/16*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^2)
*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*((a^3*b - a^2*b^2 - 2*a*b^
3)*cos(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3
 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 +
 3*a*b^6 + b^7)*f)]
```

giac [A] time = 1.36, size = 171, normalized size = 1.39

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)(a+4\,b)}{\left(a^2b+2\,ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{ab\tan\left(fx+e\right)^3+4\,b^2\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)+3\,ab\tan\left(fx+e\right)+4\,b^2\tan\left(fx+e\right)}{\left(a^2b+2\,ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 4*b)/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b + b^2)) + (a*b*tan(f*x
 + e)^3 + 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) + 3*a*b*tan(f*x + e) + 4*
b^2*tan(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2))/f
```

maple [B] time = 0.62, size = 238, normalized size = 1.93

$$\frac{a\left(\tan^3\left(fx+e\right)\right)}{8f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}+\frac{\left(\tan^3\left(fx+e\right)\right)b}{2f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{}{8b\left(a+b\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/8/f/(a+b+b*tan(f*x+e)^2)^2*a/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/2/f/(a+b+b*ta
n(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3*b-1/8*a*tan(f*x+e)/b/(a+b)/f/(a+
b+b*tan(f*x+e)^2)^2+1/2*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8/f/(a^
2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a+1/2/f
/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.46, size = 187, normalized size = 1.52

$$\frac{\frac{(a+4\,b)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^2b+2\,ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{\left(ab+4\,b^2\right)\tan\left(fx+e\right)^3-\left(a^2-3\,ab-4\,b^2\right)\tan\left(fx+e\right)}{a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5+\left(a^2b^3+2\,ab^4+b^5\right)\tan\left(fx+e\right)^4+2\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\tan\left(fx+e\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a + 4*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b + 2*a*b^2 + b
^3)*sqrt((a + b)*b)) + ((a*b + 4*b^2)*tan(f*x + e)^3 - (a^2 - 3*a*b - 4*b^2
)*tan(f*x + e))/(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 + (a^2*b^3 +
 2*a*b^4 + b^5)*tan(f*x + e)^4 + 2*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*ta
n(f*x + e)^2))/f
```

mupad [B] time = 5.10, size = 125, normalized size = 1.02

$$\frac{\frac{\tan\left(e+f\,x\right)^3\,(a+4\,b)}{8\,(a+b)^2}-\frac{\tan\left(e+f\,x\right)\,(a-4\,b)}{8\,b\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^4\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a+b}}\right)\,(a+4\,b)}{8\,b^{3/2}\,f\,{(a+b)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((tan(e + f*x)^3*(a + 4*b))/(8*(a + b)^2) - (tan(e + f*x)*(a - 4*b))/(8*b*(
a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e
 + f*x)^4)) + (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 4*b))/(8*b^(
3/2)*f*(a + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

3.214 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,f(a+b)^{5/2}}+\frac{3\tan(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\tan(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] 3/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(5/2)/f/b^(1/2)+1/4*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x
+e)^2)
```

Rubi [A] time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 199, 205}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,f(a+b)^{5/2}}+\frac{3\tan(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\tan(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*f)
+ Tan[e + f*x]/(4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (3*Tan[e + f*x]
)/(8*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{3\operatorname{Sub}}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,(a+b)^{5/2}f} + \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+}{8(a+b)^2f\left(a+b+\right.}$$

Mathematica [C] time = 2.54, size = 265, normalized size = 2.50

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\sec(2e)\left(a(5a+2b)\sin(2fx)-\left(5a^2+16ab+8b^2\right)\sin(2e)\right)(a\cos(2(e+fx))+a+2b)}{a^2} + \frac{4b(a+b)\sec(2e)}{}\right.}{64f(a+b)^2\left(a+b\sec^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-3*ArcTan[(Sec[f*x]*(Cos[2
*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*b*(a + b)
*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/a^2 + ((a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[2*e]*(-((5*a^2 + 16*a*b + 8*b^2)*Sin[2*e]) + a*(5*a + 2*b)*Si
n[2*f*x]))/a^2))/(64*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.96, size = 580, normalized size = 5.47

$$\left[-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-ab-b^2}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left((a+2\,b\right.}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(\right.}\right.}{32\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^4+2\left(a^4b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a*b - b^2
)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^
2*b + 7*a*b^2 + 2*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f
*x + e))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a
^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*f), -1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
```

```
^2 + b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^2*b + 7*a*b^2 + 2*b^3)*cos(
f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f*x + e))/((a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*f)]
```

giac [A] time = 0.42, size = 130, normalized size = 1.23

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,b\tan(fx+e)^3+5\,a\tan(fx+e)+5\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2(a^2+2\,ab+b^2)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^3 + 5*
a*tan(f*x + e) + 5*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*(a^2 + 2*a
*b + b^2)))/f
```

maple [A] time = 0.79, size = 97, normalized size = 0.92

$$\frac{\tan(fx+e)}{4\,(a+b)\,f\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{3\tan(fx+e)}{8\,(a+b)^2\,f\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{3\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{8f\,(a+b)^2\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/4*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b
+b*tan(f*x+e)^2)+3/8/f/(a+b)^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b
)^(1/2))
```

maxima [A] time = 0.46, size = 156, normalized size = 1.47

$$\frac{\frac{3\,b\tan(fx+e)^3+5\,(a+b)\tan(fx+e)}{(a^2b^2+2\,ab^3+b^4)\tan(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4+2\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b*tan(f*x + e)^3 + 5*(a + b)*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^
4)*tan(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 2*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^2) + 3*arctan(b*tan(f*x + e)/sqrt((
a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)))/f
```

mupad [B] time = 5.00, size = 112, normalized size = 1.06

$$\frac{\frac{5\tan(e+f\,x)}{8\,(a+b)}+\frac{3\,b\tan(e+f\,x)^3}{8\,(a+b)^2}}{f\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{8\,\sqrt{b}\,f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((5*tan(e + f*x))/(8*(a + b)) + (3*b*tan(e + f*x)^3)/(8*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (3
*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(8*b^(1/2)*f*(a + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

3.215 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b\tan(e+}{4af(a+b)\left(a+b\tan\right.}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e}{4af(a+b)\left(a+b\,\mathrm{t}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{\text{Subst}\left(\int\frac{1}{\left(1+x^2\right)\left(a+b+bx^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{\text{Subst}\left(\int\frac{4a+b-3bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^2}\,dx,x,\tan(e+fx\right.$$

$$=-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}+$$

$$=-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}+$$

$$=\frac{x}{a^3}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f}-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+\right.}$$

Mathematica [C] time = 5.57, size = 332, normalized size = 2.31

$$\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b\left(\left(9a^2+28ab+16b^2\right)\sin(2e)-3a(3a+2b)\sin(2fx)\right)(a\cos(2(e+fx))+a+2b)}{f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b\left(15a^2+20ab+8b^2\right)}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
```

```
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.78, size = 819, normalized size = 5.69

$$\left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos\left(fx+e\right)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos\left(fx+e\right)^2+32\left(a^2b^2+2ab^3+b^4\right)fx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

giac [A] time = 1.42, size = 205, normalized size = 1.42

$$-\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7ab^2\tan\left(fx+e\right)^3+4b^3\tan\left(fx+e\right)^3+9a^2b\tan\left(fx+e\right)+13ab^2\tan\left(fx+e\right)+4b^3}{\left(a^4+2a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

maple [B] time = 1.12, size = 321, normalized size = 2.23

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8fa\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{b^3\left(\tan^3\left(fx+e\right)\right)}{2f\,a^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{}{8a\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a*b^2/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/2/f/a^2*
b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a/
(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/2/f/a^2*b^2/(a+b+b*tan(f*x+e)^2)^2/(a+b)*t
an(f*x+e)-15/8/f/a*b/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((
a+b)*b)^(1/2))-5/2/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x
+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b^3/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(t
an(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.46, size = 231, normalized size = 1.60

$$-\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{(a+b)b}}+\frac{\left(7ab^2+4b^3\right)\tan\left(fx+e\right)^3+\left(9a^2b+13ab^2+4b^3\right)\tan\left(fx+e\right)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4+\left(a^4b^2+2a^3b^3+a^2b^4\right)\tan\left(fx+e\right)^4+2\left(a^5b+3a^4b^2+3a^3b^3+a^2b^4\right)\tan\left(fx\right.}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

mupad [B] time = 9.33, size = 3271, normalized size = 22.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a
^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b
^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 +
a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b
^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e +
f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(
64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6
 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9
*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7
+ 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2
))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3)
 - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^
4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a
```

```
^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*
x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2
*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6
 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^
8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3
+ 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)
*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3
+ 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*
a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^
5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^
2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))
 + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*
a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6
*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b
+ a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1
/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 +
3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4
 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7
*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^
(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105
*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e
 + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(
32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*
a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^
6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b
 + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b
+ a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b
^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b
^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*
(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^
5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))))*(-b*(a + b)^5)^(1/2)*(20*a*b +
15*a^2 + 8*b^2)*1i)/(8*f*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
+ 10*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

3.216 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=201

$$\frac{x(a-6b)}{2a^4}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b(2a+3b)\tan(e+fx)}{4a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{b^{3/2}\left(35a^2+56ab+24\right.}{8a^4f(a}$$

```
[Out] 1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x
+e)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan
(f*x+e)^2)^2+1/4*b*(2*a+3*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+
1/8*b*(4*a+3*b)*(a+4*b)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.34, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{5/2}}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b(2a+3b)\tan(e+}{4a^2f(a+b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*f) + (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(2*a + 3*b)*Tan[e + f*x])
/(4*a^2*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(4*a + 3*b)*(a + 4*b)*
Tan[e + f*x])/(8*a^3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a+b-5bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4}{8a^3(a}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4}{8a^3(a}$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)\sin}{2af\left(a+b+b\tan\right.}$$

Mathematica [A] time = 3.47, size = 156, normalized size = 0.78

$$\frac{\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\sin(2(e+fx))\left(\frac{2b^3(5a\cos(2(e+fx))+3a+8b)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2} + \frac{13ab^2}{(a+b)^2(a\cos(2(e+fx))+a+2b)} + 2\right) +}{8a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(e + f*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^2*
```

```
(a + 2*b + a*Cos[2*(e + f*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cos[2*(e + f*x)])
)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))*Sin[2*(e + f*x)])/(8*a^4*f)
```

fricas [B] time = 0.60, size = 970, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 32*
(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^3*b^2
 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x + (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (3
5*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*
b^3 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(4*(a^5 + 2*
a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f
*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), 1/16*(8*(a^5 - 4
*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 16*(a^4*b - 4*a^3*b^2
 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 8*(a^3*b^2 - 4*a^2*b^3 - 11*a
*b^4 - 6*b^5)*f*x - (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^4*b + 56*a^3*b^
2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3 + 24*a*b^4)*cos
(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(4*(a^5 + 2*a^4*b + a^3*b^2)
*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + (4*a
^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^8 + 2*a^7*b
 + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x +
e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

giac [A] time = 0.34, size = 239, normalized size = 1.19

$$\frac{\frac{\left(35\,a^2b^2+56\,ab^3+24\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{ab+b^2}}+\frac{11\,ab^3\tan\left(fx+e\right)^3+8\,b^4\tan\left(fx+e\right)^3+13\,a^2b^2\tan\left(fx+e\right)+21\,ab^3\tan\left(fx+e\right)+8\,b^4}{\left(a^5+2\,a^4b+a^3b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(a
*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 8*b^4*tan(f*x + e)^3 + 13*a^2*b^2*t
an(f*x + e) + 21*a*b^3*tan(f*x + e) + 8*b^4*tan(f*x + e))/((a^5 + 2*a^4*b +
 a^3*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 4*(f*x + e)*(a - 6*b)/a^4 + 4*tan
(f*x + e)/((tan(f*x + e)^2 + 1)*a^3))/f
```

maple [A] time = 1.61, size = 366, normalized size = 1.82

$$\frac{11b^3\left(\tan^3\left(fx+e\right)\right)}{8f\,a^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}+\frac{b^4\left(\tan^3\left(fx+e\right)\right)}{f\,a^3\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}+\frac{}{8f\,a^2\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 11/8/f/a^2*b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/f/a^3*
b^4/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3+13/8/f/a^2*b^2/(a+b
+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)+1/f/a^3*b^3/(a+b+b*tan(f*x+e)^2)^2/(a+b
)*tan(f*x+e)+35/8/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+
e)*b/((a+b)*b)^(1/2))+7/f/a^3*b^3/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(ta
n(f*x+e)*b/((a+b)*b)^(1/2))+3/f/a^4*b^4/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arc
tan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a^3*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2
/f/a^3*arctan(tan(f*x+e))-3/f/a^4*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.46, size = 338, normalized size = 1.68

$$\frac{\frac{\left(35\,a^2b^2+56\,ab^3+24\,b^4\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{(a+b)b}}+\frac{\left(4\,a^2b^2+19\,ab^3+12\,b^4\right)\tan\left(fx+e\right)^5+\left(8\,a^3b+37\,a^2b^2+56\,ab^3+24\,b^4\right)\tan\left(fx+e\right)^3+(4}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+\left(a^5b^2+2\,a^4b^3+a^3b^4\right)\tan\left(fx+e\right)^6+\left(2\,a^6b+7\,a^5b^2+8\,a^4b^3+3\,a^3b^4\right)\mathrm{ta}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)) + ((4*a^2*b^2 + 19*a*b^3 + 12
*b^4)*tan(f*x + e)^5 + (8*a^3*b + 37*a^2*b^2 + 56*a*b^3 + 24*b^4)*tan(f*x +
 e)^3 + (4*a^4 + 16*a^3*b + 37*a^2*b^2 + 37*a*b^3 + 12*b^4)*tan(f*x + e))/(
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3 + a^
3*b^4)*tan(f*x + e)^6 + (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 3*a^3*b^4)*tan(f
*x + e)^4 + (a^7 + 6*a^6*b + 12*a^5*b^2 + 10*a^4*b^3 + 3*a^3*b^4)*tan(f*x +
 e)^2) + 4*(f*x + e)*(a - 6*b)/a^4)/f
```

mupad [B] time = 9.97, size = 3708, normalized size = 18.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^5*(19*a*b^3 + 12*b^4 + 4*a^2*b^2))/(8*a^3*(a + b)^2) + (tan(
e + f*x)*(25*a*b^2 + 12*a^2*b + 4*a^3 + 12*b^3))/(8*a^3*(a + b)) + (b*tan(e
 + f*x)^3*(56*a*b^2 + 37*a^2*b + 8*a^3 + 24*b^3))/(8*a^3*(a + b)^2))/(f*(2*
a*b + tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4*(2*
a*b + 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((((((6*a^8*b^7 + (49*a^9*b^6)/
2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^
13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i)*(512*
a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 25
6*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*
(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^
7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b +
 a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4) - ((((
(6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 -
2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e
+ f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^1
1*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 +
 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (tan(e + f*x)*(4800*a*b^
8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 1
6*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i
- b*6i)*1i)/(4*a^4))/(((405*a*b^8)/4 + 27*b^9 + (261*a^2*b^7)/2 + (1877*a^3
*b^6)/32 - (49*a^4*b^5)/64 - (35*a^5*b^4)/16)/(4*a^12*b + a^13 + a^9*b^4 +
4*a^10*b^3 + 6*a^11*b^2) + (((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 +
(45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a
^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9
```

*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*
a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*
a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 +
1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 +
4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (((((6*a^8*b^7 + (49*a^9*
b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b
+ a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(a*1i - b*6i)*
(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3
+ 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^
2)))*(a*1i - b*6i))/(4*a^4) + (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a
^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^
9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4)))*(a
*1i - b*6i)*1i)/(2*a^4*f) - (atan(((((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 +
7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(3
2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (((6*a^8*b^7 + (49*
a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^
12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(-b^3*(a +
b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*
a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a
^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4
+ 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*
b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))
(-b^3(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(16*(5*a^8*b + a^9 +
a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)) + (((tan(e + f*x)*(4800*a*
b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 +
16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (((
6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2
*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e +
f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304
*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(
512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4
*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*
b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3
+ 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(16*
(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))/(((405*a*
b^8)/4 + 27*b^9 + (261*a^2*b^7)/2 + (1877*a^3*b^6)/32 - (49*a^4*b^5)/64 - (
35*a^5*b^4)/16)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (((
tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^
4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^
3 + 6*a^8*b^2)) - (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^
4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6
*a^11*b^2) - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2
)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b
^3 + 256*a^13*b^2))/(512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a
+ b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5
*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a
^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^
7*b^2)) + (((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*
b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b
^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5
+ (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4
*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35
*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4
+ 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3
+ 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^
2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9
+ a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(
56*a*b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^

```
6*b^3 + 10*a^7*b^2))))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i
)/(8*f*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.217 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=269

$$\frac{(3a-8b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{3x\left(a^2-4ab+16b^2\right)}{8a^5}-\frac{3b^{5/2}\left(21a^2+36ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f(a+b)^{5/2}}+\frac{3b(a+}{8a^4f(}$$

```
[Out] 3/8*(a^2-4*a*b+16*b^2)*x/a^5-3/8*b^(5/2)*(21*a^2+36*a*b+16*b^2)*arctan(b^(1
/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(5/2)/f+1/8*(3*a-8*b)*cos(f*x+e)*sin(
f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*
tan(f*x+e)^2)^2+1/8*b*(3*a^2-7*a*b-12*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*ta
n(f*x+e)^2)^2+3/8*b*(a+2*b)*(a^2-4*a*b-4*b^2)*tan(f*x+e)/a^4/(a+b)^2/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.38, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$-\frac{3b^{5/2}\left(21a^2+36ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f(a+b)^{5/2}}+\frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b\left(3a^2-7ab-12b^{\cdot}\right.}{8a^3f(a+b)\left(a+b\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 4*a*b + 16*b^2)*x)/(8*a^5) - (3*b^(5/2)*(21*a^2 + 36*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*(a + b)^(5/2)*f) + ((3
*a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*
(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e +
f*x]^2)^2) + (3*b*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Tan[e + f*x])/(8*a^4*(a +
 b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-3a+b-7bx^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \ldots\right.}{}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b\left(3a^2-\ldots\right.}{8a^3(a+b)\ldots}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b\left(3a^2-\ldots\right.}{8a^3(a+b)\ldots}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b\left(3a^2-\ldots\right.}{8a^3(a+b)\ldots}$$

$$= \frac{3\left(a^2-4ab+16b^2\right)x}{8a^5} - \frac{3b^{5/2}\left(21a^2+36ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} + \frac{(3a-\ldots}{8a^2\ldots}$$

Mathematica [C] time = 6.53, size = 1430, normalized size = 5.32

$$\frac{\left(21a^2+36ba+16b^2\right)(\cos(2e+2fx)a+a+2b)^3\left(\frac{3b^3\tan^{-1}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)\right.}{64a^5\sqrt{a+b}\,f\sqrt{b\cos(4e)-ib\sin(4e)}}\right.}{(a+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((21*a^2 + 36*a*b + 16*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*((3*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin
[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(
-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^5*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - (((3*I)/64)*b^3*ArcTan[Sec[f*x]*(
Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])
/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*
x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e +
 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(144*a^6*f*x*Cos[2*e] + 96*a^5*b*f*x*Cos[2
*e] + 912*a^4*b^2*f*x*Cos[2*e] + 6720*a^3*b^3*f*x*Cos[2*e] + 16512*a^2*b^4*
f*x*Cos[2*e] + 16896*a*b^5*f*x*Cos[2*e] + 6144*b^6*f*x*Cos[2*e] + 96*a^6*f*
x*Cos[2*f*x] + 480*a^4*b^2*f*x*Cos[2*f*x] + 4416*a^3*b^3*f*x*Cos[2*f*x] + 6
912*a^2*b^4*f*x*Cos[2*f*x] + 3072*a*b^5*f*x*Cos[2*f*x] + 96*a^6*f*x*Cos[4*e
 + 2*f*x] + 480*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 4416*a^3*b^3*f*x*Cos[4*e + 2
*f*x] + 6912*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 3072*a*b^5*f*x*Cos[4*e + 2*f*x]
 + 24*a^6*f*x*Cos[2*e + 4*f*x] - 48*a^5*b*f*x*Cos[2*e + 4*f*x] + 216*a^4*b^
2*f*x*Cos[2*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[2*e + 4*f*x] + 384*a^2*b^4*f*x
*Cos[2*e + 4*f*x] + 24*a^6*f*x*Cos[6*e + 4*f*x] - 48*a^5*b*f*x*Cos[6*e + 4*
f*x] + 216*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[6*e + 4*f*x]
+ 384*a^2*b^4*f*x*Cos[6*e + 4*f*x] + 816*a^3*b^3*Sin[2*e] + 2848*a^2*b^4*Si
n[2*e] + 3968*a*b^5*Sin[2*e] + 1792*b^6*Sin[2*e] + 44*a^6*Sin[2*f*x] + 104*
a^5*b*Sin[2*f*x] - 180*a^4*b^2*Sin[2*f*x] - 1696*a^3*b^3*Sin[2*f*x] - 3264*
a^2*b^4*Sin[2*f*x] - 1664*a*b^5*Sin[2*f*x] + 44*a^6*Sin[4*e + 2*f*x] + 104*
a^5*b*Sin[4*e + 2*f*x] - 180*a^4*b^2*Sin[4*e + 2*f*x] - 608*a^3*b^3*Sin[4*e
 + 2*f*x] - 192*a^2*b^4*Sin[4*e + 2*f*x] + 128*a*b^5*Sin[4*e + 2*f*x] + 38*
a^6*Sin[2*e + 4*f*x] + 60*a^5*b*Sin[2*e + 4*f*x] - 170*a^4*b^2*Sin[2*e + 4*
f*x] - 640*a^3*b^3*Sin[2*e + 4*f*x] - 400*a^2*b^4*Sin[2*e + 4*f*x] + 38*a^6
*Sin[6*e + 4*f*x] + 60*a^5*b*Sin[6*e + 4*f*x] - 170*a^4*b^2*Sin[6*e + 4*f*x
] - 368*a^3*b^3*Sin[6*e + 4*f*x] - 176*a^2*b^4*Sin[6*e + 4*f*x] + 12*a^6*Si
n[4*e + 6*f*x] + 8*a^5*b*Sin[4*e + 6*f*x] - 20*a^4*b^2*Sin[4*e + 6*f*x] - 1
6*a^3*b^3*Sin[4*e + 6*f*x] + 12*a^6*Sin[8*e + 6*f*x] + 8*a^5*b*Sin[8*e + 6*
f*x] - 20*a^4*b^2*Sin[8*e + 6*f*x] - 16*a^3*b^3*Sin[8*e + 6*f*x] + a^6*Sin[
6*e + 8*f*x] + 2*a^5*b*Sin[6*e + 8*f*x] + a^4*b^2*Sin[6*e + 8*f*x] + a^6*Si
n[10*e + 8*f*x] + 2*a^5*b*Sin[10*e + 8*f*x] + a^4*b^2*Sin[10*e + 8*f*x]))/(
2048*a^5*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.84, size = 1129, normalized size = 4.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x
 + e)^4 + 24*(a^5*b - 2*a^4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*co
s(f*x + e)^2 + 12*(a^4*b^2 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x
 + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^
```

```
4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e)^2)*
sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*
cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) + 4*(2*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7
+ (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10
*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3
 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*sin(f*x + e))/((a^9 + 2*a^8*b + a^7*
b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 +
(a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), 1/16*(6*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28
*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x + e)^4 + 12*(a^5*b - 2*a^4*b^2 + 9*a^3*b
^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*cos(f*x + e)^2 + 6*(a^4*b^2 - 2*a^3*b^3 + 9
*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21
*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2
*b^4 + 16*a*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^6
+ 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3
*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos
(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*
sin(f*x + e))/((a^9 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^
7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

giac [A] time = 0.66, size = 491, normalized size = 1.83

$$-\frac{\frac{3\left(21\,a^2b^3+36\,ab^4+16\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+2\,a^6b+a^5b^2\right)\sqrt{ab+b^2}}-\frac{3\,a^3b^2\tan\left(fx+e\right)^7-6\,a^2b^3\tan\left(fx+e\right)^7-36\,ab^4\tan\left(fx+e\right)^7-24\,b^5\tan\left(fx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 2*a^6*b + a^5*b^2)*sqr
t(a*b + b^2)) - (3*a^3*b^2*tan(f*x + e)^7 - 6*a^2*b^3*tan(f*x + e)^7 - 36*a
*b^4*tan(f*x + e)^7 - 24*b^5*tan(f*x + e)^7 + 6*a^4*b*tan(f*x + e)^5 - a^3*
b^2*tan(f*x + e)^5 - 73*a^2*b^3*tan(f*x + e)^5 - 144*a*b^4*tan(f*x + e)^5 -
 72*b^5*tan(f*x + e)^5 + 3*a^5*tan(f*x + e)^3 + 10*a^4*b*tan(f*x + e)^3 - 2
4*a^3*b^2*tan(f*x + e)^3 - 136*a^2*b^3*tan(f*x + e)^3 - 180*a*b^4*tan(f*x +
 e)^3 - 72*b^5*tan(f*x + e)^3 + 5*a^5*tan(f*x + e) + 8*a^4*b*tan(f*x + e) -
 18*a^3*b^2*tan(f*x + e) - 69*a^2*b^3*tan(f*x + e) - 72*a*b^4*tan(f*x + e)
- 24*b^5*tan(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2) - 3*(a^2 - 4*a*b + 16*b^2)*(f*
x + e)/a^5)/f
```

maple [A] time = 1.67, size = 470, normalized size = 1.75

$$-\frac{15b^4\left(\tan^3\left(fx+e\right)\right)}{8f\,a^3\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{3b^5\left(\tan^3\left(fx+e\right)\right)}{2f\,a^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{}{8f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -15/8/f/a^3*b^4/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-3/2/f/a
^4*b^5/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-17/8/f/a^3*b^3/(
a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)-3/2/f/a^4*b^4/(a+b+b*tan(f*x+e)^2)^2
/(a+b)*tan(f*x+e)-63/8/f/a^3*b^3/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan
```

```
(f*x+e)*b/((a+b)*b)^(1/2))-27/2/f/a^4*b^4/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*a
rctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-6/f/a^5*b^5/(a^2+2*a*b+b^2)/((a+b)*b)^(
1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/8/f/a^3/(tan(f*x+e)^2+1)^2*tan(
f*x+e)^3-3/2/f/a^4/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-3/2/f/a^4/(tan(f*x+e)^
2+1)^2*tan(f*x+e)*b+5/8/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)+6/f/a^5*arctan(
tan(f*x+e))*b^2+3/8/f/a^3*arctan(tan(f*x+e))-3/2/f/a^4*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.47, size = 464, normalized size = 1.72

$$-\frac{\frac{3\left(21\,a^2b^3+36\,ab^4+16\,b^5\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^7+2\,a^6b+a^5b^2\right)\sqrt{(a+b)b}}-\frac{3\left(a^3b^2-2\,a^2b^3-12\,ab^4-8\,b^5\right)\tan\left(fx+e\right)^7+\left(6\,a^4b-a^3b^2-73\,a^2b^3-144\,ab^4-72\,b^5\right)\tan\left(fx+e\right)^5+\left(3\,a^5\cdots\right.}{\left(a^6b^2+2\,a^5b^3+a^4b^4\right)\tan\left(fx+e\right)^8+a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+2\left(a^7b+4\,a^6b^2+5\,a^5b^3+2\,a^4b^4\right)\mathrm{t}\cdots}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)) - (3*(a^3*b^2 - 2*a^2*b^3
- 12*a*b^4 - 8*b^5)*tan(f*x + e)^7 + (6*a^4*b - a^3*b^2 - 73*a^2*b^3 - 144*
a*b^4 - 72*b^5)*tan(f*x + e)^5 + (3*a^5 + 10*a^4*b - 24*a^3*b^2 - 136*a^2*b
^3 - 180*a*b^4 - 72*b^5)*tan(f*x + e)^3 + (5*a^5 + 8*a^4*b - 18*a^3*b^2 - 6
9*a^2*b^3 - 72*a*b^4 - 24*b^5)*tan(f*x + e))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^
4)*tan(f*x + e)^8 + a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 + 2*(a^
7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^6 + (a^8 + 8*a^7*b +
19*a^6*b^2 + 18*a^5*b^3 + 6*a^4*b^4)*tan(f*x + e)^4 + 2*(a^8 + 5*a^7*b + 9*
a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^2) - 3*(a^2 - 4*a*b + 16*b^2)
*(f*x + e)/a^5)/f
```

mupad [B] time = 9.59, size = 4158, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^
3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))
/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^10*b
^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^1
5*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^1
4*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*
(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b
^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^
2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5
*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5
 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b +
21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3
 + 10*a^8*b^2)) + (((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9
 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 +
9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) + (3*
((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b
^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*
b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2
 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 +
1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3
+ 6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b
^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^1
0 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)
```

```
*(36*a*b + 21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 +
 10*a^7*b^3 + 10*a^8*b^2)))/((756*a*b^11 + 216*b^12 + (1755*a^2*b^10)/2 + (
1215*a^3*b^9)/4 - (1701*a^4*b^8)/32 - (351*a^5*b^7)/64 + (1215*a^6*b^6)/128
 - (567*a^7*b^5)/256)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2
) - (3*((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3
*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/
(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^10*b^
8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15
*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14
*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*(
512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^
3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2
)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*
(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5
+ 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 2
1*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 1
0*a^8*b^2)) + (3*((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 +
 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*
a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) + (3*((
12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4
 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^
3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 +
 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 15
36*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 +
6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2
)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10
+ a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(
36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a
^7*b^3 + 10*a^8*b^2))))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*3
i)/(8*f*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)) -
 (atan(((((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)
/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*
b^4 + 4*a^13*b^3 + 6*a^14*b^2) - (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i
)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14
*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a
^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) - (tan(e + f*x)*(18432*a*b
^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^
6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 +
 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(16*a^5) - (((3*
((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b
^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*
b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b
^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^
15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a
^2*1i - a*b*4i + b^2*16i))/(16*a^5) + (tan(e + f*x)*(18432*a*b^10 + 4608*b^
11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b
^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 +
6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(16*a^5))/((756*a*b^11 + 216*
b^12 + (1755*a^2*b^10)/2 + (1215*a^3*b^9)/4 - (1701*a^4*b^8)/32 - (351*a^5*
b^7)/64 + (1215*a^6*b^6)/128 - (567*a^7*b^5)/256)/(4*a^15*b + a^16 + a^12*b
^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a
^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2
)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) - (3*tan(e + f*x)*
(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 +
 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 +
 a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5)
- (tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 +
 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4
*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2
```

```
*16i))/(16*a^5) + (3*((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (
87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b +
a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(a^2*1i - a*b*
4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4
 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a
^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) + (tan(e + f*x
)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7
+ 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12
 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5
)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(8*a^5*f) - ((tan(e + f*x)*(48*a*b^3 -
3*a^3*b - 5*a^4 + 24*b^4 + 21*a^2*b^2))/(8*a^4*(a + b)) + (tan(e + f*x)^5*(
144*a*b^4 - 6*a^4*b + 72*b^5 + 73*a^2*b^3 + a^3*b^2))/(8*a^4*(a + b)^2) + (
tan(e + f*x)^3*(180*a*b^4 - 10*a^4*b - 3*a^5 + 72*b^5 + 136*a^2*b^3 + 24*a^
3*b^2))/(8*a^4*(a + b)^2) + (3*b*tan(e + f*x)^7*(12*a*b^3 - a^3*b + 8*b^4 +
 2*a^2*b^2))/(8*a^4*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^4*(6*a*b + a^2 + 6
*b^2) + a^2 + b^2 + tan(e + f*x)^6*(2*a*b + 4*b^2) + b^2*tan(e + f*x)^8 + t
an(e + f*x)^2*(6*a*b + 2*a^2 + 4*b^2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.218 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=352

$$\frac{5(a-2b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{b^{7/2}\left(99a^2+176ab+80b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f(a+b)^{5/2}}+\frac{\left(15a^2-34ab+80b^2\right)\sin}{48a^3f\left(a+b\tan^2(\right.}$$

```
[Out] 1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*x/a^6+1/8*b^(7/2)*(99*a^2+176*a*b+80
*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^6/(a+b)^(5/2)/f+1/48*(15*a^2
-34*a*b+80*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+5/24*(a-
2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^5*
sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/48*b*(15*a^3-29*a^2*b+64*a*b^2+120*
b^3)*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/16*b*(5*a^4-8*a^3*b+17
*a^2*b^2+116*a*b^3+80*b^4)*tan(f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.48, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{7/2}\left(99a^2+176ab+80b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f(a+b)^{5/2}}+\frac{b\left(17a^2b^2-8a^3b+5a^4+116ab^3+80b^4\right)\tan(e+fx)}{16a^5f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b\left(-29\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*x)/(16*a^6) + (b^(7/2)*(99*a^2 + 1
76*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*(a + b)
^(5/2)*f) + ((15*a^2 - 34*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*
f*(a + b + b*Tan[e + f*x]^2)^2) + (5*(a - 2*b)*Cos[e + f*x]^3*Sin[e + f*x])
/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^5*Sin[e + f*x])/(6
*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(15*a^3 - 29*a^2*b + 64*a*b^2 + 120
*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(5
*a^4 - 8*a^3*b + 17*a^2*b^2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(16*a^5*(a
+ b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-5a+b-9bx^2}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right.}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right.}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right.}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right.}$$

$$= \frac{\left(5a^3-18a^2b+48ab^2-160b^3\right)x}{16a^6} + \frac{b^{7/2}\left(99a^2+176ab+80b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+f}{\sqrt{a+b}}\right.}{8a^6(a+b)^{5/2}f}$$

Mathematica [C] time = 6.61, size = 1770, normalized size = 5.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((99*a^2 + 176*a*b + 80*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^
6*(-1/64*(b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]
]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^6*Sqrt[
a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Sin[2*e])/(a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Si
n[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2
*f*x])*Sec[2*e]*Sec[e + f*x]^6*(720*a^7*f*x*Cos[2*e] + 768*a^6*b*f*x*Cos[2*
e] + 1296*a^5*b^2*f*x*Cos[2*e] - 8352*a^4*b^3*f*x*Cos[2*e] - 64128*a^3*b^4*
f*x*Cos[2*e] - 158976*a^2*b^5*f*x*Cos[2*e] - 165888*a*b^6*f*x*Cos[2*e] - 61
440*b^7*f*x*Cos[2*e] + 480*a^7*f*x*Cos[2*f*x] + 192*a^6*b*f*x*Cos[2*f*x] +
96*a^5*b^2*f*x*Cos[2*f*x] - 4608*a^4*b^3*f*x*Cos[2*f*x] - 41856*a^3*b^4*f*x
*Cos[2*f*x] - 67584*a^2*b^5*f*x*Cos[2*f*x] - 30720*a*b^6*f*x*Cos[2*f*x] + 4
80*a^7*f*x*Cos[4*e + 2*f*x] + 192*a^6*b*f*x*Cos[4*e + 2*f*x] + 96*a^5*b^2*f
```

```
*x*Cos[4*e + 2*f*x] - 4608*a^4*b^3*f*x*Cos[4*e + 2*f*x] - 41856*a^3*b^4*f*x
*Cos[4*e + 2*f*x] - 67584*a^2*b^5*f*x*Cos[4*e + 2*f*x] - 30720*a*b^6*f*x*Co
s[4*e + 2*f*x] + 120*a^7*f*x*Cos[2*e + 4*f*x] - 192*a^6*b*f*x*Cos[2*e + 4*f
*x] + 408*a^5*b^2*f*x*Cos[2*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[2*e + 4*f*x]
- 6528*a^3*b^4*f*x*Cos[2*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[2*e + 4*f*x] + 1
20*a^7*f*x*Cos[6*e + 4*f*x] - 192*a^6*b*f*x*Cos[6*e + 4*f*x] + 408*a^5*b^2*
f*x*Cos[6*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[6*e + 4*f*x] - 6528*a^3*b^4*f*x
*Cos[6*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[6*e + 4*f*x] - 6048*a^3*b^4*Sin[2*
e] - 21312*a^2*b^5*Sin[2*e] - 29952*a*b^6*Sin[2*e] - 13824*b^7*Sin[2*e] + 2
62*a^7*Sin[2*f*x] + 524*a^6*b*Sin[2*f*x] - 26*a^5*b^2*Sin[2*f*x] + 1728*a^4
*b^3*Sin[2*f*x] + 14976*a^3*b^4*Sin[2*f*x] + 28416*a^2*b^5*Sin[2*f*x] + 145
92*a*b^6*Sin[2*f*x] + 262*a^7*Sin[4*e + 2*f*x] + 524*a^6*b*Sin[4*e + 2*f*x]
 - 26*a^5*b^2*Sin[4*e + 2*f*x] + 1728*a^4*b^3*Sin[4*e + 2*f*x] + 6912*a^3*b
^4*Sin[4*e + 2*f*x] + 5376*a^2*b^5*Sin[4*e + 2*f*x] + 768*a*b^6*Sin[4*e + 2
*f*x] + 238*a^7*Sin[2*e + 4*f*x] + 304*a^6*b*Sin[2*e + 4*f*x] - 250*a^5*b^2
*Sin[2*e + 4*f*x] + 1556*a^4*b^3*Sin[2*e + 4*f*x] + 5904*a^3*b^4*Sin[2*e +
4*f*x] + 3744*a^2*b^5*Sin[2*e + 4*f*x] + 238*a^7*Sin[6*e + 4*f*x] + 304*a^6
*b*Sin[6*e + 4*f*x] - 250*a^5*b^2*Sin[6*e + 4*f*x] + 1556*a^4*b^3*Sin[6*e +
 4*f*x] + 3888*a^3*b^4*Sin[6*e + 4*f*x] + 2016*a^2*b^5*Sin[6*e + 4*f*x] + 8
7*a^7*Sin[4*e + 6*f*x] + 46*a^6*b*Sin[4*e + 6*f*x] - 9*a^5*b^2*Sin[4*e + 6*
f*x] + 192*a^4*b^3*Sin[4*e + 6*f*x] + 160*a^3*b^4*Sin[4*e + 6*f*x] + 87*a^7
*Sin[8*e + 6*f*x] + 46*a^6*b*Sin[8*e + 6*f*x] - 9*a^5*b^2*Sin[8*e + 6*f*x]
+ 192*a^4*b^3*Sin[8*e + 6*f*x] + 160*a^3*b^4*Sin[8*e + 6*f*x] + 13*a^7*Sin[
6*e + 8*f*x] + 16*a^6*b*Sin[6*e + 8*f*x] - 7*a^5*b^2*Sin[6*e + 8*f*x] - 10*
a^4*b^3*Sin[6*e + 8*f*x] + 13*a^7*Sin[10*e + 8*f*x] + 16*a^6*b*Sin[10*e + 8
*f*x] - 7*a^5*b^2*Sin[10*e + 8*f*x] - 10*a^4*b^3*Sin[10*e + 8*f*x] + a^7*Si
n[8*e + 10*f*x] + 2*a^6*b*Sin[8*e + 10*f*x] + a^5*b^2*Sin[8*e + 10*f*x] + a
^7*Sin[12*e + 10*f*x] + 2*a^6*b*Sin[12*e + 10*f*x] + a^5*b^2*Sin[12*e + 10*
f*x]))/(12288*a^6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [A] time = 0.71, size = 1296, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(5*a^7 - 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2
*b^5)*f*x*cos(f*x + e)^4 + 12*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^
4 - 272*a^2*b^5 - 160*a*b^6)*f*x*cos(f*x + e)^2 + 6*(5*a^5*b^2 - 8*a^4*b^3
+ 17*a^3*b^4 - 82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x + 3*(99*a^2*b^5 + 176*
a*b^6 + 80*b^7 + (99*a^4*b^3 + 176*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2
*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*b^6)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log
(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 -
 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-
b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) + 2*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^
2 - 2*a^4*b^3)*cos(f*x + e)^7 + (15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^
3 + 80*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 26
6*a^3*b^4 + 180*a^2*b^5)*cos(f*x + e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3
*b^4 + 116*a^2*b^5 + 80*a*b^6)*cos(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b
 + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x +
e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), 1/48*(3*(5*a^7 - 8*a^6*b + 17*a^
5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2*b^5)*f*x*cos(f*x + e)^4 + 6*(5*a
^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^4 - 272*a^2*b^5 - 160*a*b^6)*f*x*c
os(f*x + e)^2 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 - 82*a^2*b^5 - 272*a*
b^6 - 160*b^7)*f*x - 3*(99*a^2*b^5 + 176*a*b^6 + 80*b^7 + (99*a^4*b^3 + 176
*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*
b^6)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (8*(a^7 + 2*a^6*b + a^
```

```
5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^2 - 2*a^4*b^3)*cos(f*x + e)^7 + (
15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^3 + 80*a^3*b^4)*cos(f*x + e)^5 +
2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 266*a^3*b^4 + 180*a^2*b^5)*cos(f*x
+ e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 + 116*a^2*b^5 + 80*a*b^6)*co
s(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*
(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6
*b^4)*f)]
```

giac [A] time = 0.36, size = 369, normalized size = 1.05

$$\frac{6\left(99\,a^{2}b^{4}+176\,ab^{5}+80\,b^{6}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{8}+2\,a^{7}b+a^{6}b^{2}\right)\sqrt{ab+b^{2}}}+\frac{6\left(19\,ab^{5}\tan\left(fx+e\right)^{3}+16\,b^{6}\tan\left(fx+e\right)^{3}+21\,a^{2}b^{4}\tan\left(fx+e\right)+37\,ab^{5}\tan\left(f\right.\right.}{\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 2*a^7*b + a^6*b^2)*sq
rt(a*b + b^2)) + 6*(19*a*b^5*tan(f*x + e)^3 + 16*b^6*tan(f*x + e)^3 + 21*a^
2*b^4*tan(f*x + e) + 37*a*b^5*tan(f*x + e) + 16*b^6*tan(f*x + e))/((a^7 + 2
*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 3*(5*a^3 - 18*a^2*b + 48*
a*b^2 - 160*b^3)*(f*x + e)/a^6 + (15*a^2*tan(f*x + e)^5 - 54*a*b*tan(f*x +
e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 144*a*b*tan(f*x + e
)^3 + 288*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 90*a*b*tan(f*x + e) +
144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

maple [A] time = 1.60, size = 636, normalized size = 1.81

$$\frac{19b^{5}\left(\tan^{3}\left(fx+e\right)\right)}{8f\,a^{4}\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{2b^{6}\left(\tan^{3}\left(fx+e\right)\right)}{f\,a^{5}\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{}{8f\,a^{4}\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 19/8/f/a^4*b^5/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3+2/f*b^6/
a^5/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3+21/8/f/a^4*b^4/(a+b
+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)+2/f*b^5/a^5/(a+b+b*tan(f*x+e)^2)^2/(a+b
)*tan(f*x+e)+99/8/f/a^4*b^4/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+
e)*b/((a+b)*b)^(1/2))+22/f/a^5*b^5/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(t
an(f*x+e)*b/((a+b)*b)^(1/2))+10/f*b^6/a^6/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*a
rctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+5/16/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e
)^5-9/8/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b+3/f/a^5/(tan(f*x+e)^2+1)^3*
tan(f*x+e)^5*b^2+6/f/a^5/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b^2+5/6/f/a^3/(tan
(f*x+e)^2+1)^3*tan(f*x+e)^3-3/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b-15/8/
f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b+3/f/a^5/(tan(f*x+e)^2+1)^3*tan(f*x+e)
*b^2+11/16/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)-10/f/a^6*arctan(tan(f*x+e))*
b^3+5/16/f/a^3*arctan(tan(f*x+e))-9/8/f/a^4*arctan(tan(f*x+e))*b+3/f/a^5*ar
ctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.48, size = 611, normalized size = 1.74

$$\frac{6\left(99\,a^{2}b^{4}+176\,ab^{5}+80\,b^{6}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^{8}+2\,a^{7}b+a^{6}b^{2}\right)\sqrt{(a+b)b}}+\frac{3\left(5\,a^{4}b^{2}-8\,a^{3}b^{3}+17\,a^{2}b^{4}+116\,ab^{5}+80\,b^{6}\right)\tan\left(fx+e\right)^{9}+2\left(15\,a^{5}b+11\,a^{4}b^{2}-5\,a^{3}b^{3}+368\,a^{2}b^{4}+87\right.}{\left(a^{7}b^{2}+2\,a^{6}b^{3}+a^{5}b^{4}\right)\tan\left(fx+e\right)^{10}+a^{9}+4\,a^{8}b+6\,a^{7}b^{2}+4\,a^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^8 + 2*a^7*b + a^6*b^2)*sqrt((a + b)*b)) + (3*(5*a^4*b^2 - 8*a^3*b
^3 + 17*a^2*b^4 + 116*a*b^5 + 80*b^6)*tan(f*x + e)^9 + 2*(15*a^5*b + 11*a^4
*b^2 - 5*a^3*b^3 + 368*a^2*b^4 + 876*a*b^5 + 480*b^6)*tan(f*x + e)^7 + (15*
a^6 + 86*a^5*b + 3*a^4*b^2 + 240*a^3*b^3 + 1982*a^2*b^4 + 3168*a*b^5 + 1440
*b^6)*tan(f*x + e)^5 + 2*(20*a^6 + 41*a^5*b - 15*a^4*b^2 + 197*a^3*b^3 + 98
0*a^2*b^4 + 1236*a*b^5 + 480*b^6)*tan(f*x + e)^3 + 3*(11*a^6 + 14*a^5*b - 6
*a^4*b^2 + 56*a^3*b^3 + 221*a^2*b^4 + 236*a*b^5 + 80*b^6)*tan(f*x + e))/((a
^7*b^2 + 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^10 + a^9 + 4*a^8*b + 6*a^7*b^2 +
 4*a^6*b^3 + a^5*b^4 + (2*a^8*b + 9*a^7*b^2 + 12*a^6*b^3 + 5*a^5*b^4)*tan(f
*x + e)^8 + (a^9 + 10*a^8*b + 27*a^7*b^2 + 28*a^6*b^3 + 10*a^5*b^4)*tan(f*x
 + e)^6 + (3*a^9 + 18*a^8*b + 37*a^7*b^2 + 32*a^6*b^3 + 10*a^5*b^4)*tan(f*x
 + e)^4 + (3*a^9 + 14*a^8*b + 24*a^7*b^2 + 18*a^6*b^3 + 5*a^5*b^4)*tan(f*x
+ e)^2) + 3*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*(f*x + e)/a^6)/f
```

mupad [B] time = 10.11, size = 4594, normalized size = 13.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(156*a*b^4 + 3*a^4*b + 11*a^5 + 80*b^5 + 65*a^2*b^3 - 9*a^3*
b^2))/(16*a^5*(a + b)) + (tan(e + f*x)^7*(876*a*b^5 + 15*a^5*b + 480*b^6 +
368*a^2*b^4 - 5*a^3*b^3 + 11*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^3
*(1236*a*b^5 + 41*a^5*b + 20*a^6 + 480*b^6 + 980*a^2*b^4 + 197*a^3*b^3 - 15
*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^5*(3168*a*b^5 + 86*a^5*b + 15
*a^6 + 1440*b^6 + 1982*a^2*b^4 + 240*a^3*b^3 + 3*a^4*b^2))/(48*a^5*(a + b)^
2) + (b*tan(e + f*x)^9*(116*a*b^4 + 5*a^4*b + 80*b^5 + 17*a^2*b^3 - 8*a^3*b
^2))/(16*a^5*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^6*(8*a*b + a^2 + 10*b^2)
+ a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2) + b^2*tan(e + f*x)^10 + tan(e
+ f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e + f*x)^4*(12*a*b + 3*a^2 + 10*b^2)
)) - (atan(-((((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*
b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4
*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(a*b^2
*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a^13*b^6 + 1638
4*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(4096*a^6*(4*
a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18
i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a*b^12 + 51200*b^1
3 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a
^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^
13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i)*1i)/(32*a^6) - (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a
^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)
/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2)
 + (tan(e + f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7
 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a
^17*b^2))/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2))
)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) + (tan(e + f*x)*(19
9680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^
9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 2
5*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*
(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*1i)/(32*a^6))/((3350*a*b^14 + 1
000*b^15 + (7315*a^2*b^13)/2 + (4597*a^3*b^12)/4 - (4325*a^4*b^11)/32 + (10
281*a^5*b^10)/128 + (8973*a^6*b^9)/512 - (25551*a^7*b^8)/1024 + (4235*a^8*b
^7)/512 - (2475*a^9*b^6)/1024)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6
```

```
*a^17*b^2) + (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*
b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4
*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(a*b^2
*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a^13*b^6 + 1638
4*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(4096*a^6*(4*
a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18
i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a*b^12 + 51200*b^1
3 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a
^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^
13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i))/(32*a^6) + (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14
*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4
- (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) +
(tan(e + f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 +
9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17
*b^2))/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(
a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) + (tan(e + f*x)*(19968
0*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 -
 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a
^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*
b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6)))*(a*b^2*48i - a^2*b*18i
 + a^3*5i - b^3*160i)*1i)/(16*a^6*f) - (atan((((-b^7*(a + b)^5)^(1/2)*((tan
(e + f*x)*(199680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 +
 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 8
0*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6
*a^12*b^2)) + (((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*b
^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*
a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) + (tan(e + f*x)*(-b^7*(
a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*(2048*a^12*b^7 + 9216*a^13*b
^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(128
*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)*(5*a^10*b + a^11 +
a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11
*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^
8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/16 + 5*b^2)*1i)/(5*a^10*b + a^11 +
a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2) + ((-b^7*(a + b)^5)^(1/2)*((
tan(e + f*x)*(199680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^1
0 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5
- 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3
+ 6*a^12*b^2)) - (((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^1
5*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/
(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(-b^
7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*(2048*a^12*b^7 + 9216*a^1
3*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(
128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)*(5*a^10*b + a^11
 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2)))*(-b^7*(a + b)^5)^(1/2)*
(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10
*a^8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/16 + 5*b^2)*1i)/(5*a^10*b + a^11
 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))/((3350*a*b^14 + 1000*b^1
5 + (7315*a^2*b^13)/2 + (4597*a^3*b^12)/4 - (4325*a^4*b^11)/32 + (10281*a^5
*b^10)/128 + (8973*a^6*b^9)/512 - (25551*a^7*b^8)/1024 + (4235*a^8*b^7)/512
 - (2475*a^9*b^6)/1024)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b
^2) + ((-b^7*(a + b)^5)^(1/2)*((tan(e + f*x)*(199680*a*b^12 + 51200*b^13 +
287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b
^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b
 + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)) + (((20*a^12*b^9 + 79*a^13*b
^8 + (457*a^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (
7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 +
6*a^17*b^2) + (tan(e + f*x)*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 +
5*b^2)*(2048*a^12*b^7 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6
```

```
144*a^16*b^3 + 1024*a^17*b^2))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^
3 + 6*a^12*b^2)*(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^
9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b +
 a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/
16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b
^2) - ((-b^7*(a + b)^5)^(1/2)*((tan(e + f*x)*(199680*a*b^12 + 51200*b^13 +
287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b
^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b
 + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)) - (((20*a^12*b^9 + 79*a^13*b
^8 + (457*a^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (
7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 +
6*a^17*b^2) - (tan(e + f*x)*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 +
5*b^2)*(2048*a^12*b^7 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6
144*a^16*b^3 + 1024*a^17*b^2))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^
3 + 6*a^12*b^2)*(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^
9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b +
 a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))*(11*a*b + (99*a^2)/
16 + 5*b^2))/(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b
^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*2i)/(f*(5*a^10*
b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=204

$$\frac{x}{a^4}-\frac{b(11a+6b)\tan(c+dx)}{24a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^2}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{16a^3d(a+b)^3\left(a+b\tan^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(35a^3+70a^2b+56a\right.}{16a^4}$$

```
[Out] x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctan(b^(1/2)*tan(d*x+c)/(a+b
)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*
x+c)^2)^3-1/24*b*(11*a+6*b)*tan(d*x+c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^2
-1/16*b*(19*a^2+22*a*b+8*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.33, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(70a^2b+35a^3+56ab^2+16b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{16a^3d(a+b)^3\left(a+b\tan^2(c+dx)+b\right)}-\frac{b(1}{24a^2d(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Ta
n[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(6*a*
(a + b)*d*(a + b + b*Tan[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tan[c + d*x])/(24
*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^
2)*Tan[c + d*x])/(16*a^3*(a + b)^3*d*(a + b + b*Tan[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{6a+b-5bx^2}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(c+dx\right)}{6a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} +$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} -$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} -$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\tan(c}{6a(a+b)d\left(a+b+\right.}$$

Mathematica [C] time = 6.86, size = 1411, normalized size = 6.92

$$\left(35a^3+70ba^2+56b^2a+16b^3\right)(\cos(2c+2dx)a+a+2b)^4\left(\frac{b\tan^{-1}\left(\sec(dx)\left(\frac{\cos(2c)}{2\sqrt{a+b}\sqrt{b\cos(4c)-ib\sin(4c)}}-\frac{i\sin(2c)}{2\sqrt{a+b}\sqrt{b\cos(4c)-ib\sin(4c)}}\right)\right.}{256a^4\sqrt{a+b}\,d\sqrt{b\cos(4c)-ib\sin}}\right.$$

(a

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cos[2*c + 2*d*x])^4*S
ec[c + d*x]^8*((b*ArcTan[Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c]
- I*b*Sin[4*c]]) - ((I/2)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[
```

```
4*c]]))*(-(a*Sin[d*x]) - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Cos[2*c])/(256*a
^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/256)*b*ArcTan[Sec[d
*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/2)*Sin[
2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]))*(-(a*Sin[d*x]) - 2*b*S
in[d*x] + a*Sin[2*c + d*x])]*Sin[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] -
 I*b*Sin[4*c]])))/((a + b)^3*(a + b*Sec[c + d*x]^2)^4) + ((a + 2*b + a*Cos[
2*c + 2*d*x])*Sec[2*c]*Sec[c + d*x]^8*(480*a^6*d*x*Cos[2*c] + 3168*a^5*b*d*
x*Cos[2*c] + 8928*a^4*b^2*d*x*Cos[2*c] + 14112*a^3*b^3*d*x*Cos[2*c] + 13248
*a^2*b^4*d*x*Cos[2*c] + 6912*a*b^5*d*x*Cos[2*c] + 1536*b^6*d*x*Cos[2*c] + 3
60*a^6*d*x*Cos[2*d*x] + 2232*a^5*b*d*x*Cos[2*d*x] + 5688*a^4*b^2*d*x*Cos[2*
d*x] + 7272*a^3*b^3*d*x*Cos[2*d*x] + 4608*a^2*b^4*d*x*Cos[2*d*x] + 1152*a*b
^5*d*x*Cos[2*d*x] + 360*a^6*d*x*Cos[4*c + 2*d*x] + 2232*a^5*b*d*x*Cos[4*c +
 2*d*x] + 5688*a^4*b^2*d*x*Cos[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cos[4*c + 2*
d*x] + 4608*a^2*b^4*d*x*Cos[4*c + 2*d*x] + 1152*a*b^5*d*x*Cos[4*c + 2*d*x]
+ 144*a^6*d*x*Cos[2*c + 4*d*x] + 720*a^5*b*d*x*Cos[2*c + 4*d*x] + 1296*a^4*
b^2*d*x*Cos[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[2*c + 4*d*x] + 288*a^2*b^4*
d*x*Cos[2*c + 4*d*x] + 144*a^6*d*x*Cos[6*c + 4*d*x] + 720*a^5*b*d*x*Cos[6*c
 + 4*d*x] + 1296*a^4*b^2*d*x*Cos[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[6*c +
4*d*x] + 288*a^2*b^4*d*x*Cos[6*c + 4*d*x] + 24*a^6*d*x*Cos[4*c + 6*d*x] + 7
2*a^5*b*d*x*Cos[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cos[4*c + 6*d*x] + 24*a^3*b^3
*d*x*Cos[4*c + 6*d*x] + 24*a^6*d*x*Cos[8*c + 6*d*x] + 72*a^5*b*d*x*Cos[8*c
+ 6*d*x] + 72*a^4*b^2*d*x*Cos[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cos[8*c + 6*d*x
] + 870*a^5*b*Sin[2*c] + 4292*a^4*b^2*Sin[2*c] + 8792*a^3*b^3*Sin[2*c] + 99
36*a^2*b^4*Sin[2*c] + 5824*a*b^5*Sin[2*c] + 1408*b^6*Sin[2*c] - 870*a^5*b*S
in[2*d*x] - 3792*a^4*b^2*Sin[2*d*x] - 6432*a^3*b^3*Sin[2*d*x] - 4608*a^2*b^
4*Sin[2*d*x] - 1248*a*b^5*Sin[2*d*x] + 435*a^5*b*Sin[4*c + 2*d*x] + 2124*a^
4*b^2*Sin[4*c + 2*d*x] + 3972*a^3*b^3*Sin[4*c + 2*d*x] + 3072*a^2*b^4*Sin[4
*c + 2*d*x] + 864*a*b^5*Sin[4*c + 2*d*x] - 435*a^5*b*Sin[2*c + 4*d*x] - 137
4*a^4*b^2*Sin[2*c + 4*d*x] - 1248*a^3*b^3*Sin[2*c + 4*d*x] - 384*a^2*b^4*Si
n[2*c + 4*d*x] + 87*a^5*b*Sin[6*c + 4*d*x] + 366*a^4*b^2*Sin[6*c + 4*d*x] +
 408*a^3*b^3*Sin[6*c + 4*d*x] + 144*a^2*b^4*Sin[6*c + 4*d*x] - 87*a^5*b*Sin
[4*c + 6*d*x] - 116*a^4*b^2*Sin[4*c + 6*d*x] - 44*a^3*b^3*Sin[4*c + 6*d*x])
)/(3072*a^4*(a + b)^3*d*(a + b*Sec[c + d*x]^2)^4)
```

fricas [B] time = 0.61, size = 1323, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(192*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 576*
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 576*(a^4*b^2
 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 192*(a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3
)*cos(d*x + c)^6 + 35*a^3*b^3 + 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*
b + 70*a^4*b^2 + 56*a^3*b^3 + 16*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 +
70*a^3*b^3 + 56*a^2*b^4 + 16*a*b^5)*cos(d*x + c)^2)*sqrt(-b/(a + b))*log(((
a^2 + 8*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a*b + 4*b^2)*cos(d*x + c)^2 + 4*
((a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^3 - (a*b + b^2)*cos(d*x + c))*sqrt(-b/(
a + b))*sin(d*x + c) + b^2)/(a^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^2 + b^
2)) - 4*((87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b
^2 + 83*a^3*b^3 + 30*a^2*b^4)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 +
 8*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^
3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x
 + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d), 1/96*(96*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 288*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3
 + a^2*b^4)*d*x*cos(d*x + c)^4 + 288*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b
^5)*d*x*cos(d*x + c)^2 + 96*(a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d*x + 3*(
```

```
(35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3)*cos(d*x + c)^6 + 35*a^3*b^3 +
 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*b + 70*a^4*b^2 + 56*a^3*b^3 + 1
6*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 + 70*a^3*b^3 + 56*a^2*b^4 + 16*a*
b^5)*cos(d*x + c)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(d*x + c)^2 -
 b)*sqrt(b/(a + b))/(b*cos(d*x + c)*sin(d*x + c))) - 2*((87*a^5*b + 116*a^4
*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b^2 + 83*a^3*b^3 + 30*a^2*b^4
)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 + 8*a*b^5)*cos(d*x + c))*sin(
d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9
*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7
*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5
*b^5 + a^4*b^6)*d)]
```

giac [A] time = 1.05, size = 324, normalized size = 1.59

$$-\frac{\frac{3\left(35\,a^{3}b+70\,a^{2}b^{2}+56\,ab^{3}+16\,b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{7}+3\,a^{6}b+3\,a^{5}b^{2}+a^{4}b^{3}\right)\sqrt{ab+b^{2}}}+\frac{57\,a^{2}b^{3}\tan(dx+c)^{5}+66\,ab^{4}\tan(dx+c)^{5}+24\,b^{5}\tan(dx+c)^{5}+136\,a^{3}b^{2}\tan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*(pi*floor((d*x + c)/pi
 + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b + b^2)))/((a^7 + 3*a^6*b +
3*a^5*b^2 + a^4*b^3)*sqrt(a*b + b^2)) + (57*a^2*b^3*tan(d*x + c)^5 + 66*a*b
^4*tan(d*x + c)^5 + 24*b^5*tan(d*x + c)^5 + 136*a^3*b^2*tan(d*x + c)^3 + 28
0*a^2*b^3*tan(d*x + c)^3 + 192*a*b^4*tan(d*x + c)^3 + 48*b^5*tan(d*x + c)^3
 + 87*a^4*b*tan(d*x + c) + 252*a^3*b^2*tan(d*x + c) + 267*a^2*b^3*tan(d*x +
 c) + 126*a*b^4*tan(d*x + c) + 24*b^5*tan(d*x + c))/((a^6 + 3*a^5*b + 3*a^4
*b^2 + a^3*b^3)*(b*tan(d*x + c)^2 + a + b)^3) - 48*(d*x + c)/a^4)/d
```

maple [B] time = 0.82, size = 649, normalized size = 3.18

$$-\frac{19b^{3}\left(\tan^{5}(dx+c)\right)}{16da\left(a+b+b\left(\tan^{2}(dx+c)\right)\right)^{3}\left(a^{3}+3a^{2}b+3b^{2}a+b^{3}\right)}-\frac{11b^{4}\left(\tan^{5}(dx+c)\right)}{8d\,a^{2}\left(a+b+b\left(\tan^{2}(dx+c)\right)\right)^{3}\left(a^{3}+3a^{2}b+3b^{2}a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)^2)^4,x)
```

```
[Out] -19/16/d/a*b^3/(a+b+b*tan(d*x+c)^2)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^
5-11/8/d/a^2*b^4/(a+b+b*tan(d*x+c)^2)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c
)^5-1/2/d/a^3*b^5/(a+b+b*tan(d*x+c)^2)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+
c)^5-17/6/d/a*b^2/(a+b+b*tan(d*x+c)^2)^3/(a^2+2*a*b+b^2)*tan(d*x+c)^3-3/d/a
^2*b^3/(a+b+b*tan(d*x+c)^2)^3/(a^2+2*a*b+b^2)*tan(d*x+c)^3-1/d/a^3*b^4/(a+b
+b*tan(d*x+c)^2)^3/(a^2+2*a*b+b^2)*tan(d*x+c)^3-29/16*b*tan(d*x+c)/a/(a+b)/
d/(a+b+b*tan(d*x+c)^2)^3-13/8/d/a^2*b^2/(a+b+b*tan(d*x+c)^2)^3/(a+b)*tan(d*
x+c)-1/2/d/a^3*b^3/(a+b+b*tan(d*x+c)^2)^3/(a+b)*tan(d*x+c)-35/16/d/a*b/(a^3
+3*a^2*b+3*a*b^2+b^3)/((a+b)*b)^(1/2)*arctan(tan(d*x+c)*b/((a+b)*b)^(1/2))-
35/8/d/a^2*b^2/(a^3+3*a^2*b+3*a*b^2+b^3)/((a+b)*b)^(1/2)*arctan(tan(d*x+c)*
b/((a+b)*b)^(1/2))-7/2/d/a^3*b^3/(a^3+3*a^2*b+3*a*b^2+b^3)/((a+b)*b)^(1/2)*
arctan(tan(d*x+c)*b/((a+b)*b)^(1/2))-1/d/a^4*b^4/(a^3+3*a^2*b+3*a*b^2+b^3)/
((a+b)*b)^(1/2)*arctan(tan(d*x+c)*b/((a+b)*b)^(1/2))+1/d/a^4*arctan(tan(d*x
+c))
```

maxima [B] time = 0.45, size = 401, normalized size = 1.97

$$-\frac{\frac{3\left(35\,a^{3}b+70\,a^{2}b^{2}+56\,ab^{3}+16\,b^{4}\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{(a+b)b}}\right)}{\left(a^{7}+3\,a^{6}b+3\,a^{5}b^{2}+a^{4}b^{3}\right)\sqrt{(a+b)b}}+\frac{3\left(19\,a^{2}b^{3}+22\,ab^{4}+8\,b^{5}\right)\tan(dx+c)^{5}+8\left(17\,a^{3}b^{2}+35\,a^{2}b^{3}+\cdots\right.}{a^{9}+6\,a^{8}b+15\,a^{7}b^{2}+20\,a^{6}b^{3}+15\,a^{5}b^{4}+6\,a^{4}b^{5}+a^{3}b^{6}+\left(a^{6}b^{3}+3\,a^{5}b^{4}+3\,a^{4}b^{5}+a^{3}b^{6}\right)\tan(dx+c)^{6}\cdots}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(b*tan(d*x + c)/
sqrt((a + b)*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)) +
(3*(19*a^2*b^3 + 22*a*b^4 + 8*b^5)*tan(d*x + c)^5 + 8*(17*a^3*b^2 + 35*a^2*
b^3 + 24*a*b^4 + 6*b^5)*tan(d*x + c)^3 + 3*(29*a^4*b + 84*a^3*b^2 + 89*a^2*
b^3 + 42*a*b^4 + 8*b^5)*tan(d*x + c))/(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*
b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6 + (a^6*b^3 + 3*a^5*b^4 + 3*a^4*b^5 +
 a^3*b^6)*tan(d*x + c)^6 + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*tan(d*x + c)^4 + 3*(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a^5*b^4 +
 5*a^4*b^5 + a^3*b^6)*tan(d*x + c)^2) - 48*(d*x + c)/a^4)/d
```

mupad [B] time = 9.46, size = 4506, normalized size = 22.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x)^2)^4,x)
```

```
[Out] atan((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4
+ (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15
+ a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (tan(c
 + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 5
1200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(
6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b
^2)))/(2*a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*
a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^11*b + a^
12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 - (
(((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*
a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b
^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (tan(c + d*x)
*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^
12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*
b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/(
2*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6
 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^11*b + a^12 + a^
6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4)/((((((2*a
^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b
^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 + 6
*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (tan(c + d*x)*(2048
*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5
 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a^
12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))*1i)/(2*
a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 +
 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3)*1i)/(256*(6*a^11*b + a^12 + a
^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 + (((((2*
a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*
b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 +
6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (tan(c + d*x)*(204
8*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^
5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a
^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))*1i)/(2
*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6
+ 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3)*1i)/(256*(6*a^11*b + a^12 +
a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 + ((25*a
*b^7)/4 + b^8 + (131*a^2*b^6)/8 + (721*a^3*b^5)/32 + (525*a^4*b^4)/32 + (66
5*a^5*b^3)/128)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*
a^12*b^3 + 15*a^13*b^2)))/(a^4*d) - ((tan(c + d*x)^3*(18*a*b^3 + 6*b^4 + 17
```

*a^2*b^2))/(6*a^3*(a + b)^2) + (tan(c + d*x)^5*(22*a*b^4 + 8*b^5 + 19*a^2*b
^3))/(16*a^3*(a + b)^3) + (tan(c + d*x)*(26*a*b^2 + 29*a^2*b + 8*b^3))/(16*
a^3*(a + b)))/(d*(tan(c + d*x)^4*(3*a*b^2 + 3*b^3) + 3*a*b^2 + 3*a^2*b + ta
n(c + d*x)^2*(6*a*b^2 + 3*a^2*b + 3*b^3) + a^3 + b^3 + b^3*tan(c + d*x)^6))
 + (atan((((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216
*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(1
28*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^
10*b^2)) - ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b
^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)
/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a
^13*b^2) - (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3
 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6
+ 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(4096*(
6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b
^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a
^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a^10
*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21
*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*1i)/(32*(7*a^10*b + a^1
1 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2
)) + ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*
b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(128*(6
*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^
2)) + ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4
 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)/(6*a
^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b
^2) + (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16
b^3)(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 512
00*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(4096*(6*a^1
1*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)*(
7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^
3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a^10*b +
a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*
b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*1i)/(32*(7*a^10*b + a^11 + a
^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))/(
((25*a*b^7)/4 + b^8 + (131*a^2*b^6)/8 + (721*a^3*b^5)/32 + (525*a^4*b^4)/32
 + (665*a^5*b^3)/128)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4
 + 20*a^12*b^3 + 15*a^13*b^2) - ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*
a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b
^4 + 1481*a^6*b^3))/(128*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^
4 + 20*a^9*b^3 + 15*a^10*b^2)) - ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^
9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^1
3*b^3)/4 + 4*a^14*b^2)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^
4 + 20*a^12*b^3 + 15*a^13*b^2) - (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b
^2 + 70*a^2*b + 35*a^3 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10
*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1
024*a^15*b^2))/(4096*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 +
20*a^9*b^3 + 15*a^10*b^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b
^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3
+ 16*b^3))/(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7
*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/
(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a
^8*b^3 + 21*a^9*b^2)) + ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 +
512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 148
1*a^6*b^3))/(128*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a
^9*b^3 + 15*a^10*b^2)) + ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2
 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4
 + 4*a^14*b^2)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a
^12*b^3 + 15*a^13*b^2) + (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*
a^2*b + 35*a^3 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 5

```
6320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15
*b^2))/(4096*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b
^3 + 15*a^10*b^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*
a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3
))/(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 3
5*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a
^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 +
 21*a^9*b^2))))*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3
)*1i)/(16*d*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^
4 + 35*a^8*b^3 + 21*a^9*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

3.220 $\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\cot(}{}$$

```
[Out] -a^3*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^3*(-a*tan(d*
x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d-1
/6*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4121, 3658, 3473, 3475}

$$-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^3\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

```
[Out] -((a^3*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^3*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^3*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d) - (a^3*Tan[c + d*x]^5*Sqrt[-(a*Tan[c + d*x]^2)])/
(6*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2} dx = \int \left(-a\tan^2(c+dx)\right)^{7/2} dx$$
$$= -\left(\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^7(c+dx)\,dx\right)$$
$$= -\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d} + \left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan$$
$$= \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d} - \left(a^3 \mathrm{co}\right.$$
$$= -\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3 \tan}{}$$
$$= -\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^3\tan(c+dx)\sqrt{-a\tan^2(}}{2d}$$

Mathematica [A] time = 2.16, size = 70, normalized size = 0.52

$$\frac{\cot^7(c+dx)\left(-a\tan^2(c+dx)\right)^{7/2}\left(2\tan^6(c+dx) - 3\tan^4(c+dx) + 6\tan^2(c+dx) + 12\log(\cos(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(7/2),x]
[Out] (Cot[c + d*x]^7*(-(a*Tan[c + d*x]^2))^(7/2)*(12*Log[Cos[c + d*x]] + 6*Tan[c
 + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

fricas [A] time = 0.45, size = 100, normalized size = 0.75

$$-\frac{\left(12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right) + 18\,a^3\cos\left(dx+c\right)^4 - 9\,a^3\cos\left(dx+c\right)^2 + 2\,a^3\right)\sqrt{\frac{a\cos(dx+c)^2 - a}{\cos(dx+c)^2}}}{12\,d\cos\left(dx+c\right)^5\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
[Out] -1/12*(12*a^3*cos(d*x + c)^6*log(-cos(d*x + c)) + 18*a^3*cos(d*x + c)^4 - 9
*a^3*cos(d*x + c)^2 + 2*a^3)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d
*cos(d*x + c)^5*sin(d*x + c))
```

giac [A] time = 0.71, size = 217, normalized size = 1.62

$$-\frac{6\sqrt{-a}\,a^3\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} + 2\right) - 6\sqrt{-a}\,a^3\log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} - 2\right) + }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
[Out] -1/12*(6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 + 2) - 6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^
2 - 2) + (11*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^3*sqrt(-a)
```

```
*a^3 - 90*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a^
3 + 276*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^3 -
408*sqrt(-a)*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^3
)/d
```

maple [A] time = 2.33, size = 168, normalized size = 1.25

$$\frac{\left(12\left(\cos^{6}(dx+c)\right)\ln\left(-\frac{-\sin(dx+c)-1+\cos(dx+c)}{\sin(dx+c)}\right)-12\left(\cos^{6}(dx+c)\right)\ln\left(\frac{2}{1+\cos(dx+c)}\right)+12\left(\cos^{6}(dx+c)\right)\ln\left(-\frac{-1+\cos}{}\right.}{12d\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(7/2),x)
```

```
[Out] 1/12/d*(12*cos(d*x+c)^6*ln(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))-12*cos(d
*x+c)^6*ln(2/(1+cos(d*x+c)))+12*cos(d*x+c)^6*ln(-(-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))-11*cos(d*x+c)^6+18*cos(d*x+c)^4-9*cos(d*x+c)^2+2)*cos(d*x+c)*(
-a*sin(d*x+c)^2/cos(d*x+c)^2)^(7/2)/sin(d*x+c)^7
```

maximа [A] time = 0.45, size = 81, normalized size = 0.60

$$-\frac{2\sqrt{-a}\,a^{3}\tan(dx+c)^{6}-3\sqrt{-a}\,a^{3}\tan(dx+c)^{4}+6\sqrt{-a}\,a^{3}\tan(dx+c)^{2}-6\sqrt{-a}\,a^{3}\log\left(\tan(dx+c)^{2}+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(2*sqrt(-a)*a^3*tan(d*x + c)^6 - 3*sqrt(-a)*a^3*tan(d*x + c)^4 + 6*sq
rt(-a)*a^3*tan(d*x + c)^2 - 6*sqrt(-a)*a^3*log(tan(d*x + c)^2 + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a-\frac{a}{\cos(c+d\,x)^{2}}\right)^{7/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.221 $\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=101

$$-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(co}{d}$$

```
[Out] -a^2*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^2*(-a*tan(d*
x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^2*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] -((a^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^2*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^2*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2} dx = \int \left(-a\tan^2(c+dx)\right)^{5/2} dx$$
$$= \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx$$
$$= \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c$$
$$= -\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} + \left(a^2\cot(\right.$$
$$= -\frac{a^2\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+}}{2d}$$

Mathematica [A] time = 0.54, size = 60, normalized size = 0.59

$$-\frac{\cot^5(c+dx)\left(-a\tan^2(c+dx)\right)^{5/2}\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] -1/4*(Cot[c + d*x]^5*(-(a*Tan[c + d*x]^2))^(5/2)*(4*Log[Cos[c + d*x]] + 2*T
an[c + d*x]^2 - Tan[c + d*x]^4))/d
```

fricas [A] time = 0.89, size = 87, normalized size = 0.86

$$-\frac{\left(4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{4\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*a^2*cos(d*x + c)^2 - a^2)
*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c)
)
```

giac [A] time = 0.49, size = 182, normalized size = 1.80

$$-\frac{2\sqrt{-a}\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-2\sqrt{-a}\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{3\left(\mathrm{t}\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
+ 2) - 2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 - 2) + (3*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a
^2 - 20*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^2 +
44*sqrt(-a)*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^2)
/d
```

maple [A] time = 1.99, size = 158, normalized size = 1.56

$$-\frac{\left(4\left(\cos^4(dx+c)\right)\ln\left(-\frac{-\sin(dx+c)-1+\cos(dx+c)}{\sin(dx+c)}\right)+4\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4\ln\left(\frac{2}{1+\cos(dx+c)}\right.\right.}{4d\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/4/d*(4*cos(d*x+c)^4*ln(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))+4*cos(d*x
+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*ln(2/(1+cos(d*x+c)))*cos
(d*x+c)^4-3*cos(d*x+c)^4+4*cos(d*x+c)^2-1)*cos(d*x+c)*(-a*sin(d*x+c)^2/cos(
d*x+c)^2)^(5/2)/sin(d*x+c)^5
```

maxima [A] time = 0.45, size = 62, normalized size = 0.61

$$\frac{\sqrt{-a}\,a^2\tan(dx+c)^4-2\sqrt{-a}\,a^2\tan(dx+c)^2+2\sqrt{-a}\,a^2\log\left(\tan(dx+c)^2+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a)*a^2*tan(d*x + c)^4 - 2*sqrt(-a)*a^2*tan(d*x + c)^2 + 2*sqrt(-
a)*a^2*log(tan(d*x + c)^2 + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec^2(c+dx)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(5/2), x)
```

3.222 $\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=64

$$-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a*(-a*tan(d*x+c)
^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a*Tan[
c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2} dx = \int \left(-a\tan^2(c+dx)\right)^{3/2} dx$$
$$= -\left(\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx\right)$$
$$= -\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c$$
$$= -\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a\tan(c+dx)\sqrt{-a\tan^2(c-}}{2d}$$

Mathematica [**A**] time = 0.11, size = 48, normalized size = 0.75

$$\frac{\cot^3(c+dx)\left(-a\tan^2(c+dx)\right)^{3/2}\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

```
[Out] (Cot[c + d*x]^3*(-(a*Tan[c + d*x]^2))^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c +
d*x]^2))/(2*d)
```

fricas [**A**] time = 1.33, size = 68, normalized size = 1.06

$$-\frac{\left(2\,a\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+a\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(d*x + c)^2*log(-cos(d*x + c)) + a)*sqrt((a*cos(d*x + c)^2 - a
)/cos(d*x + c)^2)/(d*cos(d*x + c)*sin(d*x + c))
```

giac [**B**] time = 1.15, size = 137, normalized size = 2.14

$$-\frac{\sqrt{-a}\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-\sqrt{-a}\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{\left(\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 + 2)
 - sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2) +
((tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a - 6*sqrt(-a
)*a)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2))/d
```

maple [**B**] time = 2.34, size = 148, normalized size = 2.31

$$\frac{\left(2\left(\cos^2\left(dx+c\right)\right)\ln\left(-\frac{-\sin(dx+c)-1+\cos(dx+c)}{\sin(dx+c)}\right)+2\left(\cos^2\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-2\ln\left(\frac{2}{1+\cos(dx+c)}\right)}{2d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/2/d*(2*cos(d*x+c)^2*ln(-(-sin(d*x+c)-1+cos(d*x+c))/sin(d*x+c))+2*cos(d*x+
c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-2*ln(2/(1+cos(d*x+c)))*cos(
d*x+c)^2-cos(d*x+c)^2+1)*cos(d*x+c)*(-a*sin(d*x+c)^2/cos(d*x+c)^2)^(3/2)/si
n(d*x+c)^3
```

maxima [A] time = 0.43, size = 40, normalized size = 0.62

$$-\frac{\sqrt{-a}\,a\tan\left(dx+c\right)^{2}-\sqrt{-a}\,a\log\left(\tan\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(-a)*a*tan(d*x + c)^2 - sqrt(-a)*a*log(tan(d*x + c)^2 + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a-\frac{a}{{\cos\left(c+d\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(3/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec^{2}{\left(c+dx \right)}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(3/2), x)
```

3.223 $\int \sqrt{a-a\sec^2(c+dx)}\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4121, 3658, 3475}

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a-a\sec^2(c+dx)}\,dx &= \int \sqrt{-a\tan^2(c+dx)}\,dx\\
&= \left(\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx\\
&= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 33, normalized size = 1.00

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

fricas [A] time = 0.63, size = 53, normalized size = 1.61

$$-\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log(-\cos(dx+c))}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(-cos(d*x + c)
)/(d*sin(d*x + c))
```

giac [B] time = 0.35, size = 141, normalized size = 4.27

$$-\frac{\left(\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)\operatorname{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)\operatorname{sgn}\left(-t\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x
 + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(-tan(1/2*d*x + 1/2*c)^3
 - tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(-tan(1/2*
d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))*sqrt(-a)*sgn(cos(d*x + c))/d
```

maple [B] time = 3.07, size = 109, normalized size = 3.30

$$-\frac{\left(\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+\ln\left(-\frac{-\sin(dx+c)-1+\cos(dx+c)}{\sin(dx+c)}\right)-\ln\left(\frac{2}{1+\cos(dx+c)}\right)\right)\cos(dx+c)\sqrt{-\frac{a\left(\sin^2(dx+c)\right)}{\cos(dx+c)^2}}}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/d*(ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+ln(-(-sin(d*x+c)-1+cos(d*x
+c))/sin(d*x+c))-ln(2/(1+cos(d*x+c))))*cos(d*x+c)*(-a*sin(d*x+c)^2/cos(d*x+
c)^2)^(1/2)/sin(d*x+c)
```

maxima [A] time = 0.44, size = 21, normalized size = 0.64

$$\frac{\sqrt{-a}\log\left(\tan(dx+c)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a)*log(tan(d*x + c)^2 + 1)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{a-\frac{a}{\cos(c+d\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(1/2),x)
[Out] int((a - a/cos(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sec^2\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(1/2),x)
[Out] Integral(sqrt(-a*sec(c + d*x)**2 + a), x)
```

3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] ln(sin(d*x+c))*tan(d*x+c)/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4121, 3658, 3475}

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx &= \int \frac{1}{\sqrt{-a\tan^2(c+dx)}}\,dx\\
&= \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 1.25

$$\frac{\tan(c+dx)(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c +
d*x]^2)])
```

fricas [A] time = 2.87, size = 56, normalized size = 1.75

$$-\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log\left(\frac{1}{2}\sin(dx+c)\right)}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(1/2*sin(d*x +
 c))/(a*d*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sec(dx+c)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sec(d*x + c)^2 + a), x)
```

maple [B] time = 1.96, size = 76, normalized size = 2.38

$$-\frac{\left(-\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\ln\left(\frac{2}{1+\cos(dx+c)}\right)\right)\sin(dx+c)}{d\sqrt{-\frac{a\left(\sin^2(dx+c)\right)}{\cos(dx+c)^2}}\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/d*(-ln(-(-1+cos(d*x+c))/sin(d*x+c))+ln(2/(1+cos(d*x+c))))*sin(d*x+c)/(-a
*sin(d*x+c)^2/cos(d*x+c)^2)^(1/2)/cos(d*x+c)
```

maxima [A] time = 0.43, size = 37, normalized size = 1.16

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}}-\frac{2\log(\tan(dx+c))}{\sqrt{-a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(-a) - 2*log(tan(d*x + c))/sqrt(-a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a-\frac{a}{\cos(c+d\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(1/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sec^2{(c+dx)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sec(c + d*x)**2 + a), x)
```

3.225 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{ad\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a/d/(-
a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{ad\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(2*a*d*Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(-a\tan^2(c+dx)\right)^{3/2}}\,dx$$
$$=-\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}}$$
$$=\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\int \cot(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}}$$
$$=\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}$$

Mathematica [**A**] time = 0.14, size = 57, normalized size = 0.85

$$-\frac{\tan^3(c+dx)\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d\left(-a\tan^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
d*x]^3)/(d*(-(a*Tan[c + d*x]^2))^(3/2))
```

fricas [**A**] time = 0.84, size = 94, normalized size = 1.40

$$-\frac{\left(2\left(\cos\left(dx+c\right)^3-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) - cos(d*x + c
))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^2*d*cos(d*x + c)^2 - a^2
*d)*sin(d*x + c))
```

giac [**B**] time = 0.85, size = 212, normalized size = 3.16

$$\frac{\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\sqrt{-a}\,a\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{8\sqrt{-a}\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{4\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{}{\sqrt{\cdot}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(tan(1/2*d*x + 1/2*c)^2/(sqrt(-a)*a*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1
/2*d*x + 1/2*c))) + 8*sqrt(-a)*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2*sgn(-ta
n(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) + 4*log(tan(1/2*d*x + 1/2*c)^
2)/(sqrt(-a)*a*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - (4*ta
n(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2
*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [B] time = 1.66, size = 141, normalized size = 2.10

$$\frac{\left(4\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-4\ln\left(\frac{2}{1+\cos(dx+c)}\right)\left(\cos^2(dx+c)\right)-\left(\cos^2(dx+c)\right)-4\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right.}{4d\cos(dx+c)^3\left(-\frac{a\left(\sin^2(dx+c)\right)}{\cos(dx+c)^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/4/d*(4*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d*x+c))-4*ln(2/(1+cos(d*x+c))
)*cos(d*x+c)^2-cos(d*x+c)^2-4*ln(-(-1+cos(d*x+c))/sin(d*x+c))+4*ln(2/(1+cos
(d*x+c)))-1)*sin(d*x+c)/cos(d*x+c)^3/(-a*sin(d*x+c)^2/cos(d*x+c)^2)^(3/2)
```

maxima [A] time = 0.43, size = 60, normalized size = 0.90

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}\,a}-\frac{2\,\log(\tan(dx+c))}{\sqrt{-a}\,a}+\frac{\sqrt{-a}}{a^2\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a) - 2*log(tan(d*x + c))/(sqrt(-a)*
a) + sqrt(-a)/(a^2*tan(d*x + c)^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\sec^2(c+dx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-3/2), x)
```

3.226 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^2/d/(-a*tan
(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^2*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^2*d*Sqrt[-
(a*Tan[c + d*x]^2)])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{5/2}}\,dx$$
$$= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$= -\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+dx)\,d}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+d}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

Mathematica [A] time = 0.28, size = 69, normalized size = 0.69

$$\frac{\tan^5(c+dx)\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d\left(-a\tan^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d
*x]])*Tan[c + d*x]^5)/(4*d*(-(a*Tan[c + d*x]^2))^(5/2))
```

fricas [A] time = 1.66, size = 125, normalized size = 1.25

$$\frac{\left(4\cos(dx+c)^3-4\left(\cos(dx+c)^5-2\cos(dx+c)^3+\cos(dx+c)\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-3\cos(dx+c)\right)\sqrt{\frac{a}{}}}{4\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*cos(d*x + c)^3 - 4*(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)
)*log(1/2*sin(d*x + c)) - 3*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d
*x + c)^2)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c))
```

giac [B] time = 1.11, size = 273, normalized size = 2.73

$$-\frac{\frac{64\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{32\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{\sqrt{-a}a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(64*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2*sgn(-tan(1/2*d*x +
1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - 32*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a
)*a^2*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - (sqrt(-a)*a^2*
sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^4
```

```
- 12*sqrt(-a)*a^2*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*tan(1
/2*d*x + 1/2*c)^2)/a^5 + (48*tan(1/2*d*x + 1/2*c)^4 - 12*tan(1/2*d*x + 1/2*
c)^2 + 1)/(sqrt(-a)*a^2*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
*tan(1/2*d*x + 1/2*c)^4))/d
```

maple [B] time = 1.78, size = 203, normalized size = 2.03

$$\frac{\left(32\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-32\ln\left(\frac{2}{1+\cos(dx+c)}\right)\left(\cos^4(dx+c)\right)-13\left(\cos^4(dx+c)\right)-64\left(\cos^2(dx+c)\right.}{32d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/32/d*(32*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))/sin(d*x+c))-32*ln(2/(1+cos(d*x+
c)))*cos(d*x+c)^4-13*cos(d*x+c)^4-64*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d
*x+c))+64*ln(2/(1+cos(d*x+c)))*cos(d*x+c)^2-6*cos(d*x+c)^2+32*ln(-(-1+cos(d
*x+c))/sin(d*x+c))-32*ln(2/(1+cos(d*x+c)))+11)*sin(d*x+c)/cos(d*x+c)^5/(-a*
sin(d*x+c)^2/cos(d*x+c)^2)^(5/2)
```

maxima [A] time = 0.45, size = 79, normalized size = 0.79

$$-\frac{\frac{2\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}\,a^2}-\frac{4\log(\tan(dx+c))}{\sqrt{-a}\,a^2}+\frac{2\sqrt{-a}\tan(dx+c)^2-\sqrt{-a}}{a^3\tan(dx+c)^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^2) - 4*log(tan(d*x + c))/(sqrt(
-a)*a^2) + (2*sqrt(-a)*tan(d*x + c)^2 - sqrt(-a))/(a^3*tan(d*x + c)^4))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(5/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\sec^2(c+dx)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-5/2), x)
```

3.227 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^3/d/(-a*tan
(d*x+c)^2)^(1/2)+1/6*cot(d*x+c)^5/a^3/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+
c))*tan(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^3*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^3*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + Cot[c + d*x]^5/(6*a^3*d*Sqrt[-(a*Tan[c + d*x]^
2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^3*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx = \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{7/2}}\,dx$$
$$= -\frac{\tan(c+dx)\int \cot^7(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$= -\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c+dx)\,d}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan}{}$$
$$= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\log}{}$$

Mathematica [A] time = 0.34, size = 79, normalized size = 0.59

$$-\frac{\tan^7(c+dx)\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)+12\log(\tan(c+dx))+12\log(\cos(c+dx))\right)}{12d\left(-a\tan^2(c+dx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] -1/12*((6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos
[c + d*x]] + 12*Log[Tan[c + d*x]])*Tan[c + d*x]^7)/(d*(-(a*Tan[c + d*x]^2))
^(7/2))
```

fricas [A] time = 0.64, size = 162, normalized size = 1.22

$$\frac{\left(18\cos(dx+c)^5-27\cos(dx+c)^3-12\left(\cos(dx+c)^7-3\cos(dx+c)^5+3\cos(dx+c)^3-\cos(dx+c)\right)\log\left(\frac{1}{2}\right.}{12\left(a^4d\cos(dx+c)^6-3a^4d\cos(dx+c)^4+3a^4d\cos(dx+c)^2-a^4d\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/12*(18*cos(d*x + c)^5 - 27*cos(d*x + c)^3 - 12*(cos(d*x + c)^7 - 3*cos(d*
x + c)^5 + 3*cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) + 11*cos(
d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^4*d*cos(d*x + c)^
6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

giac [B] time = 1.70, size = 275, normalized size = 2.07

$$-\frac{\frac{384\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{\sqrt{-a}a^3\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{192\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}a^3\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} + \frac{352\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-87\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+12}{\sqrt{-a}a^3\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/384*(384*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^3*sgn(-tan(1/2*d*x
+ 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - 192*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt
(-a)*a^3*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) + (352*tan(1/
2*d*x + 1/2*c)^6 - 87*tan(1/2*d*x + 1/2*c)^4 + 12*tan(1/2*d*x + 1/2*c)^2 -
1)/(sqrt(-a)*a^3*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*tan(1/
2*d*x + 1/2*c)^6) - (a^7*tan(1/2*d*x + 1/2*c)^6 - 12*a^7*tan(1/2*d*x + 1/2*
c)^4 + 87*a^7*tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a^10*sgn(-tan(1/2*d*x + 1/2
*c)^3 + tan(1/2*d*x + 1/2*c))))/d
```

maple [B] time = 1.83, size = 265, normalized size = 1.99

$$-\frac{\left(48\left(\cos^{6}(dx+c)\right)\ln\left(\frac{2}{1+\cos(dx+c)}\right)-48\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{6}(dx+c)\right)+25\left(\cos^{6}(dx+c)\right)-144\ln\left(\frac{\quad}{1+\cos\quad}\right.}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(7/2),x)
```

```
[Out] -1/48/d*(48*cos(d*x+c)^6*ln(2/(1+cos(d*x+c)))-48*ln(-(-1+cos(d*x+c))/sin(d*
x+c))*cos(d*x+c)^6+25*cos(d*x+c)^6-144*ln(2/(1+cos(d*x+c)))*cos(d*x+c)^4+14
4*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^4+144*ln(2/(1+c
os(d*x+c)))*cos(d*x+c)^2-144*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d*x+c))-3
3*cos(d*x+c)^2-48*ln(2/(1+cos(d*x+c)))+48*ln(-(-1+cos(d*x+c))/sin(d*x+c))+1
9)*sin(d*x+c)/(-a*sin(d*x+c)^2/cos(d*x+c)^2)^(7/2)/cos(d*x+c)^7
```

maxima [A] time = 0.45, size = 94, normalized size = 0.71

$$-\frac{\frac{6\log\left(\tan(dx+c)^{2}+1\right)}{\sqrt{-a}\,a^{3}}-\frac{12\log(\tan(dx+c))}{\sqrt{-a}\,a^{3}}+\frac{6\sqrt{-a}\tan(dx+c)^{4}-3\sqrt{-a}\tan(dx+c)^{2}+2\sqrt{-a}}{a^{4}\tan(dx+c)^{6}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^3) - 12*log(tan(d*x + c))/(sqr
t(-a)*a^3) + (6*sqrt(-a)*tan(d*x + c)^4 - 3*sqrt(-a)*tan(d*x + c)^2 + 2*sqr
t(-a))/(a^4*tan(d*x + c)^6))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^{2}}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\sec^{2}(c+dx)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-7/2), x)
```

3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15b^2f}+\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec}}{}$$

```
[Out] -1/15*(2*a^2-3*a*b-8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/b^2/f+1/15*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(c
os(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^2/f/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)-1/15*(a-8*b)*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1
/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*si
n(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/15*(a+4*b)*sec(f*x+e)*(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/b/f+1/5*sec(f*x+e)^3*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.69, antiderivative size = 471, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 412, 527, 524, 426, 424, 421, 419}

$$-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{15b^2f\sqrt{a\cos^2(e+fx)+b}}+\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((2*a^2 - 3*a*b - 8*b^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a +
b - a*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((2*a^2 - 3*
a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)
]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*b^2*f*Sqrt
[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*(
a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*b*f*Sqrt[b
 + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + ((a + 4*b)*Sec[e + f
*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])
/(15*b*f*Sqrt[b + a*Cos[e + f*x]^2]) + (Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f
*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(5*f*Sqrt[b + a*Cos[e +
 f*x]^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x^2)}}{\left(1-x^2\right)^{7/2}}\,dx, x, \sin(e+\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\left(1-x^2\right)^{7/2}}\,dx, x, \sin(e+\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{5f\sqrt{b+a\cos^2(e+fx)}} -$$

$$= \frac{(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{\left(2a^2-3ab-8b^2\right)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{\left(2a^2-3ab-8b^2\right)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{\left(2a^2-3ab-8b^2\right)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{\left(2a^2-3ab-8b^2\right)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 27.03, size = 0, normalized size = 0.00

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a} \sec \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

maple [C] time = 2.29, size = 6562, normalized size = 17.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a} \sec \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{{\cos\left(e+f\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^{2}{\left(e + f x \right)}} \sec^{5}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=288

$$\frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3bf}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e-\right.}}{3f}$$

```
[Out] 1/3*(a+2*b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f-1/3*(a
+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e
)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+2/3*(a+b
)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*
(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x
+e)^2)+1/3*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/
f
```

Rubi [A] time = 0.51, antiderivative size = 364, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 412, 527, 524, 426, 424, 421, 419}

$$\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a+b\sec^2(e+fx)}}{3bf\sqrt{a\cos^2(e+fx)+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a}}{3f\sqrt{a\cos^2(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2])/(3*b*f*Sqrt[b + a*Cos[e + f*x]^2]) - ((a + 2*b)*Sqrt[Cos[e + f*x]^2
]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqr
t[a + b - a*Sin[e + f*x]^2])/(3*b*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)]) + (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*
x]^2]) + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*
x]^2]*Tan[e + f*x])/(3*f*Sqrt[b + a*Cos[e + f*x]^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a\left(1-x^2\right)}}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(e+\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(e+\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{3f\sqrt{b+a\cos^2(e+fx)}} -$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} + \text{sec}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} + \text{sec}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} + \text{sec}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} - (a$$

Mathematica [F] time = 11.55, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \sec\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

maple [C] time = 1.50, size = 4739, normalized size = 16.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] 1/6/f*(6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+2*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+2*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b-8*I*cos(f*x+e)^5*a^(3/2)*b^(3/
2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^4*a^(3/2)*b^(3/2)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^3*a^(3/2)*b^(3/2)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-8*I*cos(f*x+e)^3*a^(1/2)*b^(5/2)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^2*a^(1/2)*b^(5/2)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+8*I*cos(f*x+e)^2*a^(3/2)*b^(3/2)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^4*a^(5/2)*b^(1/2)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^5*a^(5/2)*b^(1/2)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(
f*x+e)^2*a^2*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2
+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-4*cos(f*x+e)^
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+6*sin(f*x+e)*cos(f*x+e)^4*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-4*sin(f*x+e)*cos(f*x+e
)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-5*sin(f*x+e)*cos(
f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+6*sin(f*x+e)
*cos(f*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+6*sin(f
*x+e)*cos(f*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
```

```
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-4*
sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2
*b-5*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3+4*I*a^(1
/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*b^3+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-sin(f*x+e)*cos(f*x+e)^4
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-2*sin(f*x+e)*cos(f*x+e
)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-sin(f*x+e)*cos(f*x+
e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-2*sin(f*x+e)*cos(f
*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3+2*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+2*I*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)-2*I*sin(f*x+e)*cos(f*x+e)^4
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)-4*I*sin(f*
x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b
^(5/2)+2*I*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a^(5/2)*b^(1/2)-2*I*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(
```

```
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)-4*I*sin(f*x+e)*cos(f*x+e)^3*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2))*((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/cos(f*x+e)^2/sin(f*x+e)/b/(2*I*a
^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \sec\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\cos\left(e+f\,x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e+fx\right)}\, \sec^3\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

3.230 $\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=218

$$\frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\,s\right.}}{f\left(-a\sin^2(e+fx)\right.}$$

```
[Out] sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-EllipticE(sin(f*x+e)
,(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+(a+b)*EllipticF(sin(f*x+e),(a/(a+b))
^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a
*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.40, antiderivative size = 271, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4148, 6722, 1974, 412, 12, 493, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a\cos^2(e+fx)+b}}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e\right.}}{f\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(f
*Sqrt[b + a*Cos[e + f*x]^2]) - (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e
 + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) +
((a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sq
rt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(f*Sqrt[b +
a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x^2)}}{(1-x^2)^{3/2}}\,dx, x, \sin(e+\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{3/2}}\,dx, x, \sin(e+f\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{\left(a\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\sqrt{\cos^2(e+f}}{}$$

Mathematica [F] time = 11.66, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

maple [C] time = 1.48, size = 3454, normalized size = 15.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*(2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*a*b-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a*b+2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b-4*I*a
^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+sin(f*x+e)*cos(f*x+e
)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+sin(f*x+e)*cos(f*x+
e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-2*sin(f*x+e)*cos(f
*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+sin(f*x+e)*cos(f*
x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+sin(f*x+e)*cos(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+2*((2*I*a^(1/2)*b^(
```

```
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^2*a*b+2*I*sin(f*x+e)*cos(f*x+e)^2*a^(3/2)*b^(1/2)*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*sin(f*x+e)*cos(f*x+e)^2*a^(1/2
)*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*sin(f*x+e)*co
s(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*
I*sin(f*x+e)*cos(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)^3*a^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)^2*a^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*sin(f*x+e)*cos(
f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+4*I*cos(f*x+e)
^3*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*a
^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^2*a^(
3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*((b+a*cos(f*x+e)^2)/c
os(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)/(2*I*a^(1/2)*b^(1/2)-a+b)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a}\, \sec\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\cos\left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\sec\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

3.231 $\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[Out] EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 426, 424}

$$\frac{\sqrt{\cos^2(e+fx)}\,\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a+b\sec^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\sqrt{a\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(f*Sqrt[b + a*Cos[e + f*x]
^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x^2)}}{\sqrt{1-x^2}}\,dx, x, \sin(\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}}{f\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$= \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b}}{f\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Mathematica [A] time = 0.26, size = 69, normalized size = 0.86

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,E\left(e+fx\left|\frac{a}{a+b}\right.\right)}{f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] (Sqrt[2]*Cos[e + f*x]*EllipticE[e + f*x, a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2])/(f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

maple [C] time = 2.02, size = 3408, normalized size = 42.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*(2*I*sin(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))+2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e
)-EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*sin(f*x+e)+2*I*sin(f*
x+e)*a^(1/2)*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)*b^2-2*sin
(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+2*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+2*sin(f*x+e)*cos(f*x+e)*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*sin(f*x+e)*cos(f*x+e)*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b-4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
```

```
+b)^2)^(1/2))*a^2-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b
-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+2*I*sin(f*x+e)*
cos(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+
2*I*sin(f*x+e)*cos(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^3*a^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^2*a^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-EllipticE((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*b^2*sin(f*x+e)+4*I*cos(f*x+e)^3*a^(3/2)*b^(1/
2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*a^(1/2)*b^(3/2)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^2*a^(3/2)*b^(1/2)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*
x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\,\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2}}{3af\left(-\right.}$$

```
[Out] 1/3*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+1/3
*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b
*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*
sin(f*x+e)^2)
```

Rubi [A] time = 0.39, antiderivative size = 299, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4148, 6722, 1974, 417, 524, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\cos^2(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{3f\sqrt{a\cos^2(e+fx)+b}}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3af\sqrt{-a\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[
e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((2*a + b)*Sqrt[Cos[e + f*x
]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*
Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e
 + f*x]^2)/(a + b)])/(3*a*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{b+a\left(1-x^2\right.}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{a+b-ax^2}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [C] time = 8.50, size = 539, normalized size = 2.19

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{(2a+2b)\sqrt{\frac{a\cos(2e+2fx)+a+2b}{2a+2b}}\,E\left(\frac{1}{2}(2e+2fx)|\frac{2a}{2a+2b}\right)}{f\sqrt{a\cos(2e+2fx)+a+2b}} + \frac{\sin(2e+2fx)\cos(2(e+fx))\sec\left(2\left(\frac{1}{2}\left(\cos^{-1}(\cos(2e+2\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(((2*a + 2*b)*Sqrt[(a + 2*b + a*Co
s[2*e + 2*f*x])/(2*a + 2*b)]*EllipticE[(2*e + 2*f*x)/2, (2*a)/(2*a + 2*b)])
/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + (Cos[2*(e + f*x)]*(-(Sqrt[-(a + b
)^(-1)]*(-a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x])*Sqrt[a + 2*b + a
*Cos[2*e + 2*f*x]]) - I*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*
Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(
a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - I*a
*b*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a
 + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sq
rt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e +
ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(3*a^2*Sqrt[-(a + b)^(-1)]*
f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Co
s[2*e + 2*f*x]^2])))/(2*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

maple [C] time = 2.33, size = 4623, normalized size = 18.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] -1/6/f*(6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^2*b-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a^3
-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a*b^2*sin(f*x+e)+6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a*b^2-5*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*sin(f*x+e)*a^2*b-4*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
```

)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*sin(f*x+e)*a*b^2+4*I*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*a^(5/2)*b^(1/2)*sin(f*x+e)+2*I*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)*sin(f*x+e)-2*I*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)*sin(f*x+e)-2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^3-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b-2*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2-2*cos(f*x+e)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^5*a^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-
8*I*a^(3/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*a^(1/2)*b^(
5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+6*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+4*I*cos(f*x+e)*a^
(5/2)*b^(1/2)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*
I*cos(f*x+e)*a^(3/2)*b^(3/2)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))-2*I*cos(f*x+e)*a^(1/2)*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-5
2^(1/2)((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-4*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(

```
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a*b^2-2*cos(f*x+e)*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a^3-cos(f*x+e)*2^(1/
2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^3+4*I*cos(f*x+e)^5*
a^(5/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^3*a^
(5/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+8*I*cos(f*x+e)^3*a^(3
/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-8*I*cos(f*x+e)^2*a^(5/2
)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^2*a^(3/2)*
b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*a^(3/2)*b^(3
/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*a^(1/2)*b^(5/2)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos
(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\,\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=338

$$\frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(2a-b)}{}$$

```
[Out] 2/15*(2*a-b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/a/f+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)*(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)/a/f+1/15*(8*a^2+3*a*b-2*b^2)*EllipticE(sin(f*x+e),(
a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/a^2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-2/15*(2*a-b)*b*(a+b)*EllipticF(sin
(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+
e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.57, antiderivative size = 400, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 416, 528, 524, 426, 424, 421, 419}

$$\frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}}-\frac{2b(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(2*a - b)*Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a
+ b - a*Sin[e + f*x]^2])/(15*a*f*Sqrt[b + a*Cos[e + f*x]^2]) + (Cos[e + f*x
]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2)^(3/2
))/(5*a*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e
 + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*
x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a^2*f*Sqrt[b + a*Cos[e + f*x]^2]*
Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*Sqrt[Cos[e +
 f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^2*f*Sqrt[b + a*Cos[e + f*x]
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2 \sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\sqrt{b+a\left(1-\right.}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\sqrt{a+b-ax}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{3/}}{5af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 11.51, size = 0, normalized size = 0.00

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

maple [C] time = 2.54, size = 6394, normalized size = 18.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \cos\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=186

$$\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-(3a-$$

```
[Out] 1/16*(a+b)*(a^2-2*a*b+5*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)^(1/2))/b^(5/2)/f+1/16*(a^2-2*a*b+5*b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*
x+e)/b^2/f-1/24*(3*a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/6*s
ec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A] time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 416, 388, 195, 217, 206}

$$\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-(3a-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) + ((a^2 - 2*a*b + 5*b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) - ((3*a - 5*b)*Tan[e + f*x]*(a + b
 + b*Tan[e + f*x]^2)^(3/2))/(24*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*(a +
b + b*Tan[e + f*x]^2)^(3/2))/(6*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} + \frac{\text{Subst}\left(\int \left(-a\right.\right.}{}$$

$$= -\frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+fx)\,\text{ta}}{}$$

$$= \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\,\text{ta}}{}$$

$$= \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\,\text{ta}}{}$$

$$= \frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} + \frac{\left(a^2-2ab+5b\right.}{}$$

Mathematica [C] time = 11.40, size = 968, normalized size = 5.20

$$-\frac{ie^{i(e+fx)}\sqrt{ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2+4b}\left(\frac{15(a+b)\tan^{-1}\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}\right)a^2}{8b^{3/2}} + \frac{15\left(-1+e^{2i(e+fx)}\right)\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4}}{8b\left(1+e^{2i(e+fx)}\right)^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-1/15*I)*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2
*I)*(e + f*x))]*((-15*a*(-1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f
```

```
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])/(4*(1 + E^((2*I)*(e + f*x)))^2) + (1
5*a^2*(-1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2])/(8*b*(1 + E^((2*I)*(e + f*x)))^2) + (75*b*(-1 + E^((2*
I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2
])/(8*(1 + E^((2*I)*(e + f*x)))^2) + (40*(4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2)^(3/2))/(1 + E^((2*I)*(e + f*x)))^6 - (60*(4*b*E^((2
*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(1 + E^((2*I)*(e + f
*x)))^5 - (24*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/
2))/(1 + E^((2*I)*(e + f*x)))^4 + (3*(5*a + 21*b)*(4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(2*b*(1 + E^((2*I)*(e + f*x)))^4) +
 (50*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(1 +
E^((2*I)*(e + f*x)))^3 + (7*(7*a + 15*b)*(4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2)^(3/2))/(4*b*(1 + E^((2*I)*(e + f*x)))^3) - (2*(8*a
+ 35*b)*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(b
*(1 + E^((2*I)*(e + f*x)))^3) + (15*a^2*(a + b)*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]])/(8*b^(3/2)) - (15*a*(a + b)*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))
))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(4*Sqrt[
b]) + (75*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))))/Sqrt[
4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/8)*Cos[e + f*x]*
Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
+ E^((2*I)*(e + f*x)))^2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

fricas [A] time = 1.98, size = 468, normalized size = 2.52

$$\left[\frac{3\left(a^{3}-a^{2}b+3\,ab^{2}+5\,b^{3}\right)\sqrt{b}\,\cos\left(fx+e\right)^{5}\log\left(\frac{\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a-b\right)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+\cdots\right.\right.}{\cos\left(fx+e\right)^{4}}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b - 4*a*b^2 -
15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1
/96*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^
2*b - 4*a*b^2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^6, x)

maple [C] time = 2.12, size = 2518, normalized size = 13.54

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)

```
[Out] -1/48/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-18*sin(f*x+e)*
cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2-10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*cos(f*x+e)^3*a*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*
a^2*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+10*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+14*cos(f*x+e)^4*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a*b^2-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+9*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*a^2*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3+3*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^3-8*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*b^3+10*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3-4*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-15*cos(f*x
+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*cos(f*x+e)^6*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+15*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2-15*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^3+15*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+8*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*a^3-30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3+3*sin(f*x+e)*cos
(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
```

```
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+15*sin(f*x+e)
*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3)/(-1+cos(
f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)^5/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/b^2
```

maxima [A] time = 0.36, size = 317, normalized size = 1.70

$$\frac{8\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{3}}{b}+\frac{3\,(a+b)^{2}a\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{3\,(a+b)^{2}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{12\,(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{12\,(a+b)\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)^3/b + 3*(a + b)^2*a*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)^2*arcsinh(b*tan(
f*x + e)/sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*a*arcsinh(b*tan(f*x + e)/sqr
t((a + b)*b))/b^(3/2) - 12*(a + b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/
sqrt(b) + 24*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 24*sqrt(b)
*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 24*sqrt(b*tan(f*x + e)^2 + a + b
)*tan(f*x + e) - 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)/b^
2 + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)/b^2 + 24*(b*tan
(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)/b - 12*sqrt(b*tan(f*x + e)^2 + a +
b)*(a + b)*tan(f*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}{\cos\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\,\sec^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4bf}-\frac{(a-3b)\tan(e+fx)\sqrt{a-}}{8bf}$$

```
[Out] -1/8*(a-3*b)*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b
^(3/2)/f-1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*tan(f*x+
e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4146, 388, 195, 217, 206}

$$-\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4bf}-\frac{(a-3b)\tan(e+fx)\sqrt{a-}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a - 3*b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*b^(3/2)*f) - ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a-3b)\,\text{Subst}\left(\int \sqrt{a+b+b\right.}{4bf}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\right.}{4bf}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\right.}{4bf}$$

$$= -\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b)\tan(e+fx)\sqrt{a+\right.}{8bf}$$

Mathematica [C] time = 8.13, size = 390, normalized size = 3.20

$$-\frac{ie^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{\left(a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}\right)^{3/2}}{\left(1+e^{2i(e+fx)}\right)^3} - \frac{2\left(a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}\right)^{3/2}}{\left(1+e^{2i(e+fx)}\right)^4} + \frac{1}{2}(\right.}{2\sqrt{2}\,bf\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-1/2*I)*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*((-2*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2)^(3/2))/(1 + E^((2*I)*(e + f*x)))^4 + (4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2)^(3/2)/(1 + E^((2*I)*(e + f*x)))^3 + ((-a + 3*b)*(((-
1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
+ f*x)))^2])/(1 + E^((2*I)*(e + f*x)))^2 + ((a + b)*ArcTan[(Sqrt[b]*(-1 + E
^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2]])/Sqrt[b]))/2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]])
```

fricas [A] time = 0.85, size = 390, normalized size = 3.20

$$\left[-\frac{\left(a^2-2ab-3b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}}{\cos\left(fx+e\right)^4}\right.}{32b^2f\cos\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a*b + 3*b^2)*cos(f*x + e)^2 + 2*
b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f
*x + e)^3), -1/16*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*((a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

maple [C] time = 1.72, size = 1769, normalized size = 14.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-2*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*cos(f*x+e)^4*sin(f*x+e)*a^2+4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a*b
+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e)
```

```
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x+e)*b^2+2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^2-2*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a*b-3*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*b^2+((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^
2-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b+3*cos(f*x+e)^3*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(
f*x+e)^2*a*b-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-2*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)^3/b
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.36, size = 173, normalized size = 1.42

$$-\frac{\frac{(a+b)a\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{4\,a\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-4\,\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)-4\,\sqrt{b\tan\left(fx+e\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsinh(b*tan(f*x + e)
/sqrt((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b
)) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e) - 2*(b*tan(f*x + e)^2 +
a + b)^(3/2)*tan(f*x + e)/b + sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*
x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}{\cos\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}$$

```
[Out] 1/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+
1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4146, 195, 217, 206}

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,$$

$$= \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+}{2f}$$

Mathematica [B] time = 1.61, size = 210, normalized size = 2.76

$$\frac{\tan(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{\frac{b\sin^2(e+fx)}{a+b}}(a\cos(2(e+fx))+a+2b)+\sqrt{2}(a+\right.}{\sqrt{2}f\sqrt{\frac{b\sin^2(e+fx)}{a+b}}(a\cos(2(e+fx))+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*(Sqrt[2]*(a + b)
*ArcTanh[Sqrt[(b*Sin[e + f*x]^2)/(a + b)]/Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)]]*Cos[e + f*x]^2*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)] + (a +
 2*b + a*Cos[2*(e + f*x)])*Sqrt[(b*Sin[e + f*x]^2)/(a + b)])*Tan[e + f*x])/
(Sqrt[2]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sqrt[(b*Sin[e + f*x]^2)/(a
+ b)])
```

fricas [B] time = 0.74, size = 320, normalized size = 4.21

$$\left[\frac{(a+b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{\cos\left(fx+e\right)^4}\right.}{8bf\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(b*f*cos(f*x + e)), 1/4*((a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
```

```
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \sec\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

maple [C] time = 1.36, size = 1098, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-cos(f*x+e)^2*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a-cos(f*x+e)^2*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b+2*cos(f*x+e)^2*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*sin(f*x+e)*a+2*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b+((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a-((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^2*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/(b+a*cos
(f*x+e)^2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.34, size = 69, normalized size = 0.91

$$\frac{\frac{a \operatorname{arsinh}\left(\frac{b \tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + \sqrt{b} \operatorname{arsinh}\left(\frac{b \tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right) + \sqrt{b \tan\left(fx+e\right)^2 + a + b}\, \tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b)) + sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)
)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+f}}\right.}{f}$$

$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 1.08, size = 1227, normalized size = 15.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
```

```
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.38, size = 588, normalized size = 7.44

$$-\frac{\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin\left(fx+e\right)}, \sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^{2}+6ab-b^{2}}{\left(a+b\right)^{2}}}\right)a+\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin\left(fx+e\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*(EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a+EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-2*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a)*cos(f*x+e)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+
cos(f*x+e))/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[Out] 1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4146, 378, 377, 203}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\Big(\int$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\Big(\int$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+}$$

Mathematica [A] time = 0.67, size = 136, normalized size = 1.66

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{2}\sqrt{a}\sin(e+fx)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\right)}{2\sqrt{2}\sqrt{a}f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[(a + 2*b + a*Cos[2*(e
+ f*x)])/(a + b)])
```

fricas [B] time = 1.35, size = 499, normalized size = 6.09

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{-a}\,(a+b)\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) - sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)*sin(f*x + e) - (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
```

```
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

maple [C] time = 1.76, size = 1064, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*cos(f*x+e)^3*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^
2*a-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b+((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(e+f\,x\right)}^2\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\cos^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**2, x)
```

3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=140

$$\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4af}+\frac{(3a-b)\sin(e+f$$

```
[Out] 1/8*(3*a-b)*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(
3/2)/f+1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f+1/4
*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4146, 382, 378, 377, 203}

$$\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4af}+\frac{(3a-b)\sin(e+f$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a - b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(8*a^(3/2)*f) + ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(8*a*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e +
f*x]^2)^(3/2))/(4*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af} + \frac{(3a-b)\,\text{Subs}}{}$$

$$= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+}{}$$

$$= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+}{}$$

$$= \frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\cos(e+fx)\sin(}{}$$

Mathematica [A] time = 1.23, size = 152, normalized size = 1.09

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\,(3a-b)\sqrt{a+b}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}\,\sin(e+fx)(a\cos(2(e+fx))+}{8a^{3/2}f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*(3*a - b)*Sqrt[a + b]*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]] + Sqrt[a]*(4*a + b + a*Cos[2*(e + f
*x)])*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(8*a^(3/2
)*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

fricas [A] time = 0.84, size = 567, normalized size = 4.05

$$\left[\frac{\left(3\,a^2+2\,ab-b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 + 2*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + a*b)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f),
 -1/32*((3*a^2 + 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*
(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*
b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x + e
)^3 + (3*a^2 + a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(a^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a} \cos\left(fx+e\right)^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

maple [C] time = 2.31, size = 1713, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f*(-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2+3*2^(1/
2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-4*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
```

```
(a+b))^(1/2))*a*b*sin(f*x+e)+2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^3*a^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^3*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2+3
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b-3*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*cos(f*x+e)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/a/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\,\cos^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{(3a-b)(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^2f}+\frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16a^{5/2}f}$$

```
[Out] 1/16*(a+b)*(5*a^2-2*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)
^(1/2))/a^(5/2)/f+1/48*(3*a-b)*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f
*x+e)^2)^(1/2)/a^2/f+1/24*(5*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)
^2)^(1/2)/a/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.21, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 412, 527, 12, 377, 203}

$$\frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}+\frac{(3a-b)(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{48a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + ((3*a - b)*(5*a + 3*b)*Cos[e + f*x]*Sin
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^2*f) + ((5*a + b)*Cos[e + f
*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) + (Cos[e + f*x]
^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{-5(a}{(1+x^2)}\right.}{}$$

$$= \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e}{}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} +$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} +$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} +$$

$$= \frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{(3a-b)(5a+3b}{}$$

Mathematica [C] time = 16.37, size = 1902, normalized size = 9.70

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^10*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sqrt[a + b*Sec[
e + f*x]^2]*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2,
```

```
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a +
 b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]^5*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/(3*(a + b)*AppellF1[1/
2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1
[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)
*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2) - (12*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2) + (3*(a + b)*Cos[e
 + f*x]^4*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sin[e + f*x]*(-1/3*(a*f*Appell
F1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f
*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(f*(3*(a + b)
*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*Sin[e + f*x]*(-2*f*(a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*
(a + b)*(-1/3*(a*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1,
 -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/3) - Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b))
- (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/5) + 4*(a + b)*((-3*a*f*AppellF1[5/2, -1, 1
/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f
*x])/(5*(a + b)) - (6*f*Cos[e + f*x]*Sin[e + f*x]*(1 - (a*Sin[e + f*x]^2)/(
a + b))^(3/2)*(5/(6*(1 - (a*Sin[e + f*x]^2)/(a + b))) + (5*(a + b)^3*Csc[e
+ f*x]^6*((-2*a*Sin[e + f*x]^2)/(a + b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)
^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(
Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])))/(32*a^3*(1 - (a*Sin[e +
 f*x]^2)/(a + b)))))/5))))/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2
, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) - (3
*a*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

fricas [A] time = 2.01, size = 641, normalized size = 3.27

$$\left[-\frac{3\left(5a^3+3a^2b-ab^2+b^3\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14a^3b+5\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 + 2*(5*
a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), -1/192*(3*
(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x +
 e)^5 + 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

maple [C] time = 2.46, size = 2436, normalized size = 12.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/48/f*(-30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-14*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^3*a*b^2-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2-
10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b-3*2^(1/2)*((I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^
3*sin(f*x+e)+15*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-15
*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+10*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+14*((2*I*a^(1/2)*b^(1/2)+a-b)/
```

```
(a+b))^(1/2)*cos(f*x+e)^2*a^2*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^2*a*b^2+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3+
10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3-15*cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^7*a^3-10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^5*a^3+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3+15*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin
(f*x+e)-18*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f
*x+e)+9*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e))*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/a^2/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=450

$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35b^2f}+\frac{\left(a^2+11ab+8b^2\right)\tan(e+fx)}{}$$

```
[Out] -2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)/b^2/f+2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b
))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^
2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/35*(a+b)*(a^2-16*a*b-16*b^2)*EllipticF
(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/
35*(a^2+11*a*b+8*b^2)*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*
tan(f*x+e)/b/f+2/35*(4*a+3*b)*sec(f*x+e)^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^5*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.89, antiderivative size = 572, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a+b\sec^2(e+fx)}}{35b^2f\sqrt{a\cos^2(e+fx)+b}}+\frac{\left(a^2+11ab+8b^2\right)\tan(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]
*Sqrt[a + b - a*Sin[e + f*x]^2])/(35*b^2*f*Sqrt[b + a*Cos[e + f*x]^2]) + (2
*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[
e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x
]^2])/(35*b^2*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
 b)]) - ((a + b)*(a^2 - 16*a*b - 16*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticF[Arc
Sin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e
+ f*x]^2)/(a + b)])/(35*b*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2]) + ((a^2 + 11*a*b + 8*b^2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^
2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(35*b*f*Sqrt[b + a*Cos[e +
f*x]^2]) + (2*(4*a + 3*b)*Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a
+ b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(35*f*Sqrt[b + a*Cos[e + f*x]^2]) + (
b*Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*
Tan[e + f*x])/(7*f*Sqrt[b + a*Cos[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
```

```
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{\left(1-x^2\right)^3} dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a\left(1-x^2\right)\right)^{3/2}}{\left(1-x^2\right)^{9/2}} dx, x, \sin\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{\left(1-x^2\right)^{9/2}} dx, x, \sin(\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+f}{7f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(4a+3b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}}{35f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e}}{35bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 9.68, size = 0, normalized size = 0.00

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{7}+a\sec\left(fx+e\right)^{5}\right)\sqrt{b\sec\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^7 + a*sec(f*x + e)^5)*sqrt(b*sec(f*x + e)^2 + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

maple [C] time = 2.83, size = 7996, normalized size = 17.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{3/2}}{\cos\left(e+f\,x\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\sec^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**5, x)
```

3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=371

$$\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15bf}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+fx)}\,\sqrt{}}{}$$

```
[Out] 1/15*(3*a^2+13*a*b+8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/b/f-1/15*(3*a^2+13*a*b+8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(co
s(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*
x+e)^2/(a+b))^(1/2)+1/15*(a+b)*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/
2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin
(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)+2/15*(3*a+2*b)*sec(f*x+e)*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^3*(sec
(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.70, antiderivative size = 470, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a+b\sec^2(e+fx)}}{15bf\sqrt{a\cos^2(e+fx)+b}}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a +
b - a*Sin[e + f*x]^2])/(15*b*f*Sqrt[b + a*Cos[e + f*x]^2]) - ((3*a^2 + 13*a
*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]
*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*b*f*Sqrt[b
+ a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(9*a +
 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[
a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*f*Sqrt[b +
a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + (2*(3*a + 2*b)*Sec[e +
f*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x]
)/(15*f*Sqrt[b + a*Cos[e + f*x]^2]) + (b*Sec[e + f*x]^3*Sqrt[a + b*Sec[e +
f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(5*f*Sqrt[b + a*Cos[e
+ f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a(1-x^2)\right)^{3/2}}{(1-x^2)^{7/2}}\,dx, x,\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{(1-x^2)^{7/2}}\,dx, x, \text{s}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e}{5f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(3a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+f}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 16.24, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^5 + a\sec\left(fx+e\right)^3\right)\sqrt{b\sec\left(fx+e\right)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*sec(f*x + e)^5 + a*sec(f*x + e)^3)*sqrt(b*sec(f*x + e)^2 + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)

maple [C] time = 1.82, size = 6562, normalized size = 17.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)

[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)

[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**3, x)

3.243 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=290

$$\frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}+\frac{b\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin\right.}}{3f}$$

```
[Out] 2/3*(2*a+b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-2/3*(2*a
+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^
2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(a+b)*(3
*a+2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x
+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*s
in(f*x+e)^2)+1/3*b*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan
(f*x+e)/f
```

Rubi [A] time = 0.53, antiderivative size = 366, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a+b\sec^2(e+fx)}}{3f\sqrt{a\cos^2(e+fx)+b}}+\frac{b\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+f}}{3f\sqrt{a\cos^2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(2*a + b)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e +
 f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]) - (2*(2*a + b)*Sqrt[Cos[e + f*x]
^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*S
qrt[a + b - a*Sin[e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(3*a + 2*b)*Sqrt[Cos[e + f*x]^2]*Ellip
ticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*
Sin[e + f*x]^2]) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b -
a*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f*Sqrt[b + a*Cos[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{1-x^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a\left(1-x^2\right)\right)^{3/2}}{\left(1-x^2\right)^{5/2}}dx, x, \text{s}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{\left(1-x^2\right)^{5/2}}dx, x, \text{si}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 11.48, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^3+a\sec\left(fx+e\right)\right)\sqrt{b\sec\left(fx+e\right)^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^3 + a*sec(f*x + e))*sqrt(b*sec(f*x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

maple [C] time = 1.47, size = 5185, normalized size = 17.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=224

$$\frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{f}+\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)}}{f\left(-a\sin^2(e+\right.}$$

```
[Out] b*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+(a-b)*EllipticE(si
n(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+b*(a+b)*EllipticF(sin(f*x+e)
,(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.30, antiderivative size = 277, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4148, 6722, 1974, 413, 524, 426, 424, 421, 419}

$$\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a\cos^2(e+fx)+b}}+\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\,\mathrm{se}}}{f\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/
(f*Sqrt[b + a*Cos[e + f*x]^2]) + ((a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Si
n[e + f*x]^2])/(f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)]) + (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/
(f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \left(a+\frac{b}{1-x^2}\right)^{3/2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a(1-x^2)\right)^{3/2}}{(1-x^2)^{3/2}}dx, x,\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{3/2}}dx, x, \text{si}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\text{co}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{((-a}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{((-a}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{(a-}{}$$

Mathematica [F] time = 12.58, size = 0, normalized size = 0.00

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)\sec\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{b\sec\left(fx+e\right)^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)*sec(f*x + e)^2 + a*cos(f*x + e))*sqrt(b*sec(f*x +
e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

maple [C] time = 1.55, size = 3632, normalized size = 16.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^2*(-4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)*a*b^2-sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*cos(f*x+e)^2*a^2*b+sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*cos(f*x+e)^2*a*b^2-cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*sin(f*x+e)*a^2*b+cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*sin(f*x+e)*a*b^2+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b-2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3+2*cos(f*x+e)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)*b^3-4*I*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)+sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)
^2*b^3-sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e
)^2*a^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*b^3-cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*sin(f*x+e)*a^3+cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b
```

```
))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*sin(f*x+e)*b^3+2*I*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*cos(f*x+e)^2*a^(5/2)*b^(1/2)+4*I*sin(f*x+e)*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^2*a^(3/2)*b^(3/2)+2*I*sin(f*x+e)*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^2*a^(1/2)*b^(5/2)
+2*I*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*
a^(5/2)*b^(1/2)+4*I*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*cos(f*x+e)*a^(3/2)*b^(3/2)+2*I*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*cos(f*x+e)*a^(1/2)*b^(5/2)+4*I*cos(f*x+e)*a^(1/2)*b^(5/2
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^4*a^(5/2)*b^(1/2)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*cos(f*x+e)^3*a^(5/2)*b^(1/2)+4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^3*a^(3/2)*b^(3/2)-4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*cos(f*x+e)*a^(3/2)*b^(3/2))/sin(f*x+e)/(b+a*cos(f*x+e)^2)^2/(2*I*a^(1/
2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=241

$$\frac{a\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\ \cdots}{3\cdots}$$

```
[Out] 1/3*a*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+2
/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b
*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*si
n(f*x+e)^2)
```

Rubi [A] time = 0.42, antiderivative size = 294, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4148, 6722, 1974, 416, 524, 426, 424, 421, 419}

$$\frac{a\sin(e+fx)\cos^2(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{3f\sqrt{a\cos^2(e+fx)+b}}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e\cdots}{a+b}}\ \cdots}{3f\sqrt{-a\sin^2(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Si
n[e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1
- (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[
ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+\frac{b}{1-x^2}\right)^{3/2}dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a\left(1-x^2\right)\right)^{3/2}}{\sqrt{1-x^2}}dx,x\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{\sqrt{1-x^2}}dx,x,s\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [A] time = 1.88, size = 179, normalized size = 0.74

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(4\sqrt{2}\left(a^2+3ab+2b^2\right)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}E\left(e+fx\middle|\frac{a}{a+b}\right)+a\sin(2(e+fx))(\right.}{6f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(4*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*S
qrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] -
 2*Sqrt[2]*b*(a + b)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF
[e + f*x, a/(a + b)] + a*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/
(6*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^3\sec\left(fx+e\right)^2+a\cos\left(fx+e\right)^3\right)\sqrt{b\sec\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^3*sec(f*x + e)^2 + a*cos(f*x + e)^3)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

maple [C] time = 1.71, size = 5069, normalized size = 21.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=319

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{a\sin(\cdots}{\cdots}$$

```
[Out] -2/15*(-2*a-3*b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/f+1/5*a*cos(f*x+e)^4*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)/f+1/15*(8*a^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(c
os(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f
*x+e)^2/(a+b))^(1/2)-1/15*b*(a+b)*(4*a+3*b)*EllipticF(sin(f*x+e),(a/(a+b))^
(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*
sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A] time = 0.64, antiderivative size = 395, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4148, 6722, 1974, 416, 528, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - 3*(a + b))*Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*
Sqrt[a + b - a*Sin[e + f*x]^2])/(15*f*Sqrt[b + a*Cos[e + f*x]^2]) + (a*Cos[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2])/(5*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[C
os[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e
+ f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a*f*Sqrt[b + a*Cos[e + f*x]^2
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(4*a + 3*b)*Sqrt[Cos[e
 + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*
x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a*f*Sqrt[b + a*Cos[e + f*x]
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\left(b+a\left(1\right.\right.\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\left(a+b-\right.\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e}}{5f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [C] time = 8.98, size = 350, normalized size = 1.10

$$\frac{\cos^3(e+fx)\csc(2(e+fx))\left(a+b\sec^2(e+fx)\right)^{3/2}\left(a\left(a\sqrt{-\frac{1}{a+b}}\sin^2(2(e+fx))\sqrt{a\cos(2(e+fx))+a+2b}\,(3a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]^3*Csc[2*(e + f*x)]*(a + b*Sec[e + f*x]^2)^(3/2)*((-8*I)*Sqrt[
2]*b*(8*a^2 + 13*a*b + 3*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticE[I*Arc
Sinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a
+ b)/b]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*((16*I)*Sqrt[2]*b*(2*a + 3*b)*
Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt
[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a + b)/b]*Sqrt[(a*Sin[e + f*x]^2
)/(a + b)] + a*Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a
 + 12*b + 3*a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]^2)))/(30*a^2*Sqrt[-(a + b)
^(-1)]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^5\sec\left(fx+e\right)^2+a\cos\left(fx+e\right)^5\right)\sqrt{b\sec\left(fx+e\right)^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^5*sec(f*x + e)^2 + a*cos(f*x + e)^5)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

maple [C] time = 2.10, size = 6396, normalized size = 20.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(e+f\,x\right)}^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.247 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=243

$$\frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{192b^2f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b\,\text{t}}}{128b^2f}$$

```
[Out] 1/128*(a+b)^2*(3*a^2-10*a*b+35*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f
*x+e)^2)^(1/2))/b^(5/2)/f+1/128*(a+b)*(3*a^2-10*a*b+35*b^2)*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-10*a*b+35*b^2)*tan(f*x+e)*(a+b+b*
tan(f*x+e)^2)^(3/2)/b^2/f-1/48*(3*a-7*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5
/2)/b^2/f+1/8*sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/b/f
```

Rubi [A] time = 0.22, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4146, 416, 388, 195, 217, 206}

$$\frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{192b^2f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b\,\text{t}}}{128b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) + ((a + b)*(3*a^2 - 10*a*b + 35*b
^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(128*b^2*f) + ((3*a^2 - 10
*a*b + 35*b^2)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(192*b^2*f) -
 ((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(48*b^2*f) + (
Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf} + \frac{\text{Subst}\left(\int\left(-a\right.\right.}{}$$

$$= -\frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\,\text{ta}}{}$$

$$= \frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-}{}$$

$$= \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(}{}$$

$$= \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(}{}$$

$$= \frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} + \frac{(a+b)\left(3a\right.}{}$$

Mathematica [C] time = 10.85, size = 512, normalized size = 2.11

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{3\left(3a^2-10ab+35b^2\right)(a+b)^2\log\left(\frac{4if\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}-4\sqrt{b}f\left(-1+e^{2i(e\cdots}\right.}{1+e^{2i(e+fx)}}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-9*a^3*(1 +
 E^((2*I)*(e + f*x)))^6 + 3*a^2*b*(1 + E^((2*I)*(e + f*x)))^4*(5 + 18*E^((2
*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x))) + a*b^2*(1 + E^((2*I)*(e + f*x)))^2
*(145 + 948*E^((2*I)*(e + f*x)) + 2758*E^((4*I)*(e + f*x)) + 948*E^((6*I)*(
e + f*x)) + 145*E^((8*I)*(e + f*x))) + b^3*(105 + 910*E^((2*I)*(e + f*x)) +
 3591*E^((4*I)*(e + f*x)) + 8644*E^((6*I)*(e + f*x)) + 3591*E^((8*I)*(e + f
*x)) + 910*E^((10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x)))))/(1 + E^((2*I)
*(e + f*x)))^8 - (3*(a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*Log[(-4*Sqrt[b]*(-1
 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(96*S
qrt[2]*b^(5/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [A] time = 8.55, size = 566, normalized size = 2.33

$$\left[\frac{3\left(3a^4-4a^3b+18a^2b^2+60ab^3+35b^4\right)\sqrt{b}\cos\left(fx+e\right)^7\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\right.}{\text{co}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*sqrt(b)*cos(f
*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x +
 e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((
9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 +
 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos
(f*x + e)^7), 1/768*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*s
qrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e)^7 - 2*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*
cos(f*x + e)^6 - 2*(3*a^2*b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4
- 8*(9*a*b^3 + 7*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^6, x)
```

maple [C] time = 2.80, size = 3343, normalized size = 13.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/384/f*sin(f*x+e)*(-24*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b+70*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*b^4-70*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^4-56*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^2*b^4+18*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4+210
*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^4-9*sin(f*x+e)*cos(f*x+e)^8*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4-105*sin(f*x+e)*cos(f*x+e)^8
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4+48*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^4-148*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*cos(f*x+e)^4*a*b^3-120*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^2*a*b^3+148*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a*b^3+
215*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^3+3*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3*b-107*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^2*b^2-215*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^6*a*b^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^7*a^3*b+107*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^2*b
^2+78*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b^2-78*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b^2+120*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^3-9*cos(f*x+e)^9*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^4+9*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^4+105*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-10
5*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+56*cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+108*sin(f*x+e)*cos(f*x+e)^8*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a^2*b^2+360*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3+12*s
```

```
in(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*
b-54*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a^2*b^2-180*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a*b^3-48*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+15*cos(f*x+
e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+145*cos(f*x+e)^9*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+105*cos(f*x+e)^9*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b^3-15*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^3*b-145*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
2*b^2-105*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3)*((b+a
*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos
(f*x+e)^5/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2
```

maxima [A] time = 0.36, size = 415, normalized size = 1.71

$$\frac{\frac{48\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{5}{2}}\tan\left(fx+e\right)^{3}}{b}+\frac{9\,(a+b)^{3}a\,\mathrm{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{9\,(a+b)^{3}\,\mathrm{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{48\,(a+b)^{2}a\,\mathrm{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{48\,(\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/384*(48*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)^3/b + 9*(a + b)^3*a
*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 9*(a + b)^3*arcsinh(b*ta
n(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*a*arcsinh(b*tan(f*x + e)
/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b
)*b))/sqrt(b) + 144*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(
b) + 144*(a + b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 96*(b*ta
n(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e) + 144*sqrt(b*tan(f*x + e)^2 + a +
b)*(a + b)*tan(f*x + e) - 24*(b*tan(f*x + e)^2 + a + b)^(5/2)*(a + b)*tan(f
*x + e)/b^2 + 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)/b^2
 + 9*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(f*x + e)/b^2 + 128*(b*tan
(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b - 32*(b*tan(f*x + e)^2 + a + b)^(
3/2)*(a + b)*tan(f*x + e)/b - 48*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*t
an(f*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6bf}-\frac{(a-5b)\tan(e+fx)\left(a\right.}{24}$$

```
[Out] -1/16*(a-5*b)*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2)
)/b^(3/2)/f-1/16*(a-5*b)*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f-1/
24*(a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f+1/6*tan(f*x+e)*(a+b+b*
tan(f*x+e)^2)^(5/2)/b/f
```

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4146, 388, 195, 217, 206}

$$-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6bf}-\frac{(a-5b)\tan(e+fx)\left(a\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 5*b)*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*b^(3/2)*f) - ((a - 5*b)*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(16*b*f) - ((a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]
^2)^(3/2))/(24*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b*
f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+b+bx^2\right)^{3/2}dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a-5b)\,\text{Subst}\left(\int\left(a+b+\right.\right.}{6b}$$

$$=-\frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf}+\frac{\tan(e+fx)\left(a+\right.}{}$$

$$=-\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}-\frac{(a-5b)\tan}{}$$

$$=-\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}-\frac{(a-5b)\tan}{}$$

$$=-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{(a-5b)(a+b)\tan(e}{}$$

Mathematica [C] time = 9.17, size = 400, normalized size = 2.42

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{3(a-5b)(a+b)^2\log\left(\frac{4if\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}-4\sqrt{b}f\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}-\frac{i\sqrt{}}{}\right.}{12\sqrt{2}b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(3*a^2*(1 +
E^((2*I)*(e + f*x)))^4 + 2*a*b*(1 + E^((2*I)*(e + f*x)))^2*(11 + 50*E^((2*I
)*(e + f*x)) + 11*E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x))
+ 298*E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)
))))/(1 + E^((2*I)*(e + f*x)))^6 + (3*(a - 5*b)*(a + b)^2*Log[(-4*Sqrt[b]*(
-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(12
*Sqrt[2]*b^(3/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [A] time = 1.99, size = 470, normalized size = 2.85

$$-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2b\cdots\right.}{\cos\left(fx+e\right)^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a
^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b + 22*a*b^
2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b^3)*cos(f*x + e)^2)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)
^5), -1/96*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a -
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 -
 2*((3*a^2*b + 22*a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^2*f*cos(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

maple [C] time = 2.32, size = 2519, normalized size = 15.27

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/48/f*sin(f*x+e)*(-54*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2-22*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2-17*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+17*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^4*a^2*b+22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*c
os(f*x+e)^2*a*b^2+32*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b^2-32*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+9*sin(f*
x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
```

```
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+27*
sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b
^2-18*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(
1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+3*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*cos(f*x+e)^7*a^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^
3-10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+10*cos(f*x+e)
^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-22*cos(f*x+e)^7*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-15*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b^2+22*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2*b+15*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-15*co
s(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+15*cos(f*x+e)^4*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b^3+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-30*sin(f*x+e)*cos(
f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+15*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2
)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)^3/b/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.36, size = 243, normalized size = 1.47

$$-\frac{\frac{3\,(a+b)^2 a \operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{3\,(a+b)^2 \operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{18\,(a+b)a \operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-18\,(a+b)\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(a
 + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*arcs
```

```
inh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b)) - 12*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x
 + e) - 18*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e) - 8*(b*tan(f
*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b + 2*(b*tan(f*x + e)^2 + a + b)^(3/2
)*(a + b)*tan(f*x + e)/b + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f
*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**4, x)
```

3.249 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$\frac{3(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}\,f}$$

```
[Out] 3/8*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2
)+3/8*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*tan(f*x+e)*(a+b+b*t
an(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4146, 195, 217, 206}

$$\frac{3(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(8*Sqrt[b]*f) + (3*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
8*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^{3/2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b))\,\text{Subst}\left(\int \sqrt{a\right.}{}$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+\right.}{}$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+\right.}{}$$

$$= \frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+}}{8f}$$

Mathematica [C] time = 0.25, size = 84, normalized size = 0.76

$$\frac{(a+b)^2\sin(2(e+fx))\sqrt{a+b\sec^2(e+fx)}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{b\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}\right)}{f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)^2*Hypergeometric2F1[1/2, 3, 3/2, (b*Sin[e + f*x]^2)/(a + b - a*Sin
[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*(a + 2*b + a*
Cos[2*(e + f*x)]))
```

fricas [A] time = 0.95, size = 390, normalized size = 3.51

$$\left[\frac{3\left(a^2+2ab+b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}}{\cos\left(fx+e\right)^4}\right.}{32bf\cos\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2
)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b + 3*b^2)*cos(f*x + e)^2 + 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*
x + e)^3), 1/16*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

maple [C] time = 1.50, size = 1768, normalized size = 15.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*sin(f*x+e)*(6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^2+12*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a*b+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x+
e)*b^2-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*sin(f
*x+e)*a^2-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*si
n(f*x+e)*a*b-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4
*sin(f*x+e)*b^2+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2+
3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b+7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^3*a*b+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-7*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b-3*cos(f*x+e)^2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*cos(f*x+e)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*((b+a*co
s(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*
x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.34, size = 104, normalized size = 0.94

$$\frac{\frac{3\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+3\,(a+b)\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+2\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}\tan\left(fx+e\right)+3\sqrt{b\,\mathrm{ta}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a + b
)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 2*(b*tan(f*x + e)^2 + a
 + b)^(3/2)*tan(f*x + e) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x
 + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{3/2}}{\cos\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**2, x)
```

3.250 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4128, 416, 523, 217, 206, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \text{ta}\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan\right.}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+}}\right.}{f}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan}{}$$

Mathematica [C] time = 1.87, size = 527, normalized size = 4.47

$$\sqrt{2}\,e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+ia^{3/2}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
```

```
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 1.14, size = 1457, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.35, size = 1557, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*(6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a*b+2*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^2-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b-cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b-((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*
cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

3.251 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{a}\,(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{a\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e}{2f}$$

```
[Out] b^(3/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(a+3*b
)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*a*cos
(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4146, 413, 523, 217, 206, 377, 203}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{a}\,(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{a\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/f + (a*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(}{(1+x^2}\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\,\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-l}\right.}{}$$

$$= \frac{\sqrt{a}\,(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{f}$$

Mathematica [C] time = 7.56, size = 466, normalized size = 3.76

$$\frac{e^{-i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{2e^{2i(e+fx)}\left(2a^{3/2}fx-4b^{3/2}\log\left(-\frac{e^{3ie}f\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(}}\right.}{2b^2\left(1+e^{2i(e+fx)}\right)}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((-I)*a*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a^(3/2)
```

```
*f*x + 6*Sqrt[a]*b*f*x - I*Sqrt[a]*(a + 3*b)*Log[(a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2])/E^((2*I)*e)] + I*Sqrt[a]*(a + 3*b)*Log[(a + a*E^((2*I)*(e + f*x)) + 2*
b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])/E^((2*I)*e)] - 4*b^(3/2)*Log[-1/2*(E^((3*I)*e)*(Sqrt[b]
*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]]))/Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))
/(2*Sqrt[2]*E^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 1.37, size = 1403, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28
*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^
3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e
)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*
b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e)) + 4*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a
*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/f, 1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) + 8*sqrt(-b)*b*arctan(-1/2*((a - b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*
b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, 1/8*(4*a*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/f, 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b)*b*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/
f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos \left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

maple [C] time = 1.45, size = 1512, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*sin(f*x+e)*(4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-2^(1/2)*((I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a*b*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*b^2*sin(f*x+e)+2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b
*sin(f*x+e)+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2-((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2+((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*cos(f*x+e)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b)
*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*
cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+fx\right)^2\left(a+\frac{b}{\cos\left(e+fx\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)\sin(e+fx)\cos}{}$$

```
[Out] 3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
+3/8*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*cos(f*x+e
)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4146, 378, 377, 203}

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)\sin(e+fx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*Sqrt[a]*f) + (3*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b))}{}$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(}{}$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(}{}$$

$$= \frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a+b)\cos(e+fx)\sin(e+}{8}$$

Mathematica [A] time = 1.02, size = 191, normalized size = 1.53

$$\frac{\cos(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\left(a\cos^2(e+fx)+b\right)\sqrt{a+b\sec^2(e+fx)}\left(3(a+b)^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\right.}{2\sqrt{a}f(a\cos(2(e+fx))+a+2b)^{3/2}\sqrt{\frac{a\cos(2(e+fx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b
- a*Sin[e + f*x]^2]*(3*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]] + Sqrt[a]*(4*a + 5*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]*Sqrt[(a + b -
a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3
/2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

fricas [B] time = 0.89, size = 563, normalized size = 4.50

$$\left[-\frac{3\left(a^2+2ab+b^2\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14a^3b+5a^2b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
```

```
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + 5*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)
, -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x +
e)^3 + (3*a^2 + 5*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

maple [C] time = 1.72, size = 1713, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2-2*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+12
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-3*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
```

```
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*cos(f*x+e)^3*a^2+7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3
*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2-7*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)*a*b+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-5*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^2 + a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4 \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$

Optimal. Leaf size=193

$$\frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}+\frac{\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6af}+\frac{(5a-b)\sin(e+$$

```
[Out] 1/16*(5*a-b)*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
a^(3/2)/f+1/16*(5*a-b)*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/
2)/a/f+1/24*(5*a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f+
1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/a/f
```

Rubi [A] time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4146, 382, 378, 377, 203}

$$\frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}+\frac{\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6af}+\frac{(5a-b)\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((5*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(16*a^(3/2)*f) + ((5*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a - b)*Cos[e + f*x]^3*Sin[e + f*
x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(24*a*f) + (Cos[e + f*x]^5*Sin[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af} + \frac{(5a-b)\,\text{Su}}{}$$

$$= \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\text{co}}{}$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} +$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} +$$

$$= \frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(5a-b)(a+b)\cos(e}{}$$

Mathematica [A] time = 2.01, size = 165, normalized size = 0.85

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{3\sqrt{2}\sqrt{a+b}\left(5a^2+4ab-b^2\right)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}} + \sqrt{a}\sin(e+fx)\left(a^2\cos(4(e+fx))+23\right.\right.}{48a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((3*Sqrt[2]*Sqrt[a + b]*(5*a^2 + 4
*a*b - b^2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[a]*(23*a^2 + 29*a*b + 3*b^2 + a*(9*a + 7*b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Sin[e + f*x]))/(48*a^(3/2)*f)
```

fricas [A] time = 2.24, size = 647, normalized size = 3.35

$$\left[\frac{3\left(5a^3+9a^2b+3ab^2-b^3\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14a^3b+5\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 2*(5
*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), -1/192
*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e
)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^
2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos
(f*x + e)^5 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

maple [C] time = 2.08, size = 2439, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/48/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(-3
0*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-32*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^3*a^2*b-17*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^
3*a*b^2-22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2-22*((2*
```

```
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+9*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f
*x+e)+15*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-15*cos(f*
x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+22*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+32*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^2*a^2*b+17*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^2*a*b^2+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3+10*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3-15*cos(f*x+e)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^7*a^3-10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^5*a^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3+15*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^2-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*
x+e)-54*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-18*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x
+e)+27*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+3*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e))/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/a/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.254 $\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{d}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c\right.}{4d}$$

```
[Out] a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/d+1/8*(15*a^2
+10*a*b+3*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))*b^(1/
2)/d+1/8*b*(7*a+3*b)*(a+b+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b*tan(d*x+
c)*(a+b+b*tan(d*x+c)^2)^(3/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4128, 416, 528, 523, 217, 206, 377, 203}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{8d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]])/d +
 (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a +
 b + b*Tan[c + d*x]^2]])/(8*d) + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b
*Tan[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))
/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{5/2}}{1+x^2} dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\left((a+b)(4a+3b)+b(7a+3b)x\right.}{1+x^2}\right.}{4d}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c\right.}{4d}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c\right.}{4d}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c\right.}{4d}$$

$$= \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right.}{8d}$$

Mathematica [C] time = 10.51, size = 706, normalized size = 4.25

$$\frac{e^{i(c+dx)}\cos^5(c+dx)\sqrt{4b+ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2}\left(\frac{-4ia^{5/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(c+dx)}\right)^2+4be^{2i(c+dx)}}+ae^{2i(c+dx)}+a+2b\right)+4ia^{5/2}\log}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (E^(I*(c + d*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(c + d*x)))^2)/E^((2*I)*(c + d
*x))]*Cos[c + d*x]^5*(((-I)*b*(-1 + E^((2*I)*(c + d*x)))*(9*a*(1 + E^((2*I)
*(c + d*x)))^2 + b*(3 + 14*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x)))))/(
1 + E^((2*I)*(c + d*x)))^4 + (8*a^(5/2)*d*x - (4*I)*a^(5/2)*Log[a + 2*b + a
*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*
I)*(c + d*x)))^2]] + (4*I)*a^(5/2)*Log[a + a*E^((2*I)*(c + d*x)) + 2*b*E^((
2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c
 + d*x)))^2]] - 15*a^2*Sqrt[b]*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)))
 + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(
b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))] - 10*a*b^(3/2)*Log[
(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d
*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^
((2*I)*(c + d*x))))] - 3*b^(5/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)
)) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])
/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))])/Sqrt[4*b*E^((2*I
)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])*(a + b*Sec[c + d*x]^2)^(5/2)
)/(Sqrt[2]*d*(a + 2*b + a*Cos[2*c + 2*d*x])^(5/2))
```

fricas [B] time = 2.25, size = 1611, normalized size = 9.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a^2*cos(d*x + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4
- a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*c
os(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos
(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(b)*cos(d*x + c)
^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 8*(a*b - b^2)*cos(d*x + c)^2 +
 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b)*sqrt((a*cos(d*x + c)
^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b^2)/cos(d*x + c)^4) + 4*(3*(3*a*b
 + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^3), 1/16*(2*sqrt(-a)*a^2*cos(d*x + c)^3*log(
128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
+ b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*c
os(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a
*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10
*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
c))*sqrt(-b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)
```

```
^2 + b^2)*sin(d*x + c)))*cos(d*x + c)^3 + 2*(3*(3*a*b + b^2)*cos(d*x + c)^2
 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^3), -1/32*(8*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a
*b)*cos(d*x + c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c)^2 + b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(d*x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*
a*b + 3*b^2)*sqrt(b)*cos(d*x + c)^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4
 + 8*(a*b - b^2)*cos(d*x + c)^2 + 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
 c))*sqrt(b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b
^2)/cos(d*x + c)^4) - 4*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*co
s(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3), -1/16*(
4*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x + c)^3 +
 (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 + b)/cos(
d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(d*
x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*a*b + 3*b^2)*sqrt(-b
)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(-b)*sqrt((a*
cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)^2 + b^2)*sin(d*x + c
)))*cos(d*x + c)^3 - 2*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos
(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(dx + c\right)^2 + a\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

maple [C] time = 1.80, size = 2231, normalized size = 13.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/8/d*(16*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*
a^(1/2)*b^(1/2)+a*cos(d*x+c)+b)/(1+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(d*x+c),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+30*sin(d*x+c)*cos(d*
x+c)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)+a*cos(d*x+c
)+b)/(1+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2
)*b^(1/2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*a^2*b+20*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)+a*cos(d*x+c)+b)/(1+cos(d*x+c))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(d*x+c),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b
^2+6*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/
2)*b^(1/2)+a*cos(d*x+c)+b)/(1+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x
+c),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
```

```
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3-8*sin(d*x+c)*cos(d*x+c)^4*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)+a*cos(d*x+c)+b)/(1
+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/
2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-15*sin(d*x+c)*cos(d*x+c
)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)+a*cos(d*x+c)+b
)/(1+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b
^(1/2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-10*sin(d*x+c)*cos
(d*x+c)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)+a*cos(d*
x+c)+b)/(1+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(
1/2)*b^(1/2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*sin(d*x+c
)*cos(d*x+c)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(d*x+c)-I*a^(1/2)*b^(1/2)+a*c
os(d*x+c)+b)/(1+cos(d*x+c))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(d*x+c)-
I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3+9*cos(d*
x+c)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*cos(d*x+c)^5*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^2*b-3*cos(d*x+c)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2+11*cos(d*x+c)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*
cos(d*x+c)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-11*cos(d*x+c)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(d*x+c)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^3+2*cos(d*x+c)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*cos(d*x+c)*((a*cos
(d*x+c)^2+b)/cos(d*x+c)^2)^(5/2)*sin(d*x+c)/(-1+cos(d*x+c))/(a*cos(d*x+c)^2
+b)^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(c+d\,x\right)}^2}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x)^2)^(5/2),x)
[Out] int((a + b/cos(c + d*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**(5/2),x)
[Out] Integral((a + b*sec(c + d*x)**2)**(5/2), x)
```

3.255 $\int \left(1+\sec^2(x)\right)^{3/2} dx$

Optimal. Leaf size=42

$$\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}+\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

```
[Out] 2*arcsinh(1/2*tan(x)*2^(1/2))+arctan(tan(x)/(2+tan(x)^2)^(1/2))+1/2*(2+tan(
x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4128, 416, 523, 215, 377, 203}

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}+2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] 2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sqr
t[2 + Tan[x]^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(2+x^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{6+4x^2}{\left(1+x^2\right)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{2}}\right.$$
$$= 2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$
$$= 2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

Mathematica [C] time = 0.17, size = 109, normalized size = 2.60

$$\frac{\left(\cos^2(x)+1\right)\sec(x)\sqrt{\sec^2(x)+1}\left(\sin(x)\sqrt{\cos(2x)+3} - 2i\sqrt{2}\cos^2(x)\log\left(\sqrt{\cos(2x)+3}+i\sqrt{2}\sin(x)\right)+4\right.}{(\cos(2x)+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] ((1 + Cos[x]^2)*Sec[x]*Sqrt[1 + Sec[x]^2]*(4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sin[x
])/Sqrt[3 + Cos[2*x]]]*Cos[x]^2 - (2*I)*Sqrt[2]*Cos[x]^2*Log[Sqrt[3 + Cos[2
*x]] + I*Sqrt[2]*Sin[x]] + Sqrt[3 + Cos[2*x]]*Sin[x]))/(3 + Cos[2*x])^(3/2)
```

fricas [B] time = 0.53, size = 160, normalized size = 3.81

$$\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\,\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)\cos(x) - \arctan\left(\frac{\sin(x)}{\cos(x)}\right)\cos(x) + 2\cos(x)\log\left(\cos(x)^2+\cos(x)\sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1))*cos(x) - arctan(sin(x)/cos(x))*cos(x) + 2*cos(x
)*log(cos(x)^2 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2
+ 1)/cos(x)^2) + 1) - 2*cos(x)*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - c
os(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) + 1) + sqrt((cos(x)^2 + 1)/cos(
x)^2)*sin(x))/cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec(x)^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

maple [C] time = 1.51, size = 429, normalized size = 10.21

$$\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\left(8\sin(x)\left(\cos^2(x)\right)(-1)^{\frac{3}{4}}\sqrt{\frac{i\cos(x)+1-i+\cos(x)}{\cos(x)+1}}\sqrt{-\frac{i\cos(x)-\cos(x)-1-i}{\cos(x)+1}}\ \mathrm{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(-1+\cos(x))}{\sin(x)},-i,i\right)+4\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(x)^2)^(3/2),x)
```

```
[Out] (-1/4+1/4*I)*(8*sin(x)*cos(x)^2*(-1)^(3/4)*((I*cos(x)+1-I+cos(x))/(cos(x)+1
))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(-
1+cos(x))/sin(x),-I,I)+4*sin(x)*cos(x)^2*(-1)^(3/4)*((I*cos(x)+1-I+cos(x))/
(cos(x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticPi((-1)
^(1/4)*(-1+cos(x))/sin(x),I,I)+6*sin(x)*cos(x)^2*2^(1/2)*((I*cos(x)+1-I+cos
(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticF(
(1/2+1/2*I)*2^(1/2)*(-1+cos(x))/sin(x),I)-8*sin(x)*cos(x)^2*(-1)^(1/4)*((I*
cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1
/2)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),-I,I)-4*sin(x)*cos(x)^2*(-1)^(
1/4)*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(
x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I)-I*cos(x)^3-cos(x
)^3+I*cos(x)^2+cos(x)^2-I*cos(x)-cos(x)+1+I)*sin(x)*cos(x)*((1+cos(x)^2)/co
s(x)^2)^(3/2)/(-1+cos(x))/(1+cos(x)^2)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sec(x)^2+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(\frac{1}{{\cos(x)}^2}+1\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2 + 1)^(3/2),x)
```

```
[Out] int((1/cos(x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sec^2(x)+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)**2)**(3/2),x)
```

```
[Out] Integral((sec(x)**2 + 1)**(3/2), x)
```

3.256 $\int \sqrt{1+\sec^2(x)}\,dx$

Optimal. Leaf size=24

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

```
[Out] arcsinh(1/2*tan(x)*2^(1/2))+arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4128, 402, 215, 377, 203}

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sec[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1+\sec^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{2+x^2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)\\
&= \sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\
&= \sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.05, size = 57, normalized size = 2.38

$$
\frac{\sqrt{2}\cos(x)\sqrt{\sec^2(x)+1}\left(\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{\cos(2x)+3}}\right)\right)}{\sqrt{\cos(2x)+3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sec[x]^2],x]
[Out] (Sqrt[2]*(ArcSin[Sin[x]/Sqrt[2]] + ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[3 + Cos[2*
x]]])*Cos[x]*Sqrt[1 + Sec[x]^2])/Sqrt[3 + Cos[2*x]]
```

fricas [B] time = 0.57, size = 131, normalized size = 5.46

$$
\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{1}{2}\log\left(\cos(x)^2+\cos(x)\sin(x)+(\mathrm{co}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x)) + 1/2*log(cos(x)^2 +
 cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) +
 1) - 1/2*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - cos(x)*sin(x))*sqrt((c
os(x)^2 + 1)/cos(x)^2) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{\sec(x)^2+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(x)^2 + 1), x)
```

maple [C] time = 2.70, size = 190, normalized size = 7.92

$$
\frac{(-1+i)\left((-1)^{\frac{3}{4}}\,\text{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(-1+\cos(x))}{\sin(x)},-i,i\right)+(-1)^{\frac{3}{4}}\,\text{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(-1+\cos(x))}{\sin(x)},i,i\right)+\sqrt{2}\,\text{EllipticF}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\,(\ldots}{\sin\ldots}\right.\right.}{\ldots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(x)^2)^(1/2),x)
```

```
[Out] (-1+I)*((-1)^(3/4)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),-I,I)+(-1)^(3/4
)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I)+2^(1/2)*EllipticF((1/2+1/2*
I)*2^(1/2)*(-1+cos(x))/sin(x),I)-(-1)^(1/4)*EllipticPi((-1)^(1/4)*(-1+cos(x
))/sin(x),-I,I)-(-1)^(1/4)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I))*c
os(x)*sin(x)^2*((1+cos(x)^2)/cos(x)^2)^(1/2)*((I*cos(x)+1-I+cos(x))/(cos(x)
+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)/(-1+cos(x))/(1+cos(x)^
2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{1}{\cos(x)^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int((1/cos(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(x)**2 + 1), x)
```

3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=330

$$-\frac{2(a-b)\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3b^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}(\right.}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)}}$$

```
[Out] 2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/f/
(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x
+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*s
in(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)-2/3*(a-b)*sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b^2/f
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*sec(f*x+e)^3*(a+b-a*sin(f*x+
e)^2)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 380, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 414, 527, 524, 426, 424, 421, 419}

$$-\frac{2(a-b)\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{3b^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a-}}{3b^2f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(a - b)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a
+ b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*Sqrt
[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)])/(3*b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e +
 f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) - (2*(a - b)*Sqrt[b + a*Cos[e + f*
x]^2]*Sec[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(3*b^2*f*Sq
rt[a + b*Sec[e + f*x]^2]) + (Sqrt[b + a*Cos[e + f*x]^2]*Sec[e + f*x]^3*Sqrt
[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(3*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec^3(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{3bf\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\,\text{co}}}{}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{3b^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{}}{}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{3b^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{}}{}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{3b^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{}}{}$$

$$= \frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)\sqrt{a+b-a\sin^2(e+fx)}}{3b^2f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{(}{}$$

Mathematica [F] time = 11.48, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.16, size = 4753, normalized size = 14.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] 1/3/f*(-3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*a^2*b-4*I*cos(f*x+e)^5*a^(3/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)-4*I*cos(f*x+e)^4*a^(5/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)+2*I*cos(f*x+e)^4*a^(3/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)-4*I*cos(f*x+e)^3*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)-2*I*cos(f*x+e)^2*a^(3/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)+2*I*cos(f*x+e)^2*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+4*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*sin(f*x+e)*cos(f*x+e)^3*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^3
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+sin(f*x+e)*cos(f*x+e
)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
```

```
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-sin(f*x+e)*cos(f*
x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+2*I*a^(1/2)*b^
(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^2*a^2*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^
2+cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^5
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^4*a^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^5*a^3-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+c
os(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+sin(f*x+e)*cos(f*x+
e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-sin(f*x+e)*cos(f*x
+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3+sin(f*x+e)*cos(f*
x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-sin(f*x+e)*cos(f
*x+e)^3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*I*cos(f*x+e)^4*a^(5/2)*b^(1/2)*sin(f*x+e)*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*cos(f*x+e)^4*a^(3/2)*b^
(3/2)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*I*cos(f*
x+e)^4*a^(1/2)*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))-2*I*cos(f*x+e)^3*a^(5/2)*b^(1/2)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))+2*I*cos(f*x+e)^3*a^(3/2)*b^(3/2)*sin(f*x+e)*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
```

```
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*I*cos(f*x+e)^3*a^(1/2)*b^(5/2)*s
in(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*b^3+4*I*cos(f*x+e)^5*a^(5/2)*b^(1/2)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)+4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^3*a^(3/2)*b^(3/2))/cos(f*x+e)^4/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2
)/sin(f*x+e)/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/
2)-a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=170

$$\frac{\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] -EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*a^(1/2)*(a+b)^(1
/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)+sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b/f/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 202, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4148, 6722, 1974, 414, 21, 427, 424}

$$\frac{\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{bf\sqrt{a+b\sec^2(e+fx)}}-\frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+ \ldots}{bf\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx) \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[a + b]*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]], (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)
])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[
e + f*x]^2])) + (Sqrt[b + a*Cos[e + f*x]^2]*Sec[e + f*x]*Sqrt[a + b - a*Sin
[e + f*x]^2]*Tan[e + f*x])/(b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}\sqrt{b+a\left(1-x^2\right)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\ \tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\,c}}{}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\ \tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(a\sqrt{b+}\right.}{}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\ \tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(a\sqrt{b+}\right.}{b}$$

$$= -\frac{\sqrt{a}\ \sqrt{a+b}\ \sqrt{b+a\cos^2(e+fx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\ \sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} +$$

Mathematica [F] time = 10.63, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.72, size = 3029, normalized size = 17.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] -1/2/f*(4*I*cos(f*x+e)^3*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)+4*I*cos(f*x+e)*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)+2*I*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a^(1/2)*b^(3/2)-2*I*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a^(3/2)*b^(1/2)-2*I*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(1/2)-4*cos(f*x+e)^2*sin(f*x+e
)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
```

```
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+cos(f*x+e)^2*sin(f*x+
e)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2+2*cos(f*x+e)^2*sin(f
*x+e)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b+cos(f*x+e)^2*sin(
f*x+e)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^2-4*I*cos(f*x+e)^2
*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*sin(f*x+e)*cos(f
*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+sin(f*x+e)*cos(f*
x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+2*sin(f*x+e)*cos(f
*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+sin(f*x+e)*cos(f*
x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+2*I*sin(f*x+e)*cos
(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-4*I
*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^3*a*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)^2*a^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b-2*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b+2*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*cos(f*x+e)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)/cos(f*x+e)^2/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/b/(2*I*a^(1/2)*b^(1/2)-a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(cos
(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 103, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 421, 419}

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[1 - (a*Sin[e + f*x]^2)/(a + b)])/(f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e
+ f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\left(\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

Mathematica [A] time = 0.18, size = 69, normalized size = 0.86

$$\frac{\sec(e+fx)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\,F\left(e+fx\,\middle|\,\frac{a}{a+b}\right)}{\sqrt{2}\,f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)]
*Sec[e + f*x])/(Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.55, size = 269, normalized size = 3.36

$$\frac{\left(\sin^2\left(fx+e\right)\right)\sqrt{2}\,\sqrt{\frac{i\sqrt{a}\,\sqrt{b}\,\cos\left(fx+e\right)-i\sqrt{a}\,\sqrt{b}+a\cos\left(fx+e\right)+b}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\sqrt{-\frac{2\left(i\sqrt{a}\,\sqrt{b}\,\cos\left(fx+e\right)-i\sqrt{a}\,\sqrt{b}-a\cos\left(fx+e\right)-b\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\ \mathrm{EllipticF}\Big(}{f\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{}}{a+}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*sin(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))/((b+a*cos(
f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}\,f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*(a+b)^(1/2)*(1-a*
sin(f*x+e)^2/(a+b))^(1/2)/f/a^(1/2)/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 128, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 427, 424}

$$\frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}\,f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b]*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]], (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a
]*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e +
f*x]^2])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\left(\sqrt{b+a\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sqrt{a+b}\,\sqrt{b+a\cos^2(e+fx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{a}\,f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

Mathematica [C] time = 5.19, size = 279, normalized size = 2.66

$$\frac{\sin(e+fx)\csc(2(e+fx))\left(a^2\sqrt{-\frac{1}{a+b}}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}F\left(e+fx\middle|\frac{a}{a+b}\right)-2i\csc(2(e+fx))\sqrt{\frac{a\sin^2(e+fx)}{a+b}}\sqrt{-\frac{a\cos\cdots}{}}\right.}{\sqrt{2}\,a^2 f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Csc[2*(e + f*x)]*Sin[e + f*x]*(a^2*Sqrt[-(a + b)^(-1)]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] - (2*I)*Sqrt[-((a*C
os[e + f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Csc[2*(e + f*x)]*(2*b
*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
])/Sqrt[2]], (a + b)/b] + a*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a
 + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a + b)/b])*Sqrt[(a*Sin[e + f*x]^2)
/(a + b)]))/(Sqrt[2]*a^2*Sqrt[-(a + b)^(-1)]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)

maple [C] time = 1.73, size = 2985, normalized size = 28.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)

```
[Out] -1/2/f*(2*I*sin(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))-4*I*cos(f*x+e)^2*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)+4*I*cos(f*x+e)^3*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)+2*I*sin(f*x+e)*cos(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))-2*I*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
cos(f*x+e)*sin(f*x+e)*a^(1/2)*b^(3/2)+4*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(
```

```
1/2)+a-b)/(a+b))^(1/2)+4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*
b*sin(f*x+e)-EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*sin(f*x+e)
-2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-EllipticE
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^2*sin(f*x+e)+4*I*cos(f*x+e)*a^(1/2
)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*I*a^(1/2)*b^(3/2)*2^(1/
2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-2*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*cos(f*x+e)^3*a*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^
2*a^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b-2*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/cos(f*x+e)/((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(1/2)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
/(2*I*a^(1/2)*b^(1/2)-a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)}{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=255

$$-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\right.}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e}$$

```
[Out] 1/3*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)+2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)
/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*
sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2
))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*
(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 296, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4148, 6722, 1974, 416, 524, 426, 424, 421, 419}

$$-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3
*a*f*Sqrt[a + b*Sec[e + f*x]^2]) + (2*(a - b)*Sqrt[b + a*Cos[e + f*x]^2]*El
lipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3
*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)]) - ((a - 2*b)*b*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^2*f*
Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]
^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\;\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\;\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}}{3af\sqrt{\text{co}}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{\left(2(a-b)\sqrt{b+a\cos}\right.}{3a^2f\sqrt{}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{\left(2(a-b)\sqrt{b+a\cos}\right.}{3a^2}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{2(a-b)\sqrt{b+a\cos^2}}{3a^2f\sqrt{\text{co}}}$$

Mathematica [F] time = 7.34, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.05, size = 4640, normalized size = 18.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f*(3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^2*b-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a^3+2
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b-((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)*a*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^5*a^2*b-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x
+e)-3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a*b^2-cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e
)*a^2*b+cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+
e)*a*b^2+4*I*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+2*cos(f*x+e)^2*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^3-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)
^2*a^2*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2-4*I*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)+2*I*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
```

```
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)*sin(f*x+e)+2*I*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)*sin(f*x+e)-cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^5*a^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)*b^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*I*a^(3/2)*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-2*I*cos(f*x+e)^3*a^(3/2)*b^(3/2)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^2*a^(3/2)*b^(3/2)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a^2*b*sin(f*x+e)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2^(1/2)*((I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a*b^2-cos(f*x+e)*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*a^3+cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^3+2*I*sin(f*x+e)*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*a^(5/2)*b^(1/2)+2*I*sin(f*x+e)*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*a^(1/2)*b^(5/2)-2*I
*cos(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)
*sin(f*x+e)-4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2
```

```
)*b^(1/2)+2*I*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*
b^(1/2)+2*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^
(1/2)+2*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2
))/cos(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/sin(f*x+e)/a^2/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{3}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=345

$$\frac{4(a-b)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] 4/15*(a-b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/f/(sec(f*x+e)^2*(a+b-a*sin(f
*x+e)^2))^(1/2)+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f
*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(8*a^2-7*a*b+8*b^2)*EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/15*
b*(4*a^2-3*a*b+8*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)
^2/(a+b))^(1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 395, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{\left(8a^2-7ab+8b^2\right)\sqrt{-a}}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (4*(a - b)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2])/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + (Cos[e + f*x]^2*Sqrt[b + a*
Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a*f*Sqrt[a
+ b*Sec[e + f*x]^2]) + ((8*a^2 - 7*a*b + 8*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/
(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e
 + f*x]^2)/(a + b)]) - (b*(4*a^2 - 3*a*b + 8*b^2)*Sqrt[b + a*Cos[e + f*x]^2
]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a +
b - a*Sin[e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\sqrt{b+a\left(1-x^2\right)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{\sqrt{b+a}\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\cdots}{\cdots}$$

Mathematica [F] time = 14.75, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.48, size = 6382, normalized size = 18.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{5}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=137

$$\frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan(e+fx)\sec^2(}{}$$

[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-3/8*(a-b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*sec(f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f

Rubi [A] time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4146, 416, 388, 217, 206}

$$\frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan(e+fx)\sec^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(8*b^(5/2)*f) - (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(4*b*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} + \frac{\text{Subst}\left(\int \frac{-a+3b-3(a-b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \text{ta}\right.}{4bf}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+}}{4bf}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+}}{4bf}$$

$$= \frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2}}{8b^2f}$$

Mathematica [C] time = 9.53, size = 326, normalized size = 2.38

$$\frac{e^{i(e+fx)}\sec(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{(3a^2-2ab+3b^2)\log\left(\frac{4if\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}-4\sqrt{b}f\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}} - \frac{i}{\ }\right.}{8\sqrt{2}b^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*(-3*a*(1 + E^((2*I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)) +
 3*E^((4*I)*(e + f*x)))))/(1 + E^((2*I)*(e + f*x)))^4 - ((3*a^2 - 2*a*b + 3
*b^2)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[e + f*
x])/(8*Sqrt[2]*b^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [A] time = 0.86, size = 396, normalized size = 2.89

$$\left[\frac{\left(3\,a^2-2\,ab+3\,b^2\right)\sqrt{b}\,\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{}}{\cos\left(fx+e\right)^4}\right.}{32\,b^3 f\cos\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b
^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)
^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(3*(a*b - b^2)*cos(f*x + e)^2 - 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(
f*x + e)^3), 1/16*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(3
*(a*b - b^2)*cos(f*x + e)^2 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.20, size = 1756, normalized size = 12.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f*sin(f*x+e)*(-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^2+4*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a*b-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
```

```
+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x
+e)*b^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*sin(
f*x+e)*a^2-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*s
in(f*x+e)*a*b+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^
4*sin(f*x+e)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2
-3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^3*a*b-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+3*cos(f*x+e)^2*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x
+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/b^2
```

maxima [A] time = 0.36, size = 160, normalized size = 1.17

$$\frac{\frac{2\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,\tan\left(fx+e\right)^{3}}{b}+\frac{3\,(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{3\,(a+b)\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{8\,a\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{3\sqrt{b\tan\left(fx+e\right)^{2}+a \cdots}}{b \cdots}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)^3/b + 3*(a + b)*a*arcsin
h(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)*arcsinh(b*tan(f*x + e
)/sqrt((a + b)*b))/b^(3/2) - 8*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^
(3/2) - 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)/b^2 + 8*sqrt(
b*tan(f*x + e)^2 + a + b)*tan(f*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{6}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}$$

```
[Out] -1/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f
+1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4146, 388, 217, 206}

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+\ldots$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e\ldots$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

Mathematica [C] time = 10.28, size = 326, normalized size = 4.02

$$\frac{\tan(e+fx)\sec^4(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{16b^2\tan^4(e+fx)\left(a\cos^2(e+fx)+b\right)\,{}_2F_1\left(2,3;\frac{7}{2};-\frac{b\tan^2(\ldots}{a+\ldots}\right)\ldots}{(a+b\ldots}\right.}{30\sqrt{2}f\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{b\tan\ldots}{\ldots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]^4*(1 - (a*Sin[e + f*x]^2)/
(a + b))*Tan[e + f*x]*((16*b^2*(b + a*Cos[e + f*x]^2)*Hypergeometric2F1[2,
3, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4*Sqrt[-((b*Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^3 + (15
*(3*b + a*(3 - 2*Sin[e + f*x]^2))*(ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b)
)]] - Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(
a + b)^2)]))/(a + b)))/(30*Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[(a + b
*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[e + f*x]
^2)/(a + b)))^(3/2))
```

fricas [B] time = 0.74, size = 324, normalized size = 4.00

$$\left[-\frac{(a-b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2\ldots}{\cos\left(fx+e\right)^2}}\ldots}{\cos\left(fx+e\right)^4}\right)}{8\,b^2f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
```

```
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)), -1/4*((a - b)*sqrt(-b)*arctan(-1/2*((a - b)*
cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 2*b*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x +
e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.04, size = 1086, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*sin(f*x+e)*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e
)^2*sin(f*x+e)*a-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)
^2*sin(f*x+e)*b-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a+2*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*cos(f*x+e)^2*sin(f*x+e)*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^3*a-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a+((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1
/2)/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.33, size = 74, normalized size = 0.91

$$-\frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsinh(b*tan(f*x
 + e)/sqrt((a + b)*b))/sqrt(b) - sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e
)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4146, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}
\end{aligned}$$

Mathematica [B] time = 0.12, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{b}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[b]*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

fricas [B] time = 0.60, size = 215, normalized size = 5.51

$$\left[\frac{\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)}{4\sqrt{b}f}, \frac{\sqrt{-b}\,\mathrm{arc}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)/(sqrt(b)*f)
, 1/2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))/(b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.60, size = 379, normalized size = 9.72

$$-\frac{\sqrt{2}\sqrt{\frac{i\sqrt{a}\sqrt{b}\cos(fx+e)-i\sqrt{a}\sqrt{b}+a\cos(fx+e)+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2(i\sqrt{a}\sqrt{b}\cos(fx+e)-i\sqrt{a}\sqrt{b}-a\cos(fx+e)-b)}{(1+\cos(fx+e))(a+b)}}\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\cdots}{\sin(\cdots}\right.\right.}{f\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1
/2)/cos(f*x+e)/(-1+cos(f*x+e))/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [A] time = 0.33, size = 23, normalized size = 0.59

$$\frac{\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4128, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

fricas [B] time = 0.64, size = 408, normalized size = 10.46

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^4 -\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.63, size = 380, normalized size = 9.74

$$-\frac{\sqrt{2}\sqrt{\frac{i\sqrt{a}\sqrt{b}\cos(fx+e)-i\sqrt{a}\sqrt{b}+a\cos(fx+e)+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2(i\sqrt{a}\sqrt{b}\cos(fx+e)-i\sqrt{a}\sqrt{b}-a\cos(fx+e)-b)}{(1+\cos(fx+e))(a+b)}}\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{}}{\sin(fx+e}\right.\right.}{f\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(
1/2)/cos(f*x+e)/(-1+cos(f*x+e))/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [B] time = 0.63, size = 992, normalized size = 25.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2
*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 +
 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f
*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*si
n(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a +
2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec^{2}{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

```
[Out] 1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4146, 382, 377, 203}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\right.}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \right.}{2af}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

Mathematica [A] time = 0.24, size = 126, normalized size = 1.45

$$\frac{\sqrt{a\cos(2(e+fx))+a+2b}\left(\sqrt{a}\tan(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}+(a-b)\sec(e+fx)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx}{\sqrt{-a\sin^2(e+fx)+}}\right.\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*((a - b)*ArcTan[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b - a*Sin[e + f*x]^2]]*Sec[e + f*x] + Sqrt[a]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*Tan[e + f*x]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [B] time = 0.65, size = 502, normalized size = 5.77

$$\left[\frac{8a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\sqrt{-a}(a-b)\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a^2*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)*sin(f*x + e) - (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
```

```
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.98, size = 1056, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*sin(f*x+e)*(2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(
f*x+e)-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*c
os(f*x+e)^2*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b-((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/cos(f*x+e)/((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=143

$$\frac{3(a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\left(3a^2-2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}+\frac{\sin(e-}{}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/a^(5/2)/f+3/8*(a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2
/f+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{\left(3a^2-2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}+\frac{3(a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(5/2)*f) + (3*(a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-2bx^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x,\right.}{4af}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4}$$

$$= \frac{(3a^2-2ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} + \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+}}{8a^2f}$$

Mathematica [C] time = 16.23, size = 1840, normalized size = 12.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*
Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)*
AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^5)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -
2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2,
```

```
-2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appe
llF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (12*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[
2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x
]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[
3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x])/3))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF
1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*Appel
lF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a +
 b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
)*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*(2*f*(a*AppellF1[3
/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*A
ppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos
[e + f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)
) - (4*f*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/
2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 4*(a + b)*((3
*a*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (9*(a + b)^3*f*Cot[e + f*x]*Csc[
e + f*x]^4*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*((-2*a*Sin[e + f*x]^2)/(a +
 b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]
^2)/(a + b)])))/(8*a^3)))))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b
)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
 (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2
*(e + f*x)])/((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2,
 -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/
2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Ap
pellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2))))
```

fricas [A] time = 1.19, size = 567, normalized size = 3.97

$$\left[-\frac{\left(3\,a^2-2\,ab+3\,b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
```

```
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + 3*(a^2 - a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
 + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*
x + e)^3 + 3*(a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.37, size = 1701, normalized size = 11.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*sin(f*x+e)*(2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^
2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2-3*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b^2*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-4*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
```

```
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+3*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^3*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e
)^2*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+3*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+
3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}{\left(e+fx\right)}}{\sqrt{a+b\sec^{2}{\left(e+fx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=204

$$\frac{5(a-b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24a^2f}+\frac{(a-b)\left(5a^2+2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+($$

```
[Out] 1/16*(a-b)*(5*a^2+2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/a^(7/2)/f+1/48*(15*a^2-14*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+
b*tan(f*x+e)^2)^(1/2)/a^3/f+5/24*(a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f
*x+e)^2)^(1/2)/a^2/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)
/a/f
```

Rubi [A] time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{(a-b)\left(5a^2+2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{\left(15a^2-14ab+15b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+}}{48a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b +
 b*Tan[e + f*x]^2]])/(16*a^(7/2)*f) + ((15*a^2 - 14*a*b + 15*b^2)*Cos[e + f
*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + (5*(a - b)*Co
s[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos
[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-4bx^2}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx\right.}{6af}$$

$$= \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e-}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b}{}$$

$$= \frac{(a-b)(5a^2+2ab+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} + \frac{(15a^2-14ab+15b^2)\cos(e}{}$$

Mathematica [C] time = 16.81, size = 1739, normalized size = 8.52

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e +
```

```
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)
*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]^7)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2,
-3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2
, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*App
ellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
 + f*x]^2)) - (18*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*
x]^6*Sin[e + f*x]*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*AppellF1[
3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x]))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1
/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1
[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)
*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*S
in[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*AppellF1[3/2,
 -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appe
llF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e
+ f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 2*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/2, -3,
5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 6*(a + b)*((3*a*f*Ap
pellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(f*
Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2,
1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) +
 (3*a*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e + f*x)])/((a + 2*b + a*Cos[
2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

fricas [A] time = 1.95, size = 643, normalized size = 3.15

$$\left[\frac{3\left(5a^3-3a^2b+3ab^2-5b^3\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14a^3b+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 10
*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/192
*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
 e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*c
os(f*x + e)^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^4*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.51, size = 2425, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/48/f*sin(f*x+e)*(30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-4*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)^3*a*b^2-14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)*a*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b-9*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-30*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*b^3*sin(f*x+e)-15*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^3+15*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+4*((2*I*a^(1/2)*b^(1
```

```
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^2*a*b^2-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^6*a^3-10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3+15*c
os(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+8*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^3+10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^5*a^3+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)*b^3-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+14*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*b^3*sin(f*x+e)-18*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+18*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a*b^2*sin(f*x+e)+9*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*
b*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-15*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e))/(-1+cos(f*x+e))/cos(f*x
+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{6}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.270 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{a(2a+b)\sin(e+fx)}{b^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{(2a+b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}(\sin(e\right.}{b^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-\right.}}$$

```
[Out] a*(2*a+b)*sin(f*x+e)/b^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-
(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/(a+b
)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e
)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)+sec(f*x+e)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)
```

Rubi [A] time = 0.59, antiderivative size = 367, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 414, 527, 524, 426, 424, 421, 419}

$$\frac{a(2a+b)\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{b^2f(a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}-\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{b^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*a + b)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(b^2*(a + b)*f*Sqrt[a
 + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) - ((2*a + b)*Sqrt[b +
a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a
*Sin[e + f*x]^2])/(b^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*
x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (Sqrt[b + a*Cos[e + f*x]^2]*E
llipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Si
n[e + f*x]^2]) + (Sqrt[b + a*Cos[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/(b*
f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{3/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\left(b+a(1-x^2)\right)^{3/2}}dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\left(a+b-ax^2\right)^{3/2}}dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\Big(\int}{bf\sqrt{\cos^2(e+fx)}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}}{bf\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}}{bf\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}}{bf\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{(2a+b)\sqrt{b+a\cos^2(e}}{b^2(a+b)f\sqrt{\text{co}}}$$

Mathematica [F] time = 20.82, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^5}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.22, size = 12510, normalized size = 43.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.271 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{a\sin(e+f}{bf(a+b)\sqrt{\sec^2(e+fx)\left(-a\right.}}$$

```
[Out] -a*sin(f*x+e)/b/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+EllipticE
(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b/(a+b)/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1
/2)
```

Rubi [A] time = 0.38, antiderivative size = 182, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4148, 6722, 1974, 414, 21, 426, 424}

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}\,E\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}}-\frac{a\sin(e+fx)\sqrt{a\cos^2(e+f}}{bf(a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(b*(a + b)*f*Sqrt[a + b*Sec[e
 + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + (Sqrt[b + a*Cos[e + f*x]^2]*E
llipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(
b*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}}{b(a+b)f}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}}{b(a+b)f\sqrt{c}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(\sqrt{b+a\cos^2(e+f}\right.}{b(a+b)}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx}}{b(a+b)f\sqrt{\cos}}$$

Mathematica [A] time = 2.35, size = 113, normalized size = 0.75

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{2}(a+b)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}E\left(e+fx\middle|\frac{a}{a+b}\right)-a\sin(2(e+fx))\right)}{4bf(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[2]*(a + b)*Sqrt[(a + 2
*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - a*Sin[2*(
e + f*x)]))/(4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^3}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.95, size = 6601, normalized size = 44.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.272 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}} \, dx$

Optimal. Leaf size=229

$$\frac{\sin(e+fx)}{f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-$$

```
[Out] sin(f*x+e)/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+E
llipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(co
s(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 284, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4148, 6722, 1974, 412, 493, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{f(a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+f\ldots\right.}{af\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sec[e + f*x
]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) - (Sqrt[b + a*Cos[e + f*x]^2]*Elliptic
E[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*(a +
b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*
x]^2)/(a + b)]) + (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]
], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*f*Sqrt[Cos[e + f*x]^
2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(b+a(1-x^2)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Sub}}{(a+b)f\sqrt{\cos^2}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Sub}}{af\sqrt{\cos^2(e}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(\sqrt{b+a\cos^2(e+fx)}\sqrt{a}\right.}{a(a+b)f\sqrt{\cos}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,E\left(\text{si}\right.}{a(a+b)f\sqrt{\cos^2(e+}}$$

Mathematica [C] time = 9.60, size = 822, normalized size = 3.59

$$(\cos(2e+2fx)a+a+2b)^{3/2}\sec^3(e+fx)\left(\frac{\sqrt{-\frac{1}{a+b}}\cos(2(e+fx))\left(\sqrt{-\frac{1}{a+b}}(\cos(2e+2fx)a+a)\left(-2a^2+(\cos(2e+2fx)a+a-4b)a+2b(\cos(2e+\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3*(-1/8*(((2*(a - a*Cos[
2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/(b*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x
]]) + ((2*I)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4 - (2*(a + 2*b +
a*Cos[2*e + 2*f*x]))/b]*(EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - EllipticF[I*ArcSinh[(Sqrt
```

```
[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b]))/
Sqrt[-(a + b)^(-1)])*Sin[2*e + 2*f*x])/(a*(a + b)*f*Sqrt[((a - a*Cos[2*e +
2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Cos[2*e + 2*f*x]^2]) + (Sqr
t[-(a + b)^(-1)]*Cos[2*(e + f*x)]*(Sqrt[-(a + b)^(-1)]*(a + a*Cos[2*e + 2*f
*x])*(-2*a^2 + 2*b*(a + a*Cos[2*e + 2*f*x]) + a*(a - 4*b + a*Cos[2*e + 2*f*
x])) - I*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b +
a*Cos[2*e + 2*f*x]]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*Elliptic
E[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2
]], (a + b)/b] - I*a*b*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(4*a + 4*b -
 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e +
 2*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2
*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f
*x]])/2)]*Sin[2*e + 2*f*x])/(4*a^2*b*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a +
a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[1 - Cos[2
*e + 2*f*x]^2]))/(2*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a*
b*sec(f*x + e)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.25, size = 6593, normalized size = 28.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.273 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E}{a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\mathrm{se}}}$$

```
[Out] -b*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+(a+2*b)*E
llipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(cos(
f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2
/(a+b))^(1/2)-2*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(
a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 295, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4148, 6722, 1974, 413, 524, 426, 424, 421, 419}

$$-\frac{2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos}}{a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e
 + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + ((a + 2*b)*Sqrt[b + a*Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e +
f*x]^2])/(a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqr
t[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*b*Sqrt[b + a*Cos[e + f*x]^2]*Ellipt
icF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/
(a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx}}{a(a+b)f}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(2b\sqrt{b+a\cos^2(e+\right.}{a^2f}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left((a+2b)\sqrt{b+a\,\text{co}}\right.}{a^2(a}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+2b)\sqrt{b+a\cos}}{a^2(a+b)f}$$

Mathematica [F] time = 14.37, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\ \cos\left(fx+e\right)}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a*
b*sec(f*x + e)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.50, size = 8684, normalized size = 36.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.274 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{b(a-8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a+4b)\sin(e+fx)\left(-a\sin^2(e+fx)+a-\right.}{3a^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+\right.}}$$

```
[Out] -b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)+1/3*(a+4*b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/
(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-8*
b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a
^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 399, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 413, 528, 524, 426, 424, 421, 419}

$$\frac{\left(2a^2-3ab-8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}}+\frac{(a+4b)\sin(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(a*(a + b)*f*S
qrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + ((a + 4*b)*Sqr
t[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a^2
*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((2*a^2 - 3*a*b - 8*b^2)*Sqrt[b +
a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a
*Sin[e + f*x]^2])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e +
f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*b*Sqrt[b + a*Cos
[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
 f*x]^2)/(a + b)])/(3*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]
*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx}}{a(a+b}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos}}{3a^2}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos}}{3a^2}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos}}{3a^2}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos}}{3a^2}$$

Mathematica [F] time = 16.29, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^3}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.51, size = 11939, normalized size = 35.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^3}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.275 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=436

$$\frac{(a+6b)\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{5a^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{4b\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin\right.}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+\right.}}$$

```
[Out] -b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)+1/15*(4*a^2-5*a*b-24*b^2)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/5*(a+6*b)*cos(f*x+e)^2*sin(f*x+e
)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)+1/15*(8*a^3-9*a^2*b+16*a*b^2+48*b^3)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2)
)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-4/15*b*(a^2-2*a*b+12*
b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a
^4/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 509, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4148, 6722, 1974, 413, 528, 524, 426, 424, 421, 419}

$$\frac{\left(4a^2-5ab-24b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a\cos^2(e+fx)+b}}{15a^3f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin\ldots}{\ldots}}}{15a^4f\sqrt{\cos^2(e+fx)}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^4*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(a*(a + b)*f*S
qrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + ((4*a^2 - 5*a*
b - 24*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e +
f*x]^2])/(15*a^3*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 6*b)*Cos[e +
 f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(5*a^2*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((8*a^3 - 9*a^2*b + 16*
a*b^2 + 48*b^3)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a^4*(a + b)*f*Sqrt[Cos[e + f
*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (
4*b*(a^2 - 2*a*b + 12*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[
e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^4*f*Sqrt[
Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx}}{a(a+b}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+6b)\cos^2(e+f}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)}{}$$

Mathematica [F] time = 15.47, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{5}}{b^{2}\sec\left(fx+e\right)^{4}+2\,ab\sec\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.76, size = 15199, normalized size = 34.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{5}}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.276 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}+\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b^2f(a+b)}-\frac{a\tan(e+fx)\sec^2(e+f}{bf(a+b)\sqrt{a+b\tan^2(e+f}}$$

```
[Out] -1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)
/f-a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*(3*a+
b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/(a+b)/f
```

Rubi [A] time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4146, 413, 388, 217, 206}

$$\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b^2f(a+b)}-\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}-\frac{a\tan(e+fx)\sec^2(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b +
b*Tan[e + f*x]^2]) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(2*b^2*(a + b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+(3a+b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f}$$

$$= -\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + (3$$

Mathematica [C] time = 7.70, size = 260, normalized size = 1.88

$$-\frac{\tan(e+fx)\sec^8(e+fx)(a\cos(2(e+fx))+a+2b)\left(4b\sin^2(e+fx)(a\cos(2(e+fx))+a+2b)^2\,{}_3F_2\left(2,2,3;\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/210*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^8*(-7*(a + b)*Cos[e + f
*x]^2*(8*a^2 + 20*a*b + 15*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] + a^2*Cos
[4*(e + f*x)])*Hypergeometric2F1[1, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 4*b*(a + 2*b + a*Cos[2*(e + f*x)])^2*HypergeometricPFQ[{2, 2, 3}, {1, 9/2
}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2 + 16*b*Hypergeometric2F1[2
, 3, 9/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(4*(a + b)^2 - 7*a*
(a + b)*Sin[e + f*x]^2 + 3*a^2*Sin[e + f*x]^4))*Tan[e + f*x])/((a + b)^4*f*
(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.92, size = 524, normalized size = 3.80

$$\left[-\frac{\left(\left(3\,a^{3}+2\,a^{2}b-ab^{2}\right)\cos\left(fx+e\right)^{3}+\left(3\,a^{2}b+2\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{b}\,\log\left(\frac{\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-b^{2}\right)\cos}{}\right.}{8\left(\left(a^{2}b^{3}+ab^{4}\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 2*a^2*b - a*b^2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3
)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
+ e)^4) - 4*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x +
e)^3 + (a*b^4 + b^5)*f*cos(f*x + e)), -1/4*(((3*a^3 + 2*a^2*b - a*b^2)*cos(
f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 2*(a*b^2
+ b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 + (a*b^4 + b^5
)*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.88, size = 4338, normalized size = 31.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f*(-2*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(
f*x+e)*b^(5/2)*a+13*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/s
in(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*si
n(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*b^(7/2)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)+4*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
```

```
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*sin(f*x+e)*b^(3/2)*a^2-8*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f
*x+e)*b^(3/2)*a^2+2*cos(f*x+e)*b^(7/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)-13*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4
*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+2*cos(f*x+e)^5*b^(3/2)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a^2+6*cos(f*x+e)^5*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^3-2*cos(f*x+e)^4*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a^2+4*cos(f*x+e)^3*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-6*
cos(f*x+e)^4*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+8*cos(f*x+
e)^3*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-4*cos(f*x+e)^2*b^(
5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-8*cos(f*x+e)^2*b^(3/2)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)*b^(5/2)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a-2*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a+6*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+
e)*b^(1/2)*a^3-12*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^(1/2)*a^3+4*cos
(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^(5/2)*a+4*cos(f*x+e)^2*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^(5/2)*a-8*cos(f*x+e)^2*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*sin(f*x+e)*b^(5/2)*a+6*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^(3/2)*a^2-12*cos(f*x+e)^2*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
```

```
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*sin(f*x+e)*b^(3/2)*a^2+13*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/
2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1
/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-13*cos(f*x+e)
^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(-1+cos(f*x+e))*
(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2+13*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/s
in(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*si
n(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+13*cos(f*x+e)^4*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*
x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b^2-13*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*arctanh(1/4*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/
sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-13*cos(f*x+e)^4*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(-1+cos(f*x+e))*(cos(f*x+e)*4^
(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^(1/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
^2-2*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e
)*b^(7/2)+4*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^(7/2))*sin(f*x+e)/(-1
+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/b^(5/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)
```

maxima [A] time = 0.36, size = 161, normalized size = 1.17

$$\frac{\frac{\tan\left(fx+e\right)^3}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b} - \frac{3\,(a+b)\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{4\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)} + \frac{3\,(a+b)\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b^2} - \frac{4}{\sqrt{b\,\mathrm{ta}}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)^3/(sqrt(b*tan(f*x + e)^2 + a + b)*b) - 3*(a + b)*arcsinh(
b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 4*arcsinh(b*tan(f*x + e)/sqrt((a
+ b)*b))/b^(3/2) + 2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b))
+ 3*(a + b)*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b^2) - 4*tan(f*x +
 e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^6\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.277 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x+
e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4146, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
- (a*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{bf}$$

$$= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] time = 7.92, size = 405, normalized size = 5.26

$$-\frac{\tan(e+fx)\sec^4(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\,(a\cos(2e+2fx)+a+2b)^{3/2}\left(15\sec^2(e+fx)\left(a^2\left(2\sin^4(e+fx)-\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^4*Sqrt[1 - (2*a*Si
n[e + f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*(15*ArcSin[Sqrt[-((b*Tan[e + f*x]^2
)/(a + b))]]*Sec[e + f*x]^2*(3*b^2 + a*b*(6 - 5*Sin[e + f*x]^2) + a^2*(3 -
5*Sin[e + f*x]^2 + 2*Sin[e + f*x]^4)) + 15*(a + b)*(-3*b + a*(-3 + 2*Sin[e
+ f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]
^2)/(a + b)^2)] + 4*b*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((b*Tan[e + f*x
]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)*Tan[e + f*x]^2)/(a + b)^2))^(3/2)))/((a + b)^2*(2*a + 2*b)*f*(a + b*Sec[
e + f*x]^2)^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2*b - 2*a
*Sin[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*(-((b*Tan[e + f*x]^2)
/(a + b)))^(3/2))
```

fricas [B] time = 0.64, size = 410, normalized size = 5.32

$$\left[-\frac{4\,ab\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(a^2+ab\right)\cos\left(fx+e\right)^2+ab+b^2\right)\sqrt{b}\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx\right.}{}\right.}{4\left(\left(a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(ab^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*
b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
 + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/((a^2*b^2 + a*b^3)*f*cos(f*x +
e)^2 + (a*b^3 + b^4)*f), -1/2*(2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e)*sin(f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 + b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.82, size = 3068, normalized size = 39.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(2*cos(f*x+e)^2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*a-4*cos(f*x+e)^2*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a+2*cos(
f*x+e)^2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*a^2-4*cos(f*x+e)^2*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2+2*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^
(5/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)-4*sin(f*x+e)*EllipticP
i((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
```

```
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(5/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)+2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*a-4*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(3/2)
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a+cos(f*x+e)^2*sin(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/4*(-1+cos(f*x+e))*(cos(f*x
+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+
cos(f*x+e)^2*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/4
*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a*b-cos(f*x+e)^2*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-
4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*4^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-cos(f*x+e)^2*sin(
f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*
(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1
/2)*a^2-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1/2)*a^2+
2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*a+sin(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/4*(-1+cos(f*x+e))*(cos(f*x
+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+
sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/4*(-1+cos(f*x+
e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^2-sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/8*
(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2
)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/
2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*b^(3/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a
*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/b^(3/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/(a+b)
```

maxima [A] time = 0.35, size = 78, normalized size = 1.01

$$\frac{\frac{\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)}-\frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + tan(f*x + e)/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)) - tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b
)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.278 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4146, 191}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.63, size = 57, normalized size = 1.78

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)}{2f(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*(a + b)*f*(
a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.55, size = 65, normalized size = 2.03

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)}{\left(a^{2}+ab\right)f\cos\left(fx+e\right)^{2}+\left(ab+b^{2}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e)/((a^2
 + a*b)*f*cos(f*x + e)^2 + (a*b + b^2)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*512*a^2*b^2*tan((f*x+exp(1))/2)
*sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/
2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)*sign(tan((f*x+exp(1))/2)^2-1)/(512*a^2*
b^3+512*a^3*b^2)/(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((
f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)
```

maple [A] time = 1.45, size = 59, normalized size = 1.84

$$\frac{\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)\sin\left(fx+e\right)}{f\cos\left(fx+e\right)^{3}\left(\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*(b+a*cos(f*x+e)^2)*sin(f*x+e)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(3/2)/(a+b)
```

maxima [A] time = 0.33, size = 30, normalized size = 0.94

$$\frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\left(a+b\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*f)
```

mupad [B] time = 6.29, size = 199, normalized size = 6.22

$$\frac{\sqrt{\frac{a+2\,b+a\,\cos\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(5\,a\,\sin\left(2\,e+2\,f\,x\right)+4\,a\,\sin\left(4\,e+4\,f\,x\right)+a\,\sin\left(6\,e+6\,f\,x\right)+8\right.}{f\,\left(a+b\right)\,\left(24\,a\,b+10\,a^2+16\,b^2+15\,a^2\,\cos\left(2\,e+2\,f\,x\right)+6\,a^2\,\cos\left(4\,e+4\,f\,x\right)+a^2\,\cos\left(6\,e+6\,f\,x\right)+16\,b^2\,\mathrm{cc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] (((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2)*(5*a*sin(2*e
 + 2*f*x) + 4*a*sin(4*e + 4*f*x) + a*sin(6*e + 6*f*x) + 8*b*sin(2*e + 2*f*x
) + 4*b*sin(4*e + 4*f*x)))/(f*(a + b)*(24*a*b + 10*a^2 + 16*b^2 + 15*a^2*co
s(2*e + 2*f*x) + 6*a^2*cos(4*e + 4*f*x) + a^2*cos(6*e + 6*f*x) + 16*b^2*cos
(2*e + 2*f*x) + 32*a*b*cos(2*e + 2*f*x) + 8*a*b*cos(4*e + 4*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.279 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4128, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 1.38, size = 168, normalized size = 2.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}\,b\sin}{4a^{3/2}f(a+b)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

fricas [B] time = 0.74, size = 601, normalized size = 7.81

$$\left[-\frac{8\,ab\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+ab\right)\cos\left(fx+e\right)^2+ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
```

```
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

maple [C] time = 1.91, size = 1007, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*
sin(f*x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(3/2)/(a+b)/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [B] time = 0.84, size = 2055, normalized size = 26.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

3.280 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}+\frac{b(a+3b)\tan(e+fx)}{2a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] 1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*b*(a+3*b)*tan
(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}+\frac{b(a+3b)\tan(e+fx)}{2a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) + (b*(a + 3*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e +
 f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)\,\text{S}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)\,\text{S}}{}$$

$$= \frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+}{2a^2(a+b)f\sqrt{}}$$

Mathematica [C] time = 15.32, size = 2059, normalized size = 15.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^6*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appell
```

F1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Ap
pellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(
a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*
b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1/
2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF
1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b
)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f
x)]](a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1,
3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (
6*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(
a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a +
b)*Cos[e + f*x]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f
*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
 - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*Ap
pellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]^4*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a +
b)*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3)
+ Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*Appel
lF1[5/2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
f*x]*Sin[e + f*x])/5) - 4*(a + b)*((9*a*f*AppellF1[5/2, -1, 5/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b
)) - (6*f*Cos[e + f*x]*Hypergeometric2F1[3/2, 5/2, 7/2, (a*Sin[e + f*x]^2)/
(a + b)]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
 - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b
)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x
)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))
))

fricas [B] time = 1.18, size = 699, normalized size = 5.34

$$\left[\frac{\left(a^2b-2\,ab^2-3\,b^3+\left(a^3-2\,a^2b-3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2)
*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b
^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
+ 8*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x
 + e)^2 + (a^4*b + a^3*b^2)*f), -1/8*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a
^2*b - 3*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^3 + a^2*b)*c
os(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a
^3*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.39, size = 1645, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a^2*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
```

```
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*
x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*a^2*sin(f*x+e)+4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*b^2*sin(f*x+e)-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*
a^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b+((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^2*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)
*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2+((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
2)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2
)^(3/2)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.281 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{b(a-3b)(3a+5b)\tan(e+fx)}{8a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{3\left(a^2-2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e\cdots}{\sqrt{a+b\tan^2(e\cdots}}\right.}{8a^{7/2}f}$$

```
[Out] 3/8*(a^2-2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(7/2)/f+1/8*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(
1/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(a-3*b)
*b*(3*a+5*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{3\left(a^2-2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{8a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\sin(e+fx)\cos(\cdots}{8a^2f\sqrt{a+b\tan^2(e+fx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(7/2)*f) + ((3*a - 5*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2
*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*S
qrt[a + b + b*Tan[e + f*x]^2]) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e + f*x])/(8*
a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3a^2\cdots}{(1+\cdots}\right.}{\cdots}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(\cdots}{8a^3(a+b)f\sqrt{\cdots}}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(\cdots}{8a^3(a+b)f\sqrt{\cdots}}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(\cdots}{8a^3(a+b)f\sqrt{\cdots}}$$

$$= \frac{3\left(a^2-2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} \cdots$$

Mathematica [C] time = 16.67, size = 2046, normalized size = 10.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]^10*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*((a + b)*AppellF1[
1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF
1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b
)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)*((a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*C
os[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*((a + b)*AppellF1[1/2, -3, 3/
2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3,
5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[
3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x
]^2)) + ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)]*Cos[e + f*x]^7)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
- a*Sin[e + f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF
1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*
x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e +
 f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + ((a + b)*Cos[e + f*x]^6*Sin[e
 + f*x]*((a*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - 2*f*AppellF1[3/2, -2, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
)/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a +
b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
+ (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*Ap
pellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(
a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Cos[e + f*x]*Sin[e + f*x] + (a + b)*((a*f*AppellF1[3/2, -3, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a +
 b) - 2*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2,
 -3, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin
[e + f*x])/(a + b) - (18*f*AppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 2*(a + b)*((9*a*f*A
ppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a + b)*A
ppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a
*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)])*Sin[e + f*x]^2)^2) + (a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e +
 f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*
((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

fricas [A] time = 2.52, size = 811, normalized size = 4.18

$$\left[-\frac{3\left(a^3b-a^2b^2+3\,ab^3+5\,b^4+\left(a^4-a^3b+3\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5
*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - 8*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3
*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*
b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f), -1/32*(3*(a^3*b - a^2*b^2 + 3*a
*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e))) - 4*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3*b - 5*a
^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos
(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.34, size = 2372, normalized size = 12.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*(b+a*cos(f*x+e)^2)*(2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^5*a^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b+6*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
```

```
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*a^3*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+18*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*a*b^2*sin(f*x+e)+30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-3*2^(1/2)*((
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-9*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*b^3*sin(f*x+e)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)^4*a^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b+3*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^3*a*b^2-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b+5
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+3*cos(f*x+e)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)*a*b^2-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(
f*x+e)*b^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(3/2)/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}{\left(e + fx \right)}}{\left(a + b \sec^{2}{\left(e + fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.282 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{(5a-7b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\left(15a^2-22ab+35b^2\right)\sin(e+fx)\cos(e+fx)}{48a^3f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\left(5a^3-9a^2b+15ab^2-35b\right.}{16a}$$

```
[Out] 1/16*(5*a^3-9*a^2*b+15*a*b^2-35*b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f
*x+e)^2)^(1/2))/a^(9/2)/f+1/48*(15*a^2-22*a*b+35*b^2)*cos(f*x+e)*sin(f*x+e)
/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/24*(5*a-7*b)*cos(f*x+e)^3*sin(f*x+e)/a^
2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*tan(f
*x+e)^2)^(1/2)+1/48*b*(15*a^3-17*a^2*b+25*a*b^2+105*b^3)*tan(f*x+e)/a^4/(a+
b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{\left(-9a^2b+5a^3+15ab^2-35b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{b\left(-17a^2b+15a^3+25ab^2+105b^3\right)\tan(e+fx)}{48a^4f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(15}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((5*a^3 - 9*a^2*b + 15*a*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
 + b + b*Tan[e + f*x]^2]])/(16*a^(9/2)*f) + ((15*a^2 - 22*a*b + 35*b^2)*Cos
[e + f*x]*Sin[e + f*x])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((5*a -
 7*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f
*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-5a+b-6bx^2}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+f}}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+f}}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+f}}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+}{24a^2f\sqrt{a+b+b\tan^2(e+f}}$$

$$= \frac{\left(5a^3-9a^2b+15ab^2-35b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{\left(15a^2-22ab+35b^2\right)}{48a^3f\sqrt{a+b}}$$

Mathematica [C] time = 19.73, size = 2068, normalized size = 7.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^14*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appel
lF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*A
ppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*
(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(Sqrt[a + 2
*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1
/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Appell
F1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a +
b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])
*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2,
```

```
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9)/(2*Sqrt[a + 2*b + a*Cos[2*(e +
f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3,
 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) -
(12*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a
+ b)*Cos[e + f*x]^8*Sin[e + f*x]*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8
*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
Cos[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a +
 b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*
AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^8*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a
+ b)*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8*f*AppellF1[3/2, -3, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3
) + Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*App
ellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
+ f*x]*Sin[e + f*x])/5) - 8*(a + b)*((9*a*f*AppellF1[5/2, -3, 5/2, 7/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a +
 b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e +
 f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3
, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)
 + (3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*
Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2,
 -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[
3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*
AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Si
n[e + f*x]^2))))
```

fricas [A] time = 7.33, size = 941, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 + (5*a^5 - 4
*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*(a^5 + a^
```

```
4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)^5 + (15*
a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4*b - 17*a^
3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5
*b^2)*f), -1/192*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 +
(5*a^5 - 4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) - 4*(8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b
- 7*a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*
cos(f*x + e)^3 + (15*a^4*b - 17*a^3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*
b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.76, size = 3171, normalized size = 11.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/48/f*(b+a*cos(f*x+e)^2)*(-210*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^4*sin(f*x+e)-10*a^4*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4+30*2^(1/2)*((I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4*si
n(f*x+e)+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^4+10*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^4+15*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^4-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^2*a^4+105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)*b^4+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^3*b-13*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b^2-35*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)^6*a^3*b+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^7*a^3*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3*
b-14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b^2+14*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b^2-8*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^4-15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
```

```
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a^4*sin(f*x+e)+105*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*b^4*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+17*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-7*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^3*a^3*b+13*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^3*a^2*b^2+35*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)
^3*a*b^3+7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^3*b+15*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a^3*b-17*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a^2*b^2+25*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)*a*b^3-25*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3
+12*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b*sin(f*x+e)-18*2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2*sin(f*x+e)-24*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a^3*b*sin(f*x+e)-120*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3*sin(f*x+e)+36*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a^2*b^2*sin(f*x+e)-105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4
+60*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3*sin(f*x+e))*sin(f
*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^4/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{6}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.283 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{2a(a+2b)\sin(e+fx)}{3b^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}\right.}{3b^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+}}$$

```
[Out] -2/3*a*(a+2*b)*sin(f*x+e)/b^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)-1/3*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)+2/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+
b-a*sin(f*x+e)^2)/b^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*s
in(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*EllipticF(sin(f*x+e)
,(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/(a+b)/f/(cos(f*x+e)^2)^(
1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 383, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 414, 527, 524, 426, 424, 421, 419}

$$-\frac{2a(a+2b)\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{3b^2f(a+b)^2\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx}}{3b^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(a*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*b*(a + b)*f*Sqrt[a + b*Sec[
e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*a*(a + 2*b)*Sqrt[b + a*C
os[e + f*x]^2]*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]*
Sqrt[a + b - a*Sin[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[b + a*Cos[e + f*x]^2]*E
llipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(
3*b^2*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) - (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin
[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*(a +
b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e +
 f*x]^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(b+a(1-x^2)\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(a+b-ax^2\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx}}{3b(a+b)}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{}}{3b^2(a+b)^2f\sqrt{a+}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{}}{3b^2(a+b)^2f\sqrt{a+}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{}}{3b^2(a+b)^2f\sqrt{a+}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{}}{3b^2(a+b)^2f\sqrt{a+}}$$

Mathematica [A] time = 2.76, size = 167, normalized size = 0.52

$$\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{2}(a+b)^2\left(\frac{a\cos(2(e+fx))+a+2b}{a+b}\right)^{3/2}\left(2(a+2b)E\left(e+fx\middle|\frac{a}{a+b}\right)-bF\left(e+fx\right.\right.\right.}{24b^2f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(Sqrt[2]*(a + b)^2*((a + 2*b
 + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(2*(a + 2*b)*EllipticE[e + f*x, a/(a
+ b)] - b*EllipticF[e + f*x, a/(a + b)]) - 2*a*(a^2 + 5*a*b + 5*b^2 + a*(a
+ 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(24*b^2*(a + b)^2*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^5}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.39, size = 14353, normalized size = 44.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.284 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=319

$$-\frac{(a-b)\sin(e+fx)}{3bf(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sin(e+fx)}{3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin\right.}}$$

```
[Out] -1/3*(a-b)*sin(f*x+e)/b/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
+1/3*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e
)^2))^(1/2)+1/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+
e)^2)/a/b/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*EllipticF(sin(f*x+e),(a/(a+b))^(
1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 381, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4148, 6722, 1974, 412, 527, 524, 426, 424, 421, 419}

$$-\frac{(a-b)\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{3bf(a+b)^2\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}\sqrt{a+b\sec^2(\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*(a + b)*f*Sqrt[a + b*Sec[e + f
*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a - b)*Sqrt[b + a*Cos[e + f*x]
^2]*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b -
a*Sin[e + f*x]^2]) + ((a - b)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[S
in[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a*b*(a + b)^2*f
*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2
)/(a + b)]) + (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*(a + b)*f*Sqrt[Cos[e +
 f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{S}}{3(a+b)f\sqrt{}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+}}{3b(a+b)^2f\sqrt{a+b\,\text{se}}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+}}{3b(a+b)^2f\sqrt{a+b\,\text{se}}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+}}{3b(a+b)^2f\sqrt{a+b\,\text{se}}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+}}{3b(a+b)^2f\sqrt{a+b\,\text{se}}}$$

Mathematica [C] time = 10.16, size = 1156, normalized size = 3.62

$$(\cos(2e+2fx)a+a+2b)^{5/2}\sec^5(e+fx)\left(\frac{\cos(2(e+fx))\left(2ib\left(a^2+ba+b^2\right)\sqrt{\frac{a-a\cos(2e+2fx)}{a+b}}\sqrt{4-\frac{2(\cos(2e+2fx)a+a+2b)}{b}}E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-1/24*((-2*Sqrt[-(a +
 b)^(-1)]*(-a - a*Cos[2*e + 2*f*x])*(2*a^2*(a + 3*b + a*Cos[2*e + 2*f*x]) +
 b*(2*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - 2*(a + 2*b + a*Cos[2*e + 2
*f*x])^2) + a*(4*b^2 + 5*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Co
s[2*e + 2*f*x])^2)) + (2*I)*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a +
b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e +
```

```
2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[
2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - I*b*(a + 3*b)*Sqrt[(a - a*Cos[2*e + 2
*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b +
 a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sin[2*e + 2*f*x])/(a*b^2*S
qrt[-(a + b)^(-1)]*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*
e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*e + 2
*f*x]^2]) + (Cos[2*(e + f*x)]*(-2*Sqrt[-(a + b)^(-1)]*(-a - a*Cos[2*e + 2*f
*x])*(4*b^4 - b^2*(a + 2*b + a*Cos[2*e + 2*f*x])^2 + 2*a^3*(a + 3*b + a*Cos
[2*e + 2*f*x]) + a*b*(10*b^2 + b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b
+ a*Cos[2*e + 2*f*x])^2) + a^2*(8*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x])
- (a + 2*b + a*Cos[2*e + 2*f*x])^2)) + (2*I)*b*(a^2 + a*b + b^2)*Sqrt[(a -
a*Cos[2*e + 2*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 -
(2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-
1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] + I*a*b*(-a +
b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2)*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqr
t[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*
Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(24*a^2*
b^2*Sqrt[-(a + b)^(-1)]*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*C
os[2*e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*
e + 2*f*x]^2])))/(2*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\,\sec\left(fx+e\right)^{3}}{b^{3}\sec\left(fx+e\right)^{6}+3\,ab^{2}\sec\left(fx+e\right)^{4}+3\,a^{2}b\sec\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.10, size = 10271, normalized size = 32.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.285 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=327

$$\frac{(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2(2a+b)\left(-a\sin^2(e+fx)+\right.}{3a^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e}{a+b}}}$$

```
[Out] 2/3*(2*a+b)*sin(f*x+e)/a/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)-1/3*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)-2/3*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*s
in(f*x+e)^2)/a^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*
x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(3*a+2*b)*EllipticF(sin(f
*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/(a+b)/f/(cos(f*x+
e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 389, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$\frac{(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}-\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}}{3a^2f(a+b)^2\sqrt{\cos^2(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*Sqrt[a + b*Sec[
e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*(2*a + b)*Sqrt[b + a*Cos
[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt
[a + b - a*Sin[e + f*x]^2]) - (2*(2*a + b)*Sqrt[b + a*Cos[e + f*x]^2]*Ellip
ticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a^
2*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*S
in[e + f*x]^2)/(a + b)]) + ((3*a + 2*b)*Sqrt[b + a*Cos[e + f*x]^2]*Elliptic
F[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3
*a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b -
 a*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\left(b+a(1-x^2)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+}}{3a(a+}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b}{3a(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b}{3a(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b}{3a(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b}{3a(a+b)^2f\sqrt{a}}$$

Mathematica [F] time = 12.28, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a*
b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.08, size = 14353, normalized size = 43.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.286 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{b(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2b(3a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e\right.}}$$

```
[Out] -2/3*b*(3*a+2*b)*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)-1/3*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(3*a^2+13*a*b+8*b^2)*EllipticE(sin
(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1
/2)-1/3*b*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2
/(a+b))^(1/2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 411, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4148, 6722, 1974, 413, 526, 524, 426, 424, 421, 419}

$$\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a\cos^2(e+fx)+b}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^3f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}}-\frac{2b(3a+}{3a^2f(a+b)^2\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*b*(3*a +
2*b)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b
*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + ((3*a^2 + 13*a*b + 8*b^2
)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqr
t[a + b - a*Sin[e + f*x]^2])/(3*a^3*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a
 + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(9*a + 8*b)
*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqr
t[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(b+a(1-x^2))^{5/2}}dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+b-ax^2)^{5/2}}dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+}}{3a(}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+}{3a^2(a+b)^2f\sqrt{a}}$$

Mathematica [F] time = 16.10, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)

maple [C] time = 2.61, size = 17502, normalized size = 50.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)

[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)

[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)

3.287 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=441

$$-\frac{2b(4a+3b)\sin(e+fx)\cos^2(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\sin^{-1}(\sin(\right.}{3a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\left(-a\sin^2(e\right.}$$

```
[Out] -2/3*b*(4*a+3*b)*cos(f*x+e)^2*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*
x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(a^2+11*a*b+8*b^2)*si
n(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)+2/3*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/
2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b*(a^2-16*a*b
-16*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/
2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)
```

Rubi [A] time = 0.73, antiderivative size = 512, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {4148, 6722, 1974, 413, 526, 528, 524, 426, 424, 421, 419}

$$\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a\cos^2(e+fx)+b}}{3a^3f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(}{a\text{-}}}}{3a^4f(a+b)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Cos[e + f*x]^4*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*b*(4*a +
3*b)*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a^2*(a + b)
^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + ((a^2 + 1
1*a*b + 8*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e
 + f*x]^2])/(3*a^3*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + (2*(a + 2*b)*(
a^2 - 4*a*b - 4*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a^4*(a + b)^2*f*Sqrt[Cos
[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)
]) - (b*(a^2 - 16*a*b - 16*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin
[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^4*(a
+ b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)])))))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\ \sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+}}{3a}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\ \left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

Mathematica [F] time = 12.19, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^3}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 3.52, size = 20922, normalized size = 47.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^3}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.288 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=559

$$-\frac{2b(5a+4b)\sin(e+fx)\cos^4(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e\right.}{15a^3f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+\right.}}$$

```
[Out] -2/3*b*(5*a+4*b)*cos(f*x+e)^4*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^6*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*
x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+2/15*(2*a^3-3*a^2*b-42*a*
b^2-32*b^3)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(sec(f*x+e)^2*(a+
b-a*sin(f*x+e)^2))^(1/2)+1/15*(3*a^2+61*a*b+48*b^2)*cos(f*x+e)^2*sin(f*x+e)
*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)+1/15*(8*a^4-11*a^3*b+27*a^2*b^2+184*a*b^3+128*b^4)*EllipticE(sin(f*x+e),
(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^5/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/15
*b*(4*a^3-9*a^2*b+120*a*b^2+128*b^3)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*
(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^5/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)
^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 639, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {4148, 6722, 1974, 413, 526, 528, 524, 426, 424, 421, 419}

$$\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\cos^2(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\sqrt{a\cos^2(e+fx)+b}}{15a^3f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}+\frac{2\left(-3a^2b+2a^3-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Cos[e + f*x]^6*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*b*(5*a +
4*b)*Cos[e + f*x]^4*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a^2*(a + b)
^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + (2*(2*a^3
 - 3*a^2*b - 42*a*b^2 - 32*b^3)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2])/(15*a^4*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]
) + ((3*a^2 + 61*a*b + 48*b^2)*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Si
n[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a^3*(a + b)^2*f*Sqrt[a + b*S
ec[e + f*x]^2]) + ((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b^4)*Sq
rt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a
+ b - a*Sin[e + f*x]^2])/(15*a^5*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a +
b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^3 - 9*a^2
*b + 120*a*b^2 + 128*b^3)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e
 + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^5*(a + b)*
f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*
x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{9/2}}{\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\ \text{Subst}\left(\int \frac{\left(1-x^2\right)^{9/2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+}}{3a}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\ \sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos}{3a^2(a+b)^2f\sqrt{a}}$$

Mathematica [F] time = 22.28, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{5}}{b^{3}\sec\left(fx+e\right)^{6}+3\,ab^{2}\sec\left(fx+e\right)^{4}+3\,a^{2}b\sec\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 5.25, size = 26983, normalized size = 48.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{5}}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.289 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{a(3a+5b)\tan(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{a\tan(e+fx)\sec^2(e+fx)}{3bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*(3*a
+5*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*a*sec(f*x+e)^
2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4146, 413, 385, 217, 206}

$$-\frac{a(3a+5b)\tan(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{a\tan(e+fx)\sec^2(e+fx)}{3bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)*f)
- (a*Sec[e + f*x]^2*Tan[e + f*x])/(3*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (a*(3*a + 5*b)*Tan[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b + b*Ta
n[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+3b+3(a+b)x^2}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+\right.}{3b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a}{3b^2(a+}$$

Mathematica [C] time = 11.70, size = 357, normalized size = 2.68

$$\frac{e^{i(e+fx)}\sec^5(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{ia\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(3a^2\left(1+e^{2i(e+fx)}\right)^2+ab\left(26e^{2i(e+fx)}+5e^{4i(e+fx)}+5\right)+24b^2\right.}{(a+b)^2\left(a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}\right)^2}\right.}{12\sqrt{2}\,b^{5/2}f\left(a+b\sec^2(e+fx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*((I*a*Sqrt[b]*(-1 + E^((2*I)*(e
+ f*x)))*(24*b^2*E^((2*I)*(e + f*x)) + 3*a^2*(1 + E^((2*I)*(e + f*x)))^2 +
a*b*(5 + 26*E^((2*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x)))))/((a + b)^2*(4*b*
E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2) - (3*Log[(-4*Sqrt[b
]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
+ E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*
(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[e + f*x]^5)/(12*Sqrt[2]*b^
(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 0.93, size = 688, normalized size = 5.17

$$\left[\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{b}\log\left(\frac{\left(a^2-6ab+\cdots\right.}{\cdots}\right)}{12\left(\left(a^4b^3+2a^3b^4\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^
4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^3*b + 5*a^2*b^2)*cos(f*
x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^5 + 2*a*
b^6 + b^7)*f), 1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 +
 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-b)*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) -
 2*((3*a^3*b + 5*a^2*b^2)*cos(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x
+ e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 +
 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*
cos(f*x + e)^2 + (a^2*b^5 + 2*a*b^6 + b^7)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.10, size = 3018, normalized size = 22.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3+6*cos(f*x+e)^2*sin(f*x+e)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
```

```
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*b+3*cos(f*x+e)^2*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2-6*sin(f*x+e)*cos(f*x+e)^
2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*a^3-12*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*si
n(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*b^3*sin(f*x+e)-6*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-12*sin(f*x+e)*2^(1/2)*
((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*a*b^2-6*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3+3*cos(f*x+e)^3*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^3*a^2*b-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3
```

```
-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b+4*cos(f*x+e)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+6*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*cos(f*x+e)*a*b^2-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
*b-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2)/(-1+cos(f*x+e))/cos(f*x
+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/(a^2+2*a*b+b^2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/b^2
```

maxima [B] time = 0.39, size = 275, normalized size = 2.07

$$-\frac{\left(\frac{3\tan\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b^2}+\frac{2}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b}\right)\tan\left(fx+e\right)-\frac{3\,\mathrm{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}-\frac{2\tan\left(fx\cdots\right.}{\sqrt{b\tan\left(fx+e\right)^2+\cdots}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*tan(f*x + e)^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) + 2*a/((b*tan(
f*x + e)^2 + a + b)^(3/2)*b^2) + 2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b))*ta
n(f*x + e) - 3*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) - 2*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - tan(f*x + e)/((b*tan(f*x
+ e)^2 + a + b)^(3/2)*(a + b)) + 3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a
+ b)*b^2) - 2*a*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*b^2) +
 2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) - 4*tan(f*x + e)/(sqrt
(b*tan(f*x + e)^2 + a + b)*(a + b)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{6}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.290 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] 2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4146, 378, 191}

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (2*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 4.30, size = 74, normalized size = 0.94

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)(a\cos(2(e+fx))+2a+3b)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(2*a + 3*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^4*Tan[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [A] time = 0.86, size = 134, normalized size = 1.70

$$\frac{\left(2\,a\cos\left(fx+e\right)^3+(a+3\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\sin\left(fx+e\right)}{3\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^4+2\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(a^2b^2+2\,ab^3+b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*cos(f*x + e)^3 + (a + 3*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)2/f*2*tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)^2*(-tan((f*x
+exp(1))/2)^2*(1327104*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)+2654208*a^5*b^
5*sign(tan((f*x+exp(1))/2)^2-1)+1327104*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-
```

```
1))/(2654208*a^4*b^7+7962624*a^5*b^6+7962624*a^6*b^5+2654208*a^7*b^4)-(2654
208*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)+1769472*a^5*b^5*sign(tan((f*x+exp
(1))/2)^2-1)-884736*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-1))/(2654208*a^4*b^7
+7962624*a^5*b^6+7962624*a^6*b^5+2654208*a^7*b^4))-(1327104*a^4*b^6*sign(ta
n((f*x+exp(1))/2)^2-1)+2654208*a^5*b^5*sign(tan((f*x+exp(1))/2)^2-1)+132710
4*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-1))/(2654208*a^4*b^7+7962624*a^5*b^6+7
962624*a^6*b^5+2654208*a^7*b^4))/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+ex
p(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan(
(f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan
((f*x+exp(1))/2)^2+a+b)
```

maple [A] time = 1.96, size = 76, normalized size = 0.96

$$\frac{\sin\left(fx+e\right)\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)\left(2a\left(\cos^2\left(fx+e\right)\right)+a+3b\right)}{3f\cos\left(fx+e\right)^5\left(\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2+a+3*b)/cos(f*x+e)^5/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/(a+b)^2
```

maxima [A] time = 0.36, size = 117, normalized size = 1.48

$$\frac{\frac{2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^2}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b}+\frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)) - tan(f*x + e)/((b*tan(f*x +
e)^2 + a + b)^(3/2)*b) + tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a +
b)*b))/f
```

mupad [B] time = 12.67, size = 153, normalized size = 1.94

$$-\frac{2\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\,\left(a\,1\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,4\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}\right)}{3\,f\,(a+b)^2\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -(2*(exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*
x*1i)/2)^2)^(1/2)*(a*1i + a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*1i
 + b*exp(e*2i + f*x*2i)*6i))/(3*f*(a + b)^2*(a + 2*a*exp(e*2i + f*x*2i) + a
*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.291 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] 2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/(a+b)/f/
(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4146, 192, 191}

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Tan[e + f*x]/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 6.12, size = 215, normalized size = 3.03

$$\frac{(3a+b)\sec^5(e+fx)\left(\frac{2\sqrt{2}\sin(e+fx)}{(a+b)^2\sqrt{-a\sin^2(e+fx)+a+b}} + \frac{\sqrt{2}\sin(e+fx)}{(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)(a\cos(2e+2fx)+a+2b)^{5/2}}{48af\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{\tan(e+}{8\sqrt{2}af\,(-}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a + b)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*((Sqrt[2]*Si
n[e + f*x])/((a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*Sqrt[2]*Sin[e +
 f*x])/((a + b)^2*Sqrt[a + b - a*Sin[e + f*x]^2])))/(48*a*f*(a + b*Sec[e +
f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e
 + f*x])/(8*Sqrt[2]*a*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x
]^2)^(3/2))
```

fricas [B] time = 0.70, size = 134, normalized size = 1.89

$$\frac{\left((3\,a+b)\cos\left(fx+e\right)^3 + 2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\sin\left(fx+e\right)}{3\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^4 + 2\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2 + \left(a^2b^2+2\,ab^3+b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a + b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

```
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)2/f*2*tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)^2*(-tan((f*x
+exp(1))/2)^2*(1327104*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)+2654208*a^5*b^
5*sign(tan((f*x+exp(1))/2)^2-1)+1327104*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-
1))/(2654208*a^4*b^7+7962624*a^5*b^6+7962624*a^6*b^5+2654208*a^7*b^4)-(8847
36*a^4*b^6*sign(tan((f*x+exp(1))/2)^2-1)-1769472*a^5*b^5*sign(tan((f*x+exp(
1))/2)^2-1)-2654208*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-1))/(2654208*a^4*b^7
+7962624*a^5*b^6+7962624*a^6*b^5+2654208*a^7*b^4))-(1327104*a^4*b^6*sign(ta
n((f*x+exp(1))/2)^2-1)+2654208*a^5*b^5*sign(tan((f*x+exp(1))/2)^2-1)+132710
4*a^6*b^4*sign(tan((f*x+exp(1))/2)^2-1))/(2654208*a^4*b^7+7962624*a^5*b^6+7
962624*a^6*b^5+2654208*a^7*b^4))/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+ex
p(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan(
(f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan
((f*x+exp(1))/2)^2+a+b)
```

maple [A] time = 1.61, size = 85, normalized size = 1.20

$$\frac{\sin\left(fx+e\right)\left(3a\left(\cos^{2}\left(fx+e\right)\right)+\left(\cos^{2}\left(fx+e\right)\right)b+2b\right)\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}{3f\cos\left(fx+e\right)^{5}\left(\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{2}}\right)^{\frac{5}{2}}\left(a+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)*(3*a*cos(f*x+e)^2+cos(f*x+e)^2*b+2*b)*(b+a*cos(f*x+e)^2)/c
os(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/(a+b)^2
```

maxima [A] time = 0.37, size = 61, normalized size = 0.86

$$\frac{\frac{2\,\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

mupad [B] time = 13.90, size = 172, normalized size = 2.42

$$-\frac{\left(e^{e\,4i+f\,x\,4i}-1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^{2}}}\left(a\,3i+b\,1i+a\,e^{e\,2i+f\,x\,2i}\,6i+a\,e^{e\,4i+f\,x\,4i}\,3i+b\,e^{e\,2i+f\,x\,2i}\,10i+b\,e^{e\,4i+f\,x\,4i}\,1i\right)}{3\,f\,(a+b)^{2}\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*
1i)/2)^2)^(1/2)*(a*3i + b*1i + a*exp(e*2i + f*x*2i)*6i + a*exp(e*4i + f*x*4
i)*3i + b*exp(e*2i + f*x*2i)*10i + b*exp(e*4i + f*x*4i)*1i))/(3*f*(a + b)^2
*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i
))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.292 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a}{3a^2(a+b)^2f}$$

Mathematica [C] time = 6.50, size = 1927, normalized size = 15.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
```

```
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

fricas [B] time = 1.44, size = 881, normalized size = 7.05

$$\left[-\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

maple [C] time = 1.79, size = 3024, normalized size = 24.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3+6*cos(f*x+e)^2*sin(f*x+e)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*b+3*cos(f*x+e)^2*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2-6*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
```

```
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f
*x+e)*cos(f*x+e)^2*a^3-12*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b-6*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^3*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-12*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+6*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^3*a*b^2-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)^2*a^2*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2+3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/(a^2+2*a*b+b^2)/a^2/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

3.293 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}+\frac{b(3a+5b)\tan(e+fx)}{6a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+f}{6a^3f(a+b)^2\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a-5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f
+1/6*b*(3*a^2+22*a*b+15*b^2)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^
(1/2)+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*b*(3*a+5
*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{6a^3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}+\frac{b(3a+5b)\tan(e+fx)}{6a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a - 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) + (b*(3*a + 5*b)*Tan[e + f*x])/(6*a^2*(a + b)*f*(a + b + b*Tan[e +
 f*x]^2)^(3/2)) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(6*a^3*(a + b)
^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-4bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \text{Subs}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3}{6a^3(a}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3}{6a^3(a}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3}{6a^3(a}$$

$$= \frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3}{6a^2(a+b)}$$

Mathematica [C] time = 17.33, size = 1775, normalized size = 9.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
-2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos
[e + f*x]^6*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*
AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x]))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(
a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2
*f*(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2,
-3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[
e + f*x])/(3*(a + b)) - 2*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(5*a*(
(21*a*f*AppellF1[5/2, -3, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 7/2,
7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
/5) - 6*(a + b)*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*AppellF1[5/
2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)
*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)^2)))
```

fricas [B] time = 3.36, size = 1023, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^
2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)
```

```
*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + 2*(3*a^4*
b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^2*b^3 + 15*
a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), -1/24*(3*(a^3
*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^2 - 5*a^2*b^3
)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)*cos(f*x + e)
^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e))) - 4*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5
+ 2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^
2*b^3 + 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a
^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.41, size = 4270, normalized size = 22.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-18*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a^3*b-30*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^4*sin(f*x+e)-3*a^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^4-54*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
```

```
*a^2*b^2-30*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3+15*cos(f*x+e)^2*
sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3+9*2^(1/2)*
a^3*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^2*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*b+27*2^
(1/2)*a^2*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*b
^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^4+15*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^4+6*cos(f*x+e)^2*sin(f*x+e)*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a^4-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^4-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*
a^3*b-30*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b^2-20*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^3+6*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^3*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^5*a^2*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^4*a^2*b^2+15*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4*s
in(f*x+e)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+6*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^3*b+30*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b^2+20*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*cos(f*x+e)^3*a*b^3-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^
2*a^3*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a^2*b^2+22*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^3-22*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a*b^3-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a^3*b*sin(f*x+e)+9*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b
```

```
^2*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b*sin(f*x+e)-54*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3
*sin(f*x+e)-18*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*b^4+27*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*b^3*sin(f*x+e))/(a+b)^2/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/c
os(f*x+e)^2)^(5/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}{\left(e + fx \right)}}{\left(a + b\sec^{2}{\left(e + fx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.294 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{(3a-7b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{\left(3a^2-10ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}+\frac{b\left(9a^2-18ab-35b^2\right)\mathrm{ta}}{24a^3f(a+b)\left(a+b\tan^2(e\right.}$$

```
[Out] 1/8*(3*a^2-10*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(9/2)/f+1/24*b*(9*a^3-15*a^2*b-145*a*b^2-105*b^3)*tan(f*x+e)/a^4/(a+
b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(3*a-7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f
/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+
e)^2)^(3/2)+1/24*b*(9*a^2-18*a*b-35*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(
f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.34, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{\left(3a^2-10ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}+\frac{b\left(-15a^2b+9a^3-145ab^2-105b^3\right)\tan(e+fx)}{24a^4f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{b\left(9a^2-1\right.}{24a^3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) + ((3*a - 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e +
 f*x])/(24*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^3 - 15
*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(24*a^4*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-6bx^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx\right)}{4af}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\,(9a}{24a^3(a}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\,(9a}{24a^3(a}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\,(9a}{24a^3(a}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\,(9a}{24a^3(a}$$

$$= \frac{\left(3a^2-10ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} + \frac{(3a-7b)\cos(e+fx)\sin(e+}{8a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 20.48, size = 1777, normalized size = 6.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2,
 -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f
*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1
/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^
```

```
9)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
 -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*Appe
llF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (3*Sqrt[2]*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/((a + b - a*Sin
[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[
e + f*x]^8*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (8*f*
AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(
3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -4, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]
*(2*f*(5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/
2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(3*(a + b)) - (8*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2
*(5*a*((21*a*f*AppellF1[5/2, -4, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (24*f*AppellF1[5/2, -3
, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/5) - 8*(a + b)*((3*a*f*AppellF1[5/2, -3, 7/2, 7/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (18*f*Appe
llF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
 f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*
(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

fricas [B] time = 9.32, size = 1187, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6
 - 4*a^5*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^
5*b - 4*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), -1/96*(3*(3*a^4*b^2 -
4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6 - 4*a^5*b + 18*a^4*b^2
```

```
+ 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^5*b - 4*a^4*b^2 + 18*a^3
*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.94, size = 5600, normalized size = 21.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^4}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.295 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=332

$$\frac{(5a-9b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}+\frac{\left(5a^2-10ab+21b^2\right)\sin(e\cdots}{16a^3f\left(a+b\tan^2(e+\cdots\right.}$$

```
[Out] 5/16*(a-3*b)*(a^2+7*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/a^(11/2)/f+1/48*b*(15*a^4-20*a^3*b+38*a^2*b^2+420*a*b^3+315*b^4)*tan(f*
x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/16*(5*a^2-10*a*b+21*b^2)*co
s(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/24*(5*a-9*b)*cos(f*x
+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^5*sin(f*x+
e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/48*b*(15*a^3-25*a^2*b+49*a*b^2+105*b^3)
*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.43, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}+\frac{b\left(38a^2b^2-20a^3b+15a^4+420ab^3+315b^4\right)\tan(e+fx)}{48a^5f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{b\left(-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a - 3*b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(16*a^(11/2)*f) + ((5*a^2 - 10*a*b + 21*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*a - 9*b)*Cos[
e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos
[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15
*a^3 - 25*a^2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15*a^4 - 20*a^3*b + 38*a^2*b^2 + 420*a*b
^3 + 315*b^4)*Tan[e + f*x])/(48*a^5*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-5a+b-8bx^2}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx\right.}{6af}$$

$$= \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}}{}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} + \frac{\left(5a^2-10ab+21b^2\right)\cos(e+f}{16a^3f\left(a+b+b\tan^2(e\right.}$$

Mathematica [C] time = 27.57, size = 1776, normalized size = 5.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^16*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2,
 -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^11*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e +
f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin
```

```
[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[
1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
^11)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -
5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/
2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*Ap
pellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2)) - (15*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b -
a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*Cos[e + f*x]^10*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^
(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -5,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^10*Sin[
e + f*x]*(10*f*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*Appe
llF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
 f*x]*Sin[e + f*x])/(3*(a + b)) - (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + 5*Sin[
e + f*x]^2*(a*((21*a*f*AppellF1[5/2, -5, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - 6*f*AppellF1[5
/2, -4, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*
Sin[e + f*x]) - 2*(a + b)*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*A
ppellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*
(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

fricas [A] time = 26.44, size = 1337, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7
+ (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*x
+ e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a*b^
6)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b
^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a
*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e)) + 8*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^
7 + a^6*b - 13*a^5*b^2 - 9*a^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 +
 32*a^4*b^3 + 21*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^
```

```
4*b^3 + 287*a^3*b^4 + 210*a^2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^
3 + 38*a^3*b^4 + 420*a^2*b^5 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*
x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*
a^7*b^3 + a^6*b^4)*f), -1/192*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b
^5 - 35*a*b^6 - 21*b^7 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4
 - 21*a^2*b^5)*cos(f*x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4
 - 35*a^2*b^5 - 21*a*b^6)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*(a^
7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^7 + a^6*b - 13*a^5*b^2 - 9*a
^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 + 32*a^4*b^3 + 21*a^3*b^4)*co
s(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^4*b^3 + 287*a^3*b^4 + 210*a^
2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^3 + 38*a^3*b^4 + 420*a^2*b^5
 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*
b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 3.75, size = 6934, normalized size = 20.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.296 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=179

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tan(c+dx)}{15a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^{3/2}}-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{15a^3d(a+b)^3\sqrt{a+b\tan^2(c+dx)+b}}-$$

```
[Out] arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*(33*
a^2+40*a*b+15*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)^(1/2)-1/5*
b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*tan(d*x+
c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{15a^3d(a+b)^3\sqrt{a+b\tan^2(c+dx)+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tan(c+dx)}{15a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]]/(a^(7/2)*d) -
 (b*Tan[c + d*x])/(5*a*(a + b)*d*(a + b + b*Tan[c + d*x]^2)^(5/2)) - (b*(9*
a + 5*b)*Tan[c + d*x])/(15*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^(3/2)
) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(15*a^3*(a + b)^3*d*Sqrt[a
+ b + b*Tan[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{5a+b-4bx^2}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(c+\right.}{5a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/}}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/}}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/}}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a}{15a^2(a+b)^2c}$$

Mathematica [C] time = 18.54, size = 1777, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)]*Cos[c + d*x]^6*Sin[c + d*x])/(8*Sqrt[2]*d*(a + b*Sec[c + d*x]^2)^(7
/2)*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2,
5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
-2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)*
((21*a*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^
2)/(a + b)]*Cos[c + d*x]^7*Sin[c + d*x]^2)/(8*Sqrt[2]*(a + b - a*Sin[c + d*
x]^2)^(9/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*S
in[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x
]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^7
)/(8*Sqrt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3,
7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c +
 d*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)]*Cos[c + d*x]^5*Sin[c + d*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2,
(a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]
^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin
[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*Cos
[c + d*x]^6*Sin[c + d*x]*((7*a*d*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2
, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(3*(a + b)) - 2*d*
AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos
[c + d*x]*Sin[c + d*x]))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^6*Sin[c + d*x]*(2
*d*(7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x
]^2)/(a + b)])*Cos[c + d*x]*Sin[c + d*x] + 3*(a + b)*((7*a*d*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[
c + d*x])/(3*(a + b)) - 2*d*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*
Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]) + Sin[c + d*x]^2*(7*a*(
(27*a*d*AppellF1[5/2, -3, 11/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a
+ b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (18*d*AppellF1[5/2, -2, 9/2,
 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]
)/5) - 6*(a + b)*((21*a*d*AppellF1[5/2, -2, 9/2, 7/2, Sin[c + d*x]^2, (a*Si
n[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (12*d*Appel
lF1[5/2, -1, 7/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c +
d*x]*Sin[c + d*x])/5))))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)^2)))
```

fricas [B] time = 3.72, size = 1241, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 + a^3*b^3
 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*
```

```
cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*x + c)^2
)*sqrt(-a)*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 +
32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^
2 + 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d
*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))
 + 8*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*cos(d*x + c)^5 + (75*a^4*b^2 + 9
4*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (33*a^3*b^3 + 40*a^2*b^4 + 15*a*b^
5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/
((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8
*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a
^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4
*b^6)*d), -1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 +
 a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*
x + c)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x
+ c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 +
 b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(d*x + c)^2)*sin(d*x + c))) + 4*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*
cos(d*x + c)^5 + (75*a^4*b^2 + 94*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (3
3*a^3*b^3 + 40*a^2*b^4 + 15*a*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b
)/cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*c
os(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^
4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b
^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)^{2}+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(-7/2), x)
```

maple [C] time = 2.64, size = 6116, normalized size = 34.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)^2)^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\cos\left(c+d\,x\right)}^2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x)^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(c + dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**(-7/2), x)
```

3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

```
[Out] arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4128, 377, 203}

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sec[x]^2],x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{2+x^2}}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\ &= \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.03, size = 37, normalized size = 2.64

$$\frac{\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)\sqrt{\cos(2x)+3}\,\sec(x)}{\sqrt{2}\sqrt{\sec^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sec[x]^2],x]
```

```
[Out] (ArcSin[Sin[x]/Sqrt[2]]*Sqrt[3 + Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[1 + Sec[x]
^2])
```

fricas [B] time = 0.57, size = 53, normalized size = 3.79

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sec(x)^2 + 1), x)
```

maple [C] time = 1.10, size = 142, normalized size = 10.14

$$\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sin^2(x)\right)\sqrt{\frac{i\cos(x)+1-i+\cos(x)}{\cos(x)+1}}\sqrt{-\frac{i\cos(x)-\cos(x)-1-i}{\cos(x)+1}}\left(2\,\mathrm{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(-1+\cos(x))}{\sin(x)},i,i\right)(-1)^{\frac{3}{4}}+\sqrt{2}\,\mathrm{Ellip}\right.}{\sqrt{\frac{1+\cos^2(x)}{\cos(x)^2}}\cos(x)\left(-1+\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sec(x)^2)^(1/2),x)
```

```
[Out] (-1/2+1/2*I)*sin(x)^2*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-
cos(x)-1-I)/(cos(x)+1))^(1/2)*(2*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I
,I)*(-1)^(3/4)+EllipticF((1/2+1/2*I)*2^(1/2)*(-1+cos(x))/sin(x),I)*2^(1/2)-
2*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I)*(-1)^(1/4))/((1+cos(x)^2)/c
os(x)^2)^(1/2)/cos(x)/(-1+cos(x))
```

maxima [B] time = 0.58, size = 388, normalized size = 27.71

$$-\frac{1}{2}\arctan\left(2\left(2\left(6\cos(2x)+1\right)\cos(4x)+\cos(4x)^2+36\cos(2x)^2+\sin(4x)^2+12\sin(4x)\sin(2x)+36\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*
sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(
```

```
4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(
4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 1/2*arctan2(2*(2*(6*c
os(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*
x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*
x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6*cos(2*x)
 + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)*sin(
2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x) + 6*
sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{\frac{1}{\cos(x)^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int(1/(1/cos(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sec(x)**2 + 1), x)
```

3.298 $\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$

Optimal. Leaf size=111

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m}{2};\frac{1}{2},-p;\frac{m+2}{2};\sec^2(e\right.}{fm}$$

```
[Out] AppellF1(1/2*m,1/2,-p,1+1/2*m,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*cos(f*x+e)*(d
*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p*(-tan(f*x+e)^2)^(1/2)/f/m/((1+b*sec(f*x
+e)^2/a)^p)/sin(f*x+e)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p, x]
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

Mathematica [B] time = 18.90, size = 2195, normalized size = 19.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(d*Sec[e + f*x])^m*(Sec[e
+ f*x]^2)^(-1 + m/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a +
b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a +
b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(m/2 + p))/(3*(a
+ b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-1
 + m/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 1 - m/2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF
1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1 + m/2 + p)*Appel
lF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
```

```
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*
x]^2)/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 -
 m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m
/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b
)) - (2*(1 - m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1
[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF
1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a
 + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*x]
*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3
*(a + b)*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*(1 -
 m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(
1 - p)*AppellF1[5/2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(1 - m/2)*Appe
llF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*(-2 + m)*((6*b*p*AppellF1[5/2
, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(2 - m/2)*AppellF1[5/2, 3 - m/2,
-p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*
Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

maple [F] time = 4.43, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{m}\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{p} \left(d \sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{m} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.299 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p+2,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e\right.\right.}{f}$$

```
[Out] AppellF1(1/2,2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p
*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

Rubi [A] time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e +
 f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^p}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\,\text{co}\right.}{}$$

Mathematica [B] time = 17.15, size = 1989, normalized size = 19.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]^3*(Sec[e + f*x]^
2)^(1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[
1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*
p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*
x)])^p*(Sec[e + f*x]^2)^(3/2 + p))/(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*App
ellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*T
an[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(
Sec[e + f*x]^2)^(1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF
1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*
b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(1/2 + p)*AppellF1[1/2, -
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*
Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*A
ppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2,
```

```
-1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a
 + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2
, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
 a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*(2*(2*b*p*Ap
pellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2
, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan
[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, -1/2, 2 - p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b)) + (3*AppellF1[5/2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*((6*b*p*AppellF1[5/
2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*AppellF1[5/2, 3/2, -p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/
(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

maple [F] time = 1.56, size = 0, normalized size = 0.00

$$\int\left(\sec^{3}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.300 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p+1,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e\right.\right.}{f}$$

```
[Out] AppellF1(1/2,1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p
*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

Rubi [A] time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e +
 f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos\right.}{}$$

Mathematica [B] time = 16.58, size = 1995, normalized size = 19.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]*(Sec[e + f*x]^2)^
(-1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[1/
2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*A
ppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(1/2 + p))/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3
/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e +
f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e +
f*x]^2)^(-1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2,
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*Appe
llF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
- (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1/2 + p)*AppellF1[1/2, 1/2, -p,
3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e
+ f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[
1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p
*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)]
)^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1/2, 1 -
```

```
 p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, 1/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*(2*(2*b*p*AppellF1[3/2,
1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*
AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1/2, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e
+ f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2
*b*p*((-6*b*(1 - p)*AppellF1[5/2, 1/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*Appel
lF1[5/2, 3/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/5) - (a + b)*((6*b*p*AppellF1[5/2, 3/2, 1 - p,
7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e +
 f*x])/(5*(a + b)) - (9*AppellF1[5/2, 5/2, -p, 7/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

maple [F] time = 1.45, size = 0, normalized size = 0.00

$$\int\sec\left(fx+e\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x), x)
```

3.301 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx\right.\right.}{f}$$

```
[Out] AppellF1(1/2,p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*s
in(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b))
^p)
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos
[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e + f*x
]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(a+\frac{b}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(e\right.}{}$$

Mathematica [B] time = 16.50, size = 1983, normalized size = 19.63

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 3/2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a
+ b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-1/2 + p))/(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2
, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-3/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 3
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-3/2 + p)*AppellF1[1/2, 3/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
```

```
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))/(-3*(a + b)*A
ppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2, 3/2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[
2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*(2*(-2*b*p*AppellF
1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3
*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + Tan[e + f*x]
^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 3/2, 2 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (9
*AppellF1[5/2, 5/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 3*(a + b)*((6*b*p*AppellF1[5/2, 5/2,
 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2
*Tan[e + f*x])/(5*(a + b)) - 3*AppellF1[5/2, 7/2, -p, 7/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))))/(-3*(a + b)
*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

maple [F] time = 1.91, size = 0, normalized size = 0.00

$$\int\cos\left(fx+e\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.302 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p-1,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e\right.\right.}{f}$$

```
[Out] AppellF1(1/2,-1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+
b))^p)
```

Rubi [A] time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e
+ f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p
)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+\frac{b}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};-1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\,c\right.}{}$$

Mathematica [B] time = 17.04, size = 1987, normalized size = 19.29

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 5/2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a
+ b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-3/2 + p))/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2
, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-5/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 5
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-5/2 + p)*AppellF1[1/2, 5/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 5/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
```

```
^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(-3*(a
+ b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2
, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*(2*(-2*b*p*A
ppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3
/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) +
Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 5/2, 2 - p, 7/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(
a + b)) - 3*AppellF1[5/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + 5*(a + b)*((6*b*p*AppellF1[5
/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, -p, 7/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)))
)/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

maple [F] time = 5.57, size = 0, normalized size = 0.00

$$\int\left(\cos^{3}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.303 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p-2,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e\right.\right.}{f}$$

[Out] AppellF1(1/2,-2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b))^p)

Rubi [A] time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right.}{f}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};-2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\right.}{}$$

Mathematica [B] time = 17.41, size = 1997, normalized size = 19.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Cos[e + f*x]^4*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-7/2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF
1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*
b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2,
 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a +
b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))])*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
 p)*(Sec[e + f*x]^2)^(-7/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-7/2 + p)*Appell
F1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a +
2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]^2)/(-3
*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]*((2*b*p*A
ppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
```

```
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a +
b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*
x]*(2*(-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2
*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[
e + f*x])/3) + Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 7/2, 2 -
 p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 7*(a + b
)*((6*b*p*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (27*AppellF1[5/2,
11/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/5))))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2,
 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2)^2))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

maple [F] time = 6.06, size = 0, normalized size = 0.00

$$\int\left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.304 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=216

$$\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\,\mathrm{t}}{}\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(3*a-2*b*(2+p))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15
)+sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1+p)/b/f/(5+2*p)+(3*a^2-4*a
*b*(1+p)+4*b^2*(p^2+3*p+2))*hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b)
)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*tan(f*x+e)^
2/(a+b))^p)
```

Rubi [A] time = 0.23, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4146, 416, 388, 246, 245}

$$\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\,\mathrm{t}}{}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b^
2*f*(3 + 2*p)*(5 + 2*p))) + (Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e +
 f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*
p + p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Ta
n[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b
*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+b+bx^2\right)^p\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e}{}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e}{}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e}{}$$

Mathematica [A] time = 2.11, size = 149, normalized size = 0.69

$$\frac{\tan(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(10\tan^2(e+fx)\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+15\,{}_2F_1\left(\frac{1}{2},-\right.\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(15*Hypergeometric2F1[1/2, -p, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))] + 10*Hypergeometric2F1[3/2, -p, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/2, -p, 7/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4))/(15*f*(1 + (b*Tan[e + f*x]^
2)/(a + b))^p)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

maple [F] time = 2.48, size = 0, normalized size = 0.00

$$\int \left(\sec^{6}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}}{\cos\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.305 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=129

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)}{bf(2p+3)}$$

```
[Out] tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1+p)/b/f/(3+2*p)-(a-2*b*(1+p))*hypergeom([
1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b/f
/(3+2*p)/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 388, 246, 245}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - 2
*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+b+bx^2\right)^p\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,\text{Subst}\left(\int\left(a\right.\right.}{bf(}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{\left((a-2b(1+p))\left(a+b+b\right.\right.}{}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{}$$

Mathematica [A] time = 2.26, size = 126, normalized size = 0.98

$$\frac{\tan(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left((2b(p+1)-a)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+\left(a+b\tan^2(e+\right.\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*((-a + 2*b*(1 + p))*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b + b*Tan[e + f*x]^2)
*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

maple [F] time = 1.74, size = 0, normalized size = 0.00

$$\int\left(\sec^4\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\sec \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}}{\cos\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.306 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=72

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+
e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 246, 245}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int\left(a+b+bx^2\right)^p\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a+b}\right)^p\,dx\right.}{f}\\ &= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\right.}{f}\end{aligned}$$

Mathematica [A] time = 1.01, size = 71, normalized size = 0.99

$$\frac{\tan(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec[
e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

maple [F] time = 1.31, size = 0, normalized size = 0.00

$$\int\left(\sec^2\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.307 $\int \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\right.}{f}$$

Mathematica [B] time = 6.26, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
```

```
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

3.308 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^2}\,dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan\right.}{f}$$

Mathematica [B] time = 15.77, size = 1914, normalized size = 23.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-
2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(
a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*
Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/
2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (6*(a + b)*(-2 + p)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)
^(-2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan
[e + f*x]*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[
3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (
3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e +
 f*x]*(4*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p
```

```
*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b)) - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 2*(a + b)*((6*b*p*App
ellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

maple [F] time = 2.59, size = 0, normalized size = 0.00

$$\int\left(\cos^{2}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(e+f\,x\right)^{2}\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.309 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] AppellF1(1/2,3,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\frac{(a+b+bx^2)^p}{(1+x^2)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a+b}\right)^p}{(1+x^2)^3}\,dx,x\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan\right.}{f}$$

Mathematica [B] time = 16.28, size = 1912, normalized size = 23.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^3*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-3 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-2 + p))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-3 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[
3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-3 + p)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-3 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2,
4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1
[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f
*x]^2*Tan[e + f*x]))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*
(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*Tan[e + f
*x]*(4*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*A
```

```
ppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1[3/2, 4, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) +
 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 3, 2 - p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a
+ b)) - (18*AppellF1[5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 3*(a + b)*((6*b*p*AppellF1[
5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (24*AppellF1[5/2, 5, -p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(
3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

maple [F] time = 3.02, size = 0, normalized size = 0.00

$$\int\left(\cos^4\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(e+f\,x\right)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.310 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] AppellF1(1/2,4,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\mathrm{Subst}\left(\int\frac{\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^4}dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\mathrm{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^4}dx,x\right.}{f}$$
$$=\frac{F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan\right.}{f}$$

Mathematica [B] time = 17.12, size = 1914, normalized size = 23.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^5*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-4 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4
*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-3 + p))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-4 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[
3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-4 + p)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-4 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2,
5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF
1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5,
-p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) -
 (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*Tan[e
 + f*x]*(4*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b
```

```
*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF1[3/2, 5, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 4, 2 - p, 7/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/(5*(a + b)) - (24*AppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 4*(a + b)*((6*b*p*A
ppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - 6*AppellF1[5/2, 6, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])))
)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

maple [F] time = 3.75, size = 0, normalized size = 0.00

$$\int\left(\cos^{6}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(e+fx\right)^{6}\left(a+\frac{b}{\cos\left(e+fx\right)^{2}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.311 $\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{(a-2b)\sec^4(e+fx)}{4f}-\frac{(2a-b)\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^6(e+fx)}{6f}$$

```
[Out] -a*ln(cos(f*x+e))/f-1/2*(2*a-b)*sec(f*x+e)^2/f+1/4*(a-2*b)*sec(f*x+e)^4/f+1
/6*b*sec(f*x+e)^6/f
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 76}

$$\frac{(a-2b)\sec^4(e+fx)}{4f}-\frac{(2a-b)\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - ((2*a - b)*Sec[e + f*x]^2)/(2*f) + ((a - 2*b)*
Sec[e + f*x]^4)/(4*f) + (b*Sec[e + f*x]^6)/(6*f)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)}{x^7}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)}{x^4}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a-2b}{x^3}+\frac{-2a+b}{x^2}+\frac{a}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\log(\cos(e+fx))}{f} - \frac{(2a-b)\sec^2(e+fx)}{2f} + \frac{(a-2b)\sec^4(e+fx)}{4f} +$$

Mathematica [A] time = 0.15, size = 55, normalized size = 0.76

$$\frac{b\tan^6(e+fx)}{6f} - \frac{a\left(-\tan^4(e+fx)+2\tan^2(e+fx)+4\log(\cos(e+fx))\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

```
[Out] (b*Tan[e + f*x]^6)/(6*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan
[e + f*x]^4))/(4*f)
```

fricas [A] time = 0.49, size = 69, normalized size = 0.96

$$-\frac{12\,a\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\,(2\,a-b)\cos\left(fx+e\right)^4-3\,(a-2\,b)\cos\left(fx+e\right)^2-2\,b}{12\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(2*a - b)*cos(f*x + e)^4
- 3*(a - 2*b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*
x+exp(1)))*(1+cos(f*x+exp(1)))+2))-a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(11*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))^3*a-9
0*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x
+exp(1))))^2*a+276*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+ex
p(1)))*(1+cos(f*x+exp(1))))*a+128*b-280*a)*1/24/((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2)^3)
```

maple [A] time = 0.62, size = 65, normalized size = 0.90

$$\frac{\left(\tan^4\left(fx+e\right)\right)a}{4f}-\frac{a\left(\tan^2\left(fx+e\right)\right)}{2f}-\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin^6\left(fx+e\right)\right)}{6f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x)
```

```
[Out] 1/4/f*tan(f*x+e)^4*a-1/2/f*a*tan(f*x+e)^2-a*ln(cos(f*x+e))/f+1/6/f*b*sin(f*
x+e)^6/cos(f*x+e)^6
```

maxima [A] time = 0.33, size = 95, normalized size = 1.32

$$-\frac{6\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\,(2\,a-b)\sin\left(fx+e\right)^4-3\,(7\,a-2\,b)\sin\left(fx+e\right)^2+9\,a-2\,b}{\sin\left(fx+e\right)^6-3\,\sin\left(fx+e\right)^4+3\,\sin\left(fx+e\right)^2-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a*log(sin(f*x + e)^2 - 1) - (6*(2*a - b)*sin(f*x + e)^4 - 3*(7*a -
 2*b)*sin(f*x + e)^2 + 9*a - 2*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*si
n(f*x + e)^2 - 1))/f
```

mupad [B] time = 4.70, size = 52, normalized size = 0.72

$$\frac{\frac{a\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2}-\frac{a\,\tan\left(e+f\,x\right)^2}{2}+\frac{a\,\tan\left(e+f\,x\right)^4}{4}+\frac{b\,\tan\left(e+f\,x\right)^6}{6}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*log(tan(e + f*x)^2 + 1))/2 - (a*tan(e + f*x)^2)/2 + (a*tan(e + f*x)^4)/
4 + (b*tan(e + f*x)^6)/6)/f
```

sympy [A] time = 5.17, size = 116, normalized size = 1.61

$$\begin{cases}\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^4\left(e+fx\right)}{4f}-\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^4\left(e+fx\right)\sec^2\left(e+fx\right)}{6f}-\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{6f}+\frac{b\sec^2\left(e+fx\right)}{6f} & \text{for} \\ x\left(a+b\sec^2\left(e\right)\right)\tan^5\left(e\right) & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**2/(6*f) - b*tan(e +
f*x)**2*sec(e + f*x)**2/(6*f) + b*sec(e + f*x)**2/(6*f), Ne(f, 0)), (x*(a +
 b*sec(e)**2)*tan(e)**5, True))
```

3.312 $\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx$

Optimal. Leaf size=49

$$\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^4(e+fx)}{4f}$$

```
[Out] a*ln(cos(f*x+e))/f+1/2*(a-b)*sec(f*x+e)^2/f+1/4*b*sec(f*x+e)^4/f
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 76}

$$\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + ((a - b)*Sec[e + f*x]^2)/(2*f) + (b*Sec[e + f*x]^
4)/(4*f)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a\log(\cos(e+fx))}{f}+\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{b\sec^4(e+fx)}{4f}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.88

$$\frac{a\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}+\frac{b\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
[Out] (b*Tan[e + f*x]^4)/(4*f) + (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

fricas [A] time = 0.54, size = 50, normalized size = 1.02

$$\frac{4\,a\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)+2\,(a-b)\cos\left(fx+e\right)^2+b}{4\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="fricas")
[Out] 1/4*(4*a*cos(f*x + e)^4*log(-cos(f*x + e)) + 2*(a - b)*cos(f*x + e)^2 + b)/
(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f
*x+exp(1)))*(1+cos(f*x+exp(1)))+2))+a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(-3*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))^2*a+
20*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*
x+exp(1))))*a+16*b-28*a)*1/8/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-
cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2)^2)
```

maple [A] time = 0.53, size = 50, normalized size = 1.02

$$\frac{a\left(\tan^2\left(fx+e\right)\right)}{2f}+\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin^4\left(fx+e\right)\right)}{4f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*a*tan(f*x+e)^2+a*ln(cos(f*x+e))/f+1/4/f*b*sin(f*x+e)^4/cos(f*x+e)^4
```

maxima [A] time = 0.33, size = 64, normalized size = 1.31

$$\frac{2\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-b)\sin\left(fx+e\right)^2-2\,a+b}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2 - 1) - (2*(a - b)*sin(f*x + e)^2 - 2*a + b)/(si
n(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.78, size = 46, normalized size = 0.94

$$\frac{a\tan\left(e+f\,x\right)^2}{2\,f}-\frac{a\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}+\frac{b\tan\left(e+f\,x\right)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*tan(e + f*x)^2)/(2*f) - (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f
*x)^4)/(4*f)
```

sympy [A] time = 1.67, size = 80, normalized size = 1.63

$$\begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{4f}-\frac{b\sec^2\left(e+fx\right)}{4f} & \text{for } f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**2/(4*f) - b*sec(e + f*x)**2/(4*f), Ne(f, 0)),
 (x*(a + b*sec(e)**2)*tan(e)**3, True))
```

3.313 $\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx$

Optimal. Leaf size=30

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

```
[Out] -a*ln(cos(f*x+e))/f+1/2*b*sec(f*x+e)^2/f
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 14}

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^3}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right)dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

fricas [A] time = 0.49, size = 37, normalized size = 1.23

$$-\frac{2\,a\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-b}{2\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(f*x + e)^2*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*
x+exp(1)))*(1+cos(f*x+exp(1)))+2))-a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a-2*a+4*
b)*1/4/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+co
s(f*x+exp(1)))-2))
```

maple [A] time = 0.23, size = 28, normalized size = 0.93

$$\frac{b\left(\sec^2\left(fx+e\right)\right)}{2f}+\frac{a\ln\left(\sec\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e),x)
```

```
[Out] 1/2*b*sec(f*x+e)^2/f+1/f*a*ln(sec(f*x+e))
```

maxima [A] time = 0.32, size = 33, normalized size = 1.10

$$-\frac{a\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin\left(fx+e\right)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

mupad [B] time = 4.93, size = 32, normalized size = 1.07

$$\frac{a\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}+\frac{b\tan\left(e+f\,x\right)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f*x)^2)/(2*f)
```

sympy [A] time = 0.46, size = 42, normalized size = 1.40

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^2\left(e+fx\right)}{2f} & \text{for } f\neq 0\\ x\left(a+b\sec^2\left(e\right)\right)\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**2/(2*f), Ne(f
, 0)), (x*(a + b*sec(e)**2)*tan(e), True))
```

3.314 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a+b)\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

```
[Out] -b*ln(cos(f*x+e))/f+(a+b)*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + ((a + b)*Log[Sin[e + f*x]])/f
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.57

$$\frac{a(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{b(\log(\cos(e+fx))-\log(\sin(e+fx)))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
[Out] -((b*(Log[Cos[e + f*x]] - Log[Sin[e + f*x]]))/f) + (a*(Log[Cos[e + f*x]] +
Log[Tan[e + f*x]]))/f
```

fricas [A] time = 0.51, size = 35, normalized size = 1.25

$$-\frac{b\log\left(\cos\left(fx+e\right)^{2}\right)-(a+b)\log\left(-\frac{1}{4}\cos\left(fx+e\right)^{2}+\frac{1}{4}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*(b*log(cos(f*x + e)^2) - (a + b)*log(-1/4*cos(f*x + e)^2 + 1/4))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f
*x+exp(1)))*(1+cos(f*x+exp(1)))-2))-a/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2)))
```

maple [A] time = 0.60, size = 26, normalized size = 0.93

$$\frac{b\ln\left(\tan\left(fx+e\right)\right)}{f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x)
[Out] 1/f*b*ln(tan(f*x+e))+a*ln(sin(f*x+e))/f
```

maxima [A] time = 0.34, size = 33, normalized size = 1.18

$$-\frac{b\log\left(\sin\left(fx+e\right)^{2}-1\right)-(a+b)\log\left(\sin\left(fx+e\right)^{2}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - (a + b)*log(sin(f*x + e)^2))/f
```

mupad [B] time = 4.87, size = 32, normalized size = 1.14

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a+b\right)}{f}-\frac{a\,\ln\left({\tan\left(e+f\,x\right)}^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2),x)
[Out] (log(tan(e + f*x))*(a + b))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\cot\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

3.315 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=32

$$-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] -1/2*(a+b)*csc(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 444, 43}

$$-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + b)*Csc[e + f*x]^2)/(2*f) - (a*Log[Sin[e + f*x]])/f
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x\left(b+ax^2\right)}{\left(1-x^2\right)^2}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 52, normalized size = 1.62

$$-\frac{a\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f}-\frac{b\csc^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*(b*Csc[e + f*x]^2)/f - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Lo
g[Tan[e + f*x]]))/(2*f)
```

fricas [A] time = 0.45, size = 50, normalized size = 1.56

$$-\frac{2\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)-a-b}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(f*x + e)^2 - a)*log(1/2*sin(f*x + e)) - a - b)/(f*cos(f*x +
e)^2 - f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-(1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))*b-(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)/16+(4*
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-b-a)*1/16/(1-cos(f*x+exp(1)))*(1+
cos(f*x+exp(1)))+a/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-a/4
*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.74, size = 43, normalized size = 1.34

$$-\frac{a\left(\cot^2\left(fx+e\right)\right)}{2f}-\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{b}{2f\sin\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2*a*cot(f*x+e)^2/f-a*ln(sin(f*x+e))/f-1/2/f*b/sin(f*x+e)^2
```

maxima [A] time = 0.32, size = 29, normalized size = 0.91

$$-\frac{a\log\left(\sin\left(fx+e\right)^2\right)+\frac{a+b}{\sin\left(fx+e\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2) + (a + b)/sin(f*x + e)^2)/f
```

mupad [B] time = 6.14, size = 51, normalized size = 1.59

$$\frac{a\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}-\frac{a\,\ln\left(\tan\left(e+f\,x\right)\right)}{f}-\frac{\cot\left(e+f\,x\right)^2\,\left(\frac{a}{2}+\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a*log(tan(e + f*x)))/f - (cot(e + f*x)
^2*(a/2 + b/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.316 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{(2a+b)\csc^2(e+fx)}{2f}+\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] 1/2*(2*a+b)*csc(f*x+e)^2/f-1/4*(a+b)*csc(f*x+e)^4/f+a*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 77}

$$-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{(2a+b)\csc^2(e+fx)}{2f}+\frac{a\log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[e + f*x]^2)/(2*f) - ((a + b)*Csc[e + f*x]^4)/(4*f) + (a*Log[
Sin[e + f*x]])/f
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3}dx,x,\cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}dx,x,\cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)dx,x,\cos^2(e+fx)\right)}{2f}$$
$$= \frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}$$

Mathematica [A] time = 0.17, size = 64, normalized size = 1.25

$$\frac{a\left(-\cot^4(e+fx)+2\cot^2(e+fx)+4\log(\tan(e+fx))+4\log(\cos(e+fx))\right)}{4f}-\frac{b\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/4*(b*Cot[e + f*x]^4)/f + (a*(2*Cot[e + f*x]^2 - Cot[e + f*x]^4 + 4*Log[C
os[e + f*x]] + 4*Log[Tan[e + f*x]]))/(4*f)
```

fricas [A] time = 0.46, size = 83, normalized size = 1.63

$$-\frac{2(2a+b)\cos\left(fx+e\right)^2-4\left(a\cos\left(fx+e\right)^4-2a\cos\left(fx+e\right)^2+a\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)-3a-b}{4\left(f\cos\left(fx+e\right)^4-2f\cos\left(fx+e\right)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*a + b)*cos(f*x + e)^2 - 4*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2
 + a)*log(1/2*sin(f*x + e)) - 3*a - b)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)
^2 + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*b-32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
))^2*a+128*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+384*(1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1)))*a)/4096+(-48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^2*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+12*(1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1)))*a-b-a)*1/128/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2-
a/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+a/4*ln(abs(1-cos(f*x
+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.67, size = 64, normalized size = 1.25

$$-\frac{a\left(\cot^{4}\left(fx+e\right)\right)}{4f}+\frac{a\left(\cot^{2}\left(fx+e\right)\right)}{2f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{b\left(\cos^{4}\left(fx+e\right)\right)}{4f\sin\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/4*a*cot(f*x+e)^4/f+1/2*a*cot(f*x+e)^2/f+a*ln(sin(f*x+e))/f-1/4/f*b/sin(f
*x+e)^4*cos(f*x+e)^4
```

maxima [A] time = 0.32, size = 49, normalized size = 0.96

$$\frac{2\,a\log\left(\sin\left(fx+e\right)^{2}\right)+\frac{2\,(2\,a+b)\sin\left(fx+e\right)^{2}-a-b}{\sin\left(fx+e\right)^{4}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2) + (2*(2*a + b)*sin(f*x + e)^2 - a - b)/sin(f*x
 + e)^4)/f
```

mupad [B] time = 6.10, size = 61, normalized size = 1.20

$$\frac{a\,\ln\left(\tan\left(e+f\,x\right)\right)}{f}-\frac{a\,\ln\left(\tan\left(e+f\,x\right)^{2}+1\right)}{2\,f}-\frac{-\frac{a\tan\left(e+f\,x\right)^{2}}{2}+\frac{a}{4}+\frac{b}{4}}{f\tan\left(e+f\,x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a/4 + b/4 -
(a*tan(e + f*x)^2)/2)/(f*tan(e + f*x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\cot^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.317 $\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -a*x+a*tan(f*x+e)/f-1/3*a*tan(f*x+e)^3/f+1/5*a*tan(f*x+e)^5/f+1/7*b*tan(f*x
+e)^7/f
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (a*Tan[e + f*x]^5)
/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a-ax^2+ax^4+bx^6-\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}-\\ &= -ax+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.14

$$-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*
f) + (a*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

fricas [A] time = 0.50, size = 89, normalized size = 1.39

$$-\frac{105\,afx\cos\left(fx+e\right)^7-\left((161\,a-15\,b)\cos\left(fx+e\right)^6-(77\,a-45\,b)\cos\left(fx+e\right)^4+3\,(7\,a-15\,b)\cos\left(fx+e\right)^2\right.}{105\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/105*(105*a*f*x*cos(f*x + e)^7 - ((161*a - 15*b)*cos(f*x + e)^6 - (77*a -
 45*b)*cos(f*x + e)^4 + 3*(7*a - 15*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e))
/(f*cos(f*x + e)^7)
```

giac [A] time = 7.15, size = 61, normalized size = 0.95

$$\frac{15\,b\tan\left(fx+e\right)^7+21\,a\tan\left(fx+e\right)^5-35\,a\tan\left(fx+e\right)^3-105\,\left(fx+e\right)a+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

maple [A] time = 0.58, size = 61, normalized size = 0.95

$$\frac{a\left(\frac{\left(\tan^5\left(fx+e\right)\right)}{5}-\frac{\left(\tan^3\left(fx+e\right)\right)}{3}+\tan\left(fx+e\right)-fx-e\right)+\frac{b\left(\sin^7\left(fx+e\right)\right)}{7\cos\left(fx+e\right)^7}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x)
```

```
[Out] 1/f*(a*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-f*x-e)+1/7*b*sin(f*x+e
)^7/cos(f*x+e)^7)
```

maxima [A] time = 0.42, size = 56, normalized size = 0.88

$$\frac{15\,b\tan\left(fx+e\right)^7+21\,a\tan\left(fx+e\right)^5-35\,a\tan\left(fx+e\right)^3-105\,\left(fx+e\right)a+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

mupad [B] time = 4.99, size = 51, normalized size = 0.80

$$\frac{\frac{b\tan\left(e+f\,x\right)^7}{7}+\frac{a\tan\left(e+f\,x\right)^5}{5}-\frac{a\tan\left(e+f\,x\right)^3}{3}+a\tan\left(e+f\,x\right)-a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*tan(e + f*x) - (a*tan(e + f*x)^3)/3 + (a*tan(e + f*x)^5)/5 + (b*tan(e +
f*x)^7)/7 - a*f*x)/f
```

sympy [A] time = 3.50, size = 66, normalized size = 1.03

$$a\left(\begin{cases}-x+\frac{\tan^5\left(e+fx\right)}{5f}-\frac{\tan^3\left(e+fx\right)}{3f}+\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^6\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x\tan^6\left(e\right)\sec^2\left(e\right) & \text{for}\ f=0\\ \frac{\tan^7\left(e+fx\right)}{7f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**6,x)
```

```
[Out] a*Piecewise((-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f
*x)/f, Ne(f, 0)), (x*tan(e)**6, True)) + b*Piecewise((x*tan(e)**6*sec(e)**2
, Eq(f, 0)), (tan(e + f*x)**7/(7*f), True))
```

3.318 $\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+ax+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] a*x-a*tan(f*x+e)/f+1/3*a*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+ax+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

```
[Out] a*x - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^5)/(5
*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(-a+ax^2+bx^4+\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.19

$$\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f)
+ (b*Tan[e + f*x]^5)/(5*f)
```

fricas [A] time = 0.46, size = 72, normalized size = 1.50

$$\frac{15\,afx\cos\left(fx+e\right)^5-\left((20\,a-3\,b)\cos\left(fx+e\right)^4-(5\,a-6\,b)\cos\left(fx+e\right)^2-3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*f*x*cos(f*x + e)^5 - ((20*a - 3*b)*cos(f*x + e)^4 - (5*a - 6*b)*
cos(f*x + e)^2 - 3*b)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [A] time = 1.67, size = 49, normalized size = 1.02

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,a\tan\left(fx+e\right)^3+15\left(fx+e\right)a-15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

maple [A] time = 0.58, size = 50, normalized size = 1.04

$$\frac{a\left(\frac{\left(\tan^3\left(fx+e\right)\right)}{3}-\tan\left(fx+e\right)+fx+e\right)+\frac{b\left(\sin^5\left(fx+e\right)\right)}{5\cos\left(fx+e\right)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x)
```

```
[Out] 1/f*(a*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e)+1/5*b*sin(f*x+e)^5/cos(f*x+e)^5)
```

maxima [A] time = 0.41, size = 45, normalized size = 0.94

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,a\tan\left(fx+e\right)^3+15\left(fx+e\right)a-15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

mupad [**B**] time = 4.68, size = 40, normalized size = 0.83

$$\frac{\frac{b\tan\left(e+f\,x\right)^5}{5}+\frac{a\tan\left(e+f\,x\right)^3}{3}-a\tan\left(e+f\,x\right)+a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*tan(e + f*x)^3)/3 - a*tan(e + f*x) + (b*tan(e + f*x)^5)/5 + a*f*x)/f
```

sympy [**A**] time = 2.21, size = 54, normalized size = 1.12

$$a\left(\begin{cases} x+\frac{\tan^3\left(e+fx\right)}{3f}-\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^4\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases} x\tan^4\left(e\right)\sec^2\left(e\right) & \text{for}\ f=0\\ \frac{\tan^5\left(e+fx\right)}{5f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**4,x)
```

```
[Out] a*Piecewise((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f, Ne(f, 0)), (x*tan(
e)**4, True)) + b*Piecewise((x*tan(e)**4*sec(e)**2, Eq(f, 0)), (tan(e + f*x
)**5/(5*f), True))
```

3.319 $\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx$

Optimal. Leaf size=32

$$\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -a*x+a*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a+bx^2-\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}-\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.28

$$-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*
f)
```

fricas [A] time = 0.49, size = 53, normalized size = 1.66

$$-\frac{3\,afx\cos\left(fx+e\right)^3-\left((3\,a-b)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*f*x*cos(f*x + e)^3 - ((3*a - b)*cos(f*x + e)^2 + b)*sin(f*x + e))
/(f*cos(f*x + e)^3)
```

giac [A] time = 0.75, size = 36, normalized size = 1.12

$$\frac{b\tan\left(fx+e\right)^3-3\left(fx+e\right)a+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

maple [A] time = 0.63, size = 41, normalized size = 1.28

$$\frac{a\left(\tan\left(fx+e\right)-fx-e\right)+\frac{b\left(\sin^3\left(fx+e\right)\right)}{3\cos\left(fx+e\right)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x)
```

```
[Out] 1/f*(a*(tan(f*x+e)-f*x-e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3)
```

maxima [A] time = 0.42, size = 33, normalized size = 1.03

$$\frac{b\tan\left(fx+e\right)^3-3\left(fx+e\right)a+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

mupad [B] time = 4.52, size = 29, normalized size = 0.91

$$\frac{\frac{b\tan\left(e+f\,x\right)^3}{3}+a\tan\left(e+f\,x\right)-a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*tan(e + f*x) + (b*tan(e + f*x)^3)/3 - a*f*x)/f
```

sympy [A] time = 1.60, size = 42, normalized size = 1.31

$$a\left(\begin{cases} -x + \frac{\tan\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\tan^{2}\left(e\right) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x\tan^{2}\left(e\right)\sec^{2}\left(e\right) & \text{for}\ f = 0 \\ \frac{\tan^{3}\left(e+fx\right)}{3f} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**2,x)
```

```
[Out] a*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True)) + b*Piece
wise((x*tan(e)**2*sec(e)**2, Eq(f, 0)), (tan(e + f*x)**3/(3*f), True))
```

3.320 $\int \left(a+b\sec^2(e+fx)\right) dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+b\sec^2(e+fx)\right) dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\,\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{f}\\ &= ax+\frac{b\tan(e+fx)}{f} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
[Out] a*x + (b*Tan[e + f*x])/f
```

fricas [B] time = 0.48, size = 31, normalized size = 2.07

$$\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

giac [A] time = 0.87, size = 16, normalized size = 1.07

$$ax + \frac{b \tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

maple [A] time = 0.87, size = 16, normalized size = 1.07

$$ax + \frac{b \tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

maxima [A] time = 0.33, size = 15, normalized size = 1.00

$$ax + \frac{b \tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

mupad [B] time = 4.51, size = 17, normalized size = 1.13

$$\frac{b \tan\left(e + f\,x\right) + a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + a*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

3.321 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=19

$$-\frac{(a+b)\cot(e+fx)}{f}-ax$$

```
[Out] -a*x-(a+b)*cot(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$-\frac{(a+b)\cot(e+fx)}{f}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - ((a + b)*Cot[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+b\left(1+x^2\right)}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^2}-\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -ax-\frac{(a+b)\cot(e+fx)}{f}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 2.26

$$-\frac{a\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
[Out] -((b*Cot[e + f*x])/f) - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[e + f*x]^2])/f
```

fricas [A] time = 0.49, size = 34, normalized size = 1.79

$$-\frac{afx\sin\left(fx+e\right)+(a+b)\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -(a*f*x*sin(f*x + e) + (a + b)*cos(f*x + e))/(f*sin(f*x + e))
```

giac [B] time = 0.22, size = 57, normalized size = 3.00

$$-\frac{2\left(fx+e\right)a-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{a+b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/2*(2*(f*x + e)*a - a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e) + (a
+ b)/tan(1/2*f*x + 1/2*e))/f
```

maple [A] time = 0.89, size = 33, normalized size = 1.74

$$\frac{a\left(-\cot\left(fx+e\right)-fx-e\right)-b\cot\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
[Out] 1/f*(a*(-cot(f*x+e)-f*x-e)-b*cot(f*x+e))
```

maxima [A] time = 0.43, size = 25, normalized size = 1.32

$$-\frac{\left(fx+e\right)a+\frac{a+b}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -((f*x + e)*a + (a + b)/tan(f*x + e))/f
```

mupad [B] time = 4.50, size = 19, normalized size = 1.00

$$-a\,x-\frac{\cot\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] - a*x - (cot(e + f*x)*(a + b))/f
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}\left(e + fx\right)\right) \cot^{2}\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.322 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=33

$$-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\cot(e+fx)}{f}+ax$$

```
[Out] a*x+a*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\cot(e+fx)}{f}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] a*x + (a*Cot[e + f*x])/f - ((a + b)*Cot[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\left(1+x^2\right)}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4}-\frac{a}{x^2}+\frac{a}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+\right.}{f}\\
&= ax+\frac{a\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 1.55

$$-\frac{a\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}-\frac{b\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/3*(b*Cot[e + f*x]^3)/f - (a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1
/2, -Tan[e + f*x]^2])/(3*f)
```

fricas [B] time = 0.49, size = 76, normalized size = 2.30

$$\frac{(4a+b)\cos\left(fx+e\right)^3-3a\cos\left(fx+e\right)+3\left(afx\cos\left(fx+e\right)^2-afx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((4*a + b)*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*(a*f*x*cos(f*x + e)^2
- a*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

giac [B] time = 0.26, size = 119, normalized size = 3.61

$$\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24\left(fx+e\right)a-15a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{15a\tan\left(\frac{1}{2}f\right.}{}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + b*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a
- 15*a*tan(1/2*f*x + 1/2*e) - 3*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x
+ 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e)^2 - a - b)/tan(1/2*f*x + 1/2*e)^3)/f
```

maple [A] time = 0.74, size = 48, normalized size = 1.45

$$\frac{a\left(-\frac{\left(\cot^3\left(fx+e\right)\right)}{3}+\cot\left(fx+e\right)+fx+e\right)-\frac{b\left(\cos^3\left(fx+e\right)\right)}{3\sin\left(fx+e\right)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-1/3*b/sin(f*x+e)^3*cos(f*x+e)^3
)
```

maxima [A] time = 0.42, size = 41, normalized size = 1.24

$$\frac{3\left(fx+e\right)a+\frac{3a\tan\left(fx+e\right)^2-a-b}{\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a + (3*a*tan(f*x + e)^2 - a - b)/tan(f*x + e)^3)/f
```

mupad [B] time = 4.67, size = 35, normalized size = 1.06

$$a\,x-\frac{-a\,\tan\left(e+f\,x\right)^2+\frac{a}{3}+\frac{b}{3}}{f\,\tan\left(e+f\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] a*x - (a/3 + b/3 - a*tan(e + f*x)^2)/(f*tan(e + f*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.323 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-ax$$

```
[Out] -a*x-a*cot(f*x+e)/f+1/3*a*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - (a*Cot[e + f*x])/f + (a*Cot[e + f*x]^3)/(3*f) - ((a + b)*Cot[e + f
*x]^5)/(5*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\left(1+x^2\right)}{x^6\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^6}-\frac{a}{x^4}+\frac{a}{x^2}-\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{a\,\text{Subst}\left(\int \frac{}{1+}\right.}{}\\
&= -ax-\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 51, normalized size = 1.00

$$-\frac{a\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f}-\frac{b\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/5*(b*Cot[e + f*x]^5)/f - (a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3
/2, -Tan[e + f*x]^2])/(5*f)
```

fricas [B] time = 0.49, size = 110, normalized size = 2.16

$$-\frac{(23\,a+3\,b)\cos\left(fx+e\right)^5-35\,a\cos\left(fx+e\right)^3+15\,a\cos\left(fx+e\right)+15\left(afx\cos\left(fx+e\right)^4-2\,afx\cos\left(fx+\right.\right.}{15\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*((23*a + 3*b)*cos(f*x + e)^5 - 35*a*cos(f*x + e)^3 + 15*a*cos(f*x + e
) + 15*(a*f*x*cos(f*x + e)^4 - 2*a*f*x*cos(f*x + e)^2 + a*f*x)*sin(f*x + e)
)/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

giac [B] time = 0.40, size = 182, normalized size = 3.57

$$3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-480\left(fx+e\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 + 3*b*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1
/2*f*x + 1/2*e)^3 - 15*b*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a + 330*a*t
an(1/2*f*x + 1/2*e) + 30*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*
e)^4 + 30*b*tan(1/2*f*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 - 15*b*tan
(1/2*f*x + 1/2*e)^2 + 3*a + 3*b)/tan(1/2*f*x + 1/2*e)^5)/f
```

maple [A] time = 0.84, size = 63, normalized size = 1.24

$$\frac{a\left(-\frac{\left(\cot^5\left(fx+e\right)\right)}{5}+\frac{\left(\cot^3\left(fx+e\right)\right)}{3}-\cot\left(fx+e\right)-fx-e\right)-\frac{b\left(\cos^5\left(fx+e\right)\right)}{5\sin\left(fx+e\right)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-1/5*b/sin(f*x+
e)^5*cos(f*x+e)^5)
```

maxima [A] time = 0.42, size = 52, normalized size = 1.02

$$-\frac{15\left(fx+e\right)a+\frac{15\,a\tan\left(fx+e\right)^4-5\,a\tan\left(fx+e\right)^2+3\,a+3\,b}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a + (15*a*tan(f*x + e)^4 - 5*a*tan(f*x + e)^2 + 3*a + 3
*b)/tan(f*x + e)^5)/f
```

mupad [B] time = 4.89, size = 46, normalized size = 0.90

$$-a\,x-\frac{a\,\tan\left(e+f\,x\right)^4-\frac{a\,\tan\left(e+f\,x\right)^2}{3}+\frac{a}{5}+\frac{b}{5}}{f\,\tan\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] - a*x - (a/5 + b/5 - (a*tan(e + f*x)^2)/3 + a*tan(e + f*x)^4)/(f*tan(e + f*
x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)\cot^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

3.324 $\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{\left(a^2-4ab+b^2\right)\sec^4(e+fx)}{4f}-\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(a-b)\sec^6(e+fx)}{3f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{b^2\sec^8(e+}{8f}$$

```
[Out] -a^2*ln(cos(f*x+e))/f-a*(a-b)*sec(f*x+e)^2/f+1/4*(a^2-4*a*b+b^2)*sec(f*x+e)
^4/f+1/3*(a-b)*b*sec(f*x+e)^6/f+1/8*b^2*sec(f*x+e)^8/f
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{\left(a^2-4ab+b^2\right)\sec^4(e+fx)}{4f}-\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(a-b)\sec^6(e+fx)}{3f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{b^2\sec^8(e+}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) - (a*(a - b)*Sec[e + f*x]^2)/f + ((a^2 - 4*a*b
 + b^2)*Sec[e + f*x]^4)/(4*f) + ((a - b)*b*Sec[e + f*x]^6)/(3*f) + (b^2*Sec
[e + f*x]^8)/(8*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)^2}{x^9}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)^2}{x^5}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^5}+\frac{2(a-b)b}{x^4}+\frac{a^2-4ab+b^2}{x^3}-\frac{2a(a-b)}{x^2}+\frac{a^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{\left(a^2-4ab+b^2\right)\sec^4(e$$

Mathematica [A] time = 0.43, size = 126, normalized size = 1.26

$$-\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(-6\left(a^2-4ab+b^2\right)\sec^4(e+fx)+24a^2\log(\cos(e+fx))-8b(a-b)\sec^6(e}{6f(a\cos(2e+2fx)+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -1/6*(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(24*a^2*Log[Cos[e + f*x]] + 2
4*a*(a - b)*Sec[e + f*x]^2 - 6*(a^2 - 4*a*b + b^2)*Sec[e + f*x]^4 - 8*(a -
b)*b*Sec[e + f*x]^6 - 3*b^2*Sec[e + f*x]^8))/(f*(a + 2*b + a*Cos[2*e + 2*f*
x])^2)
```

fricas [A] time = 0.56, size = 99, normalized size = 0.99

$$-\frac{24\,a^2\cos\left(fx+e\right)^8\log\left(-\cos\left(fx+e\right)\right)+24\left(a^2-ab\right)\cos\left(fx+e\right)^6-6\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-8\left(ab-}{24\,f\cos\left(fx+e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(24*a^2*cos(f*x + e)^8*log(-cos(f*x + e)) + 24*(a^2 - a*b)*cos(f*x +
e)^6 - 6*(a^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^2
- 3*b^2)/(f*cos(f*x + e)^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(
f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))-a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(25*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))^4
*a^2-248*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+
cos(f*x+exp(1))))^3*a^2+984*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-c
os(f*x+exp(1)))*(1+cos(f*x+exp(1))))^2*a^2+256*((1-cos(f*x+exp(1)))/(1+cos(
```

```
f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b^2+512*((1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b*
a-1760*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+co
s(f*x+exp(1))))*a^2+256*b^2-1024*b*a+1168*a^2)*1/48/((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2)^4)
```

maple [A] time = 0.69, size = 120, normalized size = 1.20

$$\frac{\left(\tan^4\left(fx+e\right)\right)a^2}{4f}-\frac{a^2\left(\tan^2\left(fx+e\right)\right)}{2f}-\frac{a^2\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{ab\left(\sin^6\left(fx+e\right)\right)}{3f\cos\left(fx+e\right)^6}+\frac{b^2\left(\sin^6\left(fx+e\right)\right)}{8f\cos\left(fx+e\right)^8}+\frac{b^2\left(\sin^6\right.}{24f\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x)
```

```
[Out] 1/4/f*tan(f*x+e)^4*a^2-1/2/f*a^2*tan(f*x+e)^2-a^2*ln(cos(f*x+e))/f+1/3/f*a*
b*sin(f*x+e)^6/cos(f*x+e)^6+1/8/f*b^2*sin(f*x+e)^6/cos(f*x+e)^8+1/24/f*b^2*
sin(f*x+e)^6/cos(f*x+e)^6
```

maxima [A] time = 0.34, size = 147, normalized size = 1.47

$$-\frac{12\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\left(a^2-ab\right)\sin\left(fx+e\right)^6-6\left(11\,a^2-8\,ab-b^2\right)\sin\left(fx+e\right)^4+4\left(15\,a^2-8\,ab-b^2\right)\sin\left(fx+e\right)^2-18\,a^2+8\,ab+b^2}{\sin\left(fx+e\right)^8-4\,\sin\left(fx+e\right)^6+6\,\sin\left(fx+e\right)^4-4\,\sin\left(fx+e\right)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*a^2*log(sin(f*x + e)^2 - 1) - (24*(a^2 - a*b)*sin(f*x + e)^6 - 6*
(11*a^2 - 8*a*b - b^2)*sin(f*x + e)^4 + 4*(15*a^2 - 8*a*b - b^2)*sin(f*x +
e)^2 - 18*a^2 + 8*a*b + b^2)/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x
 + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.51, size = 124, normalized size = 1.24

$$\frac{\tan\left(e+f\,x\right)^4\,\left(\frac{(a+b)^2}{4}+\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f}-\frac{\tan\left(e+f\,x\right)^2\,\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,\left(a+b\right)\right)}{f}-\frac{\tan\left(e+f\,x\right)^6\,\left(\frac{b^2}{6}-\frac{b\,(a+b)}{3}\right)}{f}+\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^4*((a + b)^2/4 + b^2/4 - (b*(a + b))/2))/f - (tan(e + f*x)^2*
((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^6*(b^2/6 - (b*(a + b))
/3))/f + (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e + f*x)^8)/(8*f)
```

sympy [A] time = 14.18, size = 190, normalized size = 1.90

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\tan^4\left(e+fx\right)\sec^2\left(e+fx\right)}{3f}-\frac{ab\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{3f}+\frac{ab\sec^2\left(e+fx\right)}{3f} \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan^5\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) + a*b*tan(e + f*x)**4*sec(e + f*x)**2/(3*f) -
```

```
 a*b*tan(e + f*x)**2*sec(e + f*x)**2/(3*f) + a*b*sec(e + f*x)**2/(3*f) + b*
*2*tan(e + f*x)**4*sec(e + f*x)**4/(8*f) - b**2*tan(e + f*x)**2*sec(e + f*x
)**4/(12*f) + b**2*sec(e + f*x)**4/(24*f), Ne(f, 0)), (x*(a + b*sec(e)**2)*
*2*tan(e)**5, True))
```

3.325 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^3(e+fx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(2a-b)\sec^4(e+fx)}{4f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{b^2\sec^6(e+fx)}{6f}$$

```
[Out] a^2*ln(cos(f*x+e))/f+1/2*a*(a-2*b)*sec(f*x+e)^2/f+1/4*(2*a-b)*b*sec(f*x+e)^
4/f+1/6*b^2*sec(f*x+e)^6/f
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 76}

$$\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(2a-b)\sec^4(e+fx)}{4f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{b^2\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (a^2*Log[Cos[e + f*x]])/f + (a*(a - 2*b)*Sec[e + f*x]^2)/(2*f) + ((2*a - b)
*b*Sec[e + f*x]^4)/(4*f) + (b^2*Sec[e + f*x]^6)/(6*f)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^7}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a^2\log(\cos(e+fx))}{f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{(2a-b)b\sec^4(e+fx)}{4f}$$

Mathematica [A] time = 0.25, size = 107, normalized size = 1.39

$$\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12a^2\log(\cos(e+fx))+3b(2a-b)\sec^4(e+fx)+6a(a-2b)\sec^2(e+fx)+2\right.}{3f(a\cos(2e+2fx)+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a^2*Log[Cos[e + f*x]] + 6*a*(a
 - 2*b)*Sec[e + f*x]^2 + 3*(2*a - b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x]^
6))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
```

fricas [A] time = 0.48, size = 79, normalized size = 1.03

$$\frac{12\,a^2\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+2\,b^2}{12\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(a^2 - 2*a*b)*cos(f*x +
e)^4 + 3*(2*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))+1/(1-cos
(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))+a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(-11*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))
^3*a^2+90*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1
+cos(f*x+exp(1))))^2*a^2+48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-c
os(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b^2+96*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b*a-228*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a^2+3
```

```
2*b^2-192*b*a+184*a^2)*1/24/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-c
os(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2)^3)
```

maple [A] time = 0.62, size = 103, normalized size = 1.34

$$\frac{a^2\left(\tan^2\left(fx+e\right)\right)}{2f}+\frac{a^2\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{ab\left(\sin^4\left(fx+e\right)\right)}{2f\cos\left(fx+e\right)^4}+\frac{b^2\left(\sin^4\left(fx+e\right)\right)}{6f\cos\left(fx+e\right)^6}+\frac{b^2\left(\sin^4\left(fx+e\right)\right)}{12f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*a^2*tan(f*x+e)^2+a^2*ln(cos(f*x+e))/f+1/2/f*a*b*sin(f*x+e)^4/cos(f*x+
e)^4+1/6/f*b^2*sin(f*x+e)^4/cos(f*x+e)^6+1/12/f*b^2*sin(f*x+e)^4/cos(f*x+e)
^4
```

maxima [A] time = 0.33, size = 114, normalized size = 1.48

$$\frac{6\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\left(a^2-2\,ab\right)\sin\left(fx+e\right)^4-3\left(4\,a^2-6\,ab-b^2\right)\sin\left(fx+e\right)^2+6\,a^2-6\,ab-b^2}{\sin\left(fx+e\right)^6-3\,\sin\left(fx+e\right)^4+3\,\sin\left(fx+e\right)^2-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(6*a^2*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 2*a*b)*sin(f*x + e)^4 - 3*(
4*a^2 - 6*a*b - b^2)*sin(f*x + e)^2 + 6*a^2 - 6*a*b - b^2)/(sin(f*x + e)^6
- 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

mupad [B] time = 4.54, size = 92, normalized size = 1.19

$$\frac{\tan\left(e+f\,x\right)^2\,\left(\frac{{\left(a+b\right)}^2}{2}+\frac{b^2}{2}-b\,\left(a+b\right)\right)}{f}-\frac{\tan\left(e+f\,x\right)^4\,\left(\frac{b^2}{4}-\frac{b\,\left(a+b\right)}{2}\right)}{f}-\frac{a^2\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}+\frac{b^2\tan\left(e+f\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^2*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^4*(b^2
/4 - (b*(a + b))/2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e +
 f*x)^6)/(6*f)
```

sympy [A] time = 5.08, size = 128, normalized size = 1.66

$$\begin{cases}-\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{2f}-\frac{ab\sec^2\left(e+fx\right)}{2f}+\frac{b^2\tan^2\left(e+fx\right)\sec^4\left(e+fx\right)}{6f}-\frac{b^2\sec^4\left(e+fx\right)}{12f} \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan^3\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(2*f) - a*b*sec(e + f*x)**2/(2*f) +
 b**2*tan(e + f*x)**2*sec(e + f*x)**4/(6*f) - b**2*sec(e + f*x)**4/(12*f),
Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e)**3, True))
```

3.326 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx$

Optimal. Leaf size=48

$$-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

```
[Out] -a^2*ln(cos(f*x+e))/f+a*b*sec(f*x+e)^2/f+1/4*b^2*sec(f*x+e)^4/f
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) + (a*b*Sec[e + f*x]^2)/f + (b^2*Sec[e + f*x]^4
)/(4*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^5}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 82, normalized size = 1.71

$$-\frac{\sec^4(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(4a^2\cos^4(e+fx)\log(\cos(e+fx))-4ab\cos^2(e+fx)-b^2\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-b^2 - 4*a*b*Cos[e + f*x]^2 + 4*a^2*Cos[e + f*
x]^4*Log[Cos[e + f*x]])*Sec[e + f*x]^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2
))
```

fricas [A] time = 0.46, size = 53, normalized size = 1.10

$$-\frac{4\,a^2\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)-4\,ab\cos\left(fx+e\right)^2-b^2}{4\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(f*x + e)^4*log(-cos(f*x + e)) - 4*a*b*cos(f*x + e)^2 - b^2)
/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(
f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))-a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(3*((1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))^2*
a^2-12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+co
s(f*x+exp(1))))*a^2+16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*
x+exp(1)))*(1+cos(f*x+exp(1))))*a*b+8*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b^2+12*a^2-32*a*b)*1/8/((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)
))-2)^2)
```

maple [A] time = 0.34, size = 46, normalized size = 0.96

$$\frac{b^2\left(\sec^4\left(fx+e\right)\right)}{4f}+\frac{ab\left(\sec^2\left(fx+e\right)\right)}{f}+\frac{a^2\ln\left(\sec\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x)
```

```
[Out] 1/4*b^2*sec(f*x+e)^4/f+a*b*sec(f*x+e)^2/f+1/f*a^2*ln(sec(f*x+e))
```

maxima [A] time = 0.33, size = 67, normalized size = 1.40

$$-\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\,ab\sin\left(fx+e\right)^2-4\,ab-b^2}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log(sin(f*x + e)^2 - 1) + (4*a*b*sin(f*x + e)^2 - 4*a*b - b^2)/
(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 4.50, size = 61, normalized size = 1.27

$$\frac{a^2 \ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}-\frac{\tan\left(e+f\,x\right)^2\,\left(\frac{b^2}{2}-b\,\left(a+b\right)\right)}{f}+\frac{b^2\tan\left(e+f\,x\right)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (tan(e + f*x)^2*(b^2/2 - b*(a + b)))/
f + (b^2*tan(e + f*x)^4)/(4*f)
```

sympy [A] time = 1.67, size = 61, normalized size = 1.27

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{ab\sec^2\left(e+fx\right)}{f}+\frac{b^2\sec^4\left(e+fx\right)}{4f} & \text{for } f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e),x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*sec(e + f*x)**2/f + b*
*2*sec(e + f*x)**4/(4*f), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e), True))
```

3.327 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\log(\sin(e+fx))}{f}-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

```
[Out] -b*(2*a+b)*ln(cos(f*x+e))/f+(a+b)^2*ln(sin(f*x+e))/f+1/2*b^2*sec(f*x+e)^2/f
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2\log(\sin(e+fx))}{f}-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cos[e + f*x]])/f) + ((a + b)^2*Log[Sin[e + f*x]])/f + (b
^2*Sec[e + f*x]^2)/(2*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^3\left(1-x^2\right)}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{(a+b)^2\log(\sin(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 84, normalized size = 1.58

$$\frac{2(a\cos(e+fx)+b\sec(e+fx))^2\left(2\cos^2(e+fx)\left((a+b)^2\log(\sin(e+fx))-b(2a+b)\log(\cos(e+fx))\right)+b^2\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(b^2 + 2*Cos[e + f*x]^2*(-(b*(2*a + b)*Log[Cos[e + f*x]]) + (a + b)^2*Lo
g[Sin[e + f*x]]))*(a*Cos[e + f*x] + b*Sec[e + f*x])^2)/(f*(a + 2*b + a*Cos[
2*(e + f*x)])^2)
```

fricas [A] time = 0.52, size = 79, normalized size = 1.49

$$-\frac{\left(2\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(\cos\left(fx+e\right)^2\right)-\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^2+\frac{1}{4}\right)-b^2}{2\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((2*a*b + b^2)*cos(f*x + e)^2*log(cos(f*x + e)^2) - (a^2 + 2*a*b + b^2
)*cos(f*x + e)^2*log(-1/4*cos(f*x + e)^2 + 1/4) - b^2)/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos
(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))+(-2*a*b-b^2)/4*ln(abs((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(2*(
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+ex
p(1))))*a*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*
(1+cos(f*x+exp(1))))*b^2-4*a*b+2*b^2)*1/4/((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))
```

maple [A] time = 0.94, size = 60, normalized size = 1.13

$$\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}+\frac{2ab\ln\left(\tan\left(fx+e\right)\right)}{f}+\frac{b^2}{2f\cos\left(fx+e\right)^2}+\frac{b^2\ln\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] a^2*ln(sin(f*x+e))/f+2/f*a*b*ln(tan(f*x+e))+1/2/f*b^2/cos(f*x+e)^2+1/f*b^2*
ln(tan(f*x+e))
```

maxima [A] time = 0.34, size = 64, normalized size = 1.21

$$-\frac{\left(2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{b^2}{\sin\left(fx+e\right)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (a^2 + 2*a*b + b^2)*log(sin(f
*x + e)^2) + b^2/(sin(f*x + e)^2 - 1))/f
```

mupad [B] time = 4.48, size = 58, normalized size = 1.09

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a^2+2\,a\,b+b^2\right)}{f}-\frac{a^2\,\ln\left({\tan\left(e+f\,x\right)}^2+1\right)}{2\,f}+\frac{b^2\,{\tan\left(e+f\,x\right)}^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x))*(2*a*b + a^2 + b^2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(
2*f) + (b^2*tan(e + f*x)^2)/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x), x)
```

3.328 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=57

$$-\frac{\left(a^2-b^2\right)\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}$$

```
[Out] -1/2*(a+b)^2*csc(f*x+e)^2/f-b^2*ln(cos(f*x+e))/f-(a^2-b^2)*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$-\frac{\left(a^2-b^2\right)\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + b)^2*Csc[e + f*x]^2)/(2*f) - (b^2*Log[Cos[e + f*x]])/f - ((a^2 - b^2
)*Log[Sin[e + f*x]])/f
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x\left(1-x^2\right)^2}dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2}+\frac{a^2-b^2}{-1+x}+\frac{b^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)^2\csc^2(e+fx)}{2f} - \frac{b^2\log(\cos(e+fx))}{f} - \frac{\left(a^2-b^2\right)\log(\sin(e}{f}$$

Mathematica [A] time = 0.18, size = 81, normalized size = 1.42

$$-\frac{2\left(a\cos^2(e+fx)+b\right)^2\left(2\left(a^2-b^2\right)\log(\sin(e+fx))+(a+b)^2\csc^2(e+fx)+2b^2\log(\cos(e+fx))\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cos[e + f*x]^2)^2*((a + b)^2*Csc[e + f*x]^2 + 2*b^2*Log[Cos[e +
f*x]] + 2*(a^2 - b^2)*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])
^2)
```

fricas [A] time = 0.46, size = 100, normalized size = 1.75

$$\frac{a^2+2\,ab+b^2-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(\cos\left(fx+e\right)^2\right)-\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2-a^2+b^2\right)\log\left(-\frac{1}{4}\cos\right.}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + 2*a*b + b^2 - (b^2*cos(f*x + e)^2 - b^2)*log(cos(f*x + e)^2) - (
(a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2)*log(-1/4*cos(f*x + e)^2 + 1/4))/(f*
cos(f*x + e)^2 - f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos
(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+a^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))+(-((1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b^
2-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f
*x+exp(1))))*b*a-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(
1)))*(1+cos(f*x+exp(1))))*a^2)/16)
```

maple [A] time = 1.18, size = 78, normalized size = 1.37

$$-\frac{a^2\left(\cot^2\left(fx+e\right)\right)}{2f}-\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{ab}{f\sin\left(fx+e\right)^2}-\frac{b^2}{2f\sin\left(fx+e\right)^2}+\frac{b^2\ln\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*a^2*cot(f*x+e)^2/f-a^2*ln(sin(f*x+e))/f-1/f*a*b/sin(f*x+e)^2-1/2/f*b^2
/sin(f*x+e)^2+1/f*b^2*ln(tan(f*x+e))
```

maxima [A] time = 0.33, size = 60, normalized size = 1.05

$$-\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2+2\,ab+b^2}{\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - b^2)*log(sin(f*x + e)^2) + (a^2
+ 2*a*b + b^2)/sin(f*x + e)^2)/f
```

mupad [B] time = 4.58, size = 68, normalized size = 1.19

$$\frac{a^2\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}-\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a^2-b^2\right)}{f}-\frac{\cot\left(e+f\,x\right)^2\,\left(\frac{a^2}{2}+a\,b+\frac{b^2}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (log(tan(e + f*x))*(a^2 - b^2))/f - (
cot(e + f*x)^2*(a*b + a^2/2 + b^2/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**3, x)
```

3.329 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{a^2\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a(a+b)\csc^2(e+fx)}{f}$$

```
[Out] a*(a+b)*csc(f*x+e)^2/f-1/4*(a+b)^2*csc(f*x+e)^4/f+a^2*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$\frac{a^2\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a(a+b)\csc^2(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + b)*Csc[e + f*x]^2)/f - ((a + b)^2*Csc[e + f*x]^4)/(4*f) + (a^2*Log[
Sin[e + f*x]])/f
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{x\left(b+ax^2\right)^2}{\left(1-x^2\right)^3}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3}-\frac{2a(a+b)}{(-1+x)^2}-\frac{a^2}{-1+x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= \frac{a(a+b)\csc^2(e+fx)}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a^2\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A]   time = 0.25, size = 77, normalized size = 1.51

$$-\frac{\left(a\cos^2(e+fx)+b\right)^2\left(-4a^2\log(\sin(e+fx))+(a+b)^2\csc^4(e+fx)-4a(a+b)\csc^2(e+fx)\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-4*a*(a + b)*Csc[e + f*x]^2 + (a + b)^2*Csc[e
+ f*x]^4 - 4*a^2*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2))
```

fricas [A]   time = 0.50, size = 97, normalized size = 1.90

$$-\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2-4\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - 3*a^2 - 2*a*b + b^2 - 4*(a^2*cos(f*x +
 e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*log(1/2*sin(f*x + e)))/(f*cos(f*x + e)^
4 - 2*f*cos(f*x + e)^2 + f)
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*b^2-64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^2*b*a-32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-128*(1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))*b^2+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))*b*a+384*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2)/4096+(-48*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-4*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*b^2+8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a+12*(1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))*a^2-b^2-2*b*a-a^2)*1/128/((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1))))^2-a^2/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))
+a^2/4*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A]   time = 0.84, size = 87, normalized size = 1.71

$$-\frac{a^2\left(\cot^4\left(fx+e\right)\right)}{4f}+\frac{a^2\left(\cot^2\left(fx+e\right)\right)}{2f}+\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{ab\left(\cos^4\left(fx+e\right)\right)}{2f\sin\left(fx+e\right)^4}-\frac{b^2}{4f\sin\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/4*a^2*cot(f*x+e)^4/f+1/2*a^2*cot(f*x+e)^2/f+a^2*ln(sin(f*x+e))/f-1/2/f*a
*b/sin(f*x+e)^4*cos(f*x+e)^4-1/4/f*b^2/sin(f*x+e)^4
```

maxima [A] time = 0.34, size = 61, normalized size = 1.20

$$\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\left(a^2+ab\right)\sin\left(fx+e\right)^2-a^2-2\,ab-b^2}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(sin(f*x + e)^2) + (4*(a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*
b - b^2)/sin(f*x + e)^4)/f
```

mupad [B] time = 4.61, size = 83, normalized size = 1.63

$$\frac{a^2\,\ln\left(\tan\left(e+f\,x\right)\right)}{f}-\frac{\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan\left(e+f\,x\right)^2\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{f\,\tan\left(e+f\,x\right)^4}-\frac{a^2\,\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)))/f - ((a*b)/2 + a^2/4 + b^2/4 - tan(e + f*x)^2*(a^2/
2 - b^2/2))/(f*tan(e + f*x)^4) - (a^2*log(tan(e + f*x)^2 + 1))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**5, x)
```

3.330 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^6(e+fx)\,dx$

Optimal. Leaf size=95

$$\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[Out] -a^2*x+a^2*tan(f*x+e)/f-1/3*a^2*tan(f*x+e)^3/f+1/5*a^2*tan(f*x+e)^5/f+1/7*b
*(2*a+b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 203}

$$\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e +
 f*x]^5)/(5*f) + (b*(2*a + b)*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/
(9*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a^2-a^2x^2+a^2x^4+b(2a+b)x^6+b^2x^8-\frac{a^2}{1+x^2}\right)dx, x, \tan(e\right.}{f}$$

$$= \frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan^7(e}{7f}$$

$$= -a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b}{}$$

Mathematica [B] time = 2.07, size = 275, normalized size = 2.89

$$-\frac{4\sec^9(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(\left(231a^2-270ab+5b^2\right)\tan(e)\cos^7(e+fx)-3\left(21a^2-90ab+25b^2\right)\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^9*(315*a^2*f*x*Cos[e + f*x]^9 - 3
5*b^2*Sec[e]*Sin[f*x] - 5*(18*a - 19*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] -
3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^4*Sec[e]*Sin[f*x] + (231*a^2 - 27
0*a*b + 5*b^2)*Cos[e + f*x]^6*Sec[e]*Sin[f*x] - (483*a^2 - 90*a*b - 10*b^2)
*Cos[e + f*x]^8*Sec[e]*Sin[f*x] - 35*b^2*Cos[e + f*x]*Tan[e] - 5*(18*a - 19
*b)*b*Cos[e + f*x]^3*Tan[e] - 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^5*T
an[e] + (231*a^2 - 270*a*b + 5*b^2)*Cos[e + f*x]^7*Tan[e]))/(315*f*(a + 2*b
 + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.49, size = 137, normalized size = 1.44

$$-\frac{315\,a^2fx\cos\left(fx+e\right)^9-\left(\left(483\,a^2-90\,ab-10\,b^2\right)\cos\left(fx+e\right)^8-\left(231\,a^2-270\,ab+5\,b^2\right)\cos\left(fx+e\right)^6+3\right.}{315\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/315*(315*a^2*f*x*cos(f*x + e)^9 - ((483*a^2 - 90*a*b - 10*b^2)*cos(f*x +
 e)^8 - (231*a^2 - 270*a*b + 5*b^2)*cos(f*x + e)^6 + 3*(21*a^2 - 90*a*b + 2
5*b^2)*cos(f*x + e)^4 + 5*(18*a*b - 19*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

giac [A] time = 4.74, size = 98, normalized size = 1.03

$$\frac{35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+45\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)^3-}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 45*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 31
5*a^2*tan(f*x + e))/f
```

maple [A] time = 0.77, size = 105, normalized size = 1.11

$$\frac{a^2\left(\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^3(fx+e)\right)}{3}+\tan\left(fx+e\right)-fx-e\right)+\frac{2ab\left(\sin^7(fx+e)\right)}{7\cos\left(fx+e\right)^7}+b^2\left(\frac{\sin^7(fx+e)}{9\cos\left(fx+e\right)^9}+\frac{2\left(\sin^7(fx+e)\right)}{63\cos\left(fx+e\right)^7}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x)
```

```
[Out] 1/f*(a^2*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-f*x-e)+2/7*a*b*sin(f
*x+e)^7/cos(f*x+e)^7+b^2*(1/9*sin(f*x+e)^7/cos(f*x+e)^9+2/63*sin(f*x+e)^7/c
os(f*x+e)^7))
```

maxima [A] time = 0.44, size = 84, normalized size = 0.88

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\left(2\,ab+b^2\right)\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)^3-315\left(fx+e\right)a^2+}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b + b^2)*tan(f*x + e)^7 + 63*a^2*tan
(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 315*a^2*tan(f*x
+ e))/f
```

mupad [B] time = 4.54, size = 126, normalized size = 1.33

$$\frac{\tan\left(e+f\,x\right)\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan\left(e+f\,x\right)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\tan\left(e+f\,x\right)^5\left(\frac{(a+b)^2}{5}+\frac{b^2}{5}-\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*((a + b)^2/3
 + b^2/3 - (2*b*(a + b))/3) + tan(e + f*x)^5*((a + b)^2/5 + b^2/5 - (2*b*(a
 + b))/5) - tan(e + f*x)^7*(b^2/7 - (2*b*(a + b))/7) + (b^2*tan(e + f*x)^9)
/9 - a^2*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\tan^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**6,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**6, x)
```

3.331 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}+a^2x+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] a^2*x-a^2*tan(f*x+e)/f+1/3*a^2*tan(f*x+e)^3/f+1/5*b*(2*a+b)*tan(f*x+e)^5/f+
1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 203}

$$\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}+a^2x+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) + (b*(2*a + b)*Ta
n[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \left(-a^2+a^2x^2+b(2a+b)x^4+b^2x^6+\frac{a^2}{1+x^2}\right)dx,x,\tan(e+$$

$$=-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\,\mathrm{t}}{}$$

$$=a^2x-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+$$

Mathematica [B] time = 1.10, size = 395, normalized size = 5.13

$$\frac{\sec(e)\sec^7(e+fx)\left(4480a^2\sin(2e+fx)-3780a^2\sin(2e+3fx)+2100a^2\sin(4e+3fx)-1540a^2\sin(4e+5fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] (Sec[e]*Sec[e + f*x]^7*(3675*a^2*f*x*Cos[f*x] + 3675*a^2*f*x*Cos[2*e + f*x]
 + 2205*a^2*f*x*Cos[2*e + 3*f*x] + 2205*a^2*f*x*Cos[4*e + 3*f*x] + 735*a^2*
f*x*Cos[4*e + 5*f*x] + 735*a^2*f*x*Cos[6*e + 5*f*x] + 105*a^2*f*x*Cos[6*e +
 7*f*x] + 105*a^2*f*x*Cos[8*e + 7*f*x] - 5320*a^2*Sin[f*x] + 1680*a*b*Sin[f
*x] + 840*b^2*Sin[f*x] + 4480*a^2*Sin[2*e + f*x] - 1260*a*b*Sin[2*e + f*x]
+ 420*b^2*Sin[2*e + f*x] - 3780*a^2*Sin[2*e + 3*f*x] + 924*a*b*Sin[2*e + 3*
f*x] - 168*b^2*Sin[2*e + 3*f*x] + 2100*a^2*Sin[4*e + 3*f*x] - 840*a*b*Sin[4
*e + 3*f*x] - 420*b^2*Sin[4*e + 3*f*x] - 1540*a^2*Sin[4*e + 5*f*x] + 168*a*
b*Sin[4*e + 5*f*x] + 84*b^2*Sin[4*e + 5*f*x] + 420*a^2*Sin[6*e + 5*f*x] - 4
20*a*b*Sin[6*e + 5*f*x] - 280*a^2*Sin[6*e + 7*f*x] + 84*a*b*Sin[6*e + 7*f*x
] + 12*b^2*Sin[6*e + 7*f*x]))/(13440*f)
```

fricas [A] time = 0.50, size = 113, normalized size = 1.47

$$\frac{105\,a^2fx\cos\left(fx+e\right)^7-\left(2\left(70\,a^2-21\,ab-3\,b^2\right)\cos\left(fx+e\right)^6-\left(35\,a^2-84\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-6\left(7\,ab-\right.\right.}{105\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^2*f*x*cos(f*x + e)^7 - (2*(70*a^2 - 21*a*b - 3*b^2)*cos(f*x +
e)^6 - (35*a^2 - 84*a*b + 3*b^2)*cos(f*x + e)^4 - 6*(7*a*b - 4*b^2)*cos(f*x
 + e)^2 - 15*b^2)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

giac [A] time = 5.15, size = 84, normalized size = 1.09

$$\frac{15\,b^2\tan\left(fx+e\right)^7+42\,ab\tan\left(fx+e\right)^5+21\,b^2\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+105\left(fx+e\right)a^2-105\,a^2\tan}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^
5 + 35*a^2*tan(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

maple [A] time = 0.60, size = 94, normalized size = 1.22

$$\frac{a^2\left(\frac{\left(\tan^3\left(fx+e\right)\right)}{3}-\tan\left(fx+e\right)+fx+e\right)+\frac{2ab\left(\sin^5\left(fx+e\right)\right)}{5\cos\left(fx+e\right)^5}+b^2\left(\frac{\sin^5\left(fx+e\right)}{7\cos\left(fx+e\right)^7}+\frac{2\left(\sin^5\left(fx+e\right)\right)}{35\cos\left(fx+e\right)^5}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x)
```

```
[Out] 1/f*(a^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e)+2/5*a*b*sin(f*x+e)^5/cos(f*x+e
)^5+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/35*sin(f*x+e)^5/cos(f*x+e)^5))
```

maxima [A] time = 0.43, size = 71, normalized size = 0.92

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab+b^2\right)\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+105\left(fx+e\right)a^2-105\,a^2\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + b^2)*tan(f*x + e)^5 + 35*a^2*tan
(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

mupad [B] time = 4.58, size = 97, normalized size = 1.26

$$\frac{\tan\left(e+f\,x\right)^3\,\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)-\tan\left(e+f\,x\right)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan\left(e+f\,x\right)^5\,\left(\frac{b^2}{5}-\frac{2\,b\,(a+b)}{5}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^3*((a + b)^2/3 + b^2/3 - (2*b*(a + b))/3) - tan(e + f*x)*((a
+ b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^5*(b^2/5 - (2*b*(a + b))/5) + (b
^2*tan(e + f*x)^7)/7 + a^2*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**4, x)
```

3.332 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^2(e+fx)\,dx$

Optimal. Leaf size=59

$$\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -a^2*x+a^2*tan(f*x+e)/f+1/3*b*(2*a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 203}

$$\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f + (b*(2*a + b)*Tan[e + f*x]^3)/(3*f) + (b^2
*Tan[e + f*x]^5)/(5*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+b(2a+b)x^2+b^2x^4-\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}-\frac{a^2\,\text{Subst}}{}\\ &= -a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [B] time = 0.80, size = 281, normalized size = 4.76

$$-\frac{\sec(e)\sec^5(e+fx)\left(120a^2\sin(2e+fx)-120a^2\sin(2e+3fx)+30a^2\sin(4e+3fx)-30a^2\sin(4e+5fx)+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -1/480*(Sec[e]*Sec[e + f*x]^5*(150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e +
 f*x] + 75*a^2*f*x*Cos[2*e + 3*f*x] + 75*a^2*f*x*Cos[4*e + 3*f*x] + 15*a^2*
f*x*Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*Sin[f*x] + 80*
a*b*Sin[f*x] - 20*b^2*Sin[f*x] + 120*a^2*Sin[2*e + f*x] - 120*a*b*Sin[2*e +
 f*x] - 60*b^2*Sin[2*e + f*x] - 120*a^2*Sin[2*e + 3*f*x] + 40*a*b*Sin[2*e +
 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] + 30*a^2*Sin[4*e + 3*f*x] - 60*a*b*Sin[4*
e + 3*f*x] - 30*a^2*Sin[4*e + 5*f*x] + 20*a*b*Sin[4*e + 5*f*x] + 4*b^2*Sin[
4*e + 5*f*x]))/f
```

fricas [A] time = 0.51, size = 86, normalized size = 1.46

$$-\frac{15\,a^2fx\cos\left(fx+e\right)^5-\left(\left(15\,a^2-10\,ab-2\,b^2\right)\cos\left(fx+e\right)^4+\left(10\,ab-b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*a^2*f*x*cos(f*x + e)^5 - ((15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e)^
4 + (10*a*b - b^2)*cos(f*x + e)^2 + 3*b^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [A] time = 1.08, size = 70, normalized size = 1.19

$$\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+5\,b^2\tan\left(fx+e\right)^3-15\left(fx+e\right)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 5*b^2*tan(f*x + e)^3 -
 15*(f*x + e)*a^2 + 15*a^2*tan(f*x + e))/f
```

maple [A] time = 0.59, size = 85, normalized size = 1.44

$$\frac{a^2\left(\tan\left(fx+e\right)-fx-e\right)+\frac{2ab\left(\sin^3\left(fx+e\right)\right)}{3\cos\left(fx+e\right)^3}+b^2\left(\frac{\sin^3\left(fx+e\right)}{5\cos\left(fx+e\right)^5}+\frac{2\left(\sin^3\left(fx+e\right)\right)}{15\cos\left(fx+e\right)^3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x)
```

```
[Out] 1/f*(a^2*(tan(f*x+e)-f*x-e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+e)^3+b^2*(1/5*sin(
f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

maxima [A] time = 0.44, size = 58, normalized size = 0.98

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\left(2\,ab+b^2\right)\tan\left(fx+e\right)^3-15\left(fx+e\right)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b + b^2)*tan(f*x + e)^3 - 15*(f*x + e)*
a^2 + 15*a^2*tan(f*x + e))/f
```

mupad [B] time = 4.66, size = 69, normalized size = 1.17

$$\frac{\tan\left(e+f\,x\right)\,\left({\left(a+b\right)}^2+b^2-2\,b\,\left(a+b\right)\right)-{\tan\left(e+f\,x\right)}^3\,\left(\frac{b^2}{3}-\frac{2\,b\,\left(a+b\right)}{3}\right)+\frac{b^2\,{\tan\left(e+f\,x\right)}^5}{5}-a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*(b^2/3 - (2*
b*(a + b))/3) + (b^2*tan(e + f*x)^5)/5 - a^2*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**2, x)
```

3.333 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}
$$

Mathematica [B] time = 0.37, size = 106, normalized size = 2.65

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a^2fx\cos^3(e+fx)+2b(3a+b)\sec(e)\sin(fx)\cos^2(e+fx)+b^2\tan(e)\cos\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.54, size = 58, normalized size = 1.45

$$\frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

giac [A] time = 0.22, size = 53, normalized size = 1.32

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

maple [A] time = 1.03, size = 48, normalized size = 1.20

$$\frac{a^2\left(fx+e\right)+2ab\tan\left(fx+e\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.34, size = 44, normalized size = 1.10

$$a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

mupad [B] time = 4.58, size = 42, normalized size = 1.05

$$\frac{\frac{b^2 \tan\left(e+f\,x\right)^3}{3} - \tan\left(e+f\,x\right)\,\left(b^2-2\,b\,\left(a+b\right)\right)+a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

3.334 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=36

$$a^2(-x)-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -a^2*x-(a+b)^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 203}

$$a^2(-x)-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - ((a + b)^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^2}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e\right.}{f}\\ &= -a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}\end{aligned}$$

Mathematica [B] time = 0.78, size = 82, normalized size = 2.28

$$-\frac{4\sec(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(a^2fx\cos(e+fx)-\sin(fx)\left((a+b)^2\csc(e)\cot(e+fx)+b^2\sec(e)\right)\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]*(a^2*f*x*Cos[e + f*x] - ((a + b)^
2*Cot[e + f*x]*Csc[e] + b^2*Sec[e])*Sin[f*x]))/(f*(a + 2*b + a*Cos[2*(e + f
*x)])^2)
```

fricas [A] time = 0.58, size = 67, normalized size = 1.86

$$-\frac{a^2fx\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(a^2+2\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-b^2}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*f*x*cos(f*x + e)*sin(f*x + e) + (a^2 + 2*a*b + 2*b^2)*cos(f*x + e)^2
- b^2)/(f*cos(f*x + e)*sin(f*x + e))
```

giac [A] time = 0.39, size = 49, normalized size = 1.36

$$-\frac{\left(fx+e\right)a^2-b^2\tan\left(fx+e\right)+\frac{a^2+2\,ab+b^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

maple [A] time = 0.93, size = 66, normalized size = 1.83

$$\frac{a^2\left(-\cot\left(fx+e\right)-fx-e\right)-2ab\cot\left(fx+e\right)+b^2\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)}-2\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-cot(f*x+e)-f*x-e)-2*a*b*cot(f*x+e)+b^2*(1/sin(f*x+e)/cos(f*x+e)-
2*cot(f*x+e)))
```

maxima [A] time = 0.45, size = 46, normalized size = 1.28

$$-\frac{\left(fx+e\right)a^2-b^2\tan\left(fx+e\right)+\frac{a^2+2\,ab+b^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

mupad [B] time = 4.64, size = 44, normalized size = 1.22

$$\frac{b^2\tan\left(e+f\,x\right)}{f}-a^2\,x-\frac{a^2+2\,a\,b+b^2}{f\tan\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
[Out] (b^2*tan(e + f*x))/f - a^2*x - (2*a*b + a^2 + b^2)/(f*tan(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**2, x)
```

3.335 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}+a^2x-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

```
[Out] a^2*x+(a^2-b^2)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 203}

$$\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}+a^2x-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + ((a^2 - b^2)*Cot[e + f*x])/f - ((a + b)^2*Cot[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^2}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^4}+\frac{-a^2+b^2}{x^2}+\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\left(a^2-b^2\right)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= a^2x+\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] time = 0.94, size = 160, normalized size = 3.56

$$\csc(e)\csc^3(e+fx)\left(-12a^2\sin(2e+fx)+8a^2\sin(2e+3fx)-9a^2fx\cos(2e+fx)-3a^2fx\cos(2e+3fx)+3a^2f\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^3*(9*a^2*f*x*Cos[f*x] - 9*a^2*f*x*Cos[2*e + f*x] - 3*a
^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*x] + 12
*b^2*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 12*a*b*Sin[2*e + f*x] + 8*a^2*Sin[2
*e + 3*f*x] + 4*a*b*Sin[2*e + 3*f*x] - 4*b^2*Sin[2*e + 3*f*x]))/(24*f)
```

fricas [B] time = 0.49, size = 98, normalized size = 2.18

$$\frac{2\left(2a^2+ab-b^2\right)\cos\left(fx+e\right)^3-3\left(a^2-b^2\right)\cos\left(fx+e\right)+3\left(a^2fx\cos\left(fx+e\right)^2-a^2fx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(2*a^2 + a*b - b^2)*cos(f*x + e)^3 - 3*(a^2 - b^2)*cos(f*x + e) + 3*
(a^2*f*x*cos(f*x + e)^2 - a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*si
n(f*x + e))
```

giac [B] time = 0.57, size = 187, normalized size = 4.16

$$\frac{a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24\left(fx+e\right)a^2-15a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6a}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*tan(1/2*f*x + 1/2*e)^3 + b^2*tan(1
/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a^2 - 15*a^2*tan(1/2*f*x + 1/2*e) - 6*a*b*
tan(1/2*f*x + 1/2*e) + 9*b^2*tan(1/2*f*x + 1/2*e) + (15*a^2*tan(1/2*f*x + 1
/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 - 9*b^2*tan(1/2*f*x + 1/2*e)^2 - a^2
 - 2*a*b - b^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

maple [A] time = 1.28, size = 73, normalized size = 1.62

$$\frac{a^2\left(-\frac{\left(\cot^3\left(fx+e\right)\right)}{3}+\cot\left(fx+e\right)+fx+e\right)-\frac{2ab\left(\cos^3\left(fx+e\right)\right)}{3\sin\left(fx+e\right)^3}+b^2\left(-\frac{2}{3}-\frac{\left(\csc^2\left(fx+e\right)\right)}{3}\right)\cot\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-2/3*a*b/sin(f*x+e)^3*cos(f*x+
e)^3+b^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

maxima [A] time = 0.46, size = 59, normalized size = 1.31

$$\frac{3\left(fx+e\right)a^2+\frac{3\left(a^2-b^2\right)\tan\left(fx+e\right)^2-a^2-2ab-b^2}{\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2 + (3*(a^2 - b^2)*tan(f*x + e)^2 - a^2 - 2*a*b - b^2)/t
an(f*x + e)^3)/f
```

mupad [B] time = 4.61, size = 53, normalized size = 1.18

$$a^2\,x-\frac{\frac{2\,a\,b}{3}-{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(a^2-b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\,{\mathrm{tan}\left(e+f\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*a*b)/3 - tan(e + f*x)^2*(a^2 - b^2) + a^2/3 + b^2/3)/(f*tan(e +
 f*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left (e + f x \right)}\right)^{2} \cot^{4}{\left (e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**4, x)
```

3.336 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=65

$$\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-a^2x-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

```
[Out] -a^2*x-a^2*cot(f*x+e)/f+1/3*(a^2-b^2)*cot(f*x+e)^3/f-1/5*(a+b)^2*cot(f*x+e)
^5/f
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{4141, 1802, 203\}$

$$\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-a^2x-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[e + f*x])/f + ((a^2 - b^2)*Cot[e + f*x]^3)/(3*f) - ((a
+ b)^2*Cot[e + f*x]^5)/(5*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^2}{x^6\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{-a^2+b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{a^2}{}\\
&= -a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx}{5f}
\end{aligned}$$

Mathematica [B] time = 1.03, size = 256, normalized size = 3.94

$$\frac{\csc(e)\csc^5(e+fx)\left(180a^2\sin(2e+fx)-140a^2\sin(2e+3fx)-90a^2\sin(4e+3fx)+46a^2\sin(4e+5fx)+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^5*(-150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x]
+ 75*a^2*f*x*Cos[2*e + 3*f*x] - 75*a^2*f*x*Cos[4*e + 3*f*x] - 15*a^2*f*x*Co
s[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] + 280*a^2*Sin[f*x] + 120*a*b*S
in[f*x] + 20*b^2*Sin[f*x] + 180*a^2*Sin[2*e + f*x] - 60*b^2*Sin[2*e + f*x]
- 140*a^2*Sin[2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] - 90*a^2*Sin[4*e + 3*f
*x] - 60*a*b*Sin[4*e + 3*f*x] + 46*a^2*Sin[4*e + 5*f*x] + 12*a*b*Sin[4*e +
5*f*x] - 4*b^2*Sin[4*e + 5*f*x]))/(480*f)
```

fricas [B] time = 0.63, size = 136, normalized size = 2.09

$$-\frac{\left(23\,a^2+6\,ab-2\,b^2\right)\cos\left(fx+e\right)^5-5\left(7\,a^2-b^2\right)\cos\left(fx+e\right)^3+15\,a^2\cos\left(fx+e\right)+15\left(a^2fx\cos\left(fx+e\right)^4\right.}{15\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cos(f*x + e)^5 - 5*(7*a^2 - b^2)*cos(f*x +
e)^3 + 15*a^2*cos(f*x + e) + 15*(a^2*f*x*cos(f*x + e)^4 - 2*a^2*f*x*cos(f*x
 + e)^2 + a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 +
f)*sin(f*x + e))
```

giac [B] time = 1.01, size = 290, normalized size = 4.46

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ab\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*
tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 - 30*a*b*tan(1/2*f*x
 + 1/2*e)^3 + 5*b^2*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a^2 + 330*a^2*ta
n(1/2*f*x + 1/2*e) + 60*a*b*tan(1/2*f*x + 1/2*e) - 30*b^2*tan(1/2*f*x + 1/2
*e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 + 60*a*b*tan(1/2*f*x + 1/2*e)^4 - 30*
b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 - 30*a*b*tan(1/2
*f*x + 1/2*e)^2 + 5*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan
(1/2*f*x + 1/2*e)^5)/f
```

maple [A] time = 1.16, size = 107, normalized size = 1.65

$$\frac{a^2\left(-\frac{\left(\cot^5\left(fx+e\right)\right)}{5}+\frac{\left(\cot^3\left(fx+e\right)\right)}{3}-\cot\left(fx+e\right)-fx-e\right)-\frac{2ab\left(\cos^5\left(fx+e\right)\right)}{5\sin\left(fx+e\right)^5}+b^2\left(-\frac{\cos^3\left(fx+e\right)}{5\sin\left(fx+e\right)^5}-\frac{2\left(\cos^3\left(fx+e\right)\right)}{15\sin\left(fx+e\right)^3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-2/5*a*b/sin(
f*x+e)^5*cos(f*x+e)^5+b^2*(-1/5/sin(f*x+e)^5*cos(f*x+e)^3-2/15/sin(f*x+e)^3
*cos(f*x+e)^3))
```

maxima [A] time = 0.43, size = 72, normalized size = 1.11

$$-\frac{15\left(fx+e\right)a^2+\frac{15\,a^2\tan\left(fx+e\right)^4-5\left(a^2-b^2\right)\tan\left(fx+e\right)^2+3\,a^2+6\,ab+3\,b^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a^2 + (15*a^2*tan(f*x + e)^4 - 5*(a^2 - b^2)*tan(f*x +
e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

mupad [B] time = 4.84, size = 68, normalized size = 1.05

$$-a^2\,x-\frac{\frac{2\,a\,b}{5}+\frac{a^2}{5}+\frac{b^2}{5}-\tan\left(e+f\,x\right)^2\,\left(\frac{a^2}{3}-\frac{b^2}{3}\right)+a^2\,\tan\left(e+f\,x\right)^4}{f\,\tan\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] - a^2*x - ((2*a*b)/5 + a^2/5 + b^2/5 - tan(e + f*x)^2*(a^2/3 - b^2/3) + a^2
*tan(e + f*x)^4)/(f*tan(e + f*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=69

$$-\frac{(a+b)^2\log\left(a\cos^2(e+fx)+b\right)}{2ab^2f}+\frac{(a+2b)\log(\cos(e+fx))}{b^2f}+\frac{\sec^2(e+fx)}{2bf}$$

```
[Out] (a+2*b)*ln(cos(f*x+e))/b^2/f-1/2*(a+b)^2*ln(b+a*cos(f*x+e)^2)/a/b^2/f+1/2*s
ec(f*x+e)^2/b/f
```

Rubi [A] time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$-\frac{(a+b)^2\log\left(a\cos^2(e+fx)+b\right)}{2ab^2f}+\frac{(a+2b)\log(\cos(e+fx))}{b^2f}+\frac{\sec^2(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*Log[Cos[e + f*x]])/(b^2*f) - ((a + b)^2*Log[b + a*Cos[e + f*x]^2
])/(2*a*b^2*f) + Sec[e + f*x]^2/(2*b*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{(a+2b)\log(\cos(e+fx))}{b^2f} - \frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f} + \frac{\sec^2(e+fx)}{2bf}$$

Mathematica [A] time = 0.27, size = 99, normalized size = 1.43

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(ab\sec^2(e+fx)+(a+b)^2\left(-\log\left(-a\sin^2(e+fx)+a+b\right)\right)+2a(a+2b\right.}{4ab^2f\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(2*a*(a + 2*b)*Log[Cos[e + f
*x]] - (a + b)^2*Log[a + b - a*Sin[e + f*x]^2] + a*b*Sec[e + f*x]^2))/(4*a*
b^2*f*(a + b*Sec[e + f*x]^2))
```

fricas [A] time = 0.71, size = 84, normalized size = 1.22

$$-\frac{\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-ab}{2\,ab^2f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2*log(a*cos(f*x + e)^2 + b) - 2*(a^2
 + 2*a*b)*cos(f*x + e)^2*log(-cos(f*x + e)) - a*b)/(a*b^2*f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)2/f*(1/4/a*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))+(-b^3-3*b^2*a-3*b*a^2-a^3
)/(4*b^3*a+4*b^2*a^2)*ln(abs(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-
cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a+2*b-2*a))+(2*b+a)*1/4/b^
2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+c
os(f*x+exp(1))-2))+(-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f
```

```
*x+exp(1)))*(1+cos(f*x+exp(1))))*b-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a+8*b+2*a)*1/4/b^2/((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))
```

maple [A] time = 0.73, size = 112, normalized size = 1.62

$$-\frac{a\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\,b^2}-\frac{\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{fb}-\frac{\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2af}+\frac{\ln\left(\cos\left(fx+e\right)\right)a}{f\,b^2}+\frac{2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f/b^2*a*ln(b+a*cos(f*x+e)^2)-1/f/b*ln(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos
(f*x+e)^2)/a/f+1/f/b^2*ln(cos(f*x+e))*a+2*ln(cos(f*x+e))/b/f+1/2/f/b/cos(f*
x+e)^2
```

maxima [A] time = 0.34, size = 81, normalized size = 1.17

$$\frac{\frac{(a+2\,b)\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}-\frac{1}{b\sin\left(fx+e\right)^2-b}-\frac{\left(a^2+2\,ab+b^2\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{ab^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + 2*b)*log(sin(f*x + e)^2 - 1)/b^2 - 1/(b*sin(f*x + e)^2 - b) - (a^
2 + 2*a*b + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a*b^2))/f
```

mupad [B] time = 4.61, size = 103, normalized size = 1.49

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,a\,f}-\frac{\ln\left(b\tan\left(e+f\,x\right)^2+a+b\right)}{b\,f}-\frac{\ln\left(b\tan\left(e+f\,x\right)^2+a+b\right)}{2\,a\,f}+\frac{\tan\left(e+f\,x\right)^2}{2\,b\,f}-\frac{a\,\ln\left(b\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a*f) - log(a + b + b*tan(e + f*x)^2)/(b*f) - log
(a + b + b*tan(e + f*x)^2)/(2*a*f) + tan(e + f*x)^2/(2*b*f) - (a*log(a + b
+ b*tan(e + f*x)^2))/(2*b^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.338 $\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{2abf}-\frac{\log(\cos(e+fx))}{bf}$$

```
[Out] -ln(cos(f*x+e))/b/f+1/2*(a+b)*ln(b+a*cos(f*x+e)^2)/a/b/f
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{2abf}-\frac{\log(\cos(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/(b*f)) + ((a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 0.91

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)-2a\log(\cos(e+fx))}{2abf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
[Out] (-2*a*Log[Cos[e + f*x]] + (a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

fricas [A] time = 0.70, size = 41, normalized size = 0.91

$$\frac{(a+b)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,a\log\left(-\cos\left(fx+e\right)\right)}{2\,abf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*((a + b)*log(a*cos(f*x + e)^2 + b) - 2*a*log(-cos(f*x + e)))/(a*b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)2/f*(-1/4/b*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))-1/4/a*ln(abs((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))+
(-b^2-2*b*a-a^2)/(-4*b^2*a-4*b*a^2)*ln(abs(((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b+((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a+2*b-2*a)))
```

maple [A] time = 0.64, size = 59, normalized size = 1.31

$$\frac{\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2fb}+\frac{\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2af}-\frac{\ln\left(\cos\left(fx+e\right)\right)}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
[Out] 1/2/f/b*ln(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a/f-ln(cos(f*x+e))/b/
f
```

maxima [A] time = 0.36, size = 50, normalized size = 1.11

$$\frac{\frac{(a+b)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{ab}-\frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(a*sin(f*x + e)^2 - a - b)/(a*b) - log(sin(f*x + e)^2 - 1)/
b)/f
```

mupad [B] time = 4.49, size = 64, normalized size = 1.42

$$\frac{\ln\left(b\tan\left(e+fx\right)^2+a+b\right)}{2af}+\frac{\ln\left(b\tan\left(e+fx\right)^2+a+b\right)}{2bf}-\frac{\ln\left(\tan\left(e+fx\right)^2+1\right)}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(a + b + b*tan(e + f*x)^2)/(2*a*f) + log(a + b + b*tan(e + f*x)^2)/(2*b*
f) - log(tan(e + f*x)^2 + 1)/(2*a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.339 $\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

```
[Out] -1/2*ln(b+a*cos(f*x+e)^2)/a/f
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 260}

$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -Log[b + a*Cos[e + f*x]^2]/(2*a*f)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}\end{aligned}$$

Mathematica [A] time = 0.19, size = 26, normalized size = 1.13

$$-\frac{\log(a\cos(2(e+fx))+a+2b)}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*Log[a + 2*b + a*Cos[2*(e + f*x)]]/(a*f)
```

fricas [A] time = 0.53, size = 21, normalized size = 0.91

$$-\frac{\log\left(a\cos\left(fx+e\right)^2+b\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log(a*cos(f*x + e)^2 + b)/(a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/2/a*ln(abs((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1)))+1))-1/4/a*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a))
```

maple [A] time = 0.23, size = 37, normalized size = 1.61

$$-\frac{\ln\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)}{2fa}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f/a*ln(a+b*sec(f*x+e)^2)+1/f/a*ln(sec(f*x+e))
```

maxima [A] time = 0.34, size = 26, normalized size = 1.13

$$-\frac{\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(a*sin(f*x + e)^2 - a - b)/(a*f)
```

mupad [B] time = 4.56, size = 63, normalized size = 2.74

$$\frac{\mathrm{atanh}\left(\frac{a}{2\left(\frac{3a}{2}+2b+\frac{a\cos\left(2e+2fx\right)}{2}\right)}-\frac{a\cos\left(2e+2fx\right)}{2\left(\frac{3a}{2}+2b+\frac{a\cos\left(2e+2fx\right)}{2}\right)}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atanh(a/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)) - (a*cos(2*e + 2*f*x))
/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)))/(a*f)
```

sympy [A] time = 12.70, size = 128, normalized size = 5.57

$$\begin{cases} \frac{\tilde{\infty} x \tan(e)}{\sec^2(e)} & \text{for } a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2af} & \text{for } b = 0 \\ \frac{x\tan(e)}{a+b\sec^2(e)} & \text{for } f = 0 \\ -\frac{1}{2bf\sec^2\left(e+fx\right)} & \text{for } a = 0 \\ -\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sec\left(e+fx\right)\right)}{2af} - \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sec\left(e+fx\right)\right)}{2af} + \frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2af} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(ta
n(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0)), (x*tan(e)/(a + b*sec(e)**2), Eq(f, 0
)), (-1/(2*b*f*sec(e + f*x)**2), Eq(a, 0)), (-log(-I*sqrt(a)*sqrt(1/b) + se
c(e + f*x))/(2*a*f) - log(I*sqrt(a)*sqrt(1/b) + sec(e + f*x))/(2*a*f) + log
(tan(e + f*x)**2 + 1)/(2*a*f), True))
```

3.340 $\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\sin(e+fx))}{f(a+b)}+\frac{b\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)}$$

[Out] 1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+ln(sin(f*x+e))/(a+b)/f

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 72}

$$\frac{\log(\sin(e+fx))}{f(a+b)}+\frac{b\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]

[Out] (b*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)*f) + Log[Sin[e + f*x]]/((a + b)*f)

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)\left(b+ax^2\right)}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)}-\frac{b}{(a+b)(b+ax)}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= \frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f}+\frac{\log(\sin(e+fx))}{(a+b)f}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 43, normalized size = 0.93

$$\frac{b\log\left(-a\sin^2(e+fx)+a+b\right)+2a\log(\sin(e+fx))}{2a^2f+2abf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*a*Log[Sin[e + f*x]] + b*Log[a + b - a*Sin[e + f*x]^2])/(2*a^2*f + 2*a*b*
f)
```

fricas [A] time = 0.54, size = 42, normalized size = 0.91

$$\frac{b\log\left(a\cos\left(fx+e\right)^2+b\right)+2\,a\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\left(a^2+ab\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(a*cos(f*x + e)^2 + b) + 2*a*log(1/2*sin(f*x + e)))/((a^2 + a*b)*
f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(4*a+4*b)*ln(abs
(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))-1/2/a*ln(abs((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1)))+1))+b/(4*a^2+4*a*b)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1))))^2*a+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+
a+b))
```

maple [A] time = 0.75, size = 73, normalized size = 1.59

$$\frac{b\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2a\left(a+b\right)f}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{f\left(2a+2b\right)}+\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{f\left(2a+2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+1/f/(2*a+2*b)*ln(-1+cos(f*x+e))+1/f/(2
*a+2*b)*ln(1+cos(f*x+e))
```

maxima [A] time = 0.34, size = 50, normalized size = 1.09

$$\frac{\frac{b\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^2+ab}+\frac{\log\left(\sin\left(fx+e\right)^2\right)}{a+b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(a*sin(f*x + e)^2 - a - b)/(a^2 + a*b) + log(sin(f*x + e)^2)/(a +
 b))/f
```

mupad [B] time = 4.70, size = 65, normalized size = 1.41

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)}{f\,\left(a+b\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,a\,f}+\frac{b\,\ln\left(b\tan\left(e+f\,x\right)^2+a+b\right)}{2\,f\,\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x))/(f*(a + b)) - log(tan(e + f*x)^2 + 1)/(2*a*f) + (b*log(a
+ b + b*tan(e + f*x)^2))/(2*f*(a*b + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^2}-\frac{\csc^2(e+fx)}{2f(a+b)}-\frac{(a+2b)\log(\sin(e+fx))}{f(a+b)^2}$$

```
[Out] -1/2*csc(f*x+e)^2/(a+b)/f-1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f-(a+2*b)*
ln(sin(f*x+e))/(a+b)^2/f
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$-\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^2}-\frac{\csc^2(e+fx)}{2f(a+b)}-\frac{(a+2b)\log(\sin(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -Csc[e + f*x]^2/(2*(a + b)*f) - (b^2*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b
)^2*f) - ((a + 2*b)*Log[Sin[e + f*x]])/((a + b)^2*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\csc^2(e+fx)}{2(a+b)f}-\frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f}-\frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}$$

Mathematica [A] time = 0.23, size = 100, normalized size = 1.35

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(b^2\log\left(-a\sin^2(e+fx)+a+b\right)+a(a+b)\csc^2(e+fx)+2a(a+2b)\log\ldots\right)}{4af(a+b)^2\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a*(a + b)*Csc[e + f*x]^2 + 2*a*(a + 2
*b)*Log[Sin[e + f*x]] + b^2*Log[a + b - a*Sin[e + f*x]^2])*Sec[e + f*x]^2)/
(a*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

fricas [A] time = 0.94, size = 126, normalized size = 1.70

$$\frac{a^2+ab-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2-a^2-2\,ab\right)\log\left(\frac{1}{2}\sin\ldots\right)}{2\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2-\left(a^3+2\,a^2b+ab^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + a*b - (b^2*cos(f*x + e)^2 - b^2)*log(a*cos(f*x + e)^2 + b) - 2*(
(a^2 + 2*a*b)*cos(f*x + e)^2 - a^2 - 2*a*b)*log(1/2*sin(f*x + e)))/((a^3 +
2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))/(16*b+16*a)+(8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
))*b+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-b-a)/(16*b^2+32*b*a+16*a^2
)/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+1/2/a*ln(abs((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1)))+1))-b^2/(4*b^2*a+8*b*a^2+4*a^3)*ln(((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+e
```

```
xp(1)))*a+b+a)+(-2*b-a)/(4*b^2+8*b*a+4*a^2)*ln(abs(1-cos(f*x+exp(1)))/abs(1
+cos(f*x+exp(1)))))
```

maple [B] time = 1.03, size = 158, normalized size = 2.14

$$-\frac{b^2 \ln\left(b + a\left(\cos^2\left(fx+e\right)\right)\right)}{2a\left(a+b\right)^2 f} + \frac{1}{f\left(4a+4b\right)\left(-1+\cos\left(fx+e\right)\right)} - \frac{\ln\left(-1+\cos\left(fx+e\right)\right) a}{2f\left(a+b\right)^2} - \frac{\ln\left(-1+\cos\left(fx+\right.\right.}{f\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f+1/f/(4*a+4*b)/(-1+cos(f*x+e))-1/2
/f/(a+b)^2*ln(-1+cos(f*x+e))*a-1/f/(a+b)^2*ln(-1+cos(f*x+e))*b-1/f/(4*a+4*b
)/(1+cos(f*x+e))-1/2/f/(a+b)^2*ln(1+cos(f*x+e))*a-1/f/(a+b)^2*ln(1+cos(f*x+
e))*b
```

maxima [A] time = 0.34, size = 87, normalized size = 1.18

$$-\frac{\frac{b^2 \log\left(a \sin\left(fx+e\right)^2 - a - b\right)}{a^3+2\,a^2 b+ab^2} + \frac{(a+2\,b)\log\left(\sin\left(fx+e\right)^2\right)}{a^2+2\,ab+b^2} + \frac{1}{(a+b)\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(a*sin(f*x + e)^2 - a - b)/(a^3 + 2*a^2*b + a*b^2) + (a + 2*b)
*log(sin(f*x + e)^2)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(f*x + e)^2))/f
```

mupad [B] time = 4.94, size = 98, normalized size = 1.32

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,a\,f} - \frac{\cot\left(e+f\,x\right)^2}{2\,f\,\left(a+b\right)} - \frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a+2\,b\right)}{f\,\left(a^2+2\,a\,b+b^2\right)} - \frac{b^2\,\ln\left(b\tan\left(e+f\,x\right)^2+a+b\right)}{2\,a\,f\,\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a*f) - cot(e + f*x)^2/(2*f*(a + b)) - (log(tan(e
 + f*x))*(a + 2*b))/(f*(2*a*b + a^2 + b^2)) - (b^2*log(a + b + b*tan(e + f*
x)^2))/(2*a*f*(a + b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{f(a+b)^3}+\frac{b^3\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^3}-\frac{\csc^4(e+fx)}{4f(a+b)}+\frac{(2a+3b)\csc^2(e+fx)}{2f(a+b)^2}$$

```
[Out] 1/2*(2*a+3*b)*csc(f*x+e)^2/(a+b)^2/f-1/4*csc(f*x+e)^4/(a+b)/f+1/2*b^3*ln(b+
a*cos(f*x+e)^2)/a/(a+b)^3/f+(a^2+3*a*b+3*b^2)*ln(sin(f*x+e))/(a+b)^3/f
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{f(a+b)^3}+\frac{b^3\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^3}-\frac{\csc^4(e+fx)}{4f(a+b)}+\frac{(2a+3b)\csc^2(e+fx)}{2f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + 3*b)*Csc[e + f*x]^2)/(2*(a + b)^2*f) - Csc[e + f*x]^4/(4*(a + b)*f)
 + (b^3*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)^3*f) + ((a^2 + 3*a*b + 3*b^
2)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^3(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3} + \frac{-2a-3b}{(a+b)^2(-1+x)^2} + \frac{-a^2-3ab-3b^2}{(a+b)^3(-1+x)} - \frac{b^3}{(a+b)^3(b+ax)}\right) dx, x, \cos^2(e+\right.}{2f}$$

$$= \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2f} - \frac{\csc^4(e+fx)}{4(a+b)f} + \frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3f} + \frac{\left(a^2+3ab+\right.}{}$$

Mathematica [A] time = 0.62, size = 138, normalized size = 1.28

$$\frac{\sec^2(e+fx)(a\cos(2e+2fx)+a+2b)\left(\frac{4\left(a^2+3ab+3b^2\right)\log(\sin(e+fx))}{(a+b)^3} + \frac{2b^3\log\left(-a\sin^2(e+fx)+a+b\right)}{a(a+b)^3} - \frac{\csc^4(e+fx)}{a+b} + \frac{2(2a+3b}{(\right.}{8f\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])*((2*(2*a + 3*b)*Csc[e + f*x]^2)/(a + b)^2 -
 Csc[e + f*x]^4/(a + b) + (4*(a^2 + 3*a*b + 3*b^2)*Log[Sin[e + f*x]])/(a +
b)^3 + (2*b^3*Log[a + b - a*Sin[e + f*x]^2])/(a*(a + b)^3))*Sec[e + f*x]^2)
/(8*f*(a + b*Sec[e + f*x]^2))
```

fricas [B] time = 0.92, size = 265, normalized size = 2.45

$$\frac{3a^3+8a^2b+5ab^2-2\left(2a^3+5a^2b+3ab^2\right)\cos\left(fx+e\right)^2+2\left(b^3\cos\left(fx+e\right)^4-2b^3\cos\left(fx+e\right)^2+b^3\right)\log}{4\left(\left(a^4+3a^3b+3a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^3 + 8*a^2*b + 5*a*b^2 - 2*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)
^2 + 2*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*log(a*cos(f*x + e)
^2 + b) + 4*((a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a
*b^2 - 2*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/(
(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*b-32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
```

```
))^2*a+640*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+384*(1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1)))*a)/(4096*b^2+8192*b*a+4096*a^2)+(-144*((1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1))))^2*b^2-144*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
)))^2*b*a-48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+20*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*b^2+32*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
*b*a+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-b^2-2*b*a-a^2)/(128*b^3
+384*b^2*a+384*b*a^2+128*a^3)/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2-1
/2/a*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+b^3/(4*b^3*a+12*b^2
*a^2+12*b*a^3+4*a^4)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+(3*b^2+3*b*a+a^2
)/(4*b^3+12*b^2*a+12*b*a^2+4*a^3)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+e
xp(1)))))
```

maple [B] time = 1.04, size = 293, normalized size = 2.71

$$\frac{b^3\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2a\left(a+b\right)^3 f}-\frac{1}{2f\left(8a+8b\right)\left(-1+\cos\left(fx+e\right)\right)^2}-\frac{7a}{16f\left(a+b\right)^2\left(-1+\cos\left(fx+e\right)\right)}-\frac{}{16f\left(a+b\right)^2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2*b^3*ln(b+a*cos(f*x+e)^2)/a/(a+b)^3/f-1/2/f/(8*a+8*b)/(-1+cos(f*x+e))^2-
7/16/f/(a+b)^2/(-1+cos(f*x+e))*a-11/16/f/(a+b)^2/(-1+cos(f*x+e))*b+1/2/f/(a
+b)^3*ln(-1+cos(f*x+e))*a^2+3/2/f/(a+b)^3*ln(-1+cos(f*x+e))*a*b+3/2/f/(a+b)
^3*ln(-1+cos(f*x+e))*b^2-1/2/f/(8*a+8*b)/(1+cos(f*x+e))^2+7/16/f/(a+b)^2/(1
+cos(f*x+e))*a+11/16/f/(a+b)^2/(1+cos(f*x+e))*b+1/2/f/(a+b)^3*ln(1+cos(f*x+
e))*a^2+3/2/f/(a+b)^3*ln(1+cos(f*x+e))*a*b+3/2/f/(a+b)^3*ln(1+cos(f*x+e))*b
^2
```

maxima [A] time = 0.35, size = 145, normalized size = 1.34

$$\frac{\frac{2\,b^3\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^4+3\,a^3b+3\,a^2b^2+ab^3}+\frac{2\left(a^2+3\,ab+3\,b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,(2\,a+3\,b)\sin\left(fx+e\right)^2-a-b}{\left(a^2+2\,ab+b^2\right)\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
) + 2*(a^2 + 3*a*b + 3*b^2)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3) + (2*(2*a + 3*b)*sin(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*sin(f*x
+ e)^4))/f
```

mupad [B] time = 4.91, size = 160, normalized size = 1.48

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a^2+3\,a\,b+3\,b^2\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\ln\left({\tan\left(e+f\,x\right)}^2+1\right)}{2\,a\,f}-\frac{\ln\left(b\,{\tan\left(e+f\,x\right)}^2+a+b\right)\,\left(\frac{b}{2\,{\left(a+b\right)}^2}+\frac{1}{2\,\left(a+b\right)}+\frac{}{2\,(}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x))*(3*a*b + a^2 + 3*b^2))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3
)) - log(tan(e + f*x)^2 + 1)/(2*a*f) - (log(a + b + b*tan(e + f*x)^2)*(b/(2
*(a + b)^2) + 1/(2*(a + b)) + b^2/(2*(a + b)^3) - 1/(2*a)))/f - (cot(e + f*
x)^4*(1/(4*(a + b)) - (tan(e + f*x)^2*(a + 2*b))/(2*(a + b)^2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=83

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}-\frac{x}{a}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -x/a+(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(5/2)/f-(a+2*b)
*tan(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A] time = 0.27, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 479, 582, 522, 203, 205}

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}-\frac{x}{a}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + ((a + b)^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(5
/2)*f) - ((a + 2*b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+3(a+2b)x^2\right)}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{3bf}$$

$$= -\frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} + \frac{\text{Subst}\left(\int \frac{3(a+b)(a+2b)+3\left(a^2+3ab+3b^2\right)x^2}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \ldots\right.}{3b^2f}$$

$$= -\frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a+\ldots}{\ldots}$$

$$= -\frac{x}{a} + \frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f} - \frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf}$$

Mathematica [C] time = 2.79, size = 229, normalized size = 2.76

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\frac{(3a+7b)\sec(e)\sin(fx)\sec(e+fx)}{b^2f} - \frac{3(a+b)^{5/2}(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec\ldots}{2\sqrt{a+b}\ldots}\right.}{ab^2f\sqrt{b(\cos(e)-i\sin(e))\ldots}}\ldots\right.}{6\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-3*x)/a - (3*(a + b)^(5/2)
*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*
```

```
e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin
[2*e]))/(a*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - ((3*a + 7*b)*Sec[e]*Sec[e
 + f*x]*Sin[f*x])/(b^2*f) + (Sec[e]*Sec[e + f*x]^3*Sin[f*x])/(b*f) + (Sec[e
 + f*x]^2*Tan[e])/(b*f)))/(6*(a + b*Sec[e + f*x]^2))
```

fricas [B] time = 0.54, size = 373, normalized size = 4.49

$$\left[-\frac{12\,b^2 f x\cos\left(fx+e\right)^3 - 3\left(a^2+2\,ab+b^2\right)\sqrt{-\frac{a+b}{b}}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-\cdots}{a^2\cos\left(fx+e\right)^4\cdots}\right)\cdots}{12\,ab^2 f\cos\left(fx\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*b^2*f*x*cos(f*x + e)^3 - 3*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/b)*
cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt
(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + 4*((3*a^2 + 7*a*b)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*
cos(f*x + e)^3), -1/6*(6*b^2*f*x*cos(f*x + e)^3 + 3*(a^2 + 2*a*b + b^2)*sqr
t((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a
+ b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((3*a^2 + 7*a*b)*cos(f*
x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^3)]
```

giac [A] time = 6.31, size = 132, normalized size = 1.59

$$-\frac{\frac{3\left(fx+e\right)}{a}-\frac{3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,ab^2}-\frac{b^2\tan\left(fx+e\right)^3-3\,ab\tan\left(fx+e\right)-6\,b^2\tan\left(fx+e\right)}{b^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(f*x + e)/a - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((f*x + e)
/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2
)*a*b^2) - (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x + e) - 6*b^2*tan(f*x + e))/b
^3)/f
```

maple [B] time = 0.84, size = 186, normalized size = 2.24

$$\frac{\tan^3\left(fx+e\right)}{3bf}-\frac{a\tan\left(fx+e\right)}{f\,b^2}-\frac{2\tan\left(fx+e\right)}{bf}+\frac{a^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,b^2\sqrt{\left(a+b\right)b}}+\frac{3a\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{fb\sqrt{\left(a+b\right)b}}+\frac{3\arctan\left(\frac{\tan\left(fx+e\right)\cdots}{\sqrt{\left(a+b\right)b}}\right)}{f\sqrt{\left(a+b\right)b}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/3*tan(f*x+e)^3/b/f-1/f/b^2*a*tan(f*x+e)-2*tan(f*x+e)/b/f+1/f/b^2*a^2/((a+
b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/f/b*a/((a+b)*b)^(1/2)*ar
ctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/
((a+b)*b)^(1/2))+1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2
))-1/f/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 95, normalized size = 1.14

$$-\frac{\frac{3\left(fx+e\right)}{a}-\frac{b\tan\left(fx+e\right)^3-3\left(a+2\,b\right)\tan\left(fx+e\right)}{b^2}-\frac{3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\sqrt{\left(a+b\right)b}\,ab^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(f*x + e)/a - (b*tan(f*x + e)^3 - 3*(a + 2*b)*tan(f*x + e))/b^2 - 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sq
rt((a + b)*b)*a*b^2))/f
```

mupad [B] time = 4.92, size = 1109, normalized size = 13.36

$$\frac{\tan\left(e+fx\right)^3}{3\,b\,f}-\frac{\mathrm{atan}\left(\frac{40\,a^2\tan\left(e+f\,x\right)}{30\,a\,b+40\,a^2+10\,b^2+\frac{30\,a^3}{b}+\frac{12\,a^4}{b^2}+\frac{2\,a^5}{b^3}}+\frac{30\,a^3\tan\left(e+f\,x\right)}{30\,a\,b^2+40\,a^2\,b+30\,a^3+10\,b^3+\frac{12\,a^4}{b}+\frac{2\,a^5}{b^2}}+\frac{12\,a^4\tan\left(e+f\,x\right)}{30\,a\,b^3+30\,a^3\,b+12\,a^4+10\,b^4+\dots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) - atan((40*a^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b
^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a^3*tan(e + f*x))/(30*a
*b^2 + 40*a^2*b + 30*a^3 + 10*b^3 + (12*a^4)/b + (2*a^5)/b^2) + (12*a^4*tan
(e + f*x))/(30*a*b^3 + 30*a^3*b + 12*a^4 + 10*b^4 + 40*a^2*b^2 + (2*a^5)/b)
 + (2*a^5*tan(e + f*x))/(30*a*b^4 + 12*a^4*b + 2*a^5 + 10*b^5 + 40*a^2*b^3
+ 30*a^3*b^2) + (10*b^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b^2 + (30*a^3)/
b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a*b*tan(e + f*x))/(30*a*b + 40*a^2 +
10*b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3))/(a*f) - (tan(e + f*x)*(a
 + 2*b))/(b^2*f) - (atan((((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5
 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-
b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f
*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5))*1i
)/(2*a*b^5) + ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b +
 a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 - ((-b^5*(a + b)^
5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(8*a^2*b
^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5))
/((2*(12*a*b^4 + 6*a^4*b + a^5 + 3*b^5 + 19*a^2*b^3 + 15*a^3*b^2))/b^3 - ((
-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 1
5*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-b^5*(a + b)^5)^(1/2)*((8*a^2
*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*
(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5) + ((-b^5*(a + b)^5)
^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*
a^3*b^3 + 15*a^4*b^2))/b^3 - ((-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b
^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5
)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5)))*(-b^5*(a + b)^5)^(1/2)*1i)/(a*b^
5*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=59

$$-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{x}{a}+\frac{\tan(e+fx)}{bf}$$

```
[Out] x/a-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(3/2)/f+tan(f*x+
e)/b/f
```

Rubi [A] time = 0.17, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 479, 522, 203, 205}

$$-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{x}{a}+\frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(3/2)
*f) + Tan[e + f*x]/(b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{bf}$$
$$= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} - \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan\right.}{abf}$$
$$= \frac{x}{a} - \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

Mathematica [C] time = 1.10, size = 206, normalized size = 3.49

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sqrt{b(\sin(e)+i\cos(e))^4}\,(a\sec(e)\sin(fx)\sec(e+fx)+bfx)+(a+\right.}{2abf\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\,\left(a+b\sec^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((a + b)^2*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(b*f*x + a*Sec[e]*Sec[e + f*x]*Sin[f*x])))
/(2*a*b*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [B] time = 0.58, size = 297, normalized size = 5.03

$$\left[\frac{4bfx\cos\left(fx+e\right)+(a+b)\sqrt{-\frac{a+b}{b}}\cos\left(fx+e\right)\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left(\left(ab+2b^2\right)\cos\left(fx+e\right.\right.}{a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b^2}\right.}{4abf\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x*cos(f*x + e) + (a + b)*sqrt(-(a + b)/b)*cos(f*x + e)*log(((a^
2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((
a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*a*sin(f*x
+ e))/(a*b*f*cos(f*x + e)), 1/2*(2*b*f*x*cos(f*x + e) + (a + b)*sqrt((a + b
)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*a*sin(f*x + e))/(a*b*f*cos(f*x +
e))]
```

giac [A] time = 3.03, size = 91, normalized size = 1.54

$$\frac{\frac{fx+e}{a}+\frac{\tan\left(fx+e\right)}{b}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(a^2+2\,ab+b^2\right)}{\sqrt{ab+b^2}\,ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arct
an(b*tan(f*x + e)/sqrt(a*b + b^2)))*(a^2 + 2*a*b + b^2)/(sqrt(a*b + b^2)*a*
b))/f
```

maple [B] time = 0.68, size = 121, normalized size = 2.05

$$\frac{\tan\left(fx+e\right)}{bf}-\frac{a\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{fb\sqrt{(a+b)\,b}}-\frac{2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)\,b}}-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{fa\sqrt{(a+b)\,b}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] tan(f*x+e)/b/f-1/f/b*a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
-2/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b/a/((a+b)*b)
^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 66, normalized size = 1.12

$$\frac{\frac{fx+e}{a}+\frac{\tan\left(fx+e\right)}{b}-\frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (a^2 + 2*a*b + b^2)*arctan(b*tan(f*x + e)/s
qrt((a + b)*b))/(sqrt((a + b)*b)*a*b))/f
```

mupad [B] time = 4.68, size = 410, normalized size = 6.95

$$\frac{\mathrm{atan}\left(\frac{8\,a^2\tan\left(e+f\,x\right)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}+\frac{2\,a^3\tan\left(e+f\,x\right)}{2\,a^3+8\,a^2\,b+12\,a\,b^2+6\,b^3}+\frac{6\,b^2\tan\left(e+f\,x\right)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}+\frac{12\,a\,b\tan\left(e+f\,x\right)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}\right)}{a\,f}+\frac{\tan\left(e+f\,x\right)}{b\,f}+\frac{\mathrm{atanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atan((8*a^2*tan(e + f*x))/(12*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (2*a^3*tan
(e + f*x))/(12*a*b^2 + 8*a^2*b + 2*a^3 + 6*b^3) + (6*b^2*tan(e + f*x))/(12*
a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (12*a*b*tan(e + f*x))/(12*a*b + 8*a^2 +
6*b^2 + (2*a^3)/b))/(a*f) + tan(e + f*x)/(b*f) + (atanh((6*tan(e + f*x)*(-
3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^2 + 20*a^2*b + 10*a^3 +
 6*b^3 + (2*a^4)/b) + (6*a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*
b^3)^(1/2))/(18*a*b^3 + 10*a^3*b + 2*a^4 + 6*b^4 + 20*a^2*b^2) + (2*a^2*tan
(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^4 + 2*a^4*
b + 6*b^5 + 20*a^2*b^3 + 10*a^3*b^2))*(-b^3*(a + b)^3)^(1/2))/(a*b^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,f}-\frac{x}{a}$$

```
[Out] -x/a+arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a/f/b^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4141, 1975, 481, 203, 205}

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,f}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b
]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{x}{a} + \frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,f}$$

Mathematica [C] time = 0.29, size = 184, normalized size = 4.00

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+(a+b)(\cos(2e)-i\sin(2e))\tan^{-1}\left(\cdots\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b
*(Cos[e] - I*Sin[e])^4] + (a + b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e])))/(a*Sqrt[a + b]*f*(a + b*Sec[e + f
*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.54, size = 226, normalized size = 4.91

$$\left[-\frac{4\,fx-\sqrt{-\frac{a+b}{b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{4\,af}, \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x - sqrt(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*c
os(f*x + e))*sqrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/(a*f), -1/2*(2*f*x + sqrt((a + b)/b)*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*
x + e))))/(a*f)]
```

giac [A] time = 0.72, size = 69, normalized size = 1.50

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+b)}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*(a + b)/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

maple [A] time = 0.72, size = 75, normalized size = 1.63

$$\frac{\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)\,b}}+\frac{b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{fa\sqrt{(a+b)\,b}}-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b/a/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 45, normalized size = 0.98

$$\frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] ((a + b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x
+ e)/a)/f
```

mupad [B] time = 4.72, size = 126, normalized size = 2.74

$$-\frac{\mathrm{atan}\left(\frac{2\,a\,b^2\,\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\,\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{a\,f}-\frac{\mathrm{atanh}\left(\frac{2\,a\,b^2\,\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{2\,a^2\,b^2+2\,a\,b^3}\right)\,\sqrt{-b\,\left(a+b\right)}}{a\,b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] - atan((2*a*b^2*tan(e + f*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(e + f*x))/
(2*a*b^2 + 2*a^2*b))/(a*f) - (atanh((2*a*b^2*tan(e + f*x)*(- a*b - b^2)^(1/
2))/(2*a*b^3 + 2*a^2*b^2))*(-b*(a + b))^(1/2))/(a*b*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sec^2(e+fx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\ &= \frac{x}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx,x,\cot(e+fx)\right)}{af}\\ &= \frac{x}{a}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [C] time = 0.23, size = 182, normalized size = 4.04

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+b(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\ldots}{\ldots}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [A] time = 0.59, size = 231, normalized size = 5.13

$$\left[\frac{4\,fx + \sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\,af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

giac [A] time = 0.30, size = 68, normalized size = 1.51

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

maple [A] time = 0.58, size = 48, normalized size = 1.07

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{fa\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a*arctan(
tan(f*x+e))
```

maxima [A] time = 0.45, size = 44, normalized size = 0.98

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\sqrt{\left(a+b\right)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

mupad [B] time = 4.86, size = 460, normalized size = 10.22

$$\frac{x}{a}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2b^3\tan\left(e+fx\right)-\frac{\left(2a^2b^2-\frac{\tan\left(e+fx\right)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\left(a+b\right)}}{4\left(a^2+ba\right)}\right)\sqrt{-b\left(a+b\right)}}{2\left(a^2+ba\right)}\right)\sqrt{-b\left(a+b\right)}\,1i}{a^2+ba}+\frac{\left(2b^3\tan\left(e+fx\right)+\frac{\left(2a^2b^2+\frac{\tan\left(e+fx\right)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\left(a\right.}}{4\left(a^2+ba\right)}\cdots}{2\left(a^2+ba\right)}\cdots}{a^2+ba}}{\frac{\left(2b^3\tan\left(e+fx\right)-\frac{\left(2a^2b^2-\frac{\tan\left(e+fx\right)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\left(a+b\right)}}{4\left(a^2+ba\right)}\right)\sqrt{-b\left(a+b\right)}}{2\left(a^2+ba\right)}\right)\sqrt{-b\left(a+b\right)}}{a^2+ba}-\frac{\left(2b^3\tan\left(e+fx\right)+\frac{\left(2a^2b^2+\frac{\tan\left(e+fx\right)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\left(a\right.}}{4\left(a^2+ba\right)}\cdots}{2\left(a^2+ba\right)}\cdots}{a^2+ba}}\right.}{f\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}-\frac{x}{a}$$

```
[Out] -x/a+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(3/2)/f-cot(f*x
+e)/(a+b)/f
```

Rubi [A] time = 0.17, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 480, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(3
/2)*f) - Cot[e + f*x]/((a + b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b\left(1+x^2\right)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-2b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$
$$= -\frac{\cot(e+fx)}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+\right.}{a(a+b)f}$$
$$= -\frac{x}{a} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

Mathematica [C] time = 1.35, size = 204, normalized size = 3.29

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(b^2(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2e+fx)-(a+2b)\sin(f\right.}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{2af(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}\,\left(a+b\,\mathrm{se}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b^2*ArcTan[(Sec[f*x]*(
Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*((a + b)*f*x - a*Csc[e]*Csc[e + f*x]*Sin[f*
x])))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^
4])
```

fricas [B] time = 0.50, size = 310, normalized size = 5.00

$$\left[-\frac{4(a+b)fx\sin(fx+e) - b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4 - 2(3ab+4b^2)\cos(fx+e)^2 - 4\left((a^2+3ab+2b^2)\cos(fx+e)^3 - (ab+b\right.}{a^2\cos(fx+e)^4 + 2ab\cos(fx+e)^2 + b^2}\right.}{4(a^2+ab)f\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a + b)*f*x*sin(f*x + e) - b*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b
+ 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x +
e) + 4*a*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), -1/2*(2*(a + b)*f*x*si
n(f*x + e) + b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*a*cos(f*x + e
))/((a^2 + a*b)*f*sin(f*x + e))]
```

giac [A] time = 1.06, size = 91, normalized size = 1.47

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{(a^2+ab)\sqrt{ab+b^2}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*b^2/((a^2 + a*b)*sqrt(a*b + b^2)) - (f*x + e)/a - 1/((a + b)*tan(f*x +
 e)))/f
```

maple [A] time = 0.74, size = 73, normalized size = 1.18

$$\frac{b^2\arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f(a+b)a\sqrt{(a+b)b}}-\frac{1}{f(a+b)\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/(a+b)*b^2/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/(a
+b)/tan(f*x+e)-1/f/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.45, size = 66, normalized size = 1.06

$$\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+ab)\sqrt{(a+b)b}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + a*b)*sqrt((a + b)*b)) -
 (f*x + e)/a - 1/((a + b)*tan(f*x + e)))/f
```

mupad [B] time = 6.25, size = 637, normalized size = 10.27

$$-\frac{a b^2+2a^2b+a^3+a^3\tan(e+fx)\,\mathrm{atan}(\tan(e+fx))+b^3\tan(e+fx)\,\mathrm{atan}(\tan(e+fx))+3ab^2\tan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] -(a*b^2 + 2*a^2*b + a^3 - atan((a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4
 - a^3*b^3)^(3/2)*1i + b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^
3)^(3/2)*2i + b^7*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2
)*2i + a*b^6*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*10i
 + a^6*b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i + a^
2*b^5*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*21i + a^3*
b^4*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*24i + a^4*b^
3*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*16i + a^5*b^2*
tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*6i)/(3*a*b^9 + 1
8*a^2*b^8 + 46*a^3*b^7 + 65*a^4*b^6 + 55*a^5*b^5 + 28*a^6*b^4 + 8*a^7*b^3 +
 a^8*b^2))*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i +
a^3*tan(e + f*x)*atan(tan(e + f*x)) + b^3*tan(e + f*x)*atan(tan(e + f*x)) +
 3*a*b^2*tan(e + f*x)*atan(tan(e + f*x)) + 3*a^2*b*tan(e + f*x)*atan(tan(e
+ f*x)))/(a^4*f*tan(e + f*x) + a*b^3*f*tan(e + f*x) + 3*a^3*b*f*tan(e + f*x
) + 3*a^2*b^2*f*tan(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}+\frac{(a+2b)\cot(e+fx)}{f(a+b)^2}+\frac{x}{a}$$

```
[Out] x/a-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(5/2)/f+(a+2*b)*
cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f
```

Rubi [A] time = 0.25, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 480, 583, 522, 203, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}+\frac{(a+2b)\cot(e+fx)}{f(a+b)^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(5/2)
*f) + ((a + 2*b)*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3(a+2b)-3bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)-3b(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+\right.}{3(a+b)^2 f}$$

$$= \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} - \frac{b^3\,\text{Subst}\left(\right.}{}$$

$$= \frac{x}{a} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2} f} + \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

Mathematica [C] time = 3.38, size = 390, normalized size = 4.53

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{1}{8}\sqrt{a+b}\,\csc(e)\sqrt{b(\cos(e)-i\sin(e))^4}\,\csc^3(e+fx)\left(-12a^2\sin(2e+fx)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*b^3*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(9*(a + b)^2*f*x*Cos[f*
```

```
x] - 9*(a + b)^2*f*x*Cos[2*e + f*x] - 3*a^2*f*x*Cos[2*e + 3*f*x] - 6*a*b*f*
x*Cos[2*e + 3*f*x] - 3*b^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x
] + 6*a*b*f*x*Cos[4*e + 3*f*x] + 3*b^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*
x] - 24*a*b*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 18*a*b*Sin[2*e + f*x] + 8*a^
2*Sin[2*e + 3*f*x] + 14*a*b*Sin[2*e + 3*f*x]))/8))/(6*a*(a + b)^(5/2)*f*(a
+ b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

fricas [B] time = 0.53, size = 533, normalized size = 6.20

$$\left[\frac{4\left(4a^2+7ab\right)\cos\left(fx+e\right)^3+3\left(b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2\cdots}{a^2\cos(\cdots}\right.}{12\left(\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)^2 - b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a^2 + 2*a*b)*cos(f*x + e) + 12*((
a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f*x)*sin(f*x +
e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
*sin(f*x + e)), 1/6*(2*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)
^2 - b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/
(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a^2 + 2*a*b)*cos(
f*x + e) + 6*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*
f*x)*sin(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^
2*b + a*b^2)*f)*sin(f*x + e))]
```

giac [A] time = 0.33, size = 140, normalized size = 1.63

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b^3}{\left(a^3+2a^2b+ab^2\right)\sqrt{ab+b^2}}-\frac{3\left(fx+e\right)}{a}-\frac{3a\tan\left(fx+e\right)^2+6b\tan\left(fx+e\right)^2-a-b}{\left(a^2+2ab+b^2\right)\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b + b^2)) - 3*(f*x + e)/a -
 (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan
(f*x + e)^3))/f
```

maple [A] time = 1.06, size = 110, normalized size = 1.28

$$-\frac{b^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^2a\sqrt{\left(a+b\right)b}}-\frac{1}{3f\left(a+b\right)\tan\left(fx+e\right)^3}+\frac{a}{f\left(a+b\right)^2\tan\left(fx+e\right)}+\frac{2b}{f\left(a+b\right)^2\tan\left(fx+e\right)}+\frac{\arctan\left(\mathrm{t}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/(a+b)^2*b^3/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/3
/f/(a+b)/tan(f*x+e)^3+1/f/(a+b)^2/tan(f*x+e)*a+2/f/(a+b)^2/tan(f*x+e)*b+1/f
/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.46, size = 106, normalized size = 1.23

$$-\frac{\frac{3\,b^3\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^3+2\,a^2b+ab^2\right)\sqrt{(a+b)b}}-\frac{3\left(fx+e\right)}{a}-\frac{3\,(a+2\,b)\tan\left(fx+e\right)^2-a-b}{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 2*a^2*b + a*b^2)
*sqrt((a + b)*b)) - 3*(f*x + e)/a - (3*(a + 2*b)*tan(f*x + e)^2 - a - b)/((
a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

mupad [B] time = 8.97, size = 2644, normalized size = 30.74

result too large to display

Verification of antidervative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atan((10*b^12*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b
^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 +
22*a^9*b^3 + 2*a^10*b^2) + (80*a*b^11*tan(e + f*x))/(80*a*b^11 + 10*b^12 +
290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*
a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (290*a^2*b^10*tan(e + f*
x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a
^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2)
 + (630*a^3*b^9*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3
*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4
+ 22*a^9*b^3 + 2*a^10*b^2) + (912*a^4*b^8*tan(e + f*x))/(80*a*b^11 + 10*b^1
2 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 +
330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (922*a^5*b^7*tan(e +
 f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 92
2*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b
^2) + (660*a^6*b^6*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*
a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b
^4 + 22*a^9*b^3 + 2*a^10*b^2) + (330*a^7*b^5*tan(e + f*x))/(80*a*b^11 + 10*
b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6
 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (110*a^8*b^4*tan(
e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 +
 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^1
0*b^2) + (22*a^9*b^3*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 63
0*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8
*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (2*a^10*b^2*tan(e + f*x))/(80*a*b^11 + 10
*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^
6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2))/(a*f) - (1/(3*(a
+ b)) - (tan(e + f*x)^2*(a + 2*b))/(a + b)^2)/(f*tan(e + f*x)^3) - (atan(((
(-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 +
280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a
^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 - ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12
+ 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 64
4*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 - (tan(e
+ f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 +
2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 +
 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5))
)/(2*a*(a + b)^5))*1i)/(a*(a + b)^5) + ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*
x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^
5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/
```

```
2 + ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*
a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*
b^4 + 22*a^11*b^3 + 2*a^12*b^2 + (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a
^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712
*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*
a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5))*1i)/(a*(a + b)^5
))/(26*a*b^11 + 4*b^12 + 72*a^2*b^10 + 110*a^3*b^9 + 100*a^4*b^8 + 54*a^5*b
^7 + 16*a^6*b^6 + 2*a^7*b^5 + ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*
b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 4
22*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 - ((-b^
5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 +
 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*
a^11*b^3 + 2*a^12*b^2 - (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 +
 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8
+ 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3
+ 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5)))/(a*(a + b)^5) - ((-b^5*(
a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3
*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5
+ 20*a^9*b^4 + 2*a^10*b^3))/2 + ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^
3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b
^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 + (tan(e + f*x)*
(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^
5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a
^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*
(a + b)^5)))/(a*(a + b)^5)))*(-b^5*(a + b)^5)^(1/2)*1i)/(a*f*(a + b)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=120

$$-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{f(a+b)^3}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{7/2}}-\frac{\cot^5(e+fx)}{5f(a+b)}+\frac{(a+2b)\cot^3(e+fx)}{3f(a+b)^2}-\frac{x}{a}$$

```
[Out] -x/a+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(7/2)/f-(a^2+3*
a*b+3*b^2)*cot(f*x+e)/(a+b)^3/f+1/3*(a+2*b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(
f*x+e)^5/(a+b)/f
```

Rubi [A] time = 0.34, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 480, 583, 522, 203, 205}

$$-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{f(a+b)^3}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{7/2}}-\frac{\cot^5(e+fx)}{5f(a+b)}+\frac{(a+2b)\cot^3(e+fx)}{3f(a+b)^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(7
/2)*f) - ((a^2 + 3*a*b + 3*b^2)*Cot[e + f*x])/((a + b)^3*f) + ((a + 2*b)*Co
t[e + f*x]^3)/(3*(a + b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5(a+2b)-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{\text{Subst}\left(\int \frac{-15(a^2+3ab+3b^2)-15b(a+2b)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \text{ta}\right.}{15(a+b)^2f}$$

$$= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{a} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f} - \frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+}{3(a+b)^2f}$$

Mathematica [C] time = 2.84, size = 671, normalized size = 5.59

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\csc(e)\csc^5(e+fx)\left(180a^3\sin(2e+fx)-140a^3\sin(2e+3fx)-90a^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-480*b^4*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + Csc[e]*Csc[e + f*x]^5*(-150*(a + b)^3*f
*x*Cos[f*x] + 150*(a + b)^3*f*x*Cos[2*e + f*x] + 75*a^3*f*x*Cos[2*e + 3*f*x
] + 225*a^2*b*f*x*Cos[2*e + 3*f*x] + 225*a*b^2*f*x*Cos[2*e + 3*f*x] + 75*b^
3*f*x*Cos[2*e + 3*f*x] - 75*a^3*f*x*Cos[4*e + 3*f*x] - 225*a^2*b*f*x*Cos[4*
e + 3*f*x] - 225*a*b^2*f*x*Cos[4*e + 3*f*x] - 75*b^3*f*x*Cos[4*e + 3*f*x] -
 15*a^3*f*x*Cos[4*e + 5*f*x] - 45*a^2*b*f*x*Cos[4*e + 5*f*x] - 45*a*b^2*f*x
*Cos[4*e + 5*f*x] - 15*b^3*f*x*Cos[4*e + 5*f*x] + 15*a^3*f*x*Cos[6*e + 5*f*
x] + 45*a^2*b*f*x*Cos[6*e + 5*f*x] + 45*a*b^2*f*x*Cos[6*e + 5*f*x] + 15*b^3
*f*x*Cos[6*e + 5*f*x] + 280*a^3*Sin[f*x] + 780*a^2*b*Sin[f*x] + 680*a*b^2*S
in[f*x] + 180*a^3*Sin[2*e + f*x] + 540*a^2*b*Sin[2*e + f*x] + 480*a*b^2*Sin
[2*e + f*x] - 140*a^3*Sin[2*e + 3*f*x] - 420*a^2*b*Sin[2*e + 3*f*x] - 400*a
*b^2*Sin[2*e + 3*f*x] - 90*a^3*Sin[4*e + 3*f*x] - 240*a^2*b*Sin[4*e + 3*f*x
] - 180*a*b^2*Sin[4*e + 3*f*x] + 46*a^3*Sin[4*e + 5*f*x] + 132*a^2*b*Sin[4*
e + 5*f*x] + 116*a*b^2*Sin[4*e + 5*f*x])))/(960*a*(a + b)^3*f*(a + b*Sec[e
+ f*x]^2))
```

fricas [B] time = 0.62, size = 833, normalized size = 6.94

$$\left[-\frac{4\left(23\,a^3+66\,a^2b+58\,ab^2\right)\cos\left(fx+e\right)^5-20\left(7\,a^3+21\,a^2b+20\,ab^2\right)\cos\left(fx+e\right)^3-15\left(b^3\cos\left(fx+e\right)^4-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 20*(7*a^3 + 21*a^
2*b + 20*a*b^2)*cos(f*x + e)^3 - 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e
)^2 + b^3)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(
3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (
a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
+ e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(a^3 + 3*a^2*b + 3*
a*b^2)*cos(f*x + e) + 60*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^
4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*
cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), -1/30*(2*(23*a^3 + 66*
a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 10*(7*a^3 + 21*a^2*b + 20*a*b^2)*cos(f*x
 + e)^3 + 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*sqrt(b/(a +
b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)
+ 30*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x)
*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

giac [B] time = 0.84, size = 222, normalized size = 1.85

$$\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b^4}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\sqrt{ab+b^2}}-\frac{15\left(fx+e\right)}{a}-\frac{15\,a^2\tan\left(fx+e\right)^4+45\,ab\tan\left(fx+e\right)^4+45\,b^2\tan\left(fx+e\right)^4-5\,a^2\tan\left(fx+e\right)^2-15\,ab\tan\left(f\right.}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b + b^2)) - 15
*(f*x + e)/a - (15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 45*b^2*tan(
f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 - 10*b^2*tan(f*x
+ e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x +
e)^5))/f
```

maple [A] time = 1.25, size = 173, normalized size = 1.44

$$\frac{b^4 \arctan\left(\frac{\tan(fx+e)b}{\sqrt{(a+b)b}}\right)}{f(a+b)^3 a\sqrt{(a+b)b}}-\frac{1}{5f(a+b)\tan(fx+e)^5}+\frac{a}{3f(a+b)^2\tan(fx+e)^3}+\frac{2b}{3f(a+b)^2\tan(fx+e)^3}-\frac{}{f(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/(a+b)^3*b^4/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/5/
f/(a+b)/tan(f*x+e)^5+1/3/f/(a+b)^2/tan(f*x+e)^3*a+2/3/f/(a+b)^2/tan(f*x+e)^
3*b-1/f/(a+b)^3/tan(f*x+e)*a^2-3/f/(a+b)^3/tan(f*x+e)*a*b-3/f/(a+b)^3/tan(f
*x+e)*b^2-1/f/a*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 161, normalized size = 1.34

$$\frac{\frac{15\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{(a+b)b}}-\frac{15\,(fx+e)}{a}-\frac{15\,(a^2+3\,ab+3\,b^2)\tan(fx+e)^4-5\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*sqrt((a + b)*b)) - 15*(f*x + e)/a - (15*(a^2 + 3*a*b + 3*b^2)
*tan(f*x + e)^4 - 5*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b +
3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^5))/f
```

mupad [B] time = 10.17, size = 4324, normalized size = 36.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atan(((((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15 +
 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*a^
8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 + 210
*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b
^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14168*a
^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^11*b^8
 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*a^16*b
^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18 + 248*a
^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a^7*b^13 +
 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^11*b^9 + 8
8088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5 + 1080*a^16
*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a + b)^7))*(-b^7
*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7) + (((tan(e + f*x)*(48*a*b^17 + 4*b^18 +
 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b
```

^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 273
0*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 + (
(-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5
*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 +
22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14
*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (tan(e + f*x)*(-b^7*(a + b
)^7)^(1/2)*(16*a^2*b^18 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25
480*a^6*b^14 + 58968*a^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*
a^10*b^10 + 131560*a^11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*
b^6 + 5320*a^15*b^5 + 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a +
 b)^7)))/(2*a*(a + b)^7))*(-b^7*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7))/(60*a*b
^16 + 6*b^17 + 272*a^2*b^15 + 738*a^3*b^14 + 1332*a^4*b^13 + 1680*a^5*b^12
+ 1512*a^6*b^11 + 972*a^7*b^10 + 438*a^8*b^9 + 132*a^9*b^8 + 24*a^10*b^7 +
2*a^11*b^6 + (((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*
b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12
888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6
 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8
*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 1
4168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^
11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*
a^16*b^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18 +
 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a^7*
b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^11*b
^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5 + 108
0*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a + b)^7))
(-b^7(a + b)^7)^(1/2))/(a*(a + b)^7) - (((tan(e + f*x)*(48*a*b^17 + 4*b^1
8 + 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^
6*b^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 +
2730*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2
+ ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*
a^5*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10
 + 22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a
^14*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (tan(e + f*x)*(-b^7*(a
+ b)^7)^(1/2)*(16*a^2*b^18 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 +
 25480*a^6*b^14 + 58968*a^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 1544
40*a^10*b^10 + 131560*a^11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^
14*b^6 + 5320*a^15*b^5 + 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(
a + b)^7)))/(2*a*(a + b)^7))*(-b^7*(a + b)^7)^(1/2))/(a*(a + b)^7)))*(-b^7*
(a + b)^7)^(1/2)*1i)/(a*f*(a + b)^7) - (1/(5*(a + b)) + (tan(e + f*x)^4*(3*
a*b + a^2 + 3*b^2))/(a + b)^3 - (tan(e + f*x)^2*(a + 2*b))/(3*(a + b)^2))/(
f*tan(e + f*x)^5) - atan((14*b^17*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952
*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11
 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^
11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (952*a^
2*b^15*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 +
 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a
^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 24
0*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (3388*a^3*b^14*tan(e + f*x))/(168*
a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5
*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8
736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 +
 2*a^15*b^2) + (8484*a^4*b^13*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2
*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 2
5722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b
^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (15848*a^5*
b^12*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8
484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8
*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*
a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (22808*a^6*b^11*tan(e + f*x))/(168*a

```
*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*
b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 87
36*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 +
2*a^15*b^2) + (25722*a^7*b^10*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2
*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 2
5722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b
^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (22878*a^8*
b^9*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 84
84*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*
b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a
^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (16016*a^9*b^8*tan(e + f*x))/(168*a*b
^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^
12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736
*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*
a^15*b^2) + (8736*a^10*b^7*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^
15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 2572
2*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6
+ 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (3640*a^11*b^6
*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*
a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9
 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13
*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (1120*a^12*b^5*tan(e + f*x))/(168*a*b^16
 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12
+ 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^
10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^1
5*b^2) + (240*a^13*b^4*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 +
 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^
7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 11
20*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (32*a^14*b^3*tan(e
 + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^
13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 160
16*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 +
 32*a^14*b^3 + 2*a^15*b^2) + (2*a^15*b^2*tan(e + f*x))/(168*a*b^16 + 14*b^1
7 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a
^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 +
3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) +
(168*a*b^16*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b
^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22
878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5
 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.350 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)}{2f}-\frac{(a+b)^2}{2a^2bf\left(a\cos^2(e+fx)+b\right)}-\frac{\log(\cos(e+fx))}{b^2f}$$

```
[Out] -1/2*(a+b)^2/a^2/b/f/(b+a*cos(f*x+e)^2)-ln(cos(f*x+e))/b^2/f-1/2*(1/a^2-1/b
^2)*ln(b+a*cos(f*x+e)^2)/f
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)}{2f}-\frac{(a+b)^2}{2a^2bf\left(a\cos^2(e+fx)+b\right)}-\frac{\log(\cos(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a + b)^2/(2*a^2*b*f*(b + a*Cos[e + f*x]^2)) - Log[Cos[e + f*x]]/(b^2*f) -
 ((a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/(2*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)^2}{2a^2bf\left(b+a\cos^2(e+fx)\right)} - \frac{\log(\cos(e+fx))}{b^2f} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+\right.}{2f}$$

Mathematica [A] time = 0.43, size = 109, normalized size = 1.42

$$-\frac{\sec^4(e+fx)(a\cos(2e+2fx)+a+2b)^2\left(\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)+\frac{(a+b)^2}{a^2b\left(a\cos^2(e+fx)+b\right)}+\frac{2\log(\cos(e+f}{b^2}\right.}{8f\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*e + 2*f*x])^2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*x
]^2)) + (2*Log[Cos[e + f*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x
]^2])*Sec[e + f*x]^4)/(f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.66, size = 118, normalized size = 1.53

$$-\frac{a^2b + 2ab^2 + b^3 - \left(a^2b - b^3 + \left(a^3 - ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2 + b\right) + 2\left(a^3\cos\left(fx+e\right)^2 + a^2b\right.}{2\left(a^3b^2f\cos\left(fx+e\right)^2 + a^2b^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b + 2*a*b^2 + b^3 - (a^2*b - b^3 + (a^3 - a*b^2)*cos(f*x + e)^2)*
log(a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(-cos(f*x + e
)))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)2/f*(-1/4/b^2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+1/4/a^2*ln(abs((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+
2))+(-b^3-b^2*a+b*a^2+a^3)/(4*b^3*a^2+4*b^2*a^3)*ln(abs(((1-cos(f*x+exp(1))
```

```
)/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b+((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))
*a+2*b-2*a))+(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1))
)*(1+cos(f*x+exp(1))))*b^3+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-co
s(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b^2*a-((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b*a^2-((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a^3+2*b^3
-6*b^2*a-6*b*a^2+2*a^3)*1/4/b^2/a^2/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*b+((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a+2*b-2*a))
```

maple [A] time = 1.02, size = 126, normalized size = 1.64

$$\frac{\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2fb^2}-\frac{1}{2fb\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}-\frac{1}{fa\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}-\frac{\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2a^2f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/b^2*ln(b+a*cos(f*x+e)^2)-1/2/f/b/(b+a*cos(f*x+e)^2)-1/f/a/(b+a*cos(f*
x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f-1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-ln(co
s(f*x+e))/b^2/f
```

maxima [A] time = 0.35, size = 98, normalized size = 1.27

$$\frac{\frac{a^2+2\,ab+b^2}{a^3b\sin\left(fx+e\right)^2-a^3b-a^2b^2}-\frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}+\frac{\left(a^2-b^2\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^2b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)/(a^3*b*sin(f*x + e)^2 - a^3*b - a^2*b^2) - log(sin
(f*x + e)^2 - 1)/b^2 + (a^2 - b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^2*b^2))
/f
```

mupad [B] time = 4.65, size = 170, normalized size = 2.21

$$\frac{\ln\left(b\tan\left(e+f\,x\right)^2+a+b\right)}{2\,b^2\,f}-\frac{\ln\left(b\tan\left(e+f\,x\right)^2+a+b\right)}{2\,a^2\,f}+\frac{a^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}+\frac{}{2\,f\,\left(a^2\,b^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(a + b + b*tan(e + f*x)^2)/(2*b^2*f) - log(a + b + b*tan(e + f*x)^2)/(2*
a^2*f) + a^2/(2*f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + b^2/(2*f*(a*b
^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + log(tan(e + f*x)^2 + 1)/(2*a^2*f) +
 (a*b)/(f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

3.351 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

```
[Out] 1/2*(a+b)/a^2/f/(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$\frac{a+b}{2a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*f*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^2
*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x(1-x^2)}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2} - \frac{1}{a(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{a+b}{2a^2f\left(b+a\cos^2(e+fx)\right)} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

Mathematica [A] time = 0.72, size = 81, normalized size = 1.59

$$\frac{(a+2b)\log(a\cos(2(e+fx))+a+2b)+a\cos(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+2(a+b)}{2a^2f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x
)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*x)
]))
```

fricas [A] time = 0.54, size = 53, normalized size = 1.04

$$\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+a+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + a + b)/(a^3*f*cos(f
*x + e)^2 + a^2*b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-1/2/a^2*ln(abs((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))+1))+1/4/a^2*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+
(-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+6*(1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))*a-b-a)*1/4/a^2/(((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x
```

```
+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a
+b+a))
```

maple [A] time = 1.00, size = 68, normalized size = 1.33

$$\frac{\ln\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}{2a^{2}f}+\frac{1}{2fa\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}+\frac{b}{2a^{2}f\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2*ln(b+a*cos(f*x+e)^2)/a^2/f+1/2/f/a/(b+a*cos(f*x+e)^2)+1/2*b/a^2/f/(b+a*
cos(f*x+e)^2)
```

maxima [A] time = 0.36, size = 59, normalized size = 1.16

$$-\frac{\frac{a+b}{a^{3}\sin\left(fx+e\right)^{2}-a^{3}-a^{2}b}-\frac{\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a
 - b)/a^2)/f
```

mupad [B] time = 4.59, size = 97, normalized size = 1.90

$$-\frac{\mathrm{atanh}\left(\frac{4\,b^{2}\tan\left(e+f\,x\right)^{2}}{8\,b^{2}+\frac{8\,b^{3}}{a}+4\,b^{2}\tan\left(e+f\,x\right)^{2}+\frac{8\,b^{3}\tan\left(e+f\,x\right)^{2}}{a}}\right)}{a^{2}\,f}-\frac{a+b}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^{2}+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] - atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 +
(8*b^3*tan(e + f*x)^2)/a))/(a^2*f) - (a + b)/(2*a*b*f*(a + b + b*tan(e + f*
x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.352 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{b}{2a^2f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

```
[Out] -1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{b}{2a^2f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -b/(2*a^2*f*(b + a*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^2*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{b}{2a^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

Mathematica [A] time = 0.48, size = 79, normalized size = 1.61

$$-\frac{(a+2b)\log(a\cos(2(e+fx))+a+2b)+a\cos(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+2b}{2a^2f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x)
]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(a^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

fricas [A] time = 0.53, size = 52, normalized size = 1.06

$$-\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + b)/(a^3*f*cos(f*x
+ e)^2 + a^2*b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/2/a^2*ln(abs((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))+1))-1/4/a^2*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+(
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+2*((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1))))^2*b*a+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+2*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-4*(1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))*b*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+b^2+2*b*a+a^2)/(
4*b*a^2+4*a^3)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+e
```

```
xp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*
b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a))
```

maple [A] time = 0.33, size = 59, normalized size = 1.20

$$-\frac{\ln\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)}{2f\,a^2}+\frac{1}{2fa\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/a^2*ln(a+b*sec(f*x+e)^2)+1/2/f/a/(a+b*sec(f*x+e)^2)+1/f/a^2*ln(sec(f
*x+e))
```

maxima [A] time = 0.36, size = 57, normalized size = 1.16

$$\frac{\frac{b}{a^3\sin\left(fx+e\right)^2-a^3-a^2b}-\frac{\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a - b)/a
^2)/f
```

mupad [B] time = 4.48, size = 90, normalized size = 1.84

$$\frac{\mathrm{atanh}\left(\frac{4\,b^2\tan\left(e+f\,x\right)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan\left(e+f\,x\right)^2+\frac{8\,b^3\tan\left(e+f\,x\right)^2}{a}}\right)}{a^2\,f}+\frac{1}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8
*b^3*tan(e + f*x)^2)/a))/(a^2*f) + 1/(2*a*f*(a + b + b*tan(e + f*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.353 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2f(a+b)\left(a\cos^2(e+fx)+b\right)}+\frac{b(2a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^2}+\frac{\log(\sin(e+fx))}{f(a+b)^2}$$

```
[Out] 1/2*b^2/a^2/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*b*(2*a+b)*ln(b+a*cos(f*x+e)^2)/a
^2/(a+b)^2/f+ln(sin(f*x+e))/(a+b)^2/f
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 88}

$$\frac{b^2}{2a^2f(a+b)\left(a\cos^2(e+fx)+b\right)}+\frac{b(2a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^2}+\frac{\log(\sin(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^2/(2*a^2*(a + b)*f*(b + a*Cos[e + f*x]^2)) + (b*(2*a + b)*Log[b + a*Cos[e
 + f*x]^2])/(2*a^2*(a + b)^2*f) + Log[Sin[e + f*x]]/((a + b)^2*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b^2}{a(a+b)(b+ax)^2} - \frac{b(2a+b)}{a(a+b)^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{b^2}{2a^2(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f} + \frac{\log(\sin(e}{(a+b}$$

Mathematica [A] time = 0.33, size = 112, normalized size = 1.35

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{b^2(a+b)}{a^2\left(-a\sin^2(e+fx)+a+b\right)} + \frac{b(2a+b)\log\left(-a\sin^2(e+fx)+a+b\right)}{a^2} + 2\log(\sin(e+fx))\right)}{8f(a+b)^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(2*Log[Sin[e + f*x]] + (b*
(2*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (b^2*(a + b))/(a^2*(a + b -
a*Sin[e + f*x]^2))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [A] time = 0.72, size = 138, normalized size = 1.66

$$\frac{ab^2 + b^3 + \left(2\,ab^2 + b^3 + \left(2\,a^2b + ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2 + b\right) + 2\left(a^3\cos\left(fx+e\right)^2 + a^2b\right)\log\left(\right.}{2\left(\left(a^5 + 2\,a^4b + a^3b^2\right)f\cos\left(fx+e\right)^2 + \left(a^4b + 2\,a^3b^2 + a^2b^3\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2 + b^3 + (2*a*b^2 + b^3 + (2*a^2*b + a*b^2)*cos(f*x + e)^2)*log(a
*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(1/2*sin(f*x + e))
)/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^
3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(4*a^2+8*a*b+4*b
^2)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))-1/2/a^2*ln(abs((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(2*a*b+b^2)/(4*a^4+8*a^3*b+4*a^2*b^2)
*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+((1-cos(f*x+exp(1)))/(1+c
```

```
os(f*x+exp(1))))^2*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+2*(1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a+b)+(-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^2*a*b-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+4*(1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*b^2-2*a*b-b^2)/(4*a^3+4*a^2*b)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))
^2*a+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*(1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1)))*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a+b))
```

maple [A] time = 1.09, size = 155, normalized size = 1.87

$$\frac{b\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{f\left(a+b\right)^2 a}+\frac{b^2\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\left(a+b\right)^2 a^2}+\frac{b^2}{2f\left(a+b\right)^2 a\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}+\frac{}{2a^2\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*b/(a+b)^2/a*ln(b+a*cos(f*x+e)^2)+1/2/f*b^2/(a+b)^2/a^2*ln(b+a*cos(f*x+e
)^2)+1/2/f*b^2/(a+b)^2/a/(b+a*cos(f*x+e)^2)+1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(
f*x+e)^2)+1/2/f/(a+b)^2*ln(-1+cos(f*x+e))+1/2/f/(a+b)^2*ln(1+cos(f*x+e))
```

maxima [A] time = 0.35, size = 117, normalized size = 1.41

$$\frac{\frac{b^2}{a^4+2\,a^3b+a^2b^2-\left(a^4+a^3b\right)\sin\left(fx+e\right)^2}+\frac{\left(2\,ab+b^2\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^4+2\,a^3b+a^2b^2}+\frac{\log\left(\sin\left(fx+e\right)^2\right)}{a^2+2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x + e)^2) + (2*a*b
+ b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 2*a^3*b + a^2*b^2) + log(sin(f*
x + e)^2)/(a^2 + 2*a*b + b^2))/f
```

mupad [B] time = 4.85, size = 106, normalized size = 1.28

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)}{f\,\left(a^2+2\,a\,b+b^2\right)}-\frac{\ln\left({\tan\left(e+f\,x\right)}^2+1\right)}{2\,a^2\,f}-\frac{b}{2\,a\,f\,\left(a+b\right)\,\left(b\,{\tan\left(e+f\,x\right)}^2+a+b\right)}+\frac{b\,\ln\left(b\,{\tan\left(e+f\,x\right)}^2+\right.}{2\,a^2\,f\,(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x))/(f*(2*a*b + a^2 + b^2)) - log(tan(e + f*x)^2 + 1)/(2*a^2*
f) - b/(2*a*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + (b*log(a + b + b*tan(e
+ f*x)^2)*(2*a + b))/(2*a^2*f*(a + b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.354 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=111

$$-\frac{b^3}{2a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b^2(3a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^3}-\frac{\csc^2(e+fx)}{2f(a+b)^2}-\frac{(a+3b)\log(\sin(e+fx))}{f(a+b)^3}$$

```
[Out] -1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^2/f-1/2*b^
2*(3*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^3/f-(a+3*b)*ln(sin(f*x+e))/(a+b)^3
/f
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$-\frac{b^3}{2a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b^2(3a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^3}-\frac{\csc^2(e+fx)}{2f(a+b)^2}-\frac{(a+3b)\log(\sin(e+fx))}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -b^3/(2*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)
^2*f) - (b^2*(3*a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a^2*(a + b)^3*f) - ((a
 + 3*b)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2} + \frac{a+3b}{(a+b)^3(-1+x)} - \frac{b^3}{a(a+b)^2(b+ax)^2} + \frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right) dx, x, \cos^2(\right.}{2f}$$

$$= -\frac{b^3}{2a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\csc^2(e+fx)}{2(a+b)^2f} - \frac{b^2(3a+b)\log\left(b+a\cos^2(\right.}{2a^2(a+b)^3f}$$

Mathematica [A] time = 1.31, size = 130, normalized size = 1.17

$$-\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{b^2\left(\frac{2b(a+b)}{a\cos(2(e+fx))+a+2b}+(3a+b)\log\left(-a\sin^2(e+fx)+a+b\right)\right)}{a^2} + (a+b)\csc^2(e+fx) \right.}{8f(a+b)^3\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*(e + f*x)])^2*((a + b)*Csc[e + f*x]^2 + 2*(a + 3*b
)*Log[Sin[e + f*x]] + (b^2*((2*b*(a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) +
(3*a + b)*Log[a + b - a*Sin[e + f*x]^2]))/a^2)*Sec[e + f*x]^4)/((a + b)^3*f
*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.94, size = 312, normalized size = 2.81

$$\frac{a^3b + a^2b^2 + ab^3 + b^4 + \left(a^4 + a^3b - ab^3 - b^4\right)\cos\left(fx+e\right)^2 - \left(\left(3\,a^2b^2 + ab^3\right)\cos\left(fx+e\right)^4 - 3\,ab^3 - b^4 - \left(3\,a\right.\right.}{2\left(\left(a^6 + 3\,a^5b + 3\,a^4b^2 + a^3b^3\right)f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*b + a^2*b^2 + a*b^3 + b^4 + (a^4 + a^3*b - a*b^3 - b^4)*cos(f*x +
e)^2 - ((3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - 3*a*b^3 - b^4 - (3*a^2*b^2 - 2
*a*b^3 - b^4)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 2*((a^4 + 3*a^3*b
)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - (a^4 + 2*a^3*b - 3*a^2*b^2)*cos(f*x
+ e)^2)*log(1/2*sin(f*x + e)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
```

```
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))/(16*b^2+32*b*a+16*a^2)+(8*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^3*b^4+32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^3*a+
36*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^2*a^2+16*((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1))))^3*b*a^3+4*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^3*a^4+16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^4-16*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*b^3*a-27*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*b^2*a^2-22*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a^3-11*((
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^4+8*(1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1)))*b^4+32*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3*a+30*(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2*a^2+16*(1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1)))*b*a^3+10*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^4-3*b^2*a^2-6*
b*a^3-3*a^4)/(48*b^3*a^2+144*b^2*a^3+144*b*a^4+48*a^5)/(((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^3*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a+2*
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^2*a+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*a)+1/2/a^2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))+1))+(-3*b-a)/(4*b^3+12*b^2*a+12*b*a^2+4*a^3)*ln(abs(1-cos(f*x+exp(
1)))/abs(1+cos(f*x+exp(1))))+(-b^3-3*b^2*a)/(4*b^3*a^2+12*b^2*a^3+12*b*a^4+
4*a^5)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a))
```

maple [B] time = 1.27, size = 240, normalized size = 2.16

$$-\frac{3b^2\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\left(a+b\right)^3a}-\frac{b^3\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\left(a+b\right)^3a^2}-\frac{b^3}{2f\left(a+b\right)^3a\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}-\frac{}{2a^2\left(a+b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -3/2/f*b^2/(a+b)^3/a*ln(b+a*cos(f*x+e)^2)-1/2/f*b^3/(a+b)^3/a^2*ln(b+a*cos(
f*x+e)^2)-1/2/f*b^3/(a+b)^3/a/(b+a*cos(f*x+e)^2)-1/2*b^4/a^2/(a+b)^3/f/(b+a
*cos(f*x+e)^2)+1/4/f/(a+b)^2/(-1+cos(f*x+e))-1/2/f/(a+b)^3*ln(-1+cos(f*x+e)
)*a-3/2/f/(a+b)^3*ln(-1+cos(f*x+e))*b-1/4/f/(a+b)^2/(1+cos(f*x+e))-1/2/f/(a
+b)^3*ln(1+cos(f*x+e))*a-3/2/f/(a+b)^3*ln(1+cos(f*x+e))*b
```

maxima [A] time = 0.36, size = 192, normalized size = 1.73

$$-\frac{\frac{\left(3\,ab^2+b^3\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3}+\frac{\left(a+3\,b\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{a^3+a^2b-\left(a^3-b^3\right)\sin\left(fx+e\right)^2}{\left(a^5+2\,a^4b+a^3b^2\right)\sin\left(fx+e\right)^4-\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3) + (a + 3*b)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) - (a^3 + a^2*b - (a^3 - b^3)*sin(f*x + e)^2)/((a^5 + 2*a^4*b + a^3*b^2)
*sin(f*x + e)^4 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

mupad [B] time = 5.05, size = 160, normalized size = 1.44

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,a^2\,f}-\frac{\frac{1}{2\,\left(a+b\right)}+\frac{\tan\left(e+f\,x\right)^2\,\left(a\,b-b^2\right)}{2\,a\,\left(a+b\right)^2}}{f\,\left(b\tan\left(e+f\,x\right)^4+\left(a+b\right)\,\tan\left(e+f\,x\right)^2\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a+3\,b\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{b^2\,\ln\left(b\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a^2*f) - (1/(2*(a + b)) + (tan(e + f*x)^2*(a*b -
 b^2))/(2*a*(a + b)^2))/(f*(tan(e + f*x)^2*(a + b) + b*tan(e + f*x)^4)) - (
log(tan(e + f*x))*(a + 3*b))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^2*log
(a + b + b*tan(e + f*x)^2)*(3*a + b))/(2*a^2*f*(a + b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

3.355 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=140

$$\frac{b^4}{2a^2f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{b^3(4a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^4}+\frac{\left(a^2+4ab+6b^2\right)\log(\sin(e+fx))}{f(a+b)^4}-\frac{\csc^4(}{4f(}$$

[Out] 1/2*b^4/a^2/(a+b)^3/f/(b+a*cos(f*x+e)^2)+(a+2*b)*csc(f*x+e)^2/(a+b)^3/f-1/4 *csc(f*x+e)^4/(a+b)^2/f+1/2*b^3*(4*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^4/f+ (a^2+4*a*b+6*b^2)*ln(sin(f*x+e))/(a+b)^4/f

Rubi [A] time = 0.20, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{b^4}{2a^2f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+4ab+6b^2\right)\log(\sin(e+fx))}{f(a+b)^4}+\frac{b^3(4a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^4}-\frac{\csc^4(}{4f(}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]

[Out] b^4/(2*a^2*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) + ((a + 2*b)*Csc[e + f*x]^2) /((a + b)^3*f) - Csc[e + f*x]^4/(4*(a + b)^2*f) + (b^3*(4*a + b)*Log[b + a* Cos[e + f*x]^2])/(2*a^2*(a + b)^4*f) + ((a^2 + 4*a*b + 6*b^2)*Log[Sin[e + f *x]])/((a + b)^4*f)

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^9}{(1-x^2)^3(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^3(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)^3} - \frac{2(a+2b)}{(a+b)^3(-1+x)^2} + \frac{-a^2-4ab-6b^2}{(a+b)^4(-1+x)} + \frac{b^4}{a(a+b)^3(b+ax)^2} - \frac{b^3(4a+b}{a(a+b)^4(b+}\right.\right.}{2f}$$
$$= \frac{b^4}{2a^2(a+b)^3f\left(b+a\cos^2(e+fx)\right)} + \frac{(a+2b)\csc^2(e+fx)}{(a+b)^3f} - \frac{\csc^4(e+fx)}{4(a+b)^2f} + \frac{b^3(}{}$$

Mathematica [A] time = 1.84, size = 162, normalized size = 1.16

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{2b^4(a+b)}{a^2\left(-a\sin^2(e+fx)+a+b\right)} + \frac{2b^3(4a+b)\log\left(-a\sin^2(e+fx)+a+b\right)}{a^2} + 4\left(a^2+4ab+6b\right.\right.}{16f(a+b)^4\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(4*(a + b)*(a + 2*b)*Csc[e
 + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 4*a*b + 6*b^2)*Log[Sin[e +
f*x]] + (2*b^3*(4*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (2*b^4*(a + b
))/(a^2*(a + b - a*Sin[e + f*x]^2))))/(16*(a + b)^4*f*(a + b*Sec[e + f*x]^2
)^2)
```

fricas [B] time = 1.56, size = 557, normalized size = 3.98

$$\frac{3a^4b + 10a^3b^2 + 7a^2b^3 + 2ab^4 + 2b^5 - 2\left(2a^5 + 6a^4b + 4a^3b^2 - ab^4 - b^5\right)\cos\left(fx+e\right)^4 + \left(3a^5 + 6a^4b - 5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5 - 2*(2*a^5 + 6*a^4*
b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4 + (3*a^5 + 6*a^4*b - 5*a^3*b^2
- 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2 + 2*((4*a^2*b^3 + a*b^4)*cos(
f*x + e)^6 + 4*a*b^4 + b^5 - (8*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (
4*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*
((a^5 + 4*a^4*b + 6*a^3*b^2)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 - 6*a^2*b^3)*cos(f*x + e)^4 + (a^5 + 2*a^4*
b - 2*a^3*b^2 - 12*a^2*b^3)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^7 +
4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^
6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7
+ 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^2*b^5-5*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))
^2*b^4*a-4*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^3*a^2-2*(1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))*b^5-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
*b^4*a+8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3*a^2-b^5-5*b^4*a-4*b^3*
a^2)/(4*b^4*a^2+16*b^3*a^3+24*b^2*a^4+16*b*a^5+4*a^6)/(((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))*a+b+a)+(-288*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2-192*((
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a-48*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^2*a^2+28*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2+40*(1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))*a^2-b^2-2*b*a-a^2)/(128*b^4+512*b^3*a+768*b^2*a^2+512*b*a^3+128*a^
4)/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2+(-32*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^2*b^2-64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a
-32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+896*(1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1)))*b^2+1280*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a+3
84*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2)/(4096*b^4+16384*b^3*a+24576
*b^2*a^2+16384*b*a^3+4096*a^4)-1/2/a^2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1)))+1))+(b^4+4*b^3*a)/(4*b^4*a^2+16*b^3*a^3+24*b^2*a^4+16*b*a^5+4*a^
6)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+(6*b^2+4*b*a+a^2)/(4*b^4+16*b^3*a+
24*b^2*a^2+16*b*a^3+4*a^4)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))
))
```

maple [B] time = 1.22, size = 374, normalized size = 2.67

$$\frac{2b^3\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{f\left(a+b\right)^4a}+\frac{b^4\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\left(a+b\right)^4a^2}+\frac{b^4}{2f\left(a+b\right)^4a\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}+\frac{}{2f\left(a+b\right)^4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 2/f*b^3/(a+b)^4/a*ln(b+a*cos(f*x+e)^2)+1/2/f*b^4/(a+b)^4/a^2*ln(b+a*cos(f*x
+e)^2)+1/2/f*b^4/(a+b)^4/a/(b+a*cos(f*x+e)^2)+1/2/f*b^5/(a+b)^4/a^2/(b+a*co
s(f*x+e)^2)-1/16/f/(a+b)^2/(-1+cos(f*x+e))^2-7/16/f/(a+b)^3/(-1+cos(f*x+e))
*a-15/16/f/(a+b)^3/(-1+cos(f*x+e))*b+1/2/f/(a+b)^4*ln(-1+cos(f*x+e))*a^2+2/
f/(a+b)^4*ln(-1+cos(f*x+e))*a*b+3/f/(a+b)^4*ln(-1+cos(f*x+e))*b^2-1/16/f/(a
+b)^2/(1+cos(f*x+e))^2+7/16/f/(a+b)^3/(1+cos(f*x+e))*a+15/16/f/(a+b)^3/(1+c
os(f*x+e))*b+1/2/f/(a+b)^4*ln(1+cos(f*x+e))*a^2+2/f/(a+b)^4*ln(1+cos(f*x+e)
)*a*b+3/f/(a+b)^4*ln(1+cos(f*x+e))*b^2
```

maxima [B] time = 0.34, size = 279, normalized size = 1.99

$$\frac{\frac{2\left(4ab^3+b^4\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4}+\frac{2\left(a^2+4ab+6b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{2\left(2a^4+4a^3b-b^4\right)\sin\left(fx+e\right)^4+a^4+2a^3b+a^2b^2-\left(5a^4+13a^3b+8a^2b^2\right)\sin}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sin\left(fx+e\right)^6-\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4) + 2*(a^2 + 4*a*b + 6*b^2)*log(sin(f*x + e)^2)/(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*(2*a^4 + 4*a^3*b - b^4)*sin
(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - (5*a^4 + 13*a^3*b + 8*a^2*b^2)*sin(
f*x + e)^2)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^6 - (a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sin(f*x + e)^4))/f
```

mupad [B] time = 6.09, size = 206, normalized size = 1.47

$$\frac{\frac{\tan\left(e+f\,x\right)^2\,\left(2\,a+5\,b\right)}{4\,{\left(a+b\right)}^2}-\frac{1}{4\,\left(a+b\right)}+\frac{\tan\left(e+f\,x\right)^4\,\left(a^2\,b+3\,a\,b^2-b^3\right)}{2\,a\,{\left(a+b\right)}^3}}{f\,\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^6+\left(a+b\right)\,{\mathrm{tan}\left(e+f\,x\right)}^4\right)}-\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)}{2\,a^2\,f}+\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)\right)\,\left(a^2+4\,a\,b+6\right.}{f\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(2*a + 5*b))/(4*(a + b)^2) - 1/(4*(a + b)) + (tan(e + f*x)
^4*(3*a*b^2 + a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^4*(a + b) + b
*tan(e + f*x)^6)) - log(tan(e + f*x)^2 + 1)/(2*a^2*f) + (log(tan(e + f*x))*
(4*a*b + a^2 + 6*b^2))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (b
^3*log(a + b + b*tan(e + f*x)^2)*(4*a + b))/(2*a^2*f*(a + b)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.356 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}-\frac{x}{a^2}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -x/a^2-1/2*(3*a-2*b)*(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2
/b^(5/2)/f+1/2*(3*a+b)*tan(f*x+e)/a/b^2/f-1/2*(a+b)*tan(f*x+e)^3/a/b/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.27, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 582, 522, 203, 205}

$$-\frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}-\frac{x}{a^2}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) - ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(2*a^2*b^(5/2)*f) + ((3*a + b)*Tan[e + f*x])/(2*a*b^2*f) - ((a + b)*
Tan[e + f*x]^3)/(2*a*b*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(3a+b)x^2\right)}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2abf}$$

$$= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{(a+b)(3a+b)+\left(3a^2\right.}{(1+x^2)\left(a+b\right.}\right.}{2}$$

$$= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\right.}{a^2f}$$

$$= -\frac{x}{a^2} - \frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f} + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a}{2abf\left(a\right.}$$

Mathematica [C] time = 4.63, size = 286, normalized size = 2.40

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\frac{(a+b)^2((a+2b)\sin(2e)-a\sin(2fx))}{a^2b^2f(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{(3a-2b)(a+b)^{3/2}(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))}{a^2b^2f}\right.}{8\left(a+b\sec\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*
Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[
2*e]))/(a^2*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x])/(b^2*f) - ((a + b)^2*((a + 2*b)*Sin
[2*e] - a*Sin[2*f*x]))/(a^2*b^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8
*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.58, size = 514, normalized size = 4.32

$$\left[-\frac{8\,ab^2fx\cos\left(fx+e\right)^3+8\,b^3fx\cos\left(fx+e\right)+\left(\left(3\,a^3+a^2b-2\,ab^2\right)\cos\left(fx+e\right)^3+\left(3\,a^2b+ab^2-2\,b^3\right)\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b^2*f*x*cos(f*x + e)^3 + 8*b^3*f*x*cos(f*x + e) + ((3*a^3 + a^2*
b - 2*a*b^2)*cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(
-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*c
os(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(
a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
b^2)) - 4*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x + e)
)/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e)), -1/4*(4*a*b^2*f*x*co
s(f*x + e)^3 + 4*b^3*f*x*cos(f*x + e) - ((3*a^3 + a^2*b - 2*a*b^2)*cos(f*x
+ e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a + b)/b)*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f
*x + e))) - 2*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x
+ e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e))]
```

giac [A] time = 8.75, size = 163, normalized size = 1.37

$$-\frac{\frac{2\left(fx+e\right)}{a^2}-\frac{2\tan\left(fx+e\right)}{b^2}+\frac{\left(3\,a^3+4\,a^2b-ab^2-2\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^2b^2}-\frac{a^2\tan\left(fx+e\right)+2\,ab\tan\left(fx+e\right)+b^2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)ab^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/a^2 - 2*tan(f*x + e)/b^2 + (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^2*b^2) - (a^2*tan(f*x + e) + 2*a*b*tan(f*x + e)
+ b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b^2))/f
```

maple [B] time = 0.77, size = 242, normalized size = 2.03

$$\frac{\tan\left(fx+e\right)}{b^2f}+\frac{a\tan\left(fx+e\right)}{2f\,b^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{\tan\left(fx+e\right)}{fb\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{\tan\left(fx+e\right)}{2af\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] tan(f*x+e)/b^2/f+1/2/f*a/b^2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/f/b*tan(f*x+
e)/(a+b+b*tan(f*x+e)^2)+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-3/2/f*a/b^2
/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f/b/((a+b)*b)^(1/2)
*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a/((a+b)*b)^(1/2)*arctan(tan(f*
x+e)*b/((a+b)*b)^(1/2))+1/f*b/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b
)*b)^(1/2))-1/f/a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.43, size = 127, normalized size = 1.07

$$\frac{\frac{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)}{ab^3\tan\left(fx+e\right)^2+a^2b^2+ab^3}-\frac{2\left(fx+e\right)}{a^2}+\frac{2\,\tan\left(fx+e\right)}{b^2}-\frac{\left(3\,a^3+4\,a^2b-ab^2-2\,b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*tan(f*x + e)/(a*b^3*tan(f*x + e)^2 + a^2*b^2 + a*b
^3) - 2*(f*x + e)/a^2 + 2*tan(f*x + e)/b^2 - (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2*b^2))/f
```

mupad [B] time = 4.99, size = 765, normalized size = 6.43

$$\frac{\tan\left(e+f\,x\right)}{b^2\,f}-\frac{\mathrm{atan}\left(\frac{5\tan\left(e+f\,x\right)}{\frac{12\,a}{b}-\frac{10\,b}{a}-\frac{15\,b^2}{2\,a^2}+\frac{9\,a^2}{2\,b^2}+5}-\frac{10\tan\left(e+f\,x\right)}{\frac{5\,a}{b}-\frac{15\,b}{2\,a}+\frac{12\,a^2}{b^2}+\frac{9\,a^3}{2\,b^3}-10}+\frac{12\,a\tan\left(e+f\,x\right)}{12\,a+5\,b-\frac{10\,b^2}{a}+\frac{9\,a^2}{2\,b}-\frac{15\,b^3}{2\,a^2}}-\frac{15\,b\tan\left(e+f\,x\right)}{2\left(\frac{5\,a^2}{b}-\frac{15\,b}{2}-10\,a+\frac{12\,a^3}{b^2}+\frac{9\,a^4}{2\,b^3}\right.}\right.}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(b^2*f) - atan((5*tan(e + f*x))/((12*a)/b - (10*b)/a - (15*b^2
)/(2*a^2) + (9*a^2)/(2*b^2) + 5) - (10*tan(e + f*x))/((5*a)/b - (15*b)/(2*a
) + (12*a^2)/b^2 + (9*a^3)/(2*b^3) - 10) + (12*a*tan(e + f*x))/(12*a + 5*b
- (10*b^2)/a + (9*a^2)/(2*b) - (15*b^3)/(2*a^2)) - (15*b*tan(e + f*x))/(2*(
(5*a^2)/b - (15*b)/2 - 10*a + (12*a^3)/b^2 + (9*a^4)/(2*b^3))) + (9*a^2*tan
(e + f*x))/(2*(12*a*b + (9*a^2)/2 + 5*b^2 - (10*b^3)/a - (15*b^4)/(2*a^2)))
)/(a^2*f) + (tan(e + f*x)*(2*a*b + a^2 + b^2))/(2*a*f*(a*b^2 + b^3 + b^3*ta
n(e + f*x)^2)) - (atan((tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5
)^(1/2)*15i)/(2*((25*a*b^3)/4 - (49*a^3*b)/2 + 9*a^4 + (15*b^4)/2 - (85*a^2
*b^2)/4 + (81*a^5)/(4*b) + (27*a^6)/(4*b^2))) - (tan(e + f*x)*(- 3*a*b^7 -
b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*35i)/(4*(9*a^3*b - (85*a*b^3)/4 + (81*a^4)
/4 + (25*b^4)/4 - (49*a^2*b^2)/2 + (15*b^5)/(2*a) + (27*a^5)/(4*b))) - (tan
(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*45i)/(4*((81*a^3*b)
/4 - (49*a*b^3)/2 + (27*a^4)/4 - (85*b^4)/4 + 9*a^2*b^2 + (25*b^5)/(4*a) +
(15*b^6)/(2*a^2))) + (a^2*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b
^5)^(1/2)*27i)/(4*(9*a^2*b^4 - (85*b^6)/4 - (49*a*b^5)/2 + (81*a^3*b^3)/4 +
 (27*a^4*b^2)/4 + (25*b^7)/(4*a) + (15*b^8)/(2*a^2))) + (a*tan(e + f*x)*(-
3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*((27*a^4*b)/4 - (49*a*b^
4)/2 - (85*b^5)/4 + 9*a^2*b^3 + (81*a^3*b^2)/4 + (25*b^6)/(4*a) + (15*b^7)/
(2*a^2))))*(-b^5*(a + b)^3)^(1/2)*(3*a - 2*b)*1i)/(2*a^2*b^5*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

3.357 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-2b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}+\frac{x}{a^2}-\frac{(a+b)\tan(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2+1/2*(a-2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/b^
(3/2)/f-1/2*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.18, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 470, 522, 203, 205}

$$\frac{(a-2b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}+\frac{x}{a^2}-\frac{(a+b)\tan(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(2*a^2*b^(3/2)*f) - ((a + b)*Tan[e + f*x])/(2*a*b*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} + \frac{((a-2b)(a \cdots}{\cdots}$$

$$= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 2.56, size = 249, normalized size = 2.77

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\left(-a^2+ab+2b^2\right)(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2e+fx\cdots}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(\cdots}\right)}{bf\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\cdots\right)}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + ((-a^2 + a*b + 2*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-
((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(b*Sq
rt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + ((a + b)*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(b*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*S
ec[e + f*x]^2)^2)
```

fricas [B] time = 0.53, size = 393, normalized size = 4.37

$$\left[\frac{8\,abfx\cos\left(fx+e\right)^{2}+8\,b^{2}fx-4\left(a^{2}+ab\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(a^{2}-2\,ab\right)\cos\left(fx+e\right)^{2}+ab-2\,b^{2}\right.}{8\left(a^{3}bf\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*cos(f*x + e)^2 + 8*b^2*f*x - 4*(a^2 + a*b)*cos(f*x + e)*sin
(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(-(a + b)/b)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^3*b
*f*cos(f*x + e)^2 + a^2*b^2*f), 1/4*(4*a*b*f*x*cos(f*x + e)^2 + 4*b^2*f*x -
 2*(a^2 + a*b)*cos(f*x + e)*sin(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 +
a*b - 2*b^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))))/(a^3*b*f*cos(f*x + e)^2 +
 a^2*b^2*f)]
```

giac [A] time = 1.76, size = 126, normalized size = 1.40

$$\frac{\frac{2\left(fx+e\right)}{a^{2}}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)\left(a^{2}-ab-2\,b^{2}\right)}{\sqrt{ab+b^{2}}\,a^{2}b}-\frac{a\tan\left(fx+e\right)+b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)ab}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)/a^2 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - a*b - 2*b^2)/(sqrt(a*b + b^2)*a^2*b) - (a
*tan(f*x + e) + b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b))/f
```

maple [B] time = 0.90, size = 168, normalized size = 1.87

$$-\frac{\tan\left(fx+e\right)}{2fb\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)}+\frac{\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2fb\sqrt{(a+b)\,b}}-\frac{\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2fa\sqrt{(a+b)\,b}}-\frac{\tan\left(fx+e\right)}{2af\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2/f/b/((a+b)*b)^(1/2)*arctan(tan
(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+
b)*b)^(1/2))-1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-1/f*b/a^2/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.43, size = 96, normalized size = 1.07

$$-\frac{\frac{(a+b)\tan\left(fx+e\right)}{ab^{2}\tan\left(fx+e\right)^{2}+a^{2}b+ab^{2}}-\frac{2\left(fx+e\right)}{a^{2}}-\frac{\left(a^{2}-ab-2\,b^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^{2}b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)*tan(f*x + e)/(a*b^2*tan(f*x + e)^2 + a^2*b + a*b^2) - 2*(f*x
+ e)/a^2 - (a^2 - a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt
((a + b)*b)*a^2*b))/f
```

mupad [B] time = 4.78, size = 285, normalized size = 3.17

$$\frac{\operatorname{atan}\left(\frac{\tan(e+fx)}{\frac{3b}{2a}-\frac{a}{2b}+1}-\frac{\tan(e+fx)}{2\left(\frac{b}{a}+\frac{3b^2}{2a^2}-\frac{1}{2}\right)}+\frac{3b\tan(e+fx)}{2\left(a+\frac{3b}{2}-\frac{a^2}{2b}\right)}\right)}{a^2 f}-\frac{\tan\left(e+fx\right)\left(a+b\right)}{2abf\left(b\tan\left(e+fx\right)^2+a+b\right)}-\frac{\operatorname{atanh}\left(\frac{3\tan(e+fx)\sqrt{-b^4-ab^3}}{2\left(\frac{ab}{4}-a^2+\frac{3b^2}{2}+\frac{a^3}{4b}\right)}-\frac{5\tan\cdots}{4(\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan(tan(e + f*x)/((3*b)/(2*a) - a/(2*b) + 1) - tan(e + f*x)/(2*(b/a + (3*b
^2)/(2*a^2) - 1/2)) + (3*b*tan(e + f*x))/(2*(a + (3*b)/2 - a^2/(2*b))))/(a^
2*f) - (tan(e + f*x)*(a + b))/(2*a*b*f*(a + b + b*tan(e + f*x)^2)) - (atanh
((3*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(2*((a*b)/4 - a^2 + (3*b^2)/2 + a^3
/(4*b))) - (5*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*(a^2/4 - a*b + b^2/4 +
 (3*b^3)/(2*a))) + (tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*((a*b)/4 - b^2 +
 b^3/(4*a) + (3*b^4)/(2*a^2))))*(-b^3*(a + b))^(1/2)*(a - 2*b))/(2*a^2*b^3*
f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.358 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,f\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -x/a^2+1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/f/b^(1/2)/(a+
b)^(1/2)+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 471, 522, 203, 205}

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,f\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^2*Sq
rt[b]*Sqrt[a + b]*f) + Tan[e + f*x]/(2*a*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(a+2b)\,\text{Subst}}{}$$

$$= -\frac{x}{a^2} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}\,f} + \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 7.68, size = 346, normalized size = 4.07

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a\cos(2(e+fx))+a+2b)}}{b^{3/2}f} - \frac{\left(a^2+8ab+8b^2\right)((a+2b)\sin(2e)-a\sin(2\ldots}{bf(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))(a\cos(2\ldots}\ldots\right)}{64\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(-((16*x + ((-a^3 + 6*a^2*
b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a +
b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a
^2) + (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)
 - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(
b^(3/2)*f))/(64*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.68, size = 458, normalized size = 5.39

$$\left[-\frac{8\left(a^2b+ab^2\right)fx\cos\left(fx+e\right)^2+8\left(ab^2+b^3\right)fx-4\left(a^2b+ab^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+2\,ab\right)\cos\right.}{8\left(\left(a^4b+a^3b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 8*(a*b^2 + b^3)*f*x - 4*(a^2*
b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
+ 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))
*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^3*b^2 + a^2*b^3)*f)
, -1/4*(4*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b^2 + b^3)*f*x - 2*(a^2
*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
 + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2
+ (a^3*b^2 + a^2*b^3)*f)]
```

giac [A] time = 1.47, size = 99, normalized size = 1.16

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)(a+2\,b)}{\sqrt{ab+b^2}\,a^2}-\frac{2\left(fx+e\right)}{a^2}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(sqrt(a*b + b^2)*a^2) - 2*(f*x + e)/a^2 + tan(f*x + e)/(
(b*tan(f*x + e)^2 + a + b)*a))/f
```

maple [A] time = 0.95, size = 108, normalized size = 1.27

$$\frac{\tan\left(fx+e\right)}{2af\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2fa\sqrt{(a+b)\,b}}+\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\,a^2\sqrt{(a+b)\,b}}-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1/2/f/a/((a+b)*b)^(1/2)*arctan(tan(
f*x+e)*b/((a+b)*b)^(1/2))+1/f*b/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a
+b)*b)^(1/2))-1/f/a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.42, size = 75, normalized size = 0.88

$$\frac{\frac{\tan\left(fx+e\right)}{ab\tan\left(fx+e\right)^2+a^2+ab}+\frac{(a+2\,b)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)/(a*b*tan(f*x + e)^2 + a^2 + a*b) + (a + 2*b)*arctan(b*tan
(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) - 2*(f*x + e)/a^2)/f
```

mupad [B] time = 5.00, size = 711, normalized size = 8.36

$$\frac{\tan\left(e+fx\right)}{2af\left(b\tan\left(e+fx\right)^2+a+b\right)}-\frac{x}{a^2}-\frac{\mathrm{atan}\left(\frac{\left(\frac{\frac{\tan\left(e+fx\right)\left(a^2b+4ab^2+8b^3\right)}{2a^2}-\frac{\left(2ab^2-\frac{\tan\left(e+fx\right)\left(16a^5b^2+32a^4b^3\right)(a+2b)\sqrt{-b(a+b)}}{8a^2\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}}{4\left(a^3b+a^2b^2\right)}\cdots\right)}{\frac{b^2+\frac{ab}{2}}{a^3}-\frac{\frac{\tan\left(e+fx\right)\left(a^2b+4ab^2+8b^3\right)}{2a^2}-\frac{\left(2ab^2-\frac{\tan\left(e+fx\right)\left(16a^5b^2+32a^4b^3\right)(a+2b)\sqrt{-b(a+b)}}{8a^2\left(a^3b+a^2b^2\right)}\right)(a+2b)\cdots}{4\left(a^3b+a^2b^2\right)}}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(2*a*f*(a + b + b*tan(e + f*x)^2)) - x/a^2 - (atan((((tan(e +
 f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^
4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2))
)*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b
))^(1/2)*1i)/(4*(a^3*b + a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b
^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b
)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/
2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2)*1i)/(4*(a^3*b + a^2
*b^2)))/(((a*b)/2 + b^2)/a^3 - (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(
2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*
(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4
*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)) +
(((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f
*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b +
 a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*
(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2))))*(a + 2*b)*(-b*(a + b))^(1/2)*1i
)/(2*f*(a^3*b + a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.359 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} - \frac{(b(3a}{}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 1.92, size = 240, normalized size = 2.61

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(2x(a\cos(2(e+fx))+a+2b)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{f(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b(3a+2b)(c}{}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

fricas [B] time = 0.56, size = 435, normalized size = 4.73

$$\left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2 - 4ab\cos\left(fx+e\right)\sin\left(fx+e\right) + 8\left(ab+b^2\right)fx + \left(\left(3a^2+2ab\right)\cos\left(fx+e\right)^2 + 3a\right.}{8\left(\left(a^4+a^3b\right)f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
```

```
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

giac [A] time = 0.24, size = 119, normalized size = 1.29

$$-\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab+2\,b^2\right)}{\left(a^3+a^2b\right)\sqrt{ab+b^2}}+\frac{b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)\left(a^2+ab\right)}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

maple [A] time = 0.92, size = 127, normalized size = 1.38

$$-\frac{b\tan\left(fx+e\right)}{2a\left(a+b\right)f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2fa\left(a+b\right)\sqrt{\left(a+b\right)b}}-\frac{b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^2\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a/(a+b)/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^2/a^2/(a+b)/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.42, size = 106, normalized size = 1.15

$$-\frac{\frac{b\tan\left(fx+e\right)}{a^3+2\,a^2b+ab^2+\left(a^2b+ab^2\right)\tan\left(fx+e\right)^2}+\frac{\left(3\,ab+2\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\left(a^3+a^2b\right)\sqrt{\left(a+b\right)b}}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

mupad [B] time = 6.39, size = 2056, normalized size = 22.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
```

```
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

3.360 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=121

$$\frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{5/2}}-\frac{x}{a^2}-\frac{(2a-b)\cot(e+fx)}{2af(a+b)^2}-\frac{b\cot(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -x/a^2+1/2*b^(3/2)*(5*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(5/2)/f-1/2*(2*a-b)*cot(f*x+e)/a/(a+b)^2/f-1/2*b*cot(f*x+e)/a/(a+b)/f/(a
+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.25, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 203, 205}

$$\frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{5/2}}-\frac{x}{a^2}-\frac{(2a-b)\cot(e+fx)}{2af(a+b)^2}-\frac{b\cot(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(3/2)*(5*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(5/2)*f) - ((2*a - b)*Cot[e + f*x])/(2*a*(a + b)^2*f) - (b*
Cot[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(a+b\left(1+x^2\right)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-3bx^2}{x^2\left(1+x^2\right)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx\right.}{2a(a+b)f}$$

$$= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{2a^2+6ab+b^2\cdots}{\left(1+x^2\right)\left(a\cdots\right.}\right.}{2a\cdots}$$

$$= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\cdots\right.}{a^2f}$$

$$= -\frac{x}{a^2} + \frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f} - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(\cdots}{2a(a+b)f\left(a+b\cdots\right.}$$

Mathematica [C] time = 3.94, size = 288, normalized size = 2.38

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b^2(a\sin(2fx)-(a+2b)\sin(2e))}{a^2f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} - \frac{b^2(5a+2b)(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\cdots}{a^2f(a+b)^{5/2}\sqrt{\cdots}}\right.}{8\left(a+b\sec^2(e\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(a^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^2*f) + (b^2*(-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x]))/(a^2*(a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[
e]))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.60, size = 604, normalized size = 4.99

$$\left[-\frac{4\left(2a^3+ab^2\right)\cos\left(fx+e\right)^3-\left(5ab^2+2b^3+\left(5a^2b+2ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3 + a*b^2)*cos(f*x + e)^3 - (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a
*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x
 + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a
^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 4*(2*a^2*b
- a*b^2)*cos(f*x + e) + 8*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^
2*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a^3
+ a*b^2)*cos(f*x + e)^3 + (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a*b^2)*cos(f*x +
e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
 b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(2*a^2*b - a*b^2)*cos(
f*x + e) + 4*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))]
```

giac [A] time = 1.31, size = 183, normalized size = 1.51

$$\frac{\frac{\left(5ab^2+2b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^4+2a^3b+a^2b^2\right)\sqrt{ab+b^2}}-\frac{2ab\tan\left(fx+e\right)^2-b^2\tan\left(fx+e\right)^2+2a^2+2ab}{\left(b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)+b\tan\left(fx+e\right)\right)\left(a^3+2a^2b+ab^2\right)}-\frac{2\left(fx+e\right)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b + b^2)) - (2
*a*b*tan(f*x + e)^2 - b^2*tan(f*x + e)^2 + 2*a^2 + 2*a*b)/((b*tan(f*x + e)^
3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^3 + 2*a^2*b + a*b^2)) - 2*(f*x + e)
/a^2)/f
```

maple [A] time = 1.06, size = 149, normalized size = 1.23

$$\frac{b^2\tan\left(fx+e\right)}{2f\left(a+b\right)^2a\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{5b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2f\left(a+b\right)^2a\sqrt{\left(a+b\right)b}}+\frac{b^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^2a^2\sqrt{\left(a+b\right)b}}-\frac{1}{f\left(a+b\right)^2\tan\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^2/(a+b)^2/a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+5/2/f*b^2/(a+b)^2/a/((a
+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b^3/(a+b)^2/a^2/((a+b
)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/(a+b)^2/tan(f*x+e)-1/f/
a^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 163, normalized size = 1.35

$$\frac{\frac{(5ab^2+2b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{(a+b)b}} - \frac{(2ab-b^2)\tan(fx+e)^2+2a^2+2ab}{(a^3b+2a^2b^2+ab^3)\tan(fx+e)^3+(a^4+3a^3b+3a^2b^2+ab^3)\tan(fx+e)} - \frac{2(fx+e)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 2*a^3
*b + a^2*b^2)*sqrt((a + b)*b)) - ((2*a*b - b^2)*tan(f*x + e)^2 + 2*a^2 + 2*
a*b)/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(f*x + e)^3 + (a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*tan(f*x + e)) - 2*(f*x + e)/a^2)/f
```

mupad [B] time = 9.30, size = 3146, normalized size = 26.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan((((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160
*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6
+ 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*(5*
a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 2956
8*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b
^4 + 1408*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5
*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10
+ 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 5
3760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b
 + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^
7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)
*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*
a^5*b^2)) + ((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 +
16160*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10
*b^6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*(a + b)^5)^(1/2
)*(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 +
 29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a
^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/
2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*
b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^
6 + 53760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*
a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b
 + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^
(1/2)*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3
+ 10*a^5*b^2)))/(80*a^5*b^9 - 208*a^3*b^11 - 416*a^4*b^10 - 32*a^2*b^12 + 1
600*a^6*b^8 + 2768*a^7*b^7 + 2272*a^8*b^6 + 944*a^9*b^5 + 160*a^10*b^4 + ((
tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160*a^6*b^10
 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6 + 3280*a^
11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*
```

```
(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 29568*a^10*b^
8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b^4 + 1408
*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)
*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10 + 138240*
a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 53760*a^15
*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b + a^7 +
a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^7 + a^2*b
^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)*(5*a + 2
*b))/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)) -
((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160*a^6*b^
10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6 + 3280*
a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*(a + b)^5)^(1/2)*(5*a + 2*b
)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 29568*a^10*
b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b^4 + 14
08*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*
b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10 + 13824
0*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 53760*a^
15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b + a^7
+ a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^7 + a^2
*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)*(5*a +
 2*b))/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))
)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*1i)/(2*f*(5*a^6*b + a^7 + a^2*b^5 + 5*
a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)) - atan((240*a^3*b^11*tan(e + f*x))/(240
*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 +
18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2)
 + (2080*a^4*b^10*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^
9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^
10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (7760*a^5*b^9*tan(e + f*x))/(240*a^3
*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 1840
0*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (
16384*a^6*b^8*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 +
16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b
^4 + 704*a^11*b^3 + 64*a^12*b^2) + (21584*a^7*b^7*tan(e + f*x))/(240*a^3*b^
11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a
^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (184
00*a^8*b^6*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 163
84*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4
+ 704*a^11*b^3 + 64*a^12*b^2) + (10160*a^9*b^5*tan(e + f*x))/(240*a^3*b^11
+ 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*
b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (3520*a
^10*b^4*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*
a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 7
04*a^11*b^3 + 64*a^12*b^2) + (704*a^11*b^3*tan(e + f*x))/(240*a^3*b^11 + 20
80*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6
+ 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (64*a^12*b^
2*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^
8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^1
1*b^3 + 64*a^12*b^2))/(a^2*f) - (1/(a + b) + (tan(e + f*x)^2*(2*a*b - b^2))
/(2*a*(a + b)^2))/(f*(b*tan(e + f*x)^3 + tan(e + f*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.361 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=160

$$-\frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{7/2}}+\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2af(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\cot^3(e+fx)}{6af(a+b)^2}-\frac{b\cot^3(\cdots}{2af(a+b)\left(a+b\cdots\right.}$$

```
[Out] x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+b
)^(7/2)/f+1/2*(2*a^2+6*a*b-b^2)*cot(f*x+e)/a/(a+b)^3/f-1/6*(2*a-3*b)*cot(f*
x+e)^3/a/(a+b)^2/f-1/2*b*cot(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.35, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 203, 205}

$$-\frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{7/2}}+\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2af(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\cot^3(e+fx)}{6af(a+b)^2}-\frac{b\cot^3(\cdots}{2af(a+b)\left(a+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(7/2)*f) + ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(2*a*(a + b)^3
*f) - ((2*a - 3*b)*Cot[e + f*x]^3)/(6*a*(a + b)^2*f) - (b*Cot[e + f*x]^3)/(
2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-3b-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+\right.}{2a(a+b)f}$$

$$= -\frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{3(2a^2\ldots}{x^{\ldots}}\right.}{\ldots}$$

$$= \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+f\ldots}{2a(a+b)f\left(a+b+b\,\text{t}\ldots\right.}$$

$$= \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+f\ldots}{2a(a+b)f\left(a+b+b\,\text{t}\ldots\right.}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3\ldots}{6\ldots}$$

Mathematica [C] time = 6.80, size = 1588, normalized size = 9.92

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^4(e+fx)\left(\frac{48(7a+2b)\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2(e+fx))a+a+2b)(\cos(2e}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((48*b^3*(7*a + 2*b)*ArcTan[
(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]
))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f
*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
+ Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(-6*(a + b)^3*(a + 6*b)*f*x*Cos[f*x] + 3*(
a - 4*b)*(a + b)^3*f*x*Cos[3*f*x] + 6*a^4*f*x*Cos[2*e - f*x] + 54*a^3*b*f*x
*Cos[2*e - f*x] + 126*a^2*b^2*f*x*Cos[2*e - f*x] + 114*a*b^3*f*x*Cos[2*e -
f*x] + 36*b^4*f*x*Cos[2*e - f*x] + 6*a^4*f*x*Cos[2*e + f*x] + 54*a^3*b*f*x*
Cos[2*e + f*x] + 126*a^2*b^2*f*x*Cos[2*e + f*x] + 114*a*b^3*f*x*Cos[2*e + f
*x] + 36*b^4*f*x*Cos[2*e + f*x] - 6*a^4*f*x*Cos[4*e + f*x] - 54*a^3*b*f*x*C
os[4*e + f*x] - 126*a^2*b^2*f*x*Cos[4*e + f*x] - 114*a*b^3*f*x*Cos[4*e + f*
x] - 36*b^4*f*x*Cos[4*e + f*x] - 3*a^4*f*x*Cos[2*e + 3*f*x] + 3*a^3*b*f*x*C
os[2*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[2*e + 3*f*x] + 33*a*b^3*f*x*Cos[2*e +
3*f*x] + 12*b^4*f*x*Cos[2*e + 3*f*x] + 3*a^4*f*x*Cos[4*e + 3*f*x] - 3*a^3*b
*f*x*Cos[4*e + 3*f*x] - 27*a^2*b^2*f*x*Cos[4*e + 3*f*x] - 33*a*b^3*f*x*Cos[
4*e + 3*f*x] - 12*b^4*f*x*Cos[4*e + 3*f*x] - 3*a^4*f*x*Cos[6*e + 3*f*x] + 3
*a^3*b*f*x*Cos[6*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[6*e + 3*f*x] + 33*a*b^3*f*
x*Cos[6*e + 3*f*x] + 12*b^4*f*x*Cos[6*e + 3*f*x] - 3*a^4*f*x*Cos[2*e + 5*f*
x] - 9*a^3*b*f*x*Cos[2*e + 5*f*x] - 9*a^2*b^2*f*x*Cos[2*e + 5*f*x] - 3*a*b^
3*f*x*Cos[2*e + 5*f*x] + 3*a^4*f*x*Cos[4*e + 5*f*x] + 9*a^3*b*f*x*Cos[4*e +
 5*f*x] + 9*a^2*b^2*f*x*Cos[4*e + 5*f*x] + 3*a*b^3*f*x*Cos[4*e + 5*f*x] - 3
*a^4*f*x*Cos[6*e + 5*f*x] - 9*a^3*b*f*x*Cos[6*e + 5*f*x] - 9*a^2*b^2*f*x*Co
s[6*e + 5*f*x] - 3*a*b^3*f*x*Cos[6*e + 5*f*x] + 3*a^4*f*x*Cos[8*e + 5*f*x]
+ 9*a^3*b*f*x*Cos[8*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[8*e + 5*f*x] + 3*a*b^3*f
*x*Cos[8*e + 5*f*x] - 12*a^4*Sin[f*x] - 60*a^3*b*Sin[f*x] - 96*a^2*b^2*Sin[
f*x] + 18*b^4*Sin[f*x] + 4*a^4*Sin[3*f*x] + 36*a^3*b*Sin[3*f*x] + 80*a^2*b^
2*Sin[3*f*x] - 6*a*b^3*Sin[3*f*x] + 6*b^4*Sin[3*f*x] + 4*a^4*Sin[2*e - f*x]
 + 76*a^3*b*Sin[2*e - f*x] + 144*a^2*b^2*Sin[2*e - f*x] + 18*b^4*Sin[2*e -
f*x] - 4*a^4*Sin[2*e + f*x] - 76*a^3*b*Sin[2*e + f*x] - 144*a^2*b^2*Sin[2*e
 + f*x] + 6*a*b^3*Sin[2*e + f*x] + 18*b^4*Sin[2*e + f*x] - 12*a^4*Sin[4*e +
 f*x] - 60*a^3*b*Sin[4*e + f*x] - 96*a^2*b^2*Sin[4*e + f*x] - 6*a*b^3*Sin[4
*e + f*x] - 18*b^4*Sin[4*e + f*x] - 12*a^4*Sin[2*e + 3*f*x] - 24*a^3*b*Sin[
2*e + 3*f*x] + 6*a*b^3*Sin[2*e + 3*f*x] - 6*b^4*Sin[2*e + 3*f*x] + 4*a^4*Si
n[4*e + 3*f*x] + 36*a^3*b*Sin[4*e + 3*f*x] + 80*a^2*b^2*Sin[4*e + 3*f*x] -
3*a*b^3*Sin[4*e + 3*f*x] - 6*b^4*Sin[4*e + 3*f*x] - 12*a^4*Sin[6*e + 3*f*x]
 - 24*a^3*b*Sin[6*e + 3*f*x] + 3*a*b^3*Sin[6*e + 3*f*x] + 6*b^4*Sin[6*e + 3
*f*x] + 8*a^4*Sin[2*e + 5*f*x] + 20*a^3*b*Sin[2*e + 5*f*x] + 3*a*b^3*Sin[2*
e + 5*f*x] - 3*a*b^3*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5*f*x] + 20*a^3*b*S
in[6*e + 5*f*x]))))/(384*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [B] time = 0.61, size = 979, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(4*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 8*(3*a^4 + 5*a^3*b -
 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(
f*x + e) + 24*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^
4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e)), 1/12*(2*(8*a^4 + 20
*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 4*(3*a^4 + 5*a^3*b - 10*a^2*b^2 + 3*a*b^
3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 7*a*b^3 - 2*b
^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x +
 e)))*sin(f*x + e) - 6*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(f*x + e) + 12*((a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*
b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f*x)*si
n(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6
 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3
*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

giac [A] time = 0.50, size = 220, normalized size = 1.38

$$-\frac{\frac{3\,b^{3}\tan\left(fx+e\right)}{\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)}+\frac{3\left(7\,ab^{3}+2\,b^{4}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{5}+3\,a^{4}b+3\,a^{3}b^{2}+a^{2}b^{3}\right)\sqrt{ab+b^{2}}}-\frac{6\left(fx+e\right)}{a^{2}}-\frac{2\left(3\,a\tan\left(fx+e\right)^{2}+9\,b\,\mathrm{t}\cdots\right.}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b\cdots\right.}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(b*tan(f*x +
e)^2 + a + b)) + 3*(7*a*b^3 + 2*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*sqrt(a*b + b^2)) - 6*(f*x + e)/a^2 - 2*(3*a*tan(f*x + e)^2 + 9*b*tan(f
*x + e)^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

maple [A] time = 1.34, size = 186, normalized size = 1.16

$$-\frac{b^{3}\tan\left(fx+e\right)}{2fa\left(a+b\right)^{3}\left(a+b+b\left(\tan^{2}\left(fx+e\right)\right)\right)}-\frac{7b^{3}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{2fa\left(a+b\right)^{3}\sqrt{\left(a+b\right)b}}-\frac{b^{4}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{\left(a+b\right)b}}\right)}{f\,a^{2}\left(a+b\right)^{3}\sqrt{\left(a+b\right)b}}-\frac{1}{3f\left(a+b\right)^{2}\tan\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b^3/a/(a+b)^3*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-7/2/f*b^3/a/(a+b)^3/((
a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^4/a^2/(a+b)^3/((a+
b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/3/f/(a+b)^2/tan(f*x+e)^3
+1/f/(a+b)^3/tan(f*x+e)*a+3/f/(a+b)^3/tan(f*x+e)*b+1/f/a^2*arctan(tan(f*x+e
))
```

maxima [A] time = 0.45, size = 235, normalized size = 1.47

$$-\frac{\frac{3\left(7\,ab^{3}+2\,b^{4}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{\left(a^{5}+3\,a^{4}b+3\,a^{3}b^{2}+a^{2}b^{3}\right)\sqrt{\left(a+b\right)b}}-\frac{3\left(2\,a^{2}b+6\,ab^{2}-b^{3}\right)\tan\left(fx+e\right)^{4}-2\,a^{3}-4\,a^{2}b-2\,ab^{2}+2\left(3\,a^{3}+11\,a^{2}b+8\,ab^{2}\right)\tan\left(fx+e\right)^{2}}{\left(a^{4}b+3\,a^{3}b^{2}+3\,a^{2}b^{3}+ab^{4}\right)\tan\left(fx+e\right)^{5}+\left(a^{5}+4\,a^{4}b+6\,a^{3}b^{2}+4\,a^{2}b^{3}+ab^{4}\right)\tan\left(fx+e\right)^{3}}-\frac{6\left(fx+e\right)}{a^{2}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(7*a*b^3 + 2*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)) - (3*(2*a^2*b + 6*a*b^2 - b^3
)*tan(f*x + e)^4 - 2*a^3 - 4*a^2*b - 2*a*b^2 + 2*(3*a^3 + 11*a^2*b + 8*a*b^
2)*tan(f*x + e)^2)/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(f*x + e)^5
+ (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*tan(f*x + e)^3) - 6*(f*x
+ e)/a^2)/f
```

mupad [B] time = 10.46, size = 4987, normalized size = 31.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(3*a + 8*b))/(3*(a + b)^2) - 1/(3*(a + b)) + (tan(e + f*x)
^4*(6*a*b^2 + 2*a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) +
 b*tan(e + f*x)^5)) + atan((560*a^3*b^16*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (7280*a^4*b^15*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42
560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120
*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (4
2560*a^5*b^14*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14
+ 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 6
99840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 3584
0*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (149184*a^6*b^1
3*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6
*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b
^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 +
 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (351904*a^7*b^12*tan(e + f*
x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 3519
04*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*
a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b
^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (593440*a^8*b^11*tan(e + f*x))/(560*a^3
*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12
+ 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 2
78768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^
16*b^3 + 64*a^17*b^2) + (741120*a^9*b^10*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (699840*a^10*b^9*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 +
42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 7411
20*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*
a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) +
(505008*a^11*b^8*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^
14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10
+ 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 3
5840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (278768*a^12
*b^7*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*
a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^1
0*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^
5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (116480*a^13*b^6*tan(e +
 f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 3
```

51904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 5050
08*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^1
5*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (35840*a^14*b^5*tan(e + f*x))/(560*a
^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^1
2 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 +
 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*
a^16*b^3 + 64*a^17*b^2) + (7680*a^15*b^4*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (1024*a^16*b^3*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42
560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120
*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (6
4*a^17*b^2*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 1
49184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 6998
40*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a
^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2))/(a^2*f) - (atan((((
tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 + 57680*a^6*b^1
5 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 550560*a^10*b^11
+ 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144*a^14*b^7 + 2
9120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) - ((-b^5*(a + b
)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^8*b^15 + 7104
0*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12*b^11 + 12291
84*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^16*b^7 + 22745
6*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 + 128*a^21*b^2
 - (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b^18 + 7936*a^
8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14 + 1886976*a^1
2*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*b^10 + 420992
0*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19*b^6 + 170240
*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4*(7*a^8*b + a
^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^
2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 3
5*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*1i)/(4*(7*a^8*
b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a
^7*b^2)) + ((tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 +
57680*a^6*b^15 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 5505
60*a^10*b^11 + 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144
*a^14*b^7 + 29120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) +
((-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^
8*b^15 + 71040*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12
*b^11 + 1229184*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^1
6*b^7 + 227456*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 +
 128*a^21*b^2 + (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b
^18 + 7936*a^8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14
+ 1886976*a^12*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*
b^10 + 4209920*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19
*b^6 + 170240*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4
*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^
3 + 21*a^7*b^2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 3
5*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*1
i)/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a
^6*b^3 + 21*a^7*b^2)))/(304*a^4*b^15 - 208*a^3*b^16 - 32*a^2*b^17 + 7040*a^
5*b^14 + 31200*a^6*b^13 + 75936*a^7*b^12 + 118944*a^8*b^11 + 126528*a^9*b^1
0 + 92640*a^10*b^9 + 46000*a^11*b^8 + 14768*a^12*b^7 + 2752*a^13*b^6 + 224*
a^14*b^5 + ((tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 +
57680*a^6*b^15 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 5505
60*a^10*b^11 + 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144
*a^14*b^7 + 29120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) -

```
((-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^
8*b^15 + 71040*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12
*b^11 + 1229184*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 + 528000*a^1
6*b^7 + 227456*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*a^20*b^3 +
 128*a^21*b^2 - (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(512*a^7*b
^18 + 7936*a^8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*a^11*b^14
+ 1886976*a^12*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 4942080*a^15*
b^10 + 4209920*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 582400*a^19
*b^6 + 170240*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^23*b^2))/(4
*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^
3 + 21*a^7*b^2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 3
5*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b))/
(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*
b^3 + 21*a^7*b^2)) - ((tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a
^5*b^16 + 57680*a^6*b^15 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b
^12 + 550560*a^10*b^11 + 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b
^8 + 88144*a^14*b^7 + 29120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^
18*b^3) + ((-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16
+ 13952*a^8*b^15 + 71040*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 9
48992*a^12*b^11 + 1229184*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b^8 +
528000*a^16*b^7 + 227456*a^17*b^6 + 71232*a^18*b^5 + 15360*a^19*b^4 + 2048*
a^20*b^3 + 128*a^21*b^2 + (tan(e + f*x)*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*
(512*a^7*b^18 + 7936*a^8*b^17 + 57600*a^9*b^16 + 259840*a^10*b^15 + 815360*
a^11*b^14 + 1886976*a^12*b^13 + 3331328*a^13*b^12 + 4576000*a^14*b^11 + 494
2080*a^15*b^10 + 4209920*a^16*b^9 + 2818816*a^17*b^8 + 1467648*a^18*b^7 + 5
82400*a^19*b^6 + 170240*a^20*b^5 + 34560*a^21*b^4 + 4352*a^22*b^3 + 256*a^2
3*b^2))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 +
 35*a^6*b^3 + 21*a^7*b^2))))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a
^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3 + 21*a^7*b^2)))*(-b^5*(a + b)^7)^(1/2)*(7*
a + 2*b))/(4*(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4
 + 35*a^6*b^3 + 21*a^7*b^2))))*(-b^5*(a + b)^7)^(1/2)*(7*a + 2*b)*1i)/(2*f*
(7*a^8*b + a^9 + a^2*b^7 + 7*a^3*b^6 + 21*a^4*b^5 + 35*a^5*b^4 + 35*a^6*b^3
 + 21*a^7*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.362 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=207

$$\frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{9/2}}+\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6af(a+b)^3}-\frac{x}{a^2}-\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2af(a+b)^4}$$

```
[Out] -x/a^2+1/2*b^(7/2)*(9*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(9/2)/f-1/2*(2*a^3+8*a^2*b+12*a*b^2-b^3)*cot(f*x+e)/a/(a+b)^4/f+1/6*(2*a
^2+6*a*b-3*b^2)*cot(f*x+e)^3/a/(a+b)^3/f-1/10*(2*a-5*b)*cot(f*x+e)^5/a/(a+b
)^2/f-1/2*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.44, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 203, 205}

$$\frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{9/2}}+\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6af(a+b)^3}-\frac{\left(8a^2b+2a^3+12ab^2-b^3\right)\cot(e+fx)}{2af(a+b)^4}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(9/2)*f) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x]
)/(2*a*(a + b)^4*f) + ((2*a^2 + 6*a*b - 3*b^2)*Cot[e + f*x]^3)/(6*a*(a + b)
^3*f) - ((2*a - 5*b)*Cot[e + f*x]^5)/(10*a*(a + b)^2*f) - (b*Cot[e + f*x]^5
)/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-5b-7bx^2}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx\right.}{2a(a+b)f}$$

$$= -\frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{5(2a^2+6a}{x^4(1}\right.}{}$$

$$= \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx}{2a(a+b)f\left(a+b+b\,\text{ta}\right.}$$

$$= -\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2a(a+b)^4f} + \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-}{1}$$

$$= -\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2a(a+b)^4f} + \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-}{1}$$

$$= -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2a(a+b)^4f} +$$

Mathematica [C] time = 7.36, size = 3028, normalized size = 14.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((9*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-1/8*(b^4*Arc
Tan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((
I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]
) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*C
os[4*e] - I*b*Sin[4*e]]) + ((I/8)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a +
 b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b
*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x
])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b
)^4*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[
e + f*x]^5*Sec[2*e]*Sec[e + f*x]^4*(75*a^5*f*x*Cos[f*x] + 900*a^4*b*f*x*Cos
[f*x] + 2850*a^3*b^2*f*x*Cos[f*x] + 3900*a^2*b^3*f*x*Cos[f*x] + 2475*a*b^4*
f*x*Cos[f*x] + 600*b^5*f*x*Cos[f*x] - 15*a^5*f*x*Cos[3*f*x] + 240*a^4*b*f*x
*Cos[3*f*x] + 1110*a^3*b^2*f*x*Cos[3*f*x] + 1740*a^2*b^3*f*x*Cos[3*f*x] + 1
185*a*b^4*f*x*Cos[3*f*x] + 300*b^5*f*x*Cos[3*f*x] - 75*a^5*f*x*Cos[2*e - f*
x] - 900*a^4*b*f*x*Cos[2*e - f*x] - 2850*a^3*b^2*f*x*Cos[2*e - f*x] - 3900*
a^2*b^3*f*x*Cos[2*e - f*x] - 2475*a*b^4*f*x*Cos[2*e - f*x] - 600*b^5*f*x*Co
s[2*e - f*x] - 75*a^5*f*x*Cos[2*e + f*x] - 900*a^4*b*f*x*Cos[2*e + f*x] - 2
850*a^3*b^2*f*x*Cos[2*e + f*x] - 3900*a^2*b^3*f*x*Cos[2*e + f*x] - 2475*a*b
^4*f*x*Cos[2*e + f*x] - 600*b^5*f*x*Cos[2*e + f*x] + 75*a^5*f*x*Cos[4*e + f
*x] + 900*a^4*b*f*x*Cos[4*e + f*x] + 2850*a^3*b^2*f*x*Cos[4*e + f*x] + 3900
*a^2*b^3*f*x*Cos[4*e + f*x] + 2475*a*b^4*f*x*Cos[4*e + f*x] + 600*b^5*f*x*C
os[4*e + f*x] + 15*a^5*f*x*Cos[2*e + 3*f*x] - 240*a^4*b*f*x*Cos[2*e + 3*f*x
] - 1110*a^3*b^2*f*x*Cos[2*e + 3*f*x] - 1740*a^2*b^3*f*x*Cos[2*e + 3*f*x] -
 1185*a*b^4*f*x*Cos[2*e + 3*f*x] - 300*b^5*f*x*Cos[2*e + 3*f*x] - 15*a^5*f*
x*Cos[4*e + 3*f*x] + 240*a^4*b*f*x*Cos[4*e + 3*f*x] + 1110*a^3*b^2*f*x*Cos[
4*e + 3*f*x] + 1740*a^2*b^3*f*x*Cos[4*e + 3*f*x] + 1185*a*b^4*f*x*Cos[4*e +
 3*f*x] + 300*b^5*f*x*Cos[4*e + 3*f*x] + 15*a^5*f*x*Cos[6*e + 3*f*x] - 240*
a^4*b*f*x*Cos[6*e + 3*f*x] - 1110*a^3*b^2*f*x*Cos[6*e + 3*f*x] - 1740*a^2*b
^3*f*x*Cos[6*e + 3*f*x] - 1185*a*b^4*f*x*Cos[6*e + 3*f*x] - 300*b^5*f*x*Cos
[6*e + 3*f*x] + 45*a^5*f*x*Cos[2*e + 5*f*x] + 120*a^4*b*f*x*Cos[2*e + 5*f*x
] + 30*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[2*e + 5*f*x] - 19
5*a*b^4*f*x*Cos[2*e + 5*f*x] - 60*b^5*f*x*Cos[2*e + 5*f*x] - 45*a^5*f*x*Cos
[4*e + 5*f*x] - 120*a^4*b*f*x*Cos[4*e + 5*f*x] - 30*a^3*b^2*f*x*Cos[4*e + 5
*f*x] + 180*a^2*b^3*f*x*Cos[4*e + 5*f*x] + 195*a*b^4*f*x*Cos[4*e + 5*f*x] +
 60*b^5*f*x*Cos[4*e + 5*f*x] + 45*a^5*f*x*Cos[6*e + 5*f*x] + 120*a^4*b*f*x*
Cos[6*e + 5*f*x] + 30*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[6*
e + 5*f*x] - 195*a*b^4*f*x*Cos[6*e + 5*f*x] - 60*b^5*f*x*Cos[6*e + 5*f*x] -
 45*a^5*f*x*Cos[8*e + 5*f*x] - 120*a^4*b*f*x*Cos[8*e + 5*f*x] - 30*a^3*b^2*
f*x*Cos[8*e + 5*f*x] + 180*a^2*b^3*f*x*Cos[8*e + 5*f*x] + 195*a*b^4*f*x*Cos
[8*e + 5*f*x] + 60*b^5*f*x*Cos[8*e + 5*f*x] - 15*a^5*f*x*Cos[4*e + 7*f*x] -
 60*a^4*b*f*x*Cos[4*e + 7*f*x] - 90*a^3*b^2*f*x*Cos[4*e + 7*f*x] - 60*a^2*b
^3*f*x*Cos[4*e + 7*f*x] - 15*a*b^4*f*x*Cos[4*e + 7*f*x] + 15*a^5*f*x*Cos[6*
e + 7*f*x] + 60*a^4*b*f*x*Cos[6*e + 7*f*x] + 90*a^3*b^2*f*x*Cos[6*e + 7*f*x
] + 60*a^2*b^3*f*x*Cos[6*e + 7*f*x] + 15*a*b^4*f*x*Cos[6*e + 7*f*x] - 15*a^
5*f*x*Cos[8*e + 7*f*x] - 60*a^4*b*f*x*Cos[8*e + 7*f*x] - 90*a^3*b^2*f*x*Cos
[8*e + 7*f*x] - 60*a^2*b^3*f*x*Cos[8*e + 7*f*x] - 15*a*b^4*f*x*Cos[8*e + 7*
f*x] + 15*a^5*f*x*Cos[10*e + 7*f*x] + 60*a^4*b*f*x*Cos[10*e + 7*f*x] + 90*a
^3*b^2*f*x*Cos[10*e + 7*f*x] + 60*a^2*b^3*f*x*Cos[10*e + 7*f*x] + 15*a*b^4*
f*x*Cos[10*e + 7*f*x] - 10*a^5*Sin[f*x] + 860*a^4*b*Sin[f*x] + 3120*a^3*b^2
*Sin[f*x] + 3600*a^2*b^3*Sin[f*x] - 300*b^5*Sin[f*x] + 46*a^5*Sin[3*f*x] -
508*a^4*b*Sin[3*f*x] - 2324*a^3*b^2*Sin[3*f*x] - 3120*a^2*b^3*Sin[3*f*x] +
75*a*b^4*Sin[3*f*x] - 150*b^5*Sin[3*f*x] - 240*a^5*Sin[2*e - f*x] - 1840*a^
```

```
4*b*Sin[2*e - f*x] - 4840*a^3*b^2*Sin[2*e - f*x] - 5040*a^2*b^3*Sin[2*e - f
*x] - 300*b^5*Sin[2*e - f*x] + 240*a^5*Sin[2*e + f*x] + 1840*a^4*b*Sin[2*e
+ f*x] + 4840*a^3*b^2*Sin[2*e + f*x] + 5040*a^2*b^3*Sin[2*e + f*x] - 75*a*b
^4*Sin[2*e + f*x] - 300*b^5*Sin[2*e + f*x] - 10*a^5*Sin[4*e + f*x] + 860*a^
4*b*Sin[4*e + f*x] + 3120*a^3*b^2*Sin[4*e + f*x] + 3600*a^2*b^3*Sin[4*e + f
*x] + 75*a*b^4*Sin[4*e + f*x] + 300*b^5*Sin[4*e + f*x] - 240*a^4*b*Sin[2*e
+ 3*f*x] - 900*a^3*b^2*Sin[2*e + 3*f*x] - 1200*a^2*b^3*Sin[2*e + 3*f*x] - 7
5*a*b^4*Sin[2*e + 3*f*x] + 150*b^5*Sin[2*e + 3*f*x] + 46*a^5*Sin[4*e + 3*f*
x] - 508*a^4*b*Sin[4*e + 3*f*x] - 2324*a^3*b^2*Sin[4*e + 3*f*x] - 3120*a^2*
b^3*Sin[4*e + 3*f*x] + 60*a*b^4*Sin[4*e + 3*f*x] + 150*b^5*Sin[4*e + 3*f*x]
 - 240*a^4*b*Sin[6*e + 3*f*x] - 900*a^3*b^2*Sin[6*e + 3*f*x] - 1200*a^2*b^3
*Sin[6*e + 3*f*x] - 60*a*b^4*Sin[6*e + 3*f*x] - 150*b^5*Sin[6*e + 3*f*x] -
48*a^5*Sin[2*e + 5*f*x] - 32*a^4*b*Sin[2*e + 5*f*x] + 340*a^3*b^2*Sin[2*e +
 5*f*x] + 864*a^2*b^3*Sin[2*e + 5*f*x] - 60*a*b^4*Sin[2*e + 5*f*x] + 30*b^5
*Sin[2*e + 5*f*x] - 90*a^5*Sin[4*e + 5*f*x] - 300*a^4*b*Sin[4*e + 5*f*x] -
300*a^3*b^2*Sin[4*e + 5*f*x] + 60*a*b^4*Sin[4*e + 5*f*x] - 30*b^5*Sin[4*e +
 5*f*x] - 48*a^5*Sin[6*e + 5*f*x] - 32*a^4*b*Sin[6*e + 5*f*x] + 340*a^3*b^2
*Sin[6*e + 5*f*x] + 864*a^2*b^3*Sin[6*e + 5*f*x] - 15*a*b^4*Sin[6*e + 5*f*x
] - 30*b^5*Sin[6*e + 5*f*x] - 90*a^5*Sin[8*e + 5*f*x] - 300*a^4*b*Sin[8*e +
 5*f*x] - 300*a^3*b^2*Sin[8*e + 5*f*x] + 15*a*b^4*Sin[8*e + 5*f*x] + 30*b^5
*Sin[8*e + 5*f*x] + 46*a^5*Sin[4*e + 7*f*x] + 172*a^4*b*Sin[4*e + 7*f*x] +
216*a^3*b^2*Sin[4*e + 7*f*x] + 15*a*b^4*Sin[4*e + 7*f*x] - 15*a*b^4*Sin[6*e
 + 7*f*x] + 46*a^5*Sin[8*e + 7*f*x] + 172*a^4*b*Sin[8*e + 7*f*x] + 216*a^3*
b^2*Sin[8*e + 7*f*x]))/(7680*a^2*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

fricas [**B**] time = 0.61, size = 1505, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(46*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 4
*(70*a^5 + 234*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5
 + 20*(6*a^5 + 10*a^4*b - 20*a^3*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3
 - 15*((9*a^2*b^3 + 2*a*b^4)*cos(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3
 - 5*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x
 + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*
b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x +
e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(2*a^4*b + 8*a^3*b^2
+ 12*a^2*b^3 - a*b^4)*cos(f*x + e) + 120*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + a*b^4)*f*x*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^
3 - 2*a*b^4 - b^5)*f*x*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*
b^3 - 7*a*b^4 - 2*b^5)*f*x*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3
+ 4*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^
3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 -
2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*
b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*
b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), -1/60*(2*(46*a^5 + 172*a^4*b +
 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 2*(70*a^5 + 234*a^4*b + 218*a^3*b
^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + 10*(6*a^5 + 10*a^4*b - 20*a^3
*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3 + 15*((9*a^2*b^3 + 2*a*b^4)*cos
(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3 - 5*a*b^4 - 2*b^5)*cos(f*x + e)
^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x
+ e)))*sin(f*x + e) + 30*(2*a^4*b + 8*a^3*b^2 + 12*a^2*b^3 - a*b^4)*cos(f*x
 + e) + 60*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*x*cos(f*x + e
)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*x*cos(f*x
 + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*x*cos
```

```
(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*x)*sin(f*x
+ e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
- (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x
 + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f
*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*
sin(f*x + e))]
```

giac [A] time = 1.16, size = 310, normalized size = 1.50

$$\frac{\frac{15\,b^4\tan\left(fx+e\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{15\left(9\,ab^4+2\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\sqrt{ab+b^2}}-\frac{30\left(fx+e\right)}{a^2}-\frac{2\left(15\,a^2\tan\left(f\right.\right.}{}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*
(b*tan(f*x + e)^2 + a + b)) + 15*(9*a*b^4 + 2*b^5)*(pi*floor((f*x + e)/pi +
 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b + b^2)) - 30*(f*x + e)/a^2 - 2*(15*
a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5*a^2*
tan(f*x + e)^2 - 20*a*b*tan(f*x + e)^2 - 15*b^2*tan(f*x + e)^2 + 3*a^2 + 6*
a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^5))/
f
```

maple [A] time = 1.35, size = 248, normalized size = 1.20

$$\frac{b^4\tan\left(fx+e\right)}{2f\left(a+b\right)^4a\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{9b^4\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{2f\left(a+b\right)^4a\sqrt{(a+b)\,b}}+\frac{b^5\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{(a+b)b}}\right)}{f\left(a+b\right)^4a^2\sqrt{(a+b)\,b}}-\frac{1}{5f\left(a+b\right)^2\tan\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^4/(a+b)^4/a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+9/2/f*b^4/(a+b)^4/a/((a
+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b^5/(a+b)^4/a^2/((a+b
)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/5/f/(a+b)^2/tan(f*x+e)^5+
1/3/f/(a+b)^3/tan(f*x+e)^3*a+1/f/(a+b)^3/tan(f*x+e)^3*b-1/f/(a+b)^4/tan(f*x
+e)*a^2-4/f/(a+b)^4/tan(f*x+e)*a*b-6/f/(a+b)^4/tan(f*x+e)*b^2-1/f/a^2*arcta
n(tan(f*x+e))
```

maxima [A] time = 0.47, size = 319, normalized size = 1.54

$$\frac{\frac{15\left(9\,ab^4+2\,b^5\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\sqrt{(a+b)b}}-\frac{15\left(2\,a^3b+8\,a^2b^2+12\,ab^3-b^4\right)\tan\left(fx+e\right)^6+10\left(3\,a^4+14\,a^3b+26\,a^2b^2+15\,ab^3\right)\tan\left(fx+e\right)^4+6\,a^4+18\,a^3}{\left(a^5b+4\,a^4b^2+6\,a^3b^3+4\,a^2b^4+ab^5\right)\tan\left(fx+e\right)^7+\left(a^6+5\,a^5b+10\,a^4b^2+10\right.}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(9*a*b^4 + 2*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 + 4
*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt((a + b)*b)) - (15*(2*a^3*b +
 8*a^2*b^2 + 12*a*b^3 - b^4)*tan(f*x + e)^6 + 10*(3*a^4 + 14*a^3*b + 26*a^2
*b^2 + 15*a*b^3)*tan(f*x + e)^4 + 6*a^4 + 18*a^3*b + 18*a^2*b^2 + 6*a*b^3 -
 2*(5*a^4 + 22*a^3*b + 29*a^2*b^2 + 12*a*b^3)*tan(f*x + e)^2)/((a^5*b + 4*a
^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*tan(f*x + e)^7 + (a^6 + 5*a^5*b + 1
```

```
0*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*tan(f*x + e)^5) - 30*(f*x + e)/
a^2)/f
```

mupad [B] time = 10.86, size = 6017, normalized size = 29.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan(((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 9
97248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^1
5 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112
*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 25
95328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*
a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 10
2400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 +
23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^
16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 +
 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*
b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 +
 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3*b^23 + 2624
*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^
8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 1207507
2*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 +
 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 +
 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))/(2*a^2) -
(((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*
a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28
745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*
b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*
a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b
^3 + 128*a^26*b^2 - (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a
^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 238141
44*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^1
4 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938
560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 +
1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a
^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) - tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b
^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18
 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12
*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 49774
08*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960
*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))/(2*a^2))/(2752*a
^4*b^20 - ((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^1
9 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^1
3*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 348
74112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8
 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 +
2688*a^25*b^3 + 128*a^26*b^2 - (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22
 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^
18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 1182438
40*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b
^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*
a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*
b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) - tan(e + f*x)*(128*a^3*b^23 +
 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 16131
84*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12
075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b
^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*
```

b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))*1i)/(2*a^2) - 32*a^2*b^22 - 144*a^3*b^21 - ((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))*1i)/(2*a^2) + 33824*a^5*b^19 + 182784*a^6*b^18 + 613648*a^7*b^17 + 1429120*a^8*b^16 + 2433024*a^9*b^15 + 3113088*a^10*b^14 + 3034768*a^11*b^13 + 2261952*a^12*b^12 + 1281952*a^13*b^11 + 543872*a^14*b^10 + 167664*a^15*b^9 + 35584*a^16*b^8 + 4672*a^17*b^7 + 288*a^18*b^6))/(a^2*f) - (1/(5*(a + b)) + (tan(e + f*x)^4*(11*a*b + 3*a^2 + 15*b^2))/(3*(a + b)^3) - (tan(e + f*x)^2*(5*a + 12*b))/(15*(a + b)^2) + (tan(e + f*x)^6*(12*a*b^3 + 2*a^3*b - b^4 + 8*a^2*b^2))/(2*a*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b) + b*tan(e + f*x)^7)) + (atan((((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3) - ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 - (tan(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)))*1i)/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)) + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3) + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 +

170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)))*1i)/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)))/(2752*a^4*b^20 - 144*a^3*b^21 - 32*a^2*b^22 + 33824*a^5*b^19 + 182784*a^6*b^18 + 613648*a^7*b^17 + 1429120*a^8*b^16 + 2433024*a^9*b^15 + 3113088*a^10*b^14 + 3034768*a^11*b^13 + 2261952*a^12*b^12 + 1281952*a^13*b^11 + 543872*a^14*b^10 + 167664*a^15*b^9 + 35584*a^16*b^8 + 4672*a^17*b^7 + 288*a^18*b^6 + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3) - ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 - (tan(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2)) - ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3) + ((-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^

```
11 + a^2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*
b^4 + 84*a^8*b^3 + 36*a^9*b^2))))/(4*(9*a^10*b + a^11 + a^2*b^9 + 9*a^3*b^8
 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 84*a^8*b^3 + 36*a^
9*b^2))))*(-b^7*(a + b)^9)^(1/2)*(9*a + 2*b)*1i)/(2*f*(9*a^10*b + a^11 + a^
2*b^9 + 9*a^3*b^8 + 36*a^4*b^7 + 84*a^5*b^6 + 126*a^6*b^5 + 126*a^7*b^4 + 8
4*a^8*b^3 + 36*a^9*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.363 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=78

$$\frac{(a+b)^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{a+b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[Out] 1/4*(a+b)^2/a^3/f/(b+a*cos(f*x+e)^2)^2+(-a-b)/a^3/f/(b+a*cos(f*x+e)^2)-1/2*
ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$\frac{(a+b)^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{a+b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (a + b)^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - (a + b)/(a^3*f*(b + a*Cos[e
+ f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(1-x^2\right)^2}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3} - \frac{2(a+b)}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx\right.}{2a^3f}$$

Mathematica [A] time = 2.15, size = 136, normalized size = 1.74

$$-\frac{2\left(a^2+4ab+3b^2\right)+a^2\cos^2(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+(a+2b)^2\log(a\cos(2(e+fx))+a}{2a^3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)
]] + a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(
e + f*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f
*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.54, size = 116, normalized size = 1.49

$$-\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2 - a^2 + 2\,ab + 3\,b^2 + 2\left(a^2\cos\left(fx+e\right)^4 + 2\,ab\cos\left(fx+e\right)^2 + b^2\right)\log\left(a\cos\left(fx+\right.\right.}{4\left(a^5f\cos\left(fx+e\right)^4 + 2\,a^4bf\cos\left(fx+e\right)^2 + a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - a^2 + 2*a*b + 3*b^2 + 2*(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f
*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/2/a^3*ln(abs((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))+1))-1/4/a^3*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+(
3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^2+6*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^4*b*a+3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^2+
```

```
12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^2-8*((1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1))))^3*b*a-20*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^
2+18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2-28*((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^2*b*a+50*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2
*a^2+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-8*(1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1)))*b*a-20*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+3*b^2
+6*b*a+3*a^2)*1/8/a^3/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)^2)
```

maple [A] time = 0.86, size = 138, normalized size = 1.77

$$\frac{1}{4fa\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}+\frac{b}{2f\,a^2\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}+\frac{b^2}{4a^3f\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{\ln\left(b+a\left(\cos^2\left(fx\right.\right.\right.}{2a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/4/f/a/(b+a*cos(f*x+e)^2)^2+1/2/f/a^2/(b+a*cos(f*x+e)^2)^2*b+1/4*b^2/a^3/f
/(b+a*cos(f*x+e)^2)^2-1/2*ln(b+a*cos(f*x+e)^2)/a^3/f-1/f/a^2/(b+a*cos(f*x+e
)^2)-b/a^3/f/(b+a*cos(f*x+e)^2)
```

maxima [A] time = 0.35, size = 112, normalized size = 1.44

$$\frac{\frac{4\left(a^2+ab\right)\sin\left(fx+e\right)^2-3\,a^2-6\,ab-3\,b^2}{a^5\sin\left(fx+e\right)^4+a^5+2\,a^4b+a^3b^2-2\left(a^5+a^4b\right)\sin\left(fx+e\right)^2}-\frac{2\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a^2 + a*b)*sin(f*x + e)^2 - 3*a^2 - 6*a*b - 3*b^2)/(a^5*sin(f*x +
e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*
sin(f*x + e)^2 - a - b)/a^3)/f
```

mupad [B] time = 4.60, size = 166, normalized size = 2.13

$$\frac{\mathrm{atanh}\left(\frac{4\,b^2\tan\left(e+f\,x\right)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan\left(e+f\,x\right)^2+\frac{8\,b^3\tan\left(e+f\,x\right)^2}{a}}\right)}{a^3\,f}+\frac{\frac{-a^3+3\,a\,b^2+2\,b^3}{4\,a^2\,b^2}-\frac{\tan\left(e+f\,x\right)^2\left(a^2-b^2\right)}{2\,a^2\,b}}{f\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8
*b^3*tan(e + f*x)^2)/a))/(a^3*f) + ((3*a*b^2 - a^3 + 2*b^3)/(4*a^2*b^2) - (
tan(e + f*x)^2*(a^2 - b^2))/(2*a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)
^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.364 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}+\frac{a+2b}{2a^3f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[Out] -1/4*b*(a+b)/a^3/f/(b+a*cos(f*x+e)^2)^2+1/2*(a+2*b)/a^3/f/(b+a*cos(f*x+e)^2
)+1/2*ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 77}

$$-\frac{b(a+b)}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}+\frac{a+2b}{2a^3f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(b*(a + b))/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) + (a + 2*b)/(2*a^3*f*(b + a
*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3(1-x^2)}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3} + \frac{a+2b}{a^2(b+ax)^2} - \frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{b(a+b)}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{a+2b}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\log\left(b+a\cos^2(e+fx\right.}{2a^3f}$$

Mathematica [A] time = 0.93, size = 131, normalized size = 1.62

$$\frac{2\left(a^2+3ab+3b^2\right)+a^2\cos^2(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+(a+2b)^2\log(a\cos(2(e+fx))+a+2b)}{2a^3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*(a + 2*b)*Co
s[2*(e + f*x)]*(1 + Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*b +
 a*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.56, size = 111, normalized size = 1.37

$$\frac{2\left(a^2+2ab\right)\cos\left(fx+e\right)^2+ab+3b^2+2\left(a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)}{4\left(a^5f\cos\left(fx+e\right)^4+2a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4 +
 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-1/2/a^3*ln(abs((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))+1))+1/4/a^3*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+
(-3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^3-9*((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1))))^4*b^2*a-9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*
```

```
b*a^2-3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^3-12*((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^3*b^3-4*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))
^3*b^2*a+28*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b*a^2+20*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^3-18*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^2*b^3+10*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2*a-22*((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a^2-34*((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^2*a^3-12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3-4*(1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2*a+28*(1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))*b*a^2+20*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3-3*b^3-9*b^2*a
-9*b*a^2-3*a^3)/(8*b*a^3+8*a^4)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)^2)
```

maple [A] time = 0.81, size = 115, normalized size = 1.42

$$-\frac{b}{4 f\,a^{2} \left(b +a \left(\cos^{2}\left(f x +e \right)\right)\right)^{2}}-\frac{b^{2}}{4 a^{3} f \left(b +a \left(\cos^{2}\left(f x +e \right)\right)\right)^{2}}+\frac{\ln \left(b +a \left(\cos^{2}\left(f x +e \right)\right)\right)}{2 a^{3} f}+\frac{1}{2 f\,a^{2} \left(b +a \left(\cos^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/4/f/a^2/(b+a*cos(f*x+e)^2)^2*b-1/4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2+1/2*ln
(b+a*cos(f*x+e)^2)/a^3/f+1/2/f/a^2/(b+a*cos(f*x+e)^2)+b/a^3/f/(b+a*cos(f*x+
e)^2)
```

maxima [A] time = 0.35, size = 113, normalized size = 1.40

$$-\frac{\frac{2 \left(a^{2}+2 a b\right) \sin\left(f x+e\right)^{2}-2 a^{2}-5 a b-3 b^{2}}{a^{5} \sin\left(f x+e\right)^{4}+a^{5}+2 a^{4} b+a^{3} b^{2}-2 \left(a^{5}+a^{4} b\right) \sin\left(f x+e\right)^{2}}-\frac{2 \log\left(a \sin\left(f x+e\right)^{2}-a-b\right)}{a^{3}}}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(a^2 + 2*a*b)*sin(f*x + e)^2 - 2*a^2 - 5*a*b - 3*b^2)/(a^5*sin(f*x
 + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log
(a*sin(f*x + e)^2 - a - b)/a^3)/f
```

mupad [B] time = 4.45, size = 153, normalized size = 1.89

$$-\frac{\frac{a^{2}+3 a b+2 b^{2}}{4 a^{2} b}+\frac{b \tan\left(e+f x\right)^{2}}{2 a^{2}}}{f \left(2 a b+a^{2}+b^{2}+\tan\left(e+f x\right)^{2} \left(2 b^{2}+2 a b\right)+b^{2} \tan\left(e+f x\right)^{4}\right)}-\frac{\operatorname{atanh}\left(\frac{4 b^{2} \tan\left(e+f x\right)^{2}}{8 b^{2}+\frac{8 b^{3}}{a}+4 b^{2} \tan\left(e+f x\right)^{2}+\frac{8 b^{3} \tan\left(e+f\right.}{a}}\right.}{a^{3} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] - ((3*a*b + a^2 + 2*b^2)/(4*a^2*b) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b
+ a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - atanh
((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*
tan(e + f*x)^2)/a))/(a^3*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.365 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=74

$$\frac{b^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[Out] 1/4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2-b/a^3/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*co
s(f*x+e)^2)/a^3/f
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$\frac{b^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - b/(a^3*f*(b + a*Cos[e + f*x]^2)) -
 Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3} - \frac{2b}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right) dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{b^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

Mathematica [**A**] time = 1.33, size = 129, normalized size = 1.74

$$-\frac{a^2\cos^2(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+(a+2b)^2\log(a\cos(2(e+fx))+a+2b)+2a\cos(2(e+fx)}{2a^3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a^2
*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e + f*x)
]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f*(a + 2*b + a*
Cos[2*(e + f*x)])^2)
```

fricas [**A**] time = 0.53, size = 102, normalized size = 1.38

$$-\frac{4\,ab\cos\left(fx+e\right)^2+3\,b^2+2\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)}{4\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b*cos(f*x + e)^2 + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*
cos(f*x + e)^2 + a^3*b^2*f)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/2/a^3*ln(abs((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))+1))-1/4/a^3*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a)+(
3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^4+12*((1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1))))^4*b^3*a+18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*
b^2*a^2+12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b*a^3+3*((1-cos(f*x+
```

```
exp(1)))/(1+cos(f*x+exp(1))))^4*a^4+12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^3*b^4+16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^3*a-24*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^2*a^2-40*((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^3*b*a^3-12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^4+1
8*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^4+8*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^2*b^3*a+12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b
^2*a^2+56*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a^3+18*((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^2*a^4+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))*b^4+16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3*a-24*(1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))*b^2*a^2-40*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
*b*a^3-12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^4+3*b^4+12*b^3*a+18*b^2
*a^2+12*b*a^3+3*a^4)/(8*b^2*a^3+16*b*a^4+8*a^5)/(((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*a+b+a)^2)
```

maple [A] time = 0.39, size = 81, normalized size = 1.09

$$\frac{1}{4 f a \left(a+b \left(\sec^2\left(f x+e\right)\right)\right)^2}-\frac{\ln\left(a+b \left(\sec^2\left(f x+e\right)\right)\right)}{2 f a^3}+\frac{1}{2 f a^2 \left(a+b \left(\sec^2\left(f x+e\right)\right)\right)}+\frac{\ln\left(\sec\left(f x+e\right)\right)}{f a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/4/f/a/(a+b*sec(f*x+e)^2)^2-1/2/f/a^3*ln(a+b*sec(f*x+e)^2)+1/2/f/a^2/(a+b*
sec(f*x+e)^2)+1/f/a^3*ln(sec(f*x+e))
```

maxima [A] time = 0.37, size = 102, normalized size = 1.38

$$\frac{\frac{4 \, ab \sin\left(fx+e\right)^2-4 \, ab-3 \, b^2}{a^5 \sin\left(fx+e\right)^4+a^5+2 \, a^4b+a^3b^2-2 \, \left(a^5+a^4b\right) \sin\left(fx+e\right)^2}-\frac{2 \, \log\left(a \sin\left(fx+e\right)^2-a-b\right)}{a^3}}{4 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a*b*sin(f*x + e)^2 - 4*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*a
^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2 -
 a - b)/a^3)/f
```

mupad [B] time = 4.40, size = 142, normalized size = 1.92

$$\frac{\frac{3\,a+2\,b}{4\,a^2}+\frac{b\,\tan\left(e+f\,x\right)^2}{2\,a^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^4\right)}+\frac{\mathrm{atanh}\left(\frac{4\,b^2\,\tan\left(e+f\,x\right)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan\left(e+f\,x\right)^2+\frac{8\,b^3\,\tan\left(e+f\,x\right)}{a}}\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((3*a + 2*b)/(4*a^2) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b + a^2 + b^2 +
tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + atanh((4*b^2*tan(e
+ f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*tan(e + f*x)^2
)/a))/(a^3*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.366 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3f(a+b)\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cos^2(e+fx\right.}{2a^3f(a+b)^3}$$

```
[Out] -1/4*b^3/a^3/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/f/(
b+a*cos(f*x+e)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^3/
f+ln(sin(f*x+e))/(a+b)^3/f
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$-\frac{b^3}{4a^3f(a+b)\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cos^2(e+fx\right.}{2a^3f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -b^3/(4*a^3*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^3*
(a + b)^2*f*(b + a*Cos[e + f*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*Co
s[e + f*x]^2])/(2*a^3*(a + b)^3*f) + Log[Sin[e + f*x]]/((a + b)^3*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b(3a^2+3ab+b^2)}{a^2(a+b)^3(b+ax)}\right) dx, x, \cos^2\right.}{2f}$$

$$= -\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{b\left(3a^2+\right.}{}$$

Mathematica [A] time = 1.00, size = 158, normalized size = 1.22

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{b^3(a+b)^2}{a^3\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{2b^2(a+b)(3a+2b)}{a^3\left(-a\sin^2(e+fx)+a+b\right)} + \frac{2b\left(3a^2+3ab+b^2\right)\log\left(-a\sin^2(e+fx\right.}{a^3}\right.}{32f(a+b)^3\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(4*Log[Sin[e + f*x]] + (2*
b*(3*a^2 + 3*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3 - (b^3*(a + b)^2
)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*(a
+ b - a*Sin[e + f*x]^2))))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 1.07, size = 307, normalized size = 2.36

$$\frac{5\,a^2b^3 + 8\,ab^4 + 3\,b^5 + 2\left(3\,a^3b^2 + 5\,a^2b^3 + 2\,ab^4\right)\cos\left(fx+e\right)^2 + 2\left(3\,a^2b^3 + 3\,ab^4 + b^5 + \left(3\,a^4b + 3\,a^3b^2 + a^2b^3\right.\right.}{4\left(\left(a^8 + 3\,a^7b + 3\,a^6b^2 + a^5b^3\right)f\cos\left(fx+e\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^2*b^3 + 8*a*b^4 + 3*b^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*cos(
f*x + e)^2 + 2*(3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*cos(f*x + e)^4 + 2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*log(a*c
os(f*x + e)^2 + b) + 4*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*log(1/2*sin(f*x + e)))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x
 + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
```

```
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(4*a^3+12*a^2*b+
12*a*b^2+4*b^3)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))-1/2/a^3*l
n(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(3*a^2*b+3*a*b^2+b^3)/(4*
a^6+12*a^5*b+12*a^4*b^2+4*a^3*b^3)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^2*a+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b-2*(1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1)))*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a+b)+(-
9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^3*b-18*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^4*a^2*b^2-12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^4*a*b^3-3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^4+36*((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^3*a^3*b+24*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^3*a^2*b^2-16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a*b^3-12*(
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^4-54*((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1))))^2*a^3*b-12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2
*b^2-8*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b^3-18*((1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1))))^2*b^4+36*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
*a^3*b+24*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2*b^2-16*(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*a*b^3-12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*
b^4-9*a^3*b-18*a^2*b^2-12*a*b^3-3*b^4)/(8*a^5+16*a^4*b+8*a^3*b^2)/(((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^2*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+2*(1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1)))*b+a+b)^2)
```

maple [B] time = 1.07, size = 304, normalized size = 2.34

$$-\frac{b^3}{4f(a+b)^3 a\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{b^4}{2f(a+b)^3 a^2\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}-\frac{b^5}{4a^3(a+b)^3 f\left(b+a\left(\cos^2\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/4/f*b^3/(a+b)^3/a/(b+a*cos(f*x+e)^2)^2-1/2/f*b^4/(a+b)^3/a^2/(b+a*cos(f*
x+e)^2)^2-1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+3/2/f*b/(a+b)^3/a*ln(b
+a*cos(f*x+e)^2)+3/2/f*b^2/(a+b)^3/a^2*ln(b+a*cos(f*x+e)^2)+1/2/f*b^3/(a+b)
^3/a^3*ln(b+a*cos(f*x+e)^2)+3/2/f*b^2/(a+b)^3/a/(b+a*cos(f*x+e)^2)+5/2/f*b^
3/(a+b)^3/a^2/(b+a*cos(f*x+e)^2)+1/f*b^4/(a+b)^3/a^3/(b+a*cos(f*x+e)^2)+1/2
/f/(a+b)^3*ln(-1+cos(f*x+e))+1/2/f/(a+b)^3*ln(1+cos(f*x+e))
```

maxima [A] time = 0.37, size = 243, normalized size = 1.87

$$\frac{\frac{2\left(3a^2b+3ab^2+b^3\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^6+3a^5b+3a^4b^2+a^3b^3}+\frac{6a^2b^2+9ab^3+3b^4-2\left(3a^2b^2+2ab^3\right)\sin\left(fx+e\right)^2}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+\left(a^7+2a^6b+a^5b^2\right)\sin\left(fx+e\right)^4-2\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\sin\left(fx+e\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3) + (6*a^2*b^2 + 9*a*b^3 + 3*b^4 - 2*(3*a^2*b^2 + 2
*a*b^3)*sin(f*x + e)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 +
(a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*sin(f*x + e)^2) + 2*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3))/f
```

mupad [B] time = 4.86, size = 190, normalized size = 1.46

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\frac{2\,b^2+5\,a\,b}{4\,a^2\,(a+b)}+\frac{b\,\tan\left(e+f\,x\right)^2\,\left(b^2+2\,a\,b\right)}{2\,a^2\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^4\right)}-\frac{\ln\left(\tan\left(e\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - ((5*a*b + 2*b^2)/(4
*a^2*(a + b)) + (b*tan(e + f*x)^2*(2*a*b + b^2))/(2*a^2*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - lo
g(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b*log(a + b + b*tan(e + f*x)^2)*(3*a*b +
 3*a^2 + b^2))/(2*a^3*f*(a + b)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.367 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{b^4}{4a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{b^3(2a+b)}{a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)}-\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cos^2(e+fx)\right.}{2a^3f(a+b)^4}$$

```
[Out] 1/4*b^4/a^3/(a+b)^2/f/(b+a*cos(f*x+e)^2)^2-b^3*(2*a+b)/a^3/(a+b)^3/f/(b+a*c
os(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^3/f-1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+a*co
s(f*x+e)^2)/a^3/(a+b)^4/f-(a+4*b)*ln(sin(f*x+e))/(a+b)^4/f
```

Rubi [A] time = 0.21, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{b^4}{4a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{b^3(2a+b)}{a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)}-\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cos^2(e+fx)\right.}{2a^3f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^4/(4*a^3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) - (b^3*(2*a + b))/(a^3*(a
+ b)^3*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)^3*f) - (b^2*(6
*a^2 + 4*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^4*f) - ((a +
4*b)*Log[Sin[e + f*x]])/((a + b)^4*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^9}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2(6a^2+4ab+}{a^2(a+b)^4(b+a}\right.\right.}{2f}$$

$$= \frac{b^4}{4a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b^3(2a+b)}{a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\csc^2(e+f}{2(a+b)^3}$$

Mathematica [A] time = 1.73, size = 176, normalized size = 1.14

$$-\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{b^4(a+b)^2}{a^3\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{4b^3(a+b)(2a+b)}{a^3\left(-a\sin^2(e+fx)+a+b\right)} + \frac{2b^2(6a^2+4ab+b^2)\log\left(-a\sin^2(e+\right.}{a^3}\right.}{32f(a+b)^4\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/32*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*Csc[e + f
*x]^2 + 4*(a + 4*b)*Log[Sin[e + f*x]] + (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[a
+ b - a*Sin[e + f*x]^2])/a^3 - (b^4*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]
^2)^2) + (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/((a +
 b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 1.95, size = 584, normalized size = 3.79

$$\frac{2\,a^4b^2 + 2\,a^3b^3 + 7\,a^2b^4 + 10\,ab^5 + 3\,b^6 + 2\left(a^6 + a^5b - 4\,a^3b^3 - 6\,a^2b^4 - 2\,ab^5\right)\cos\left(fx+e\right)^4 + \left(4\,a^5b + 4\,a^4b^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6 + 2*(a^6 + a^5*b
- 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 + (4*a^5*b + 4*a^4*b^2 +
8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)^2 - 2*((6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (6*a^4*b^2
 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (12*a^3*b^3 + 2*a^2*b^
4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 4*((a^6 + 4*
a^5*b)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - (a^6 + 2*a^5*b - 8*a^4*b^2)*c
os(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 - 4*a^3*b^3)*cos(f*x + e)^2)*log(1/2*s
in(f*x + e)))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x
+ e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*
cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
 a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((16*(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*b+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-b-a
)/(16*b^4+64*b^3*a+96*b^2*a^2+64*b*a^3+16*a^4)/(1-cos(f*x+exp(1)))*(1+cos(f
*x+exp(1)))+(3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^6+18*((1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^5*a+45*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^4*b^4*a^2+48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^3*a^3
+18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^2*a^4+12*((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^3*b^6+40*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^3*b^5*a+20*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^4*a^2-80*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^3*a^3-72*((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^3*b^2*a^4+18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^6
+44*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^5*a+46*((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^2*b^4*a^2+64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
)))^2*b^3*a^3+108*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2*a^4+12*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^6+40*(1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1)))*b^5*a+20*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^4*a^2-80*(1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3*a^3-72*(1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1)))*b^2*a^4+3*b^6+18*b^5*a+45*b^4*a^2+48*b^3*a^3+18*b^2*a^4)/(8*b^4*
a^3+32*b^3*a^4+48*b^2*a^5+32*b*a^6+8*a^7)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+
a)^2-(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))/(16*b^3+48*b^2*a+48*b*a^2+16*a
^3)+1/2/a^3*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(-4*b-a)/(4*
b^4+16*b^3*a+24*b^2*a^2+16*b*a^3+4*a^4)*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos
(f*x+exp(1))))+(-b^4-4*b^3*a-6*b^2*a^2)/(4*b^4*a^3+16*b^3*a^4+24*b^2*a^5+16
*b*a^6+4*a^7)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a))
```

maple [B] time = 1.51, size = 389, normalized size = 2.53

$$\frac{b^4}{4f(a+b)^4 a\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}+\frac{b^5}{2f(a+b)^4 a^2\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)^2}+\frac{b^6}{4f(a+b)^4 a^3\left(b+a\left(\cos^2\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/4/f*b^4/(a+b)^4/a/(b+a*cos(f*x+e)^2)^2+1/2/f*b^5/(a+b)^4/a^2/(b+a*cos(f*x
+e)^2)^2+1/4/f*b^6/(a+b)^4/a^3/(b+a*cos(f*x+e)^2)^2-3/f*b^2/(a+b)^4/a*ln(b+
a*cos(f*x+e)^2)-2/f*b^3/(a+b)^4/a^2*ln(b+a*cos(f*x+e)^2)-1/2/f*b^4/(a+b)^4/
a^3*ln(b+a*cos(f*x+e)^2)-2/f*b^3/(a+b)^4/a/(b+a*cos(f*x+e)^2)-3/f*b^4/(a+b)
^4/a^2/(b+a*cos(f*x+e)^2)-1/f*b^5/(a+b)^4/a^3/(b+a*cos(f*x+e)^2)+1/4/f/(a+b
)^3/(-1+cos(f*x+e))-1/2/f/(a+b)^4*ln(-1+cos(f*x+e))*a-2/f/(a+b)^4*ln(-1+cos
(f*x+e))*b-1/4/f/(a+b)^3/(1+cos(f*x+e))-1/2/f/(a+b)^4*ln(1+cos(f*x+e))*a-2/
f/(a+b)^4*ln(1+cos(f*x+e))*b
```

maxima [B] time = 0.35, size = 344, normalized size = 2.23

$$-\frac{\frac{2\left(6a^2b^2+4ab^3+b^4\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4}+\frac{2(a+4b)\log\left(\sin\left(fx+e\right)^2\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{2a^5+4a^4b+2a^3b^2+2\left(a^5-4a^2b^3-2ab^4\right)\sin}{\left(a^8+3a^7b+3a^6b^2+a^5b^3\right)\sin\left(fx+e\right)^6-2\left(a^8+4a^7b+6a^6b^2+4a^5b\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 2*(a + 4*b)*log(sin(f*x + e)^2)/
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*a^5 + 4*a^4*b + 2*a^3*b^2
+ 2*(a^5 - 4*a^2*b^3 - 2*a*b^4)*sin(f*x + e)^4 - (4*a^5 + 4*a^4*b - 8*a^2*b
^3 - 11*a*b^4 - 3*b^5)*sin(f*x + e)^2)/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*sin(f*x + e)^6 - 2*(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sin
(f*x + e)^4 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^
5)*sin(f*x + e)^2))/f
```

mupad [B] time = 6.03, size = 272, normalized size = 1.77

$$\frac{\frac{\tan\left(e+f\,x\right)^2\,\left(-4\,a^2\,b+7\,a\,b^2+2\,b^3\right)}{4\,a^2\,\left(a^2+2\,a\,b+b^2\right)}-\frac{1}{2\,\left(a+b\right)}+\frac{\tan\left(e+f\,x\right)^4\,\left(-a^2\,b^2+3\,a\,b^3+b^4\right)}{2\,a^2\,\left(a+b\right)\,\left(a^2+2\,a\,b+b^2\right)}}{f\,\left(\tan\left(e+f\,x\right)^2\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^4\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^6\right)}+\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,a^3\,f}-\frac{}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^2*(7*a*b^2 - 4*a^2*b + 2*b^3))/(4*a^2*(2*a*b + a^2 + b^2)) -
 1/(2*(a + b)) + (tan(e + f*x)^4*(3*a*b^3 + b^4 - a^2*b^2))/(2*a^2*(a + b)*
(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^2*(2*a*b + a^2 + b^2) + tan(e + f*x)
^4*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^6)) + log(tan(e + f*x)^2 + 1)/(2*a^3*
f) - (log(tan(e + f*x))*(a + 4*b))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^
2*b^2)) - (b^2*log(a + b + b*tan(e + f*x)^2)*(4*a*b + 6*a^2 + b^2))/(2*a^3*
f*(a + b)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=192

$$-\frac{b^5}{4a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^4(5a+2b)}{2a^3f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+5ab+10b^2\right)\log(\sin(e+fx))}{f(a+b)^5}+$$

```
[Out] -1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+1/2*b^4*(5*a+2*b)/a^3/(a+b)^4/f
/(b+a*cos(f*x+e)^2)+1/2*(2*a+5*b)*csc(f*x+e)^2/(a+b)^4/f-1/4*csc(f*x+e)^4/(
a+b)^3/f+1/2*b^3*(10*a^2+5*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^5/f+(a^2
+5*a*b+10*b^2)*ln(sin(f*x+e))/(a+b)^5/f
```

Rubi [A] time = 0.27, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$-\frac{b^5}{4a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^4(5a+2b)}{2a^3f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+5ab+10b^2\right)\log(\sin(e+fx))}{f(a+b)^5}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -b^5/(4*a^3*(a + b)^3*f*(b + a*Cos[e + f*x]^2)^2) + (b^4*(5*a + 2*b))/(2*a^
3*(a + b)^4*f*(b + a*Cos[e + f*x]^2)) + ((2*a + 5*b)*Csc[e + f*x]^2)/(2*(a
+ b)^4*f) - Csc[e + f*x]^4/(4*(a + b)^3*f) + (b^3*(10*a^2 + 5*a*b + b^2)*Lo
g[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^5*f) + ((a^2 + 5*a*b + 10*b^2)*Log[
Sin[e + f*x]])/((a + b)^5*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^{11}}{(1-x^2)^3(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x)^3(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)^3} + \frac{-2a-5b}{(a+b)^4(-1+x)^2} + \frac{-a^2-5ab-10b^2}{(a+b)^5(-1+x)} - \frac{b^5}{a^2(a+b)^3(b+ax)^3} + \frac{b^4(5a+2b}{a^2(a+b)^4(b+}\right.\right.}{2f}$$

$$= -\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)} + \frac{(2a+5}{2}$$

Mathematica [A] time = 5.26, size = 208, normalized size = 1.08

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{b^5(a+b)^2}{a^3\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{2b^4(a+b)(5a+2b)}{a^3\left(-a\sin^2(e+fx)+a+b\right)} + 4\left(a^2+5ab+10b^2\right)\log(\mathrm{si}\right.}{32f(a+b)^5\left(a+b\,\mathrm{se}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*(2*a + 5*b)*Csc
[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 5*a*b + 10*b^2)*Log[Sin[e
 + f*x]] + (2*b^3*(10*a^2 + 5*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3
 - (b^5*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^4*(a + b)*(5*a
 + 2*b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^5*f*(a + b*Sec[e +
f*x]^2)^3)
```

fricas [B] time = 3.17, size = 859, normalized size = 4.47

$$\frac{3\,a^5b^2 + 12\,a^4b^3 + 9\,a^3b^4 + 9\,a^2b^5 + 12\,ab^6 + 3\,b^7 - 2\left(2\,a^7 + 7\,a^6b + 5\,a^5b^2 - 5\,a^3b^4 - 7\,a^2b^5 - 2\,ab^6\right)\cos\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^5*b^2 + 12*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 + 12*a*b^6 + 3*b^7 - 2*
(2*a^7 + 7*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 7*a^2*b^5 - 2*a*b^6)*cos(f*x + e
)^6 + (3*a^7 + 4*a^6*b - 19*a^5*b^2 - 20*a^4*b^3 - 20*a^3*b^4 - 19*a^2*b^5
+ 4*a*b^6 + 3*b^7)*cos(f*x + e)^4 + 2*(3*a^6*b + 10*a^5*b^2 + 2*a^4*b^3 - 2
*a^2*b^5 - 10*a*b^6 - 3*b^7)*cos(f*x + e)^2 + 2*((10*a^4*b^3 + 5*a^3*b^4 +
a^2*b^5)*cos(f*x + e)^8 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(10*a^4*b^3 - 5*a^
3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (10*a^4*b^3 - 35*a^3*b^4 - 9*a^
2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(10*a^3*b^4 - 5*a^2*b^5 - 4*a*b^6 -
 b^7)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*((a^7 + 5*a^6*b + 10*a^
5*b^2)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 - 2*(a^7 + 4*a^6*b
 + 5*a^5*b^2 - 10*a^4*b^3)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 35*a
^4*b^3 + 10*a^3*b^4)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 10
*a^3*b^4)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^10 + 5*a^9*b + 10*a^8*
b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*
```

```
b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 +
 a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^
7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*
b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^
5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*b^3-96*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^2*b^2*a-96*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a^2-32*((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3+1152*(1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))*b^3+2688*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2*a+1920*(1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b*a^2+384*(1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))*a^3)/(4096*b^6+24576*b^5*a+61440*b^4*a^2+81920*b^3*a^3+61440*b^
2*a^4+24576*b*a^5+4096*a^6)+(-32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
6*b^7-224*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*b^6*a-672*((1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1))))^6*b^5*a^2-960*((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1))))^6*b^4*a^3-720*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*b^3*a
^4-336*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*b^2*a^5-112*((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^6*b*a^6-16*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^6*a^7-128*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b^7-512*((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b^6*a-384*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^5*b^5*a^2+676*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b
^4*a^3+1080*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b^3*a^4+720*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*b^2*a^5+392*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^5*b*a^6+76*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a^7-
192*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^7-576*((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^4*b^6*a-704*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^4*b^5*a^2-1777*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^4*a^3-1572*(
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^3*a^4-838*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^4*b^2*a^5-612*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
))^4*b*a^6-145*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^7-128*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^7-512*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^3*b^6*a-384*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^5*a^2+
852*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b^4*a^3+1096*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^3*b^3*a^4+672*((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1))))^3*b^2*a^5+568*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b*a^6+14
0*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^7-32*((1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1))))^2*b^7-224*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b
^6*a-672*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^5*a^2-822*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^4*a^3-536*((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1))))^2*b^3*a^4-436*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2*
a^5-312*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b*a^6-70*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^2*a^7+32*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*b^4*a^3+112*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^3*a^4+144*(1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))*b^2*a^5+80*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*b*a^6+16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^7-b^4*a^3-4*b^3*a
^4-6*b^2*a^5-4*b*a^6-a^7)/(128*b^5*a^3+640*b^4*a^4+1280*b^3*a^5+1280*b^2*a^
6+640*b*a^7+128*a^8)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b+((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a+2*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^2*b-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+(1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))*b+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)^2-1/2
```

```
/a^3*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(10*b^2+5*b*a+a^2)/
(4*b^5+20*b^4*a+40*b^3*a^2+40*b^2*a^3+20*b*a^4+4*a^5)*ln(abs(1-cos(f*x+exp(
1)))/abs(1+cos(f*x+exp(1))))+(b^5+5*b^4*a+10*b^3*a^2)/(4*b^5*a^3+20*b^4*a^4
+40*b^3*a^5+40*b^2*a^6+20*b*a^7+4*a^8)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1))))^2*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+2*(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+b+a
))
```

maple [B] time = 1.46, size = 522, normalized size = 2.72

$$\frac{5b^3\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{f\left(a+b\right)^5 a}+\frac{5b^4\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\left(a+b\right)^5 a^2}+\frac{b^5\ln\left(b+a\left(\cos^2\left(fx+e\right)\right)\right)}{2f\left(a+b\right)^5 a^3}-\frac{b^5}{4f\left(a+b\right)^5 a\left(b+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 5/f*b^3/(a+b)^5/a*ln(b+a*cos(f*x+e)^2)+5/2/f*b^4/(a+b)^5/a^2*ln(b+a*cos(f*x
+e)^2)+1/2/f*b^5/(a+b)^5/a^3*ln(b+a*cos(f*x+e)^2)-1/4/f*b^5/(a+b)^5/a/(b+a*
cos(f*x+e)^2)^2-1/2/f*b^6/(a+b)^5/a^2/(b+a*cos(f*x+e)^2)^2-1/4/f*b^7/(a+b)^
5/a^3/(b+a*cos(f*x+e)^2)^2+5/2/f*b^4/(a+b)^5/a/(b+a*cos(f*x+e)^2)+7/2/f*b^5
/(a+b)^5/a^2/(b+a*cos(f*x+e)^2)+1/f*b^6/(a+b)^5/a^3/(b+a*cos(f*x+e)^2)-1/16
/f/(a+b)^3/(-1+cos(f*x+e))^2-7/16/f/(a+b)^4/(-1+cos(f*x+e))*a-19/16/f/(a+b)
^4/(-1+cos(f*x+e))*b+1/2/f/(a+b)^5*ln(-1+cos(f*x+e))*a^2+5/2/f/(a+b)^5*ln(-
1+cos(f*x+e))*a*b+5/f/(a+b)^5*ln(-1+cos(f*x+e))*b^2-1/16/f/(a+b)^3/(1+cos(f
*x+e))^2+7/16/f/(a+b)^4/(1+cos(f*x+e))*a+19/16/f/(a+b)^4/(1+cos(f*x+e))*b+1
/2/f/(a+b)^5*ln(1+cos(f*x+e))*a^2+5/2/f/(a+b)^5*ln(1+cos(f*x+e))*a*b+5/f/(a
+b)^5*ln(1+cos(f*x+e))*b^2
```

maxima [B] time = 0.36, size = 454, normalized size = 2.36

$$\frac{\frac{2\left(10a^2b^3+5ab^4+b^5\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^8+5a^7b+10a^6b^2+10a^5b^3+5a^4b^4+a^3b^5}+\frac{2\left(a^2+5ab+10b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}+\frac{2\left(2a^6+5a^5b-5a^2b^4-2ab^5\right)\sin\left(fx+e\right)^6-a^6-3a^5b-3a^4b^2-}{\left(a^9+4a^8b+6a^7b^2+4a^6b^3+a^5b^4\right)\sin\left(fx+e\right)^8-2\left(a^9+5a^8b+\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a*sin(f*x + e)^2 - a - b)/(a^8 + 5*
a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 2*(a^2 + 5*a*b + 1
0*b^2)*log(sin(f*x + e)^2)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5) + (2*(2*a^6 + 5*a^5*b - 5*a^2*b^4 - 2*a*b^5)*sin(f*x + e)^6 - a^6
 - 3*a^5*b - 3*a^4*b^2 - a^3*b^3 - (9*a^6 + 29*a^5*b + 20*a^4*b^2 - 10*a^2*
b^4 - 13*a*b^5 - 3*b^6)*sin(f*x + e)^4 + 2*(3*a^6 + 11*a^5*b + 13*a^4*b^2 +
 5*a^3*b^3)*sin(f*x + e)^2)/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b
^4)*sin(f*x + e)^8 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4
 + a^4*b^5)*sin(f*x + e)^6 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*
a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*sin(f*x + e)^4))/f
```

mupad [B] time = 6.42, size = 327, normalized size = 1.70

$$\frac{\ln\left(\tan\left(e+fx\right)\right)\left(a^2+5ab+10b^2\right)}{f\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)}-\frac{\frac{1}{4(a+b)}-\frac{\tan\left(e+fx\right)^2(a+3b)}{2(a+b)^2}+\frac{\tan\left(e+fx\right)^4\left(-4a^3b-15a^2b^2+9ab^3+2b^4\right)}{4a^2(a+b)\left(a^2+2ab+b^2\right)}+}{f\left(\tan\left(e+fx\right)^4\left(a^2+2ab+b^2\right)+\tan\left(e+fx\right)^6\left(2b^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (log(tan(e + f*x))*(5*a*b + a^2 + 10*b^2))/(f*(5*a*b^4 + 5*a^4*b + a^5 + b^
5 + 10*a^2*b^3 + 10*a^3*b^2)) - (1/(4*(a + b)) - (tan(e + f*x)^2*(a + 3*b))
/(2*(a + b)^2) + (tan(e + f*x)^4*(9*a*b^3 - 4*a^3*b + 2*b^4 - 15*a^2*b^2))/
(4*a^2*(a + b)*(2*a*b + a^2 + b^2)) + (tan(e + f*x)^6*(4*a*b^4 + b^5 - 4*a^
2*b^3 - a^3*b^2))/(2*a^2*(a + b)^2*(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^4
*(2*a*b + a^2 + b^2) + tan(e + f*x)^6*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^8)
) - log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b^3*log(a + b + b*tan(e + f*x)^2)*
(5*a*b + 10*a^2 + b^2))/(2*a^3*f*(a + b)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.369 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{x}{a^3}-\frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b\tan^2(e+fx)-\right.}$$

```
[Out] -x/a^3+1/8*(3*a^2-4*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)
^(1/2)/a^3/b^(5/2)/f-1/4*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^2-1/
8*(3*a-4*b)*(a+b)*tan(f*x+e)/a^2/b^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.29, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 578, 522, 203, 205}

$$-\frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f}-\frac{x}{a^3}-\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b\tan^2(e+fx)-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(8*a^3*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(4*a*b*f*(a +
 b + b*Tan[e + f*x]^2)^2) - ((3*a - 4*b)*(a + b)*Tan[e + f*x])/(8*a^2*b^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(3a-b)x^2\right)}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{a^3} + \frac{\sqrt{a+b}\,\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+f}{4abf\left(a+b+b\tan^2(e+\right.}$$

Mathematica [C] time = 6.41, size = 523, normalized size = 3.56

$$-\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sec(2e)\left(3a^4\sin(2(e+2fx))-3a^4\sin(4e+2fx)-9a^4\sin(2e)+9a^4\sin\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/128*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((2*(3*a^3 - a^2*b +
4*a*b^2 + 8*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[
f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(
a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[
b*(Cos[e] - I*Sin[e])^4]) + Sec[2*e]*(8*b^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos
[2*e] + 16*a*b^2*(a + 2*b)*f*x*Cos[2*f*x] + 4*a^2*b^2*f*x*Cos[2*(e + 2*f*x)
] + 16*a^2*b^2*f*x*Cos[4*e + 2*f*x] + 32*a*b^3*f*x*Cos[4*e + 2*f*x] + 4*a^2
*b^2*f*x*Cos[6*e + 4*f*x] - 9*a^4*Sin[2*e] - 15*a^3*b*Sin[2*e] + 18*a^2*b^2
*Sin[2*e] + 72*a*b^3*Sin[2*e] + 48*b^4*Sin[2*e] + 9*a^4*Sin[2*f*x] + 13*a^3
*b*Sin[2*f*x] - 28*a^2*b^2*Sin[2*f*x] - 32*a*b^3*Sin[2*f*x] + 3*a^4*Sin[2*(
e + 2*f*x)] - 3*a^3*b*Sin[2*(e + 2*f*x)] - 6*a^2*b^2*Sin[2*(e + 2*f*x)] - 3
*a^4*Sin[4*e + 2*f*x] + a^3*b*Sin[4*e + 2*f*x] + 20*a^2*b^2*Sin[4*e + 2*f*x
] + 16*a*b^3*Sin[4*e + 2*f*x]))))/(a^3*b^2*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.64, size = 664, normalized size = 4.52

$$\left[-\frac{32\,a^2b^2fx\cos\left(fx+e\right)^4+64\,ab^3fx\cos\left(fx+e\right)^2+32\,b^4fx-\left(\left(3\,a^4-4\,a^3b+8\,a^2b^2\right)\cos\left(fx+e\right)^4+3\,a^2b^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^2*f*x*cos(f*x + e)^4 + 64*a*b^3*f*x*cos(f*x + e)^2 + 32*b^
4*f*x - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 - 4*a*b^3
 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-(a + b)/
b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4
*(3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3
)*cos(f*x + e))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f
*x + e)^2 + a^3*b^4*f), -1/16*(16*a^2*b^2*f*x*cos(f*x + e)^4 + 32*a*b^3*f*x
*cos(f*x + e)^2 + 16*b^4*f*x + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^
4 + 3*a^2*b^2 - 4*a*b^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x
 + e)^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a
+ b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(a^4 - a^3*b - 2*a^2*b^
2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3)*cos(f*x + e))*sin(f*x + e
))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3*b^4*f)]
```

giac [A] time = 6.25, size = 211, normalized size = 1.44

$$-\frac{\frac{8\left(fx+e\right)}{a^3}-\frac{\left(3\,a^3-a^2b+4\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^3b^2}+\frac{5\,a^2b\tan\left(fx+e\right)^3+ab^2\tan\left(fx+e\right)^3-4\,b^3\tan\left(fx+e\right)^3+3\,a^3\tan\left(fx+\right.}{\left(b\tan\left(fx+e\right)^2+a+b\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(f*x + e)/a^3 - (3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*(pi*floor((f*x +
e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b
^2)*a^3*b^2) + (5*a^2*b*tan(f*x + e)^3 + a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f
*x + e)^3 + 3*a^3*tan(f*x + e) + 2*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e
) - 4*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b^2))/f
```

maple [B] time = 1.09, size = 356, normalized size = 2.42

$$-\frac{5\left(\tan^3\left(fx+e\right)\right)}{8f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2 b}-\frac{\tan^3\left(fx+e\right)}{8fa\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{3a\tan\left(fx+e\right)}{8f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2 b^2}-\frac{}{4f(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -5/8/f/(a+b+b*tan(f*x+e)^2)^2/b*tan(f*x+e)^3-1/8/f/a/(a+b+b*tan(f*x+e)^2)^2
*tan(f*x+e)^3-3/8/f*a/(a+b+b*tan(f*x+e)^2)^2/b^2*tan(f*x+e)-1/4/f/(a+b+b*ta
n(f*x+e)^2)^2/b*tan(f*x+e)+5/8*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+3/8/f/
b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/8/f/a/b/((a+b)*b
)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a^2/((a+b)*b)^(1/2)*arct
an(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a^2*b/(a+b+b*tan(f*x+e)^2)^2*tan(f*x
+e)^3+1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/f/a^3*b/((a+b)*b)^(1/
2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 193, normalized size = 1.31

$$-\frac{\frac{\left(5a^2b+ab^2-4b^3\right)\tan\left(fx+e\right)^3+\left(3a^3+2a^2b-5ab^2-4b^3\right)\tan\left(fx+e\right)}{a^2b^4\tan\left(fx+e\right)^4+a^4b^2+2a^3b^3+a^2b^4+2\left(a^3b^3+a^2b^4\right)\tan\left(fx+e\right)^2}+\frac{8\left(fx+e\right)}{a^3}-\frac{\left(3a^3-a^2b+4ab^2+8b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3b^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((5*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e)^3 + (3*a^3 + 2*a^2*b - 5*a*b^
2 - 4*b^3)*tan(f*x + e))/(a^2*b^4*tan(f*x + e)^4 + a^4*b^2 + 2*a^3*b^3 + a^
2*b^4 + 2*(a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 - (3*a^3 -
a^2*b + 4*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a +
b)*b)*a^3*b^2))/f
```

mupad [B] time = 4.90, size = 615, normalized size = 4.18

$$-\frac{\mathrm{atan}\left(\frac{25\tan\left(e+fx\right)}{32\left(\frac{5b}{4a}-\frac{3a}{16b}+\frac{9a^2}{32b^2}+\frac{25}{32}\right)}-\frac{3\tan\left(e+fx\right)}{16\left(\frac{9a}{32b}+\frac{25b}{32a}+\frac{5b^2}{4a^2}-\frac{3}{16}\right)}+\frac{9\tan\left(e+fx\right)}{32\left(\frac{25b^2}{32a^2}-\frac{3b}{16a}+\frac{5b^3}{4a^3}+\frac{9}{32}\right)}+\frac{5\tan\left(e+fx\right)}{4\left(\frac{25a}{32b}-\frac{3a^2}{16b^2}+\frac{9a^3}{32b^3}+\frac{5}{4}\right)}\right)}{a^3 f}-\frac{\tan(e+f}{f\left(2ab+a^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] - atan((25*tan(e + f*x))/(32*((5*b)/(4*a) - (3*a)/(16*b) + (9*a^2)/(32*b^2)
 + 25/32)) - (3*tan(e + f*x))/(16*((9*a)/(32*b) + (25*b)/(32*a) + (5*b^2)/(
4*a^2) - 3/16)) + (9*tan(e + f*x))/(32*((25*b^2)/(32*a^2) - (3*b)/(16*a) +
```

```
(5*b^3)/(4*a^3) + 9/32)) + (5*tan(e + f*x))/(4*((25*a)/(32*b) - (3*a^2)/(16
*b^2) + (9*a^3)/(32*b^3) + 5/4)))/(a^3*f) - ((tan(e + f*x)^3*(a*b + 5*a^2 -
 4*b^2))/(8*a^2*b) - (tan(e + f*x)*(a + b)*(a*b - 3*a^2 + 4*b^2))/(8*a^2*b^
2))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*
x)^4)) - (atanh((27*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a*b^2)/25
6 - (27*b^3)/128 + (171*b^4)/(256*a) - (7*b^5)/(64*a^2) + (5*b^6)/(32*a^3)
+ (5*b^7)/(4*a^4))) - (81*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a^2
*b)/256 - (27*a*b^2)/128 + (171*b^3)/256 - (7*b^4)/(64*a) + (5*b^5)/(32*a^2
) + (5*b^6)/(4*a^3))) - (35*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(32*((171*a
^2*b)/256 - (7*a*b^2)/64 - (27*a^3)/128 + (5*b^3)/32 + (5*b^4)/(4*a) + (27*
a^4)/(256*b))) + (5*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(4*((5*a*b^2)/32 -
(7*a^2*b)/64 + (171*a^3)/256 + (5*b^3)/4 - (27*a^4)/(128*b) + (27*a^5)/(256
*b^2))) + (63*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(64*((171*a*b^2)/256 - (2
7*a^2*b)/128 + (27*a^3)/256 - (7*b^3)/64 + (5*b^4)/(32*a) + (5*b^5)/(4*a^2)
)))*(-b^5*(a + b))^(1/2)*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3*b^5*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

3.370 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=137

$$\frac{x}{a^3}+\frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b\tan^2(e+fx)+b\right)}+\frac{\left(a^2-4ab-8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}f\sqrt{a+b}}-\frac{(a+b)\tan(e+fx)}{4abf\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/b^(3
/2)/f/(a+b)^(1/2)-1/4*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a-
4*b)*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.26, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 527, 522, 203, 205}

$$\frac{\left(a^2-4ab-8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}f\sqrt{a+b}}+\frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{(a+b)\tan(e+fx)}{4abf\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(8*a^3*b^(3/2)*Sqrt[a + b]*f) - ((a + b)*Tan[e + f*x])/(4*a*b*f*(a + b + b*
Tan[e + f*x]^2)^2) + ((a - 4*b)*Tan[e + f*x])/(8*a^2*b*f*(a + b + b*Tan[e +
 f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+(a-3b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{(a+l}{}\right.}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{}$$

$$= \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,f} - \frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{}{8a^2}$$

Mathematica [C] time = 14.16, size = 1473, normalized size = 10.75

$$\frac{(\cos(2e+2fx)a+a+2b)^3\left(\frac{(3a^2+8ba+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\left(3a^2+16ba+3(a+2b)\cos(2(e+fx))a+16b^2\right)\sin(2(e+fx))}{(a+b)^2(\cos(2(e+fx))a+a+2b)^2}\right)\sec^6(e}{1024b^{5/2}f\left(b\sec^2(e+fx)+a\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)
```

fricas [B] time = 0.58, size = 763, normalized size = 5.57

$$\left[\frac{32\left(a^3b^2+a^2b^3\right)fx\cos\left(fx+e\right)^4+64\left(a^2b^3+ab^4\right)fx\cos\left(fx+e\right)^2+32\left(ab^4+b^5\right)fx+\left(\left(a^4-4a^3b-8a^2b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^2*b^3 + a*b^4)*f*x
*cos(f*x + e)^2 + 32*(a*b^4 + b^5)*f*x + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cos(f
*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8*a*b^3)*cos
(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x +
 e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f
```

```
*x + e)^2 + b^2)) - 4*((a^4*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^
3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2 + a^5*b^
3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^4*b^4 + a
^3*b^5)*f), 1/16*(16*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 32*(a^2*b^3 +
 a*b^4)*f*x*cos(f*x + e)^2 + 16*(a*b^4 + b^5)*f*x - ((a^4 - 4*a^3*b - 8*a^2
*b^2)*cos(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8
*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((a^4*b + 7*a^3*b^2
 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e)
)*sin(f*x + e))/((a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^
4)*f*cos(f*x + e)^2 + (a^4*b^4 + a^3*b^5)*f)]
```

giac [A] time = 2.37, size = 168, normalized size = 1.23

$$\frac{\frac{8\left(fx+e\right)}{a^3}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(a^2-4\,ab-8\,b^2\right)}{\sqrt{ab+b^2}\,a^3b}+\frac{ab\tan\left(fx+e\right)^3-4\,b^2\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)-5\,ab\tan\left(fx+e\right)-4\,b^2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^2a^2b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(f*x + e)/a^3 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - 4*a*b - 8*b^2)/(sqrt(a*b + b^2)*a^3*b) +
(a*b*tan(f*x + e)^3 - 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) - 5*a*b*tan(f
*x + e) - 4*b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b))/f
```

maple [B] time = 0.86, size = 264, normalized size = 1.93

$$\frac{\tan^3\left(fx+e\right)}{8fa\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{b\left(\tan^3\left(fx+e\right)\right)}{2fa^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{\tan\left(fx+e\right)}{8f\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2b}-\frac{}{8af\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/8/f/a/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-1/2/f/a^2*b/(a+b+b*tan(f*x+e)^2
)^2*tan(f*x+e)^3-1/8/f/(a+b+b*tan(f*x+e)^2)^2/b*tan(f*x+e)-5/8*tan(f*x+e)/a
/f/(a+b+b*tan(f*x+e)^2)^2-1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/8
/f/a/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a^2/((a+b
)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b/((a+b)*b)^(1/2)*a
rctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.45, size = 163, normalized size = 1.19

$$\frac{\frac{\left(ab-4\,b^2\right)\tan\left(fx+e\right)^3-\left(a^2+5\,ab+4\,b^2\right)\tan\left(fx+e\right)}{a^2b^3\tan\left(fx+e\right)^4+a^4b+2\,a^3b^2+a^2b^3+2\left(a^3b^2+a^2b^3\right)\tan\left(fx+e\right)^2}+\frac{8\left(fx+e\right)}{a^3}+\frac{\left(a^2-4\,ab-8\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(((a*b - 4*b^2)*tan(f*x + e)^3 - (a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a
^2*b^3*tan(f*x + e)^4 + a^4*b + 2*a^3*b^2 + a^2*b^3 + 2*(a^3*b^2 + a^2*b^3)
*tan(f*x + e)^2) + 8*(f*x + e)/a^3 + (a^2 - 4*a*b - 8*b^2)*arctan(b*tan(f*x
 + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3*b))/f
```

mupad [B] time = 5.40, size = 1117, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(a - 4*b))/(8*a^2) - (tan(e + f*x)*(a + b)*(a + 4*b))/(8*a
^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e +
 f*x)^4)) - atan(tan(e + f*x)/(32*(b/(4*a) - 1/32)) + tan(e + f*x)/(4*(a/(3
2*b) - 1/4)))/(a^3*f) + (atan(-(((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*
b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^
(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2
+ 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^
4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*1i)/(1
6*(a^3*b^4 + a^4*b^3)) - ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)
/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(
4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2
))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128
*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*1i)/(16*(a^3*
b^4 + a^4*b^3)))/(((a*b^2)/4 - (a^2*b)/4 + a^3/32 + b^3)/(a^6*b) + ((((-b^3
*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6
*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*
(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (ta
n(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))
^(1/2)*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + ((((-b^3*(a + b))^
(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256
*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 +
 a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)
*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*
a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3))))*(-b^3*(a + b))^(1/2)*(4*a*b
- a^2 + 8*b^2)*1i)/(8*f*(a^3*b^4 + a^4*b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

3.371 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{x}{a^3}+\frac{(3a+4b)\tan(e+fx)}{8a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{\left(3a^2+12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,f(a+b)^{3/2}}+\frac{\tan(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right.}$$

```
[Out] -x/a^3+1/8*(3*a^2+12*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/
(a+b)^(3/2)/f/b^(1/2)+1/4*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a+4*
b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.23, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4141, 1975, 471, 527, 522, 203, 205}

$$\frac{\left(3a^2+12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,f(a+b)^{3/2}}+\frac{(3a+4b)\tan(e+fx)}{8a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{x}{a^3}+\frac{\tan(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*f) + Tan[e + f*x]/(4*a*f*(a + b + b*Tan[
e + f*x]^2)^2) + ((3*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)*f*(a + b + b*Tan
[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{1-3x^2}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{x}{a^3} + \frac{\left(3a^2+12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,(a+b)^{3/2}f} + \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2}$$

Mathematica [C] time = 10.60, size = 1334, normalized size = 9.67

$$(\cos(2(e+fx))a+a+2b)^3\sec^6(e+fx)\left(-\frac{6a(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{4\left(3a^2+8ba+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{4a\sqrt{b}\left(3a^2+16ba\right.}{(}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((-6*a*(a + 2*b)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*(3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*a*Sqrt[b]
*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (2*Sqrt[b]*(3*a^3 + 14*a^
2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e + f*x)])*Sin[2
*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) - (Sqrt[b]*((2*(3
*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTan[(S
ec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))
/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(S
qrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]*(256*b^2*(a + b)^2*(3
*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[
2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*
f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*
x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x]
 + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*
a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin
[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 2880*a^3*b^3*Sin[2*e] +
5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^6*Sin[2*e] + 9*a^6*Sin
[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x] - 2112*a^3*b^3*Sin[2
*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*x] + 3*a^6*Sin[2*(e +
2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*Sin[2*(e + 2*f*x)] - 38
4*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e + 2*f*x)] - 3*a^6*Sin[4
*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2*Sin[4*e + 2*f*x] + 10
56*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e + 2*f*x] + 512*a*b^5*Sin
[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(a^3*(a + b)^2) - (2*Sqr
t[b]*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x
]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos
[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2
*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(
-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*
b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^
2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(e
 + f*x)])^2)))/(a + b)^2))/(4096*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.55, size = 860, normalized size = 6.23

$$\left[-\frac{32\left(a^4b+2a^3b^2+a^2b^3\right)fx\cos\left(fx+e\right)^4+64\left(a^3b^2+2a^2b^3+ab^4\right)fx\cos\left(fx+e\right)^2+32\left(a^2b^3+2ab^4+b^5\right)f}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^3*b^2 +
 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 32*(a^2*b^3 + 2*a*b^4 + b^5)*f*x +
 ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 12*a*b^3 + 8*
b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
 e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^4*b
 + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^
4)*cos(f*x + e))*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e
)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4
*b^4 + a^3*b^5)*f), -1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e
)^4 + 32*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^2*b^3 + 2
*a*b^4 + b^5)*f*x + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*
b^2 + 12*a*b^3 + 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)
*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)
*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*
x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((
a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^
4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*f)]
```

giac [A] time = 1.31, size = 181, normalized size = 1.31

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,a^2+12\,ab+8\,b^2\right)}{\left(a^4+a^3b\right)\sqrt{ab+b^2}}+\frac{3\,ab\tan\left(fx+e\right)^3+4\,b^2\tan\left(fx+e\right)^3+5\,a^2\tan\left(fx+e\right)+9\,ab\tan\left(fx+e\right)+4\,b^2\tan\left(fx+e\right)}{\left(a^3+a^2b\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 12*a*b + 8*b^2)/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (3*a*b*
tan(f*x + e)^3 + 4*b^2*tan(f*x + e)^3 + 5*a^2*tan(f*x + e) + 9*a*b*tan(f*x
+ e) + 4*b^2*tan(f*x + e))/((a^3 + a^2*b)*(b*tan(f*x + e)^2 + a + b)^2) - 8
*(f*x + e)/a^3)/f
```

maple [B] time = 0.72, size = 263, normalized size = 1.91

$$\frac{3b\left(\tan^3\left(fx+e\right)\right)}{8fa\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a+b\right)}+\frac{b^2\left(\tan^3\left(fx+e\right)\right)}{2f\,a^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a+b\right)}+\frac{5\tan\left(fx+e\right)}{8af\left(a+b+b\left(\tan^2\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 3/8/f/a/(a+b+b*tan(f*x+e)^2)^2*b/(a+b)*tan(f*x+e)^3+1/2/f/a^2/(a+b+b*tan(f*
x+e)^2)^2*b^2/(a+b)*tan(f*x+e)^3+5/8*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+
1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+3/8/f/a/(a+b)/((a+b)*b)^(1/2)
*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/2/f/a^2/(a+b)/((a+b)*b)^(1/2)*arcta
n(tan(f*x+e)*b/((a+b)*b)^(1/2))*b+1/f/a^3/(a+b)/((a+b)*b)^(1/2)*arctan(tan(
f*x+e)*b/((a+b)*b)^(1/2))*b^2-1/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.44, size = 191, normalized size = 1.38

$$\frac{\frac{\left(3\,a^2+12\,ab+8\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^4+a^3b\right)\sqrt{(a+b)b}}+\frac{\left(3\,ab+4\,b^2\right)\tan\left(fx+e\right)^3+\left(5\,a^2+9\,ab+4\,b^2\right)\tan\left(fx+e\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3+\left(a^3b^2+a^2b^3\right)\tan\left(fx+e\right)^4+2\left(a^4b+2\,a^3b^2+a^2b^3\right)\tan\left(fx+e\right)^2}-\frac{8\left(fx+e\right)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
+ a^3*b)*sqrt((a + b)*b)) + ((3*a*b + 4*b^2)*tan(f*x + e)^3 + (5*a^2 + 9*a*
b + 4*b^2)*tan(f*x + e))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^3*b^2 +
a^2*b^3)*tan(f*x + e)^4 + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^2) -
 8*(f*x + e)/a^3)/f
```

mupad [B] time = 7.44, size = 2405, normalized size = 17.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(5*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a*b + 4*b^2))/(8*a
^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*t
an(e + f*x)^4)) - atan((((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/
(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 +
 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))/(2*a^3)
+ (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))
/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3 - ((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a
^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 +
 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*
b^2)))/(2*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3
 + 72*a^3*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3)/(((9*a*b^3)/4 + (9*a^3
*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (((((2*a^6*b^4 +
(9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e +
 f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(
2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(320*a*b^4 + 9*a^4*b
 + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^5*b + a^6 + a^4*b^2)))/
a^3 + (((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8
 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 2
56*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) - (tan(e + f*
x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^
5*b + a^6 + a^4*b^2)))/a^3))/(a^3*f) - (atan((((-b*(a + b)^3)^(1/2)*((tan(e
 + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*
a^5*b + a^6 + a^4*b^2)) - ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2
 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^
(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 +
 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3
+ 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 +
 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2)*1i)/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3
 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b
+ 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) + ((-
b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b +
a^8 + a^6*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2
)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b
+ a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^
2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2
 + 8*b^2)*1i)/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))/(((9*a*b^3)/4
 + (9*a^3*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - ((-b*(a
+ b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 +
 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) - ((-b*(a + b)^3)^(1/2)*((2*a^
6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e +
 f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7
*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b +
 a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b +
```

```
a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a
^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(32
0*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6
 + a^4*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b
^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a
*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^
2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2
)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)
))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2))
))*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*1i)/(8*f*(a^6*b + a^3*b^4
+ 3*a^4*b^3 + 3*a^5*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e + fx\right)}{\left(a + b\sec^2\left(e + fx\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

3.372 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b\tan(e+f}{4af(a+b)\left(a+b\tan^2(\right.}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+}{4af(a+b)\left(a+b\tan^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+\right.}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(\right.}$$

Mathematica [C] time = 5.56, size = 332, normalized size = 2.31

$$\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b\left(\left(9a^2+28ab+16b^2\right)\sin(2e)-3a(3a+2b)\sin(2fx)\right)(a\cos(2(e+fx))+a+2b)}{f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{b\left(15a^2+20ab+\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
```

```
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.63, size = 819, normalized size = 5.69

$$\left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos\left(fx+e\right)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos\left(fx+e\right)^2+32\left(a^2b^2+2ab^3+b^4\right)fx+\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

giac [A] time = 0.23, size = 205, normalized size = 1.42

$$-\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7ab^2\tan\left(fx+e\right)^3+4b^3\tan\left(fx+e\right)^3+9a^2b\tan\left(fx+e\right)+13ab^2\tan\left(fx+e\right)+4b^3\tan\left(\right.}{\left(a^4+2a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

maple [B] time = 0.84, size = 321, normalized size = 2.23

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8fa\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{b^3\left(\tan^3\left(fx+e\right)\right)}{2fa^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2\left(a^2+2ab+b^2\right)}-\frac{}{8a\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a*b^2/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/2/f/a^2*
b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a/
(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/2/f/a^2*b^2/(a+b+b*tan(f*x+e)^2)^2/(a+b)*t
an(f*x+e)-15/8/f/a*b/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((
a+b)*b)^(1/2))-5/2/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x
+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b^3/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(t
an(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.47, size = 231, normalized size = 1.60

$$-\frac{\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{(a+b)b}}+\frac{\left(7\,ab^{2}+4\,b^{3}\right)\tan\left(fx+e\right)^{3}+\left(9\,a^{2}b+13\,ab^{2}+4\,b^{3}\right)\tan\left(fx+e\right)}{a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}+\left(a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4}\right)\tan\left(fx+e\right)^{4}+2\,\left(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4}\right)\tan}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

mupad [B] time = 8.43, size = 3271, normalized size = 22.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a
^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b
^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 +
a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b
^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e +
f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(
64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6
 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9
*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7
+ 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2
))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3)
 - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^
4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a
```

```
^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*
x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2
*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6
 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^
8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3
+ 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)
*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3
+ 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*
a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^
5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^
2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))
 + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*
a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6
*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b
+ a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1
/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 +
3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4
 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7
*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^
(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105
*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e
 + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(
32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*
a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^
6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b
 + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b
+ a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b
^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b
^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*
(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^
5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))))*(-b*(a + b)^5)^(1/2)*(20*a*b +
15*a^2 + 8*b^2)*1i)/(8*f*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
+ 10*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

3.373 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=181

$$-\frac{x}{a^3}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2f(a+b)^3}-\frac{b(9a+4b)\cot(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tan^{-1}\left(\right.}{8a^3f(a+b)^{7/2}}$$

```
[Out] -x/a^3+1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(7/2)/f-1/8*(8*a^2-11*a*b-4*b^2)*cot(f*x+e)/a^2/(a+b)^3/f-1/
4*b*cot(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(9*a+4*b)*cot(f*x+e)/
a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.38, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{7/2}}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2f(a+b)^3}-\frac{b(9a+4b)\cot(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*f) - ((8*a^2 - 11*a*b - 4*b^2)*Cot[e +
f*x])/(8*a^2*(a + b)^3*f) - (b*Cot[e + f*x])/(4*a*(a + b)*f*(a + b + b*Tan[
e + f*x]^2)^2) - (b*(9*a + 4*b)*Cot[e + f*x])/(8*a^2*(a + b)^2*f*(a + b + b
*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-b-5bx^2}{x^2(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e\right.}{4a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8a^2(a}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8a^2(a}$$

$$= -\frac{x}{a^3} + \frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f} - \frac{\left(8a^2-11ab-4b^2\right)\cot(e}{8a^2(a+b)^3f}$$

Mathematica [C] time = 7.09, size = 2089, normalized size = 11.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
(-1/64*(b^2*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])
)*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^2*ArcTan[Sec[f*x]*(Cos[
2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sq
rt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] +
 a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[
4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f
*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]*Sec[e + f*x]^6*(8*a^5*f*x*Cos[f*x] + 56*a
^4*b*f*x*Cos[f*x] + 184*a^3*b^2*f*x*Cos[f*x] + 296*a^2*b^3*f*x*Cos[f*x] + 2
24*a*b^4*f*x*Cos[f*x] + 64*b^5*f*x*Cos[f*x] - 12*a^5*f*x*Cos[3*f*x] - 68*a^
4*b*f*x*Cos[3*f*x] - 132*a^3*b^2*f*x*Cos[3*f*x] - 108*a^2*b^3*f*x*Cos[3*f*x
] - 32*a*b^4*f*x*Cos[3*f*x] - 8*a^5*f*x*Cos[2*e - f*x] - 56*a^4*b*f*x*Cos[2
*e - f*x] - 184*a^3*b^2*f*x*Cos[2*e - f*x] - 296*a^2*b^3*f*x*Cos[2*e - f*x]
 - 224*a*b^4*f*x*Cos[2*e - f*x] - 64*b^5*f*x*Cos[2*e - f*x] - 8*a^5*f*x*Cos
[2*e + f*x] - 56*a^4*b*f*x*Cos[2*e + f*x] - 184*a^3*b^2*f*x*Cos[2*e + f*x]
- 296*a^2*b^3*f*x*Cos[2*e + f*x] - 224*a*b^4*f*x*Cos[2*e + f*x] - 64*b^5*f*
x*Cos[2*e + f*x] + 8*a^5*f*x*Cos[4*e + f*x] + 56*a^4*b*f*x*Cos[4*e + f*x] +
 184*a^3*b^2*f*x*Cos[4*e + f*x] + 296*a^2*b^3*f*x*Cos[4*e + f*x] + 224*a*b^
4*f*x*Cos[4*e + f*x] + 64*b^5*f*x*Cos[4*e + f*x] + 12*a^5*f*x*Cos[2*e + 3*f
```

```
*x] + 68*a^4*b*f*x*Cos[2*e + 3*f*x] + 132*a^3*b^2*f*x*Cos[2*e + 3*f*x] + 10
8*a^2*b^3*f*x*Cos[2*e + 3*f*x] + 32*a*b^4*f*x*Cos[2*e + 3*f*x] - 12*a^5*f*x
*Cos[4*e + 3*f*x] - 68*a^4*b*f*x*Cos[4*e + 3*f*x] - 132*a^3*b^2*f*x*Cos[4*e
 + 3*f*x] - 108*a^2*b^3*f*x*Cos[4*e + 3*f*x] - 32*a*b^4*f*x*Cos[4*e + 3*f*x
] + 12*a^5*f*x*Cos[6*e + 3*f*x] + 68*a^4*b*f*x*Cos[6*e + 3*f*x] + 132*a^3*b
^2*f*x*Cos[6*e + 3*f*x] + 108*a^2*b^3*f*x*Cos[6*e + 3*f*x] + 32*a*b^4*f*x*C
os[6*e + 3*f*x] - 4*a^5*f*x*Cos[2*e + 5*f*x] - 12*a^4*b*f*x*Cos[2*e + 5*f*x
] - 12*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 4*a^2*b^3*f*x*Cos[2*e + 5*f*x] + 4*a^
5*f*x*Cos[4*e + 5*f*x] + 12*a^4*b*f*x*Cos[4*e + 5*f*x] + 12*a^3*b^2*f*x*Cos
[4*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[4*e + 5*f*x] - 4*a^5*f*x*Cos[6*e + 5*f*x]
 - 12*a^4*b*f*x*Cos[6*e + 5*f*x] - 12*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 4*a^2*
b^3*f*x*Cos[6*e + 5*f*x] + 4*a^5*f*x*Cos[8*e + 5*f*x] + 12*a^4*b*f*x*Cos[8*
e + 5*f*x] + 12*a^3*b^2*f*x*Cos[8*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[8*e + 5*f*
x] - 32*a^5*Sin[f*x] - 64*a^4*b*Sin[f*x] - 30*a^2*b^3*Sin[f*x] - 120*a*b^4*
Sin[f*x] - 48*b^5*Sin[f*x] + 32*a^5*Sin[3*f*x] + 64*a^4*b*Sin[3*f*x] + 26*a
^3*b^2*Sin[3*f*x] + 86*a^2*b^3*Sin[3*f*x] + 32*a*b^4*Sin[3*f*x] - 48*a^5*Si
n[2*e - f*x] - 128*a^4*b*Sin[2*e - f*x] - 128*a^3*b^2*Sin[2*e - f*x] - 30*a
^2*b^3*Sin[2*e - f*x] - 120*a*b^4*Sin[2*e - f*x] - 48*b^5*Sin[2*e - f*x] +
48*a^5*Sin[2*e + f*x] + 128*a^4*b*Sin[2*e + f*x] + 102*a^3*b^2*Sin[2*e + f*
x] - 86*a^2*b^3*Sin[2*e + f*x] - 136*a*b^4*Sin[2*e + f*x] - 48*b^5*Sin[2*e
+ f*x] - 32*a^5*Sin[4*e + f*x] - 64*a^4*b*Sin[4*e + f*x] + 26*a^3*b^2*Sin[4
*e + f*x] + 86*a^2*b^3*Sin[4*e + f*x] + 136*a*b^4*Sin[4*e + f*x] + 48*b^5*S
in[4*e + f*x] - 8*a^5*Sin[2*e + 3*f*x] - 26*a^3*b^2*Sin[2*e + 3*f*x] - 86*a
^2*b^3*Sin[2*e + 3*f*x] - 32*a*b^4*Sin[2*e + 3*f*x] + 32*a^5*Sin[4*e + 3*f*
x] + 64*a^4*b*Sin[4*e + 3*f*x] - 13*a^3*b^2*Sin[4*e + 3*f*x] - 36*a^2*b^3*S
in[4*e + 3*f*x] - 16*a*b^4*Sin[4*e + 3*f*x] - 8*a^5*Sin[6*e + 3*f*x] + 13*a
^3*b^2*Sin[6*e + 3*f*x] + 36*a^2*b^3*Sin[6*e + 3*f*x] + 16*a*b^4*Sin[6*e +
3*f*x] + 8*a^5*Sin[2*e + 5*f*x] + 13*a^3*b^2*Sin[2*e + 5*f*x] + 6*a^2*b^3*S
in[2*e + 5*f*x] - 13*a^3*b^2*Sin[4*e + 5*f*x] - 6*a^2*b^3*Sin[4*e + 5*f*x]
+ 8*a^5*Sin[6*e + 5*f*x]))/(512*a^3*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.68, size = 1060, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 4*(16*a^4*b - 1
3*a^3*b^2 + 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 - (35*a^2*b^3 + 28*a*b^4 +
8*b^5 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2
+ 28*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
+ 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*
b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(
f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) + 4*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 32*((a^5 + 3*a^4
*b + 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*
x)*sin(f*x + e))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 +
 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), -1/16*(2*(8*a^5 + 13*a^3
*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 2*(16*a^4*b - 13*a^3*b^2 + 5*a^2*b^3 + 4
*a*b^4)*cos(f*x + e)^3 + (35*a^2*b^3 + 28*a*b^4 + 8*b^5 + (35*a^4*b + 28*a^
3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 28*a^2*b^3 + 8*a*b^4)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(8*a^3*b^2 -
11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 16*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*x*cos(
f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
```

```
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

giac [A] time = 2.40, size = 257, normalized size = 1.42

$$\frac{\frac{(35a^2b^2+28ab^3+8b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+3a^5b+3a^4b^2+a^3b^3)\sqrt{ab+b^2}}+\frac{11ab^3\tan(fx+e)^3+4b^4\tan(fx+e)^3+13a^2b^2\tan(fx+e)+17ab^3\tan(fx+e)+4}{(a^5+3a^4b+3a^3b^2+a^2b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 4*b^4*tan(f*x + e)^3 + 1
3*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e) + 4*b^4*tan(f*x + e))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e
)/a^3 - 8/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

maple [B] time = 1.15, size = 337, normalized size = 1.86

$$\frac{11b^3\left(\tan^3\left(fx+e\right)\right)}{8f\left(a+b\right)^3a\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{b^4\left(\tan^3\left(fx+e\right)\right)}{2f\left(a+b\right)^3a^2\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{13b^2\tan\left(f\right.}{8f\left(a+b\right)^3\left(a+b+b\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 11/8/f*b^3/(a+b)^3/a/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+1/2/f*b^4/(a+b)^3/
a^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+13/8/f*b^2/(a+b)^3/(a+b+b*tan(f*x+e
)^2)^2*tan(f*x+e)+17/8/f*b^3/(a+b)^3/a/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+1/
2/f*b^4/(a+b)^3/a^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+35/8/f*b^2/(a+b)^3/a/
((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+7/2/f*b^3/(a+b)^3/a^2/
((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b^4/(a+b)^3/a^3/((
a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/(a+b)^3/tan(f*x+e)-1
/f/a^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.46, size = 311, normalized size = 1.72

$$\frac{\frac{(35a^2b^2+28ab^3+8b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+3a^5b+3a^4b^2+a^3b^3)\sqrt{(a+b)b}}-\frac{(8a^2b^2-11ab^3-4b^4)\tan(fx+e)^4+8a^4+16a^3b+8a^2b^2+(16a^3b+3a^2b^2-17}{(a^5b^2+3a^4b^3+3a^3b^4+a^2b^5)\tan(fx+e)^5+2(a^6b+4a^5b^2+6a^4b^3+4a^3b^4+a^2b^5)\tan(fx+e)^3+(a^7+5a}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)) - ((8*a^2*b^2 - 11
*a*b^3 - 4*b^4)*tan(f*x + e)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^2 + (16*a^3*b +
 3*a^2*b^2 - 17*a*b^3 - 4*b^4)*tan(f*x + e)^2)/((a^5*b^2 + 3*a^4*b^3 + 3*a^
3*b^4 + a^2*b^5)*tan(f*x + e)^5 + 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*
b^4 + a^2*b^5)*tan(f*x + e)^3 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 +
5*a^3*b^4 + a^2*b^5)*tan(f*x + e)) - 8*(f*x + e)/a^3)/f
```

mupad [B] time = 10.71, size = 4890, normalized size = 27.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^4*(11*a*b^3 + 4*b^4 - 8*a^2*b^2))/(8*a^2*(a + b)^3) - 1/(a +
 b) + (tan(e + f*x)^2*(13*a*b^2 - 16*a^2*b + 4*b^3))/(8*a^2*(a + b)^2))/(f*
(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b + a^2 + b^2) + b^2*ta
n(e + f*x)^5)) - atan((286720*a^6*b^15*tan(e + f*x))/(286720*a^6*b^15 + 361
9840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11
 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22166323
2*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 10
48576*a^18*b^3 + 65536*a^19*b^2) + (3619840*a^7*b^14*tan(e + f*x))/(286720*
a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 17717
2480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*
b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320
*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (21052416*a^8*b^13*tan(e +
 f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a
^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 +
334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^1
6*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (74346496*a
^9*b^12*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^
13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346
560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6
 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2
) + (177172480*a^10*b^11*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14
+ 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a
^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 1
05978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3
 + 65536*a^19*b^2) + (299796480*a^11*b^10*tan(e + f*x))/(286720*a^6*b^15 +
3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b
^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22166
3232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 +
 1048576*a^18*b^3 + 65536*a^19*b^2) + (369346560*a^12*b^9*tan(e + f*x))/(28
6720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 +
177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*
a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 78
64320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (334344192*a^13*b^8*t
an(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 7434
6496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*
b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 354457
60*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (2216
63232*a^14*b^7*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416
*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 +
 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a
^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a
^19*b^2) + (105978880*a^15*b^6*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7
*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 29979
6480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b
^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^
18*b^3 + 65536*a^19*b^2) + (35445760*a^16*b^5*tan(e + f*x))/(286720*a^6*b^1
5 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^
10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 2
21663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b
^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (7864320*a^17*b^4*tan(e + f*x))/(
286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12
+ 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 33434419
2*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 +
7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (1048576*a^18*b^3*t
an(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 7434
6496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*
```

b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 354457
60*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (6553
6*a^19*b^2*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8
*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369
346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*
b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*
b^2))/(a^3*f) - (atan((((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(131072*a^6*b^
18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15 + 205112320*a
^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b^12 + 994283520*a^13*b^11
+ 965376000*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^8 + 154054656
*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5 + 983040*a^20*b^4 + 65536*
a^21*b^3) - ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(65536*a^10*b
^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 34897920*a^13*b^14 + 115474432
*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16*b^11 + 728825856*a^17*b^1
0 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414515200*a^20*b^7 + 19506790
4*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^4 + 2097152*a^24*b^3 + 131
072*a^25*b^2 - (tan(e + f*x)*(-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^
2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 58982400*a^14*b^16 + 266076160*a
^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*b^13 + 3411279872*a^18*b^1
2 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 + 4310958080*a^21*b^9 + 288
6467584*a^22*b^8 + 1502871552*a^23*b^7 + 596377600*a^24*b^6 + 174325760*a^2
5*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 262144*a^28*b^2))/(16*(7*a^9
*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21
*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^
6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a^2 + 8*b^2)*1i)/(16*(7*a^9
*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21
*a^8*b^2)) + ((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(131072*a^6*b^18 + 20316
16*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15 + 205112320*a^10*b^14 +
462013440*a^11*b^13 + 778473472*a^12*b^12 + 994283520*a^13*b^11 + 96537600
0*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^8 + 154054656*a^17*b^7
+ 43115520*a^18*b^6 + 8135680*a^19*b^5 + 983040*a^20*b^4 + 65536*a^21*b^3)
+ ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(65536*a^10*b^17 + 9994
24*a^11*b^16 + 7405568*a^12*b^15 + 34897920*a^13*b^14 + 115474432*a^14*b^13
+ 281329664*a^15*b^12 + 517603328*a^16*b^11 + 728825856*a^17*b^10 + 789381
120*a^18*b^9 + 656195584*a^19*b^8 + 414515200*a^20*b^7 + 195067904*a^21*b^6
+ 66060288*a^22*b^5 + 15155200*a^23*b^4 + 2097152*a^24*b^3 + 131072*a^25*b
^2 + (tan(e + f*x)*(-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*(524288
*a^12*b^18 + 8126464*a^13*b^17 + 58982400*a^14*b^16 + 266076160*a^15*b^15 +
834928640*a^16*b^14 + 1932263424*a^17*b^13 + 3411279872*a^18*b^12 + 468582
4000*a^19*b^11 + 5060689920*a^20*b^10 + 4310958080*a^21*b^9 + 2886467584*a^
22*b^8 + 1502871552*a^23*b^7 + 596377600*a^24*b^6 + 174325760*a^25*b^5 + 35
389440*a^26*b^4 + 4456448*a^27*b^3 + 262144*a^28*b^2))/(16*(7*a^9*b + a^10
+ a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2))
))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35
*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a^2 + 8*b^2)*1i)/(16*(7*a^9*b + a^10
+ a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2))
)/(32768*a^4*b^17 + 499712*a^5*b^16 + 3416064*a^6*b^15 + 13829120*a^7*b^14
+ 36684800*a^8*b^13 + 66318336*a^9*b^12 + 81629184*a^10*b^11 + 64616448*a^1
1*b^10 + 25344000*a^12*b^9 - 6246400*a^13*b^8 - 14405632*a^14*b^7 - 8444928
*a^15*b^6 - 2415616*a^16*b^5 - 286720*a^17*b^4 - ((-b^3*(a + b)^7)^(1/2)*(t
an(e + f*x)*(131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232
320*a^9*b^15 + 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b
^12 + 994283520*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 3872
72704*a^16*b^8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5
+ 983040*a^20*b^4 + 65536*a^21*b^3) - ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*
a^2 + 8*b^2)*(65536*a^10*b^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 3489
7920*a^13*b^14 + 115474432*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16
*b^11 + 728825856*a^17*b^10 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414
515200*a^20*b^7 + 195067904*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^

```
4 + 2097152*a^24*b^3 + 131072*a^25*b^2 - (tan(e + f*x)*(-b^3*(a + b)^7)^(1/
2)*(28*a*b + 35*a^2 + 8*b^2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 589824
00*a^14*b^16 + 266076160*a^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*
b^13 + 3411279872*a^18*b^12 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 +
 4310958080*a^21*b^9 + 2886467584*a^22*b^8 + 1502871552*a^23*b^7 + 59637760
0*a^24*b^6 + 174325760*a^25*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 26
2144*a^28*b^2))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35
*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^
4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a
^2 + 8*b^2))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^
6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)) + ((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(
131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15
+ 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b^12 + 9942835
20*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^
8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^5 + 983040*a^20
*b^4 + 65536*a^21*b^3) + ((-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*
(65536*a^10*b^17 + 999424*a^11*b^16 + 7405568*a^12*b^15 + 34897920*a^13*b^1
4 + 115474432*a^14*b^13 + 281329664*a^15*b^12 + 517603328*a^16*b^11 + 72882
5856*a^17*b^10 + 789381120*a^18*b^9 + 656195584*a^19*b^8 + 414515200*a^20*b
^7 + 195067904*a^21*b^6 + 66060288*a^22*b^5 + 15155200*a^23*b^4 + 2097152*a
^24*b^3 + 131072*a^25*b^2 + (tan(e + f*x)*(-b^3*(a + b)^7)^(1/2)*(28*a*b +
35*a^2 + 8*b^2)*(524288*a^12*b^18 + 8126464*a^13*b^17 + 58982400*a^14*b^16
+ 266076160*a^15*b^15 + 834928640*a^16*b^14 + 1932263424*a^17*b^13 + 341127
9872*a^18*b^12 + 4685824000*a^19*b^11 + 5060689920*a^20*b^10 + 4310958080*a
^21*b^9 + 2886467584*a^22*b^8 + 1502871552*a^23*b^7 + 596377600*a^24*b^6 +
174325760*a^25*b^5 + 35389440*a^26*b^4 + 4456448*a^27*b^3 + 262144*a^28*b^2
))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35
*a^7*b^3 + 21*a^8*b^2))))/(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^
5*b^5 + 35*a^6*b^4 + 35*a^7*b^3 + 21*a^8*b^2)))*(28*a*b + 35*a^2 + 8*b^2))/
(16*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*a^
7*b^3 + 21*a^8*b^2))))*(-b^3*(a + b)^7)^(1/2)*(28*a*b + 35*a^2 + 8*b^2)*1i)
/(8*f*(7*a^9*b + a^10 + a^3*b^7 + 7*a^4*b^6 + 21*a^5*b^5 + 35*a^6*b^4 + 35*
a^7*b^3 + 21*a^8*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.374 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=230

$$\frac{x}{a^3}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2f(a+b)^3}-\frac{b(11a+4b)\cot^3(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tan^{-1}\left(\right.}{8a^3f(a+b)^{9/2}}$$

```
[Out] x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))/a^3/(a+b)^(9/2)/f+1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*cot(f*x+e)/a^2/(a
+b)^4/f-1/24*(8*a^2-39*a*b-12*b^2)*cot(f*x+e)^3/a^2/(a+b)^3/f-1/4*b*cot(f*x
+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+4*b)*cot(f*x+e)^3/a^2/(a
+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.46, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4141, 1975, 472, 579, 583, 522, 203, 205}

$$-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{9/2}}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2f(a+b)^3}+\frac{\left(32a^2b+8a^3-15ab^2-4b^3\right.}{8a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(9/2)*f) + ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b^3
)*Cot[e + f*x])/(8*a^2*(a + b)^4*f) - ((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*
x]^3)/(24*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^3)/(4*a*(a + b)*f*(a + b + b*T
an[e + f*x]^2)^2) - (b*(11*a + 4*b)*Cot[e + f*x]^3)/(8*a^2*(a + b)^2*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-7bx^2}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e\right.}{4a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8a}$$

$$= \frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2(a+b)^3f} -$$

$$= \frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2(a+b)^3f} -$$

$$= \frac{x}{a^3} - \frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} + \frac{\left(8a^3+32a^2b-15ab^2-4b\right.}{8a^2(a+b)^4f}$$

<u>**Mathematica**</u> [C] time = 7.76, size = 3340, normalized size = 14.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
((b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*
e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a
*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a + b
]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*b^3*ArcTan[Sec[f*x]*(Cos[2*e
]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[
a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*
Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]])))/((a + b)^4*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x]
)*Csc[e]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^6*(-36*a^6*f*x*Cos[f*x] - 336
*a^5*b*f*x*Cos[f*x] - 1560*a^4*b^2*f*x*Cos[f*x] - 3600*a^3*b^3*f*x*Cos[f*x]
 - 4260*a^2*b^4*f*x*Cos[f*x] - 2496*a*b^5*f*x*Cos[f*x] - 576*b^6*f*x*Cos[f*
x] + 36*a^6*f*x*Cos[3*f*x] + 240*a^5*b*f*x*Cos[3*f*x] + 408*a^4*b^2*f*x*Cos
[3*f*x] - 48*a^3*b^3*f*x*Cos[3*f*x] - 732*a^2*b^4*f*x*Cos[3*f*x] - 672*a*b^
5*f*x*Cos[3*f*x] - 192*b^6*f*x*Cos[3*f*x] + 36*a^6*f*x*Cos[2*e - f*x] + 336
*a^5*b*f*x*Cos[2*e - f*x] + 1560*a^4*b^2*f*x*Cos[2*e - f*x] + 3600*a^3*b^3*
```

f*x*Cos[2*e - f*x] + 4260*a^2*b^4*f*x*Cos[2*e - f*x] + 2496*a*b^5*f*x*Cos[2*e - f*x] + 576*b^6*f*x*Cos[2*e - f*x] + 36*a^6*f*x*Cos[2*e + f*x] + 336*a^5*b*f*x*Cos[2*e + f*x] + 1560*a^4*b^2*f*x*Cos[2*e + f*x] + 3600*a^3*b^3*f*x*Cos[2*e + f*x] + 4260*a^2*b^4*f*x*Cos[2*e + f*x] + 2496*a*b^5*f*x*Cos[2*e + f*x] + 576*b^6*f*x*Cos[2*e + f*x] - 36*a^6*f*x*Cos[4*e + f*x] - 336*a^5*b*f*x*Cos[4*e + f*x] - 1560*a^4*b^2*f*x*Cos[4*e + f*x] - 3600*a^3*b^3*f*x*Cos[4*e + f*x] - 4260*a^2*b^4*f*x*Cos[4*e + f*x] - 2496*a*b^5*f*x*Cos[4*e + f*x] - 576*b^6*f*x*Cos[4*e + f*x] - 36*a^6*f*x*Cos[2*e + 3*f*x] - 240*a^5*b*f*x*Cos[2*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[2*e + 3*f*x] + 48*a^3*b^3*f*x*Cos[2*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[2*e + 3*f*x] + 672*a*b^5*f*x*Cos[2*e + 3*f*x] + 192*b^6*f*x*Cos[2*e + 3*f*x] + 36*a^6*f*x*Cos[4*e + 3*f*x] + 240*a^5*b*f*x*Cos[4*e + 3*f*x] + 408*a^4*b^2*f*x*Cos[4*e + 3*f*x] - 48*a^3*b^3*f*x*Cos[4*e + 3*f*x] - 732*a^2*b^4*f*x*Cos[4*e + 3*f*x] - 672*a*b^5*f*x*Cos[4*e + 3*f*x] - 192*b^6*f*x*Cos[4*e + 3*f*x] - 36*a^6*f*x*Cos[6*e + 3*f*x] - 240*a^5*b*f*x*Cos[6*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[6*e + 3*f*x] + 48*a^3*b^3*f*x*Cos[6*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[6*e + 3*f*x] + 672*a*b^5*f*x*Cos[6*e + 3*f*x] + 192*b^6*f*x*Cos[6*e + 3*f*x] - 12*a^6*f*x*Cos[2*e + 5*f*x] - 144*a^5*b*f*x*Cos[2*e + 5*f*x] - 456*a^4*b^2*f*x*Cos[2*e + 5*f*x] - 624*a^3*b^3*f*x*Cos[2*e + 5*f*x] - 396*a^2*b^4*f*x*Cos[2*e + 5*f*x] - 96*a*b^5*f*x*Cos[2*e + 5*f*x] + 12*a^6*f*x*Cos[4*e + 5*f*x] + 144*a^5*b*f*x*Cos[4*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[4*e + 5*f*x] + 624*a^3*b^3*f*x*Cos[4*e + 5*f*x] + 396*a^2*b^4*f*x*Cos[4*e + 5*f*x] + 96*a*b^5*f*x*Cos[4*e + 5*f*x] - 12*a^6*f*x*Cos[6*e + 5*f*x] - 144*a^5*b*f*x*Cos[6*e + 5*f*x] - 456*a^4*b^2*f*x*Cos[6*e + 5*f*x] - 624*a^3*b^3*f*x*Cos[6*e + 5*f*x] - 396*a^2*b^4*f*x*Cos[6*e + 5*f*x] - 96*a*b^5*f*x*Cos[6*e + 5*f*x] + 12*a^6*f*x*Cos[8*e + 5*f*x] + 144*a^5*b*f*x*Cos[8*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[8*e + 5*f*x] + 624*a^3*b^3*f*x*Cos[8*e + 5*f*x] + 396*a^2*b^4*f*x*Cos[8*e + 5*f*x] + 96*a*b^5*f*x*Cos[8*e + 5*f*x] - 12*a^6*f*x*Cos[4*e + 7*f*x] - 48*a^5*b*f*x*Cos[4*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[4*e + 7*f*x] - 48*a^3*b^3*f*x*Cos[4*e + 7*f*x] - 12*a^2*b^4*f*x*Cos[4*e + 7*f*x] + 12*a^6*f*x*Cos[6*e + 7*f*x] + 48*a^5*b*f*x*Cos[6*e + 7*f*x] + 72*a^4*b^2*f*x*Cos[6*e + 7*f*x] + 48*a^3*b^3*f*x*Cos[6*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[6*e + 7*f*x] - 12*a^6*f*x*Cos[8*e + 7*f*x] - 48*a^5*b*f*x*Cos[8*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[8*e + 7*f*x] - 48*a^3*b^3*f*x*Cos[8*e + 7*f*x] - 12*a^2*b^4*f*x*Cos[8*e + 7*f*x] + 12*a^6*f*x*Cos[10*e + 7*f*x] + 48*a^5*b*f*x*Cos[10*e + 7*f*x] + 72*a^4*b^2*f*x*Cos[10*e + 7*f*x] + 48*a^3*b^3*f*x*Cos[10*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[10*e + 7*f*x] - 128*a^6*Sin[f*x] - 440*a^5*b*Sin[f*x] - 1152*a^4*b^2*Sin[f*x] - 1920*a^3*b^3*Sin[f*x] + 228*a^2*b^4*Sin[f*x] + 1320*a*b^5*Sin[f*x] + 432*b^6*Sin[f*x] + 48*a^6*Sin[3*f*x] + 104*a^5*b*Sin[3*f*x] + 640*a^4*b^2*Sin[3*f*x] + 1511*a^3*b^3*Sin[3*f*x] - 528*a^2*b^4*Sin[3*f*x] + 264*a*b^5*Sin[3*f*x] + 144*b^6*Sin[3*f*x] - 32*a^6*Sin[2*e - f*x] + 384*a^5*b*Sin[2*e - f*x] + 2048*a^4*b^2*Sin[2*e - f*x] + 3072*a^3*b^3*Sin[2*e - f*x] + 228*a^2*b^4*Sin[2*e - f*x] + 1320*a*b^5*Sin[2*e - f*x] + 432*b^6*Sin[2*e - f*x] + 32*a^6*Sin[2*e + f*x] - 384*a^5*b*Sin[2*e + f*x] - 2048*a^4*b^2*Sin[2*e + f*x] - 2919*a^3*b^3*Sin[2*e + f*x] + 642*a^2*b^4*Sin[2*e + f*x] + 1416*a*b^5*Sin[2*e + f*x] + 432*b^6*Sin[2*e + f*x] - 128*a^6*Sin[4*e + f*x] - 440*a^5*b*Sin[4*e + f*x] - 1152*a^4*b^2*Sin[4*e + f*x] - 2073*a^3*b^3*Sin[4*e + f*x] - 642*a^2*b^4*Sin[4*e + f*x] - 1416*a*b^5*Sin[4*e + f*x] - 432*b^6*Sin[4*e + f*x] - 144*a^6*Sin[2*e + 3*f*x] - 672*a^5*b*Sin[2*e + 3*f*x] - 960*a^4*b^2*Sin[2*e + 3*f*x] + 153*a^3*b^3*Sin[2*e + 3*f*x] + 528*a^2*b^4*Sin[2*e + 3*f*x] - 264*a*b^5*Sin[2*e + 3*f*x] - 144*b^6*Sin[2*e + 3*f*x] + 48*a^6*Sin[4*e + 3*f*x] + 104*a^5*b*Sin[4*e + 3*f*x] + 640*a^4*b^2*Sin[4*e + 3*f*x] + 1664*a^3*b^3*Sin[4*e + 3*f*x] - 66*a^2*b^4*Sin[4*e + 3*f*x] - 408*a*b^5*Sin[4*e + 3*f*x] - 144*b^6*Sin[4*e + 3*f*x] - 144*a^6*Sin[6*e + 3*f*x] - 672*a^5*b*Sin[6*e + 3*f*x] - 960*a^4*b^2*Sin[6*e + 3*f*x] + 66*a^2*b^4*Sin[6*e + 3*f*x] + 408*a*b^5*Sin[6*e + 3*f*x] + 144*b^6*Sin[6*e + 3*f*x] + 80*a^6*Sin[2*e + 5*f*x] + 480*a^5*b*Sin[2*e + 5*f*x] + 832*a^4*b^2*Sin[2*e + 5*f*x] + 294*a^2*b^4*Sin[2*e + 5*f*x] + 96*a*b^5*Sin[2*e + 5*f*x] - 48*a^6*Sin[4*e + 5*f*x] - 120*a^5*b*Sin[4*e + 5*f*x] - 294*a^2*b^4*Sin[4*e + 5*f*x] - 96*

```
a*b^5*Sin[4*e + 5*f*x] + 80*a^6*Sin[6*e + 5*f*x] + 480*a^5*b*Sin[6*e + 5*f*
x] + 832*a^4*b^2*Sin[6*e + 5*f*x] - 51*a^3*b^3*Sin[6*e + 5*f*x] - 132*a^2*b
^4*Sin[6*e + 5*f*x] - 48*a*b^5*Sin[6*e + 5*f*x] - 48*a^6*Sin[8*e + 5*f*x] -
 120*a^5*b*Sin[8*e + 5*f*x] + 51*a^3*b^3*Sin[8*e + 5*f*x] + 132*a^2*b^4*Sin
[8*e + 5*f*x] + 48*a*b^5*Sin[8*e + 5*f*x] + 32*a^6*Sin[4*e + 7*f*x] + 104*a
^5*b*Sin[4*e + 7*f*x] + 51*a^3*b^3*Sin[4*e + 7*f*x] + 18*a^2*b^4*Sin[4*e +
7*f*x] - 51*a^3*b^3*Sin[6*e + 7*f*x] - 18*a^2*b^4*Sin[6*e + 7*f*x] + 32*a^6
*Sin[8*e + 7*f*x] + 104*a^5*b*Sin[8*e + 7*f*x]))/(6144*a^3*(a + b)^4*f*(a +
 b*Sec[e + f*x]^2)^3)
```

fricas [B] time = 0.88, size = 1649, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(4*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 4*
(24*a^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(
f*x + e)^5 - 4*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^
5)*cos(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6
 - 63*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 -
16*a*b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos
(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
 (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*
x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(8*a^4*b^2 + 32*a
^3*b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 96*((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 -
8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2
+ 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*
a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b +
6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*
b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^
7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e)), 1/48
*(2*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 2*(24*a
^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(f*x +
 e)^5 - 2*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^5)*co
s(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 - 63
*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 - 16*a*
b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos(f*x
+ e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a
 + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(8*a^4*b^2 + 32*a^3*
b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 48*((a^6 + 4*a^5*b + 6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a
^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8
*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3
*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b + 6*a
^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2
 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b
^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b
^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))]
```

giac [A] time = 4.14, size = 314, normalized size = 1.37

$$-\frac{\frac{3\left(63a^2b^3+36ab^4+8b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\sqrt{ab+b^2}}+\frac{3\left(15ab^4\tan(fx+e)^3+4b^5\tan(fx+e)^3+17a^2b^3\tan(fx+e)+21ab^4\tan(fx\cdots\right.}{\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
4*a^4*b^3 + a^3*b^4)*sqrt(a*b + b^2)) + 3*(15*a*b^4*tan(f*x + e)^3 + 4*b^5*
tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) + 21*a*b^4*tan(f*x + e) + 4*b^5*ta
n(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*(b*tan(f*x +
 e)^2 + a + b)^2) - 24*(f*x + e)/a^3 - 8*(3*a*tan(f*x + e)^2 + 12*b*tan(f*x
 + e)^2 - a - b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
3))/f
```

maple [A] time = 1.48, size = 374, normalized size = 1.63

$$-\frac{15b^4\left(\tan^3\left(fx+e\right)\right)}{8fa\left(a+b\right)^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{b^5\left(\tan^3\left(fx+e\right)\right)}{2fa^2\left(a+b\right)^4\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{17b^3\tan\left(fx+\cdots\right.}{8f\left(a+b\right)^4\left(a+b+b\left(\tan\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -15/8/f*b^4/a/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-1/2/f*b^5/a^2/(a+
b)^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-17/8/f*b^3/(a+b)^4/(a+b+b*tan(f*x+
e)^2)^2*tan(f*x+e)-21/8/f*b^4/a/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-1
/2/f*b^5/a^2/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-63/8/f*b^3/a/(a+b)^4
/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-9/2/f*b^4/a^2/(a+b)^4
/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^5/a^3/(a+b)^4/(
(a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/3/f/(a+b)^3/tan(f*x+e
)^3+1/f/(a+b)^4/tan(f*x+e)*a+4/f/(a+b)^4/tan(f*x+e)*b+1/f/a^3*arctan(tan(f*
x+e))
```

maxima [A] time = 0.46, size = 409, normalized size = 1.78

$$-\frac{\frac{3\left(63a^2b^3+36ab^4+8b^5\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\sqrt{(a+b)b}}-\frac{3\left(8a^3b^2+32a^2b^3-15ab^4-4b^5\right)\tan\left(fx+e\right)^6-8a^5-24a^4b-24a^3b^2-8a^2b^3+\left(48a^4b+232a^3b^2+133a^2\cdots\right.}{\left(a^6b^2+4a^5b^3+6a^4b^4+4a^3b^5+a^2b^6\right)\tan\left(fx+e\right)^7+2\left(a^7b+5a^6b^2+10a^5b^3+10a^4b^4+5a^3b^5+a^2b^6\right)\mathrm{ta}\cdots}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)) -
(3*(8*a^3*b^2 + 32*a^2*b^3 - 15*a*b^4 - 4*b^5)*tan(f*x + e)^6 - 8*a^5 - 24*
a^4*b - 24*a^3*b^2 - 8*a^2*b^3 + (48*a^4*b + 232*a^3*b^2 + 133*a^2*b^3 - 63
*a*b^4 - 12*b^5)*tan(f*x + e)^4 + 8*(3*a^5 + 16*a^4*b + 23*a^3*b^2 + 10*a^2
*b^3)*tan(f*x + e)^2)/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b
^6)*tan(f*x + e)^7 + 2*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3
*b^5 + a^2*b^6)*tan(f*x + e)^5 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*tan(f*x + e)^3) - 24*(f*x + e)/a^3)/f
```

mupad [B] time = 11.98, size = 7057, normalized size = 30.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((860160*a^6*b^20*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200
*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^1
4*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^
9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 871628
80*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (145
15200*a^7*b^19*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 1153474
56*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b
^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12
+ 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 350
2829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21
*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (115347456*
a^8*b^18*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8
*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 +
9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 1606
6462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 350282956
8*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 +
 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (570587136*a^9*b^
17*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18
+ 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 957730
8160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 1606646272
0*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18
*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762
560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (1961717760*a^10*b^16*t
an(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 57
0587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160
*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^
15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8
 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*
a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (4965811200*a^11*b^15*tan(e
 + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587
136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^1
2*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b
^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1
329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22
*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (9577308160*a^12*b^14*tan(e + f
*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*
a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^
14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11
+ 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 13295
27808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4
 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (14379552768*a^13*b^13*tan(e + f*x)
)/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9
*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14
+ 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1
2106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 13295278
08*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 +
1376256*a^23*b^3 + 65536*a^24*b^2) + (17038737408*a^14*b^12*tan(e + f*x))/(
860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^
17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 1
4379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1210
6321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*
a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 137
6256*a^23*b^3 + 65536*a^24*b^2) + (16066462720*a^15*b^11*tan(e + f*x))/(860
160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17
+ 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 1437
9552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1210632
1920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^1
9*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 137625
```

6*a^23*b^3 + 65536*a^24*b^2) + (12106321920*a^16*b^10*tan(e + f*x))/(860160
*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1
961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 1437955
2768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 1210632192
0*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b
^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a
^23*b^3 + 65536*a^24*b^2) + (7294187520*a^17*b^9*tan(e + f*x))/(860160*a^6*
b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 196171
7760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*
a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^1
6*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 +
392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b
^3 + 65536*a^24*b^2) + (3502829568*a^18*b^8*tan(e + f*x))/(860160*a^6*b^20
+ 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*
a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*
b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^1
0 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 39223
2960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 +
65536*a^24*b^2) + (1329527808*a^19*b^7*tan(e + f*x))/(860160*a^6*b^20 + 145
15200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*
b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13
+ 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7
294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*
a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536
*a^24*b^2) + (392232960*a^20*b^6*tan(e + f*x))/(860160*a^6*b^20 + 14515200*
a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 +
4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 1703
8737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187
520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b
^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*
b^2) + (87162880*a^21*b^5*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^1
9 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581
1200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408
*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^1
7*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87
162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) +
(13762560*a^22*b^4*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115
347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^
11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b
^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 +
3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*
a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (137625
6*a^23*b^3*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a
^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15
+ 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16
066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829
568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5
+ 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (65536*a^24*b^2
*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 +
570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 95773081
60*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*
a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b
^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 1376256
0*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2))/(a^3*f) - (1/(3*(a + b)) -
(tan(e + f*x)^2*(3*a + 10*b))/(3*(a + b)^2) + (tan(e + f*x)^6*(15*a*b^4 +
4*b^5 - 32*a^2*b^3 - 8*a^3*b^2))/(8*a^2*(a + b)^4) + (tan(e + f*x)^4*(51*a*
b^3 - 48*a^3*b + 12*b^4 - 184*a^2*b^2))/(24*a^2*(a + b)^3))/(f*(tan(e + f*x
)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)^5*(2*a*b + 2*b^2) + b^2*tan(e + f*x)
^7)) - (atan((((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*(131072*a^6*b^23 + 2686

976*a^7*b^22 + 26214400*a^8*b^21 + 161013760*a^9*b^20 + 695239680*a^10*b^19
+ 2234314752*a^11*b^18 + 5525833728*a^12*b^17 + 10739159040*a^13*b^16 + 16
625679360*a^14*b^15 + 20693114880*a^15*b^14 + 20844212224*a^16*b^13 + 17084
284928*a^17*b^12 + 11452103680*a^18*b^11 + 6309949440*a^19*b^10 + 286651392
0*a^20*b^9 + 1069486080*a^21*b^8 + 321586176*a^22*b^7 + 74711040*a^23*b^6 +
12451840*a^24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b^3) - ((-b^5*(a + b)^9)
^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 + 1327104*a^11*b^21 + 136
31488*a^12*b^20 + 91750400*a^13*b^19 + 443154432*a^14*b^18 + 1607925760*a^1
5*b^17 + 4509663232*a^16*b^16 + 9971564544*a^17*b^15 + 17627217920*a^18*b^1
4 + 25149669376*a^19*b^13 + 29127081984*a^20*b^12 + 27445297152*a^21*b^11 +
21016346624*a^22*b^10 + 13016432640*a^23*b^9 + 6461587456*a^24*b^8 + 25337
52832*a^25*b^7 + 767361024*a^26*b^6 + 173293568*a^27*b^5 + 27525120*a^28*b^
4 + 2752512*a^29*b^3 + 131072*a^30*b^2 - (tan(e + f*x)*(-b^5*(a + b)^9)^(1/
2)*(36*a*b + 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 10747904*a^13*b^22 + 10485
7600*a^14*b^21 + 647495680*a^15*b^20 + 2839019520*a^16*b^19 + 9398648832*a^
17*b^18 + 24385683456*a^18*b^17 + 50803507200*a^19*b^16 + 86365962240*a^20*
b^15 + 121081692160*a^21*b^14 + 140895059968*a^22*b^13 + 136492089344*a^23*
b^12 + 110074265600*a^24*b^11 + 73665085440*a^25*b^10 + 40642805760*a^26*b^
9 + 18289262592*a^27*b^8 + 6604455936*a^28*b^7 + 1867776000*a^29*b^6 + 3984
58880*a^30*b^5 + 60293120*a^31*b^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(
16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a
^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 +
a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 +
84*a^9*b^3 + 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^2)*1i)/(16*(9*a^11*b +
a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^
8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)) + ((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*
(131072*a^6*b^23 + 2686976*a^7*b^22 + 26214400*a^8*b^21 + 161013760*a^9*b^2
0 + 695239680*a^10*b^19 + 2234314752*a^11*b^18 + 5525833728*a^12*b^17 + 107
39159040*a^13*b^16 + 16625679360*a^14*b^15 + 20693114880*a^15*b^14 + 208442
12224*a^16*b^13 + 17084284928*a^17*b^12 + 11452103680*a^18*b^11 + 630994944
0*a^19*b^10 + 2866513920*a^20*b^9 + 1069486080*a^21*b^8 + 321586176*a^22*b^
7 + 74711040*a^23*b^6 + 12451840*a^24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b
^3) + ((-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 +
1327104*a^11*b^21 + 13631488*a^12*b^20 + 91750400*a^13*b^19 + 443154432*a^1
4*b^18 + 1607925760*a^15*b^17 + 4509663232*a^16*b^16 + 9971564544*a^17*b^15
+ 17627217920*a^18*b^14 + 25149669376*a^19*b^13 + 29127081984*a^20*b^12 +
27445297152*a^21*b^11 + 21016346624*a^22*b^10 + 13016432640*a^23*b^9 + 6461
587456*a^24*b^8 + 2533752832*a^25*b^7 + 767361024*a^26*b^6 + 173293568*a^27
*b^5 + 27525120*a^28*b^4 + 2752512*a^29*b^3 + 131072*a^30*b^2 + (tan(e + f*
x)*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 107
47904*a^13*b^22 + 104857600*a^14*b^21 + 647495680*a^15*b^20 + 2839019520*a^
16*b^19 + 9398648832*a^17*b^18 + 24385683456*a^18*b^17 + 50803507200*a^19*b
^16 + 86365962240*a^20*b^15 + 121081692160*a^21*b^14 + 140895059968*a^22*b^
13 + 136492089344*a^23*b^12 + 110074265600*a^24*b^11 + 73665085440*a^25*b^1
0 + 40642805760*a^26*b^9 + 18289262592*a^27*b^8 + 6604455936*a^28*b^7 + 186
7776000*a^29*b^6 + 398458880*a^30*b^5 + 60293120*a^31*b^4 + 5767168*a^32*b^
3 + 262144*a^33*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b
^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/
(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*
a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^
2)*1i)/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6
+ 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))/(763699200*a^12*
b^14 - 663552*a^5*b^21 - 5955584*a^6*b^20 - 31360000*a^7*b^19 - 106229760*a
^8*b^18 - 233375744*a^9*b^17 - 293113856*a^10*b^16 - 19971072*a^11*b^15 - 3
2768*a^4*b^22 + 1804718080*a^13*b^13 + 2475196416*a^14*b^12 + 2343814144*a^
15*b^11 + 1598148608*a^16*b^10 + 785971200*a^17*b^9 + 272035840*a^18*b^8 +
62651392*a^19*b^7 + 8552448*a^20*b^6 + 516096*a^21*b^5 + ((-b^5*(a + b)^9)^
(1/2)*(tan(e + f*x)*(131072*a^6*b^23 + 2686976*a^7*b^22 + 26214400*a^8*b^21
+ 161013760*a^9*b^20 + 695239680*a^10*b^19 + 2234314752*a^11*b^18 + 552583

```
3728*a^12*b^17 + 10739159040*a^13*b^16 + 16625679360*a^14*b^15 + 2069311488
0*a^15*b^14 + 20844212224*a^16*b^13 + 17084284928*a^17*b^12 + 11452103680*a
^18*b^11 + 6309949440*a^19*b^10 + 2866513920*a^20*b^9 + 1069486080*a^21*b^8
 + 321586176*a^22*b^7 + 74711040*a^23*b^6 + 12451840*a^24*b^5 + 1310720*a^2
5*b^4 + 65536*a^26*b^3) - ((-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)
*(65536*a^10*b^22 + 1327104*a^11*b^21 + 13631488*a^12*b^20 + 91750400*a^13*
b^19 + 443154432*a^14*b^18 + 1607925760*a^15*b^17 + 4509663232*a^16*b^16 +
9971564544*a^17*b^15 + 17627217920*a^18*b^14 + 25149669376*a^19*b^13 + 2912
7081984*a^20*b^12 + 27445297152*a^21*b^11 + 21016346624*a^22*b^10 + 1301643
2640*a^23*b^9 + 6461587456*a^24*b^8 + 2533752832*a^25*b^7 + 767361024*a^26*
b^6 + 173293568*a^27*b^5 + 27525120*a^28*b^4 + 2752512*a^29*b^3 + 131072*a^
30*b^2 - (tan(e + f*x)*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*(52
4288*a^12*b^23 + 10747904*a^13*b^22 + 104857600*a^14*b^21 + 647495680*a^15*
b^20 + 2839019520*a^16*b^19 + 9398648832*a^17*b^18 + 24385683456*a^18*b^17
+ 50803507200*a^19*b^16 + 86365962240*a^20*b^15 + 121081692160*a^21*b^14 +
140895059968*a^22*b^13 + 136492089344*a^23*b^12 + 110074265600*a^24*b^11 +
73665085440*a^25*b^10 + 40642805760*a^26*b^9 + 18289262592*a^27*b^8 + 66044
55936*a^28*b^7 + 1867776000*a^29*b^6 + 398458880*a^30*b^5 + 60293120*a^31*b
^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 +
9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^
3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7
 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2)))*(36
*a*b + 63*a^2 + 8*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5
*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))
- ((-b^5*(a + b)^9)^(1/2)*(tan(e + f*x)*(131072*a^6*b^23 + 2686976*a^7*b^22
 + 26214400*a^8*b^21 + 161013760*a^9*b^20 + 695239680*a^10*b^19 + 223431475
2*a^11*b^18 + 5525833728*a^12*b^17 + 10739159040*a^13*b^16 + 16625679360*a^
14*b^15 + 20693114880*a^15*b^14 + 20844212224*a^16*b^13 + 17084284928*a^17*
b^12 + 11452103680*a^18*b^11 + 6309949440*a^19*b^10 + 2866513920*a^20*b^9 +
 1069486080*a^21*b^8 + 321586176*a^22*b^7 + 74711040*a^23*b^6 + 12451840*a^
24*b^5 + 1310720*a^25*b^4 + 65536*a^26*b^3) + ((-b^5*(a + b)^9)^(1/2)*(36*a
*b + 63*a^2 + 8*b^2)*(65536*a^10*b^22 + 1327104*a^11*b^21 + 13631488*a^12*b
^20 + 91750400*a^13*b^19 + 443154432*a^14*b^18 + 1607925760*a^15*b^17 + 450
9663232*a^16*b^16 + 9971564544*a^17*b^15 + 17627217920*a^18*b^14 + 25149669
376*a^19*b^13 + 29127081984*a^20*b^12 + 27445297152*a^21*b^11 + 21016346624
*a^22*b^10 + 13016432640*a^23*b^9 + 6461587456*a^24*b^8 + 2533752832*a^25*b
^7 + 767361024*a^26*b^6 + 173293568*a^27*b^5 + 27525120*a^28*b^4 + 2752512*
a^29*b^3 + 131072*a^30*b^2 + (tan(e + f*x)*(-b^5*(a + b)^9)^(1/2)*(36*a*b +
 63*a^2 + 8*b^2)*(524288*a^12*b^23 + 10747904*a^13*b^22 + 104857600*a^14*b^
21 + 647495680*a^15*b^20 + 2839019520*a^16*b^19 + 9398648832*a^17*b^18 + 24
385683456*a^18*b^17 + 50803507200*a^19*b^16 + 86365962240*a^20*b^15 + 12108
1692160*a^21*b^14 + 140895059968*a^22*b^13 + 136492089344*a^23*b^12 + 11007
4265600*a^24*b^11 + 73665085440*a^25*b^10 + 40642805760*a^26*b^9 + 18289262
592*a^27*b^8 + 6604455936*a^28*b^7 + 1867776000*a^29*b^6 + 398458880*a^30*b
^5 + 60293120*a^31*b^4 + 5767168*a^32*b^3 + 262144*a^33*b^2))/(16*(9*a^11*b
 + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126
*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))))/(16*(9*a^11*b + a^12 + a^3*b^9 + 9*
a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3
+ 36*a^10*b^2)))*(36*a*b + 63*a^2 + 8*b^2))/(16*(9*a^11*b + a^12 + a^3*b^9
+ 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126*a^7*b^5 + 126*a^8*b^4 + 84*a^9*
b^3 + 36*a^10*b^2))))*(-b^5*(a + b)^9)^(1/2)*(36*a*b + 63*a^2 + 8*b^2)*1i)/
(8*f*(9*a^11*b + a^12 + a^3*b^9 + 9*a^4*b^8 + 36*a^5*b^7 + 84*a^6*b^6 + 126
*a^7*b^5 + 126*a^8*b^4 + 84*a^9*b^3 + 36*a^10*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.375 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=285

$$-\frac{x}{a^3}-\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2f(a+b)^3}-\frac{b(13a+4b)\cot^5(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b^{7/2}\left(99a^2+44ab+8b^2\right)\tan^{-1}\left(\cdot\right.}{8a^3f(a+b)^{11/2}}$$

```
[Out] -x/a^3+1/8*b^(7/2)*(99*a^2+44*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(11/2)/f-1/8*(8*a^4+40*a^3*b+80*a^2*b^2-19*a*b^3-4*b^4)*cot(
f*x+e)/a^2/(a+b)^5/f+1/24*(8*a^3+32*a^2*b-51*a*b^2-12*b^3)*cot(f*x+e)^3/a^2
/(a+b)^4/f-1/40*(8*a^2-75*a*b-20*b^2)*cot(f*x+e)^5/a^2/(a+b)^3/f-1/4*b*cot(
f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(13*a+4*b)*cot(f*x+e)^5/a^2
/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.61, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 203, 205}

$$\frac{b^{7/2}\left(99a^2+44ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{11/2}}-\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2f(a+b)^3}+\frac{\left(32a^2b+8a^3-51ab^2-12b^3\right)c\cdots}{24a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(11/2)*f) - ((8*a^4 + 40*a^3*b + 80*a^2*b^2 -
 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(8*a^2*(a + b)^5*f) + ((8*a^3 + 32*a^2*b -
 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(24*a^2*(a + b)^4*f) - ((8*a^2 - 75*a*b
 - 20*b^2)*Cot[e + f*x]^5)/(40*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^5)/(4*a*(
a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(8
*a^2*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-5b-9bx^2}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+f\right.}{4a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + -$$

$$= -\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8a^2(a}$$

$$= \frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{}{4}$$

$$= -\frac{\left(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4\right)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{\left(8a^3+32a^2b-51ab^2-12b\right.}{24a^2(a+b)^4f}$$

$$= -\frac{\left(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4\right)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{\left(8a^3+32a^2b-51ab^2-12b\right.}{24a^2(a+b)^4f}$$

$$= -\frac{x}{a^3} + \frac{b^{7/2}\left(99a^2+44ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f} - \frac{\left(8a^4+40a^3b+80a^2b^2-19\right.}{8a^2(a+b}$$

<u>**Mathematica**</u> **[C]** time = 8.34, size = 976, normalized size = 3.42

$$\frac{\left(99a^2+44ba+8b^2\right)(\cos(2e+2fx)a+a+2b)^3\left(\frac{ib^4\tan^{-1}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-\right.}{64a^3\sqrt{a+b}\,f\sqrt{b\cos(4e)-ib\sin(4e)}}\right.}{(a+b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(x*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(a^3*(a + b*Sec[e
+ f*x]^2)^3) + ((11*a*Cos[e] + 26*b*Cos[e])*(a + 2*b + a*Cos[2*e + 2*f*x])^
3*Csc[e]*Csc[e + f*x]^2*Sec[e + f*x]^6)/(120*(a + b)^4*f*(a + b*Sec[e + f*x
]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Cot[e]*Csc[e + f*x]^4*Sec[e + f
*x]^6)/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + ((99*a^2 + 44*a*b + 8*b^
2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1/64*(b^4*ArcTan[Sec[f
*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[
2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*S
```

```
in[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] -
 I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt
[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e
] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[
2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^5*(a +
 b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*
x]^5*Sec[e + f*x]^6*Sin[f*x])/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + (
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]^3*Sec[e + f*x]^6*(-11*
a*Sin[f*x] - 26*b*Sin[f*x]))/(120*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3) + (
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]*Sec[e + f*x]^6*(23*a^2
*Sin[f*x] + 106*a*b*Sin[f*x] + 173*b^2*Sin[f*x]))/(120*(a + b)^5*f*(a + b*S
ec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6
*(a*b^5*Sin[2*e] + 2*b^6*Sin[2*e] - a*b^5*Sin[2*f*x]))/(16*a^3*(a + b)^4*f*
(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[2*e]*Sec[
e + f*x]^6*(-21*a^2*b^4*Sin[2*e] - 52*a*b^5*Sin[2*e] - 16*b^6*Sin[2*e] + 21
*a^2*b^4*Sin[2*f*x] + 6*a*b^5*Sin[2*f*x]))/(64*a^3*(a + b)^5*f*(a + b*Sec[e
 + f*x]^2)^3)
```

fricas [B] time = 0.97, size = 2229, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*
cos(f*x + e)^9 - 4*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945
*a^3*b^4 - 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 4*(120*a^7 + 40*a^6*b -
1416*a^5*b^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f
*x + e)^5 + 20*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^
2*b^5 + 36*a*b^6)*cos(f*x + e)^3 - 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5
)*cos(f*x + e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b
^4 - 36*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*
a^2*b^5 + 12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 3
6*a*b^6 - 8*b^7)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b
^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e
) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(8*a^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e
) + 480*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*
x*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a
*b^6)*f*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b
^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5
*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10
+ 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f
*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e)), -1/240*(2
*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*cos(f*x +
e)^9 - 2*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945*a^3*b^4 -
 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 2*(120*a^7 + 40*a^6*b - 1416*a^5*b
^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f*x + e)^5
+ 10*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^2*b^5 + 36
*a*b^6)*cos(f*x + e)^3 + 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5)*cos(f*x
+ e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b^4 - 36*a^
2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*a^2*b^5 +
12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 36*a*b^6 -
```

```
8*b^7)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(8*a
^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e) + 240
*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*x*cos(f
*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f
*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*
a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^
3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 +
10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10 + 5*a^9
*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a
^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)
^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*
b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*
b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6
*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

giac [A] time = 1.27, size = 404, normalized size = 1.42

$$\frac{\frac{15\left(99\,a^2b^4+44\,ab^5+8\,b^6\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5\right)\sqrt{ab+b^2}}+\frac{15\left(19\,ab^5\tan\left(fx+e\right)^3+4\,b^6\tan\left(fx+e\right)^3+21\,a^2b^4\tan\left(fx+e\right)+25\,ab^5\tan\left(fx+e\right)\cdots\right.}{\left(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5\right)\left(b\tan\left(fx+e\right)^2+a+b\right)\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 5*a^7*b + 10*a^6*b^2
+ 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt(a*b + b^2)) + 15*(19*a*b^5*tan(f*x
 + e)^3 + 4*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) + 25*a*b^5*tan(f*x
 + e) + 4*b^6*tan(f*x + e))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*(b*tan(f*x + e)^2 + a + b)^2) - 120*(f*x + e)/a^3 - 8*(15
*a^2*tan(f*x + e)^4 + 75*a*b*tan(f*x + e)^4 + 150*b^2*tan(f*x + e)^4 - 5*a^
2*tan(f*x + e)^2 - 25*a*b*tan(f*x + e)^2 - 20*b^2*tan(f*x + e)^2 + 3*a^2 +
6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*t
an(f*x + e)^5))/f
```

maple [A] time = 1.63, size = 437, normalized size = 1.53

$$\frac{19b^5\left(\tan^3\left(fx+e\right)\right)}{8fa\left(a+b\right)^5\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{b^6\left(\tan^3\left(fx+e\right)\right)}{2f\,a^2\left(a+b\right)^5\left(a+b+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{21b^4\tan\left(fx+\cdots\right.}{8f\left(a+b\right)^5\left(a+b+b\left(\tan\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 19/8/f*b^5/a/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+1/2/f*b^6/a^2/(a+b
)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+21/8/f*b^4/(a+b)^5/(a+b+b*tan(f*x+e
)^2)^2*tan(f*x+e)+25/8/f*b^5/a/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+1/
2/f*b^6/a^2/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+99/8/f*b^4/a/(a+b)^5/
((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+11/2/f*b^5/a^2/(a+b)^5
/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b^6/a^3/(a+b)^5/(
(a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/5/f/(a+b)^3/tan(f*x+e
)^5+1/3/f/(a+b)^4/tan(f*x+e)^3*a+4/3/f/(a+b)^4/tan(f*x+e)^3*b-1/f/(a+b)^5/t
an(f*x+e)*a^2-5/f/(a+b)^5/tan(f*x+e)*a*b-10/f/(a+b)^5/tan(f*x+e)*b^2-1/f/a^
3*arctan(tan(f*x+e))
```

maxima [A] time = 0.47, size = 520, normalized size = 1.82

$$\frac{\frac{15\left(99\,a^2b^4+44\,ab^5+8\,b^6\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5\right)\sqrt{(a+b)b}}-\frac{15\left(8\,a^4b^2+40\,a^3b^3+80\,a^2b^4-19\,ab^5-4\,b^6\right)\tan(fx+e)^8+5\left(48\,a^5b+280\,a^4b^2+680\,a^3b^3+385\right.}{\left(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7\right)\tan(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt
((a + b)*b)) - (15*(8*a^4*b^2 + 40*a^3*b^3 + 80*a^2*b^4 - 19*a*b^5 - 4*b^6)
*tan(f*x + e)^8 + 5*(48*a^5*b + 280*a^4*b^2 + 680*a^3*b^3 + 385*a^2*b^4 - 7
5*a*b^5 - 12*b^6)*tan(f*x + e)^6 + 24*a^6 + 96*a^5*b + 144*a^4*b^2 + 96*a^3
*b^3 + 24*a^2*b^4 + 8*(15*a^6 + 95*a^5*b + 258*a^4*b^2 + 291*a^3*b^3 + 113*
a^2*b^4)*tan(f*x + e)^4 - 8*(5*a^6 + 29*a^5*b + 57*a^4*b^2 + 47*a^3*b^3 + 1
4*a^2*b^4)*tan(f*x + e)^2)/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5
+ 5*a^3*b^6 + a^2*b^7)*tan(f*x + e)^9 + 2*(a^8*b + 6*a^7*b^2 + 15*a^6*b^3 +
 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)*tan(f*x + e)^7 + (a^9 + 7*a
^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*
b^7)*tan(f*x + e)^5) - 120*(f*x + e)/a^3)/f
```

mupad [B] time = 15.21, size = 7460, normalized size = 26.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan(((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 1944780
80*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^
16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^1
9*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^2
2*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^
25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^2
8*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 +
340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2
 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^
26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 +
106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 +
1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17
 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b
^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^2
9*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*
b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 917
50400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^
3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 +
 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 2413229
7728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037
952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103
040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345
536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 3166261964
8*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b
^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31
*b^3))/(2*a^3) - (((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^2
5 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 229
68795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 34659
6343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 98982
```

1665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 6215
38574336*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 8423
0471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*
a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 1310
72*a^35*b^2 - (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840
000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a
^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a
^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 319384715264
0*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 370018877
4400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 110261
0432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 742811
23840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^3
5*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3)
)*1i)/(2*a^3) - tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000
*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^2
3 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20
+ 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17
+ 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14
+ 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 +
31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 82903
0400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 +
65536*a^31*b^3))/(2*a^3))/(27354112*a^10*b^21 - ((((65536*a^10*b^27 + 1654
784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*
b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20
+ 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17
+ 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^1
4 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^1
1 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 +
8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a
^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(524288*a^12*b
^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 723
5174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450
944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 22493
25281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 40
89682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 +
1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10
+ 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 45
21984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b
^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) - tan(e + f*x)*(131072*a^6*b
^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 176881766
4*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^
13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^
16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^
19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^
22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*
b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660
800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))*1i)/(2*a^3) - 32768*a^4*
b^27 - 827392*a^5*b^26 - 9084928*a^6*b^25 - 57263104*a^7*b^24 - 221133824*a
^8*b^23 - 467977216*a^9*b^22 - ((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21
954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*
a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^
18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^
21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a
^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a
^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^
7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*
a^34*b^3 + 131072*a^35*b^2 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^1
3*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24
+ 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 +

693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))*1i)/(2*a^3) + 4041583616*a^11*b^20 + 16331772928*a^12*b^19 + 40173472768*a^13*b^18 + 71534228480*a^14*b^17 + 97563767808*a^15*b^16 + 104426556416*a^16*b^15 + 88612000768*a^17*b^14 + 59708484608*a^18*b^13 + 31782593536*a^19*b^12 + 13203725312*a^20*b^11 + 4193231872*a^21*b^10 + 984308736*a^22*b^9 + 161366016*a^23*b^8 + 16580608*a^24*b^7 + 811008*a^25*b^6))/(a^3*f) - (1/(5*(a + b)) + (tan(e + f*x)^4*(65*a*b + 15*a^2 + 113*b^2))/(15*(a + b)^3) - (tan(e + f*x)^2*(5*a + 14*b))/(15*(a + b)^2) + (tan(e + f*x)^8*(80*a^2*b^4 - 4*b^6 - 19*a*b^5 + 40*a^3*b^3 + 8*a^4*b^2))/(8*a^2*(a + b)^5) + (tan(e + f*x)^6*(48*a^4*b - 63*a*b^4 - 12*b^5 + 448*a^2*b^3 + 232*a^3*b^2))/(24*a^2*(a + b)^4))/(f*(tan(e + f*x)^5*(2*a*b + a^2 + b^2) + tan(e + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) + (atan((((tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3) - ((-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*1i)/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2)) + ((tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960

000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*
b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^
20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^
17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^
14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^1
1 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 82
9030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^
4 + 65536*a^31*b^3) + ((-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(6
5536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^
24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 +
69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 5
85044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1
000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 +
380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29
853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*
a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 + (tan(e
+ f*x)*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28
+ 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174
400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 2984509440
00*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 224932528
1280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 408968
2329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 192
7993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 2
18864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 452198
4000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 +
262144*a^38*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*
b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4
+ 165*a^11*b^3 + 55*a^12*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*
b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 +
330*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a
*b + 99*a^2 + 8*b^2)*1i)/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 5
5*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^1
0*b^4 + 165*a^11*b^3 + 55*a^12*b^2)))/(27354112*a^10*b^21 - 827392*a^5*b^26
- 9084928*a^6*b^25 - 57263104*a^7*b^24 - 221133824*a^8*b^23 - 467977216*a^
9*b^22 - 32768*a^4*b^27 + 4041583616*a^11*b^20 + 16331772928*a^12*b^19 + 40
173472768*a^13*b^18 + 71534228480*a^14*b^17 + 97563767808*a^15*b^16 + 10442
6556416*a^16*b^15 + 88612000768*a^17*b^14 + 59708484608*a^18*b^13 + 3178259
3536*a^19*b^12 + 13203725312*a^20*b^11 + 4193231872*a^21*b^10 + 984308736*a
^22*b^9 + 161366016*a^23*b^8 + 16580608*a^24*b^7 + 811008*a^25*b^6 + ((tan(
e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 31923404
8*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12
*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*
b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*
b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*
b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^
10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 1507
32800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3) - (
(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(65536*a^10*b^27 + 165478
4*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^
23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 +
170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 +
828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14
+ 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11
+ 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 85
88754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^3
3*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(-b^7*(a + b)^11
)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28 + 13369344*a^13*b^27 +
163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 3117154
3040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 69306941

```
4400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 319384
7152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 370
0188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 +
1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 +
74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 7602176
00*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2))/(16*
(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 + 330
*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*a^1
2*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165
*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^1
1*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2))/(
16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 165*a^6*b^8 +
330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a^11*b^3 + 55*
a^12*b^2)) - ((tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*
a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23
 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 +
 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 +
 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 +
 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 +
31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030
400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 +
65536*a^31*b^3) + ((-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(65536
*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 +
 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 6950
6170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 58504
4721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 10005
64490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 3807
51118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 298539
74528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32
*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 + (tan(e + f*
x)*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 + 8*b^2)*(524288*a^12*b^28 + 13
369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*
a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a
^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280
*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329
600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993
098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 21886
4025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000
*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262
144*a^38*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9
+ 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 16
5*a^11*b^3 + 55*a^12*b^2))))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10
 + 55*a^5*b^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330
*a^10*b^4 + 165*a^11*b^3 + 55*a^12*b^2)))*(-b^7*(a + b)^11)^(1/2)*(44*a*b +
 99*a^2 + 8*b^2))/(16*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b
^9 + 165*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 +
 165*a^11*b^3 + 55*a^12*b^2))))*(-b^7*(a + b)^11)^(1/2)*(44*a*b + 99*a^2 +
8*b^2)*1i)/(8*f*(11*a^13*b + a^14 + a^3*b^11 + 11*a^4*b^10 + 55*a^5*b^9 + 1
65*a^6*b^8 + 330*a^7*b^7 + 462*a^8*b^6 + 462*a^9*b^5 + 330*a^10*b^4 + 165*a
^11*b^3 + 55*a^12*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.376 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^5(e+fx)\,dx$

Optimal. Leaf size=111

$$\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

[Out] -1/3*(a+2*b)*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b^2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 88, 50, 63, 208}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]

[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec[e + f*x]^2]/f - ((a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2*f) + (a + b*Sec[e + f*x]^2)^(5/2)/(5*b^2*f)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)\sqrt{a+bx}}{b}+\frac{\sqrt{a+bx}}{x}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\text{Subst}}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec\right.}{5}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec\right.}{5}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a\right.}{}$$

Mathematica [F] time = 2.22, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5, x]
```

fricas [B] time = 3.28, size = 456, normalized size = 4.11

$$\left[\frac{15\,\sqrt{a}b^2\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(a)*b^2*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 8*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f
*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(
f*x + e)^4), 1/60*(15*sqrt(-a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b
*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 -
4*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f*x + e)^2
 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^
4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)64*(1/240*(-15*a*(-sqrt(a+b)*tan(1/2
*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2
*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^9-165*a*sqrt(a
+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*ta
n(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))
^2+a+b))^8-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))
^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+e
xp(1)))^2+a+b))^7*(540*a^2-320*b^2-100*a*b)-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(
f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a
*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^6*(660*a^2+640*b
^2-1660*a*b)-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)
))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x
+exp(1)))^2+a+b))^5*(-30*a^3+832*b^3+1250*a*b^2-2460*a^2*b)-(-sqrt(a+b)*tan
(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))
^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(135*a^5-2
880*b^5+3335*a*b^4+380*a^2*b^3-1430*a^3*b^2+700*a^4*b)-sqrt(a+b)*(-sqrt(a+b
)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp
(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4*(-8
10*a^3-2560*b^3+3910*a*b^2-900*a^2*b)-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+s
qrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+ex
p(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(-660*a^4+320*b^4-3220*a*b^3+3
140*a^2*b^2+900*a^3*b)-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a
*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1))
```

```
)^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*(-60*a^4+3200*b^4-3180*a*b^3-980*a^
2*b^2+1500*a^3*b)-sqrt(a+b)*(45*a^5+768*b^5-1539*a*b^4+1028*a^2*b^3-450*a^3
*b^2+100*a^4*b))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*t
an(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^
2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqr
t(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(
1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^5+1/32*a*atan(1/2*(-sqrt(
a+b)*tan(1/2*(f*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan
(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^
2+a+b))/sqrt(-a))/sqrt(-a))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 2.05, size = 924, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/30/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-1+cos(f*x+e))*(15*
(a+b)^(1/2)*cos(f*x+e)^5*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2))*a^(1/2)*b^2+2*(a+b)^(1/2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^2+10*(a+b)^(1/2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a*b-15*(a+b)^(1/2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^2-15*cos(f*x+e)^5*ln(-4*(-1+cos(f*x+e))*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(
1/2))*a*b^2+15*cos(f*x+e)^5*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^2+2*(
a+b)^(1/2)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*
(a+b)^(1/2)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-15
*(a+b)^(1/2)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-(
a+b)^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+10*
(a+b)^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-(a
+b)^(1/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+10*(
a+b)^(1/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-3*(
a+b)^(1/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-3*(a+
b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)/sin(f*x+e)^2/cos(
f*x+e)^4/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b)^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \tan\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^5 \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^2\left(e + fx\right)} \tan^5\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**5, x)
```

3.377 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^3(e+fx)\,dx$

Optimal. Leaf size=80

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/b/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*
a^(1/2)/f-(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - Sqrt[a + b*Sec[e
+ f*x]^2]/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,d\right.}{2\,}$$

$$= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{}$$

$$= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+\right.}{3bf}$$

Mathematica [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^3(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3, x]
```

fricas [B] time = 1.24, size = 386, normalized size = 4.82

$$\left[\frac{3\sqrt{a}\,b\cos\left(fx+e\right)^2\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*co
s(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 +
 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)
^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)) + 8*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(b*f*cos(f*x + e)^2), -1/12*(3*sqrt(-a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*co
s(f*x + e)^2 - 4*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(b*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)16*(1/12*(3*a*(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*
tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^5+(-sqrt(a+b)*tan
(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))
^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-9*a^3-48
*b^3+15*a*b^2+6*a^2*b)+sqrt(a+b)*(9*a-12*b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)
))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(
f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4+(-sqrt(a+b)*tan(1/2*(f*x
+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*ta
n(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(6*a^2+16*b^2-18*
a*b)+sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(
1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f
*x+exp(1)))^2+a+b))^2*(-6*a^2+24*b^2-6*a*b)+sqrt(a+b)*(-3*a^3+20*b^3-19*a*b
^2+6*a^2*b))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1
/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*
b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*
tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))
^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^3-1/8*a*atan(1/2*(-sqrt(a+b)*
tan(1/2*(f*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*
(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b
))/sqrt(-a))/sqrt(-a))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 1.56, size = 648, normalized size = 8.10

$$-\frac{\sqrt{4}\,\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\,\left(-1+\cos\left(fx+e\right)\right)\left(3\sqrt{a+b}\,\left(\cos^3\left(fx+e\right)\right)\sqrt{a}\,\ln\left(4\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] -1/6/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-1+cos(f*x+e))*(3*(
a+b)^(1/2)*cos(f*x+e)^3*a^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2))*b+(a+b)^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f
```

```
*x+e))^2)^(1/2)*a-3*(a+b)^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b-3*cos(f*x+e)^3*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b+3*
cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b+(a+b)^(1/2)*cos(f*x+
e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-3*(a+b)^(1/2)*cos(f*x+e)
^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+(a+b)^(1/2)*cos(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b*(a+b)^(1/2))/sin(f*x+e)^2/cos(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)/(a+b)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx + e\right)^{2} + a} \tan \left(fx + e\right)^{3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\tan \left(e+f\,x\right)}^3\,\sqrt{a+\frac{b}{{\cos \left(e+f\,x\right)}^2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^{2}{\left(e + f x \right)}} \tan^{3}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**3, x)
```

3.378 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan(e+fx)\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^(1/
2)/f
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 50, 63, 208}

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [B] time = 0.44, size = 119, normalized size = 2.20

$$\frac{\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{b}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{b}} - 2\sqrt{a}\cos(e+fx)\sinh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)\right)}{\sqrt{2}\sqrt{b}f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] ((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]]*Cos[e + f*x] + Sqrt[2]
*Sqrt[b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])*Sqrt[a + b*Sec[e + f*x]^2]
)/(Sqrt[2]*Sqrt[b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])
```

fricas [B] time = 0.69, size = 312, normalized size = 5.78

$$\left[\frac{\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8 + 256\,a^3b\cos\left(fx+e\right)^6 + 160\,a^2b^2\cos\left(fx+e\right)^4 + 32\,ab^3\cos\left(fx+e\right)^2 + b^4 - 8\left(1\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
```

```
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)8*(1/2*(b*(-sqrt(a+b)*tan(1/2*(f*x+e
xp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(
1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-b*sqrt(a+b))/(-2*sqrt
(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*
tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)
))^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))
^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+e
xp(1)))^2+a+b))^2-a+3*b)+1/4*a*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2
-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan
(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-a))/sqrt(-a))*
sign(cos(f*x+exp(1)))/f
```

maple [A] time = 0.25, size = 61, normalized size = 1.13

$$-\frac{\sqrt{a}\, \ln\left(\frac{2a+2\sqrt{a}\, \sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f}+\frac{\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] -1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))+(a+b*s
ec(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\, \tan\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e), x)
```

mupad [B] time = 5.49, size = 46, normalized size = 0.85

$$\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{f}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sqrt{a}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + b/cos(e + f*x)^2)^(1/2)/f - (a^(1/2)*atanh((a + b/cos(e + f*x)^2)^(1/2
)/a^(1/2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\tan\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x), x)
```

3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 446, 83, 63, 208}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*ArcT
anh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\right.}{2f}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}}\right.}{}$$

$$= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Mathematica [F] time = 1.61, size = 0, normalized size = 0.00

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.93, size = 963, normalized size = 13.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(a + b)*l
og(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)
^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)))/f, 1/8*(4*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
)/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x
 + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
```

```
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, -1/4*(
sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqr
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3
*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(-1/4
*sqrt(a+b)*ln(abs(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1
/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*
b*tan(1/2*(f*x+exp(1)))^2+a+b)))+1/4*sqrt(a+b)*ln(abs(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1
)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)))-a*atan
(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(
1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f
*x+exp(1)))^2+a+b))/sqrt(-a))/sqrt(-a)+sqrt(a+b)*(a+b)*ln(abs(-(a+b)*(-sqrt
(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x
+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-s
qrt(a+b)*(a-b)))/(-4*a-4*b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 1.85, size = 570, normalized size = 8.14

$$-\frac{\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\,\sqrt{4}\,\cos\left(fx+e\right)\left(2\sqrt{a}\,\ln\left(4\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\,\sqrt{a}+4a\cos\left(fx+e\right)+4\sqrt{a}\,\sqrt{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f*x+e)*(2*a^(1/2
)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*c
os(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(1/2
)+ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*
cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+ln(-2*(-1+cos(f*x+e))*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1
```

```
/2))*b-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^
(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/(-1+cos(f*x+e)))*a-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b)*(-1+cos(f*x+e))/sin(f*x+e)^2/((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b)^(1/2)
```

maxima [C] time = 0.93, size = 3317, normalized size = 47.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(a^(3/2)*log(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4
*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(
a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*
cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 4*s
qrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*
sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*
f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 16*(a^2*cos(4*f*x + 4*e
)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos
(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a + b)*sqrt(a)*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2) -
2*sqrt(a + b)*a*log(4*(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)
^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2
 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2
*a*b)*cos(2*f*x + 2*e))*(a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
 a))^2 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 +
4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*
(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*
x + 2*e))*(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^2
+ 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) - 2)^2 + 4*(a^2*cos(4*f*x + 4*e)^2
 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*
(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*s
in(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a + b)*abs(2*e^(2*I*
f*x + 2*I*e) - 2) + 4*a + 4*b)*sqrt(a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*
e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f
*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2 + 8*(a^2 + 2*a*b + b^2)*abs(2*e^
(2*I*f*x + 2*I*e) - 2))/abs(2*e^(2*I*f*x + 2*I*e) - 2)^2) - (a^(3/2) + 2*sq
rt(a)*b)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2 +
16*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^2*cos(4*f*x + 4*e
)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos
(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a*sin(2*f*x + 2*e)
*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4
```

```
*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a*cos(2*f*x + 2*e) + a
+ 2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a
*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)))*sqrt(a) + 4*sqrt(a^
2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos
(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^
2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(
2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*(a*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a*sin(1/2*arctan2(a*sin(4*f*
x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*c
os(2*f*x + 2*e) + a))^2)) + 2*(a^(3/2) + sqrt(a)*b)*log(a^2*cos(2*f*x + 2*e
)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x +
4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 +
 a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*a^(3/2)*sin(2*f*x + 2*e)*sin(1/2*arctan2(
a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(
a + 2*b)*cos(2*f*x + 2*e) + a)) + sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(
a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e
) + a))^2 + sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 +
 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 2*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4
*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a^(3/2
)*cos(2*f*x + 2*e) + a^(3/2) + 2*sqrt(a)*b)*cos(1/2*arctan2(a*sin(4*f*x + 4
*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*
f*x + 2*e) + a)) + a^2 + 4*a*b + 4*b^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e)))
/(a*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/2*(2*a+b)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/2*cot(f*x+e)^2*(a+b*sec
(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 99, 156, 63, 208}

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a + b)*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]*f) - (Cot[e +
f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \text{se}\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{bf}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}\,f} - $$

Mathematica [C] time = 5.68, size = 527, normalized size = 4.83

$$\frac{e^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{1+e^{2i(e+fx)}}{\left(-1+e^{2i(e+fx)}\right)^2} - \frac{(2a+b)\log\left(1-e^{2i(e+fx)}\right)+\sqrt{a}\sqrt{a+b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+\right.}}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*((1 + E^((2*I)*(e + f*x)))/(-1 + E^((2*I)*(e + f*x)))^2
- ((-2*I)*Sqrt[a]*Sqrt[a + b]*f*x + (2*a + b)*Log[1 - E^((2*I)*(e + f*x))]
+ Sqrt[a]*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a]*Sqrt[a +
b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*a*Log[a + b +
```

```
a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - b*Log[a + b + a*E^((2*I)*
(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2
]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

fricas [B] time = 1.27, size = 1342, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^
2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 +
4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a
 + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a + b)*cos(f*x + e)^2 - 2*a - b
)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
b^2)) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a + b)*cos(
f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f
*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^
3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + ((2*a + b)*cos(f*x +
e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a
 + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x
+ e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2
+ a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

```
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(1/16*sqrt(a*
tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))
^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)+2*(-1/16*(-(a-b)*(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*
tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-sqrt(a+b)*(a+b))/
((-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1
/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+
a+b))^2-a-b)+1/2*a*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2-sqrt(a+b)+s
qrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+ex
p(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-a))/sqrt(-a)-1/8*(2*a+b)*a
tan((-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*ta
n(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))
^2+a+b))/sqrt(-a-b))/sqrt(-a-b)+sqrt(a+b)*(-2*a-b)*ln(abs((a+b)*(-sqrt(a+b)
*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(
1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))+sqrt(a
+b)*(a-b)))/(-16*a-16*b)))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 1.71, size = 2528, normalized size = 23.19

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*(-1+cos(f*x+e))*(-4*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*cos(f*x+e)^2*(a+b)^(3/2)*a^(1/2)*b+5*ln(-2*(-1+cos(f*
x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(
f*x+e)^2/(a+b)^(1/2))*4^(1/2)*a^2*b+4*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))
*4^(1/2)*a*b^2+2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*4^(1/2)*cos(f*x+e)^2*a^3+ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*4^(1/2)
*cos(f*x+e)^2*b^3-2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*4^(1/2)*cos(f*x+e
)^2*a^3-4*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2))*cos(f*x+e)^2*(a+b)^(3/2)*a^(3/2)-ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2)
)*4^(1/2)*cos(f*x+e)^2*b^3+4*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(3/2)-4*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*4^(1/2)*(a+b)^(3/2)*b-5*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*4^(1/2)*a^2*b-4*ln(-4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*
x+e)))*4^(1/2)*a*b^2-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*4^(1/2)*b^3+2*ln(-2*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
```

```
+b)^(1/2))*4^(1/2)*a^3+ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*4^(1/2)*b^3+8*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)^2*(a+b)^(3/2)+16*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*cos(f*x+e)*(a+b)^(3/2)-2*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e
)))*4^(1/2)*a^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*cos(f
*x+e)*(a+b)^(3/2)*a+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*c
os(f*x+e)*(a+b)^(3/2)*b-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/
2)*cos(f*x+e)^2*(a+b)^(3/2)*a+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*4^(1/2)*cos(f*x+e)^2*(a+b)^(3/2)*b+5*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*4^(1/2)*cos(f*x+e)^2*a^2
*b+4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/(-1+cos(f*x+e)))*4^(1/2)*cos(f*x+e)^2*a*b^2-5*ln(-2*(-1+cos(f*x+e))*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+
b)^(1/2))*4^(1/2)*cos(f*x+e)^2*a^2*b-4*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2)
)*4^(1/2)*cos(f*x+e)^2*a*b^2+4*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(1/2)*b+8*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(3/2)*(a+b)^(3/2))*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^
2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4/(a+b)^(5/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \cot\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^3 \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^2\left(e+fx\right)} \cot^3\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

```
[Out] -1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+
b)^(3/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/8*(4*a+3*b
)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f-1/4*cot(f*x+e)^4*(a+b*sec(f
*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4139, 446, 99, 151, 156, 63, 208}

$$-\frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 12*a*b
+ 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)*
f) + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f)
- (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2\right.}{4f}$$
$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(}}{4f}$$
$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(}}{4f}$$
$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(}}{4f}$$
$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+f}}{\sqrt{a+b}}\right.}{8(a+b)^{3/2}f}$$

Mathematica [F] time = 5.08, size = 0, normalized size = 0.00

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 2.71, size = 1953, normalized size = 12.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) + ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^
2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f
*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)
, 1/16*(((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^
2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sq
rt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*c
os(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a^2 + 11*a*b + 5*
b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/32*(8*((a^
2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^
2 + 2*a*b + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2
)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a
*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^
4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(4*((a^2 + 2*a*b + b^2)*c
os(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*s
qrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt
(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*
a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 -
 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-
a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) +
2*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*
x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b +
 b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(2*(-(8192*a+
8192*b)*tan(1/2*(f*x+exp(1)))^2/(-2097152*a-2097152*b)-(-90112*a-73728*b)/(
-2097152*a-2097152*b))*sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)
))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)+2*(((-sqr
t(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*
x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*
(-4*a^3+6*b^3+6*a*b^2-4*a^2*b)+(6*a^2-4*b^2)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1
)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*
(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3+sqrt(a+b)*(-sqrt(a+b)*t
an(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)
))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*(7*a^2
+3*b^2+10*a*b)+sqrt(a+b)*(-5*a^3-5*b^3-15*a*b^2-15*a^2*b))/((-sqrt(a+b)*tan
(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))
^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a-b)^2/(
-64*a-64*b)-1/2*a*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2-sqrt(a+b)+sq
rt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp
(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-a))/sqrt(-a)+(8*a^2+3*b^2+1
2*a*b)*ln(abs((a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x
+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1
/2*(f*x+exp(1)))^2+a+b))+sqrt(a+b)*(a-b)))/sqrt(a+b)/(-64*a-64*b)+1/2*(-8*a
^2-3*b^2-12*a*b)*atan((-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f
*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan
(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-a-b))/sqrt(-a-b)/(-16*a-16*b)))*sign(cos(f
*x+exp(1)))/f
```

maple [B] time = 1.81, size = 7346, normalized size = 45.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\,\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{5}\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\cot^5\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.382 $\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$

Optimal. Leaf size=219

$$\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}-\frac{\sqrt{a}\ \text{t}}{}$$

```
[Out] 1/16*(a^3+5*a^2*b+15*a*b^2-5*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))*a^(1/2)/f-1/16*(a-b)*(a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f
+1/24*(a-5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/6*(a+b+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.44, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 478, 582, 523, 217, 206, 377, 203}

$$\frac{\left(5a^2b+a^3+15ab^2-5b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{\tan^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - b)*(a + 5*b)*Tan[e + f*
x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 5*b)*Tan[e + f*x]^3*S
qrt[a + b + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(6*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b(1+x^2)}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4(5(a+b)+(-a+5b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}dx\right.}{6f}$$
$$= \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+}}{6f}$$
$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(}{}$$
$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(}{}$$
$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(}{}$$
$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{16b^{5/2}f}$$

Mathematica [A] time = 3.57, size = 263, normalized size = 1.20

$$-\frac{\tan(e+fx)\sec^4(e+fx)\left(4\left(3a^2+12ab-7b^2\right)\cos(2(e+fx))+\left(3a^2+14ab-33b^2\right)\cos(4(e+fx))+9a^2+34\right.}{384b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -1/8*((16*Sqrt[a]*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e +
f*x]^2]] - ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Sin[e + f*x
])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(Sqrt[2]*b^2*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) - ((9*a^2 + 3
4*a*b - 59*b^2 + 4*(3*a^2 + 12*a*b - 7*b^2)*Cos[2*(e + f*x)] + (3*a^2 + 14*
a*b - 33*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*T
an[e + f*x])/(384*b^2*f)
```

fricas [A] time = 7.64, size = 1775, normalized size = 8.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(b)
*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
- 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^3*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*(a^3 + 5*a^2*b +
15*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b
^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/19
2*(48*sqrt(a)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 - 3*(a^3 + 5*a^2*b + 15*a*b
^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^
3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/96*(24*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^5 + 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(
a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(b^3*f*cos(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \tan\left(fx+e\right)^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

maple [C] time = 2.28, size = 2756, normalized size = 12.58

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

```
[Out] 1/48/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(90*sin(f*x+e)*co
s(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^3*a*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^
2*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a^2*b-10*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2-15*sin(f*x+e)*cos(f*x+e)^
6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*
x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-96*sin(f*x+e)*
cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a^2*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3+40*cos
(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-40*cos(f*x+e)^5*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)^7*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^2*b+33*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2+14*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
2*b-33*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^3+8*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-26*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^3+26*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
3+33*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-33*cos(f*x+e)
^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^6
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*a^3-30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3+15*sin(
```

```
f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-8*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)
^2)/cos(f*x+e)^5/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \tan\left(fx+e\right)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^6 \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^2\left(e + fx\right)} \tan^6\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**6, x)
```

3.383 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=165

$$-\frac{\left(a^2+6ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8bf}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a+b\tan^2}}\right.}{f}$$

```
[Out] -1/8*(a^2+6*a*b-3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/b^(3/2)/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/
f+1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.31, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 478, 582, 523, 217, 206, 377, 203}

$$-\frac{\left(a^2+6ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-a+3b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx\right.}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+}}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+}}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+}}{4f}$$

$$= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+}}\right.}{8b^{3/2}f}$$

Mathematica [A] time = 2.63, size = 208, normalized size = 1.26

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(8\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right) - \frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{b}}\right)}{4\sqrt{2}\,bf\sqrt{a\cos(2e+2fx)+a+2b}} + \tan(e+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] ((8*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
 - ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin
[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt[2]
*b*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + ((a - b + (a - 5*b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(16*b*f)
```

fricas [B] time = 2.20, size = 1621, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*b^2*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
```

```
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((a*b - 5*b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*b^2*cos(f*x + e)^3*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b -
 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b
^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x
+ e)^3), -1/32*(8*sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a
*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b
- 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4) - 4*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3), -1/16*(4*
sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b - 3*b^2)*sqrt(-b)*
arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))
)*cos(f*x + e)^3 - 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(f x+e\right)^{2}+a}\, \tan \left(f x+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

maple [C] time = 1.95, size = 2005, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/8/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(16*sin(f*x+e)*cos
(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
```

```
b^(1/2)+a-b)/(a+b))^(1/2))*a*b+sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*a^2-12*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6*sin(f*x+e)*cos(f*
x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*b^2+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2-5*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x
+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+5*cos(f*x+e)^4*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^3*a*b-5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+5*cos(f*x+e)^2
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(
-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)^3/b/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \tan\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+fx\right)^4 \sqrt{a+\frac{b}{\cos\left(e+fx\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\tan^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**4, x)
```

3.384 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=118

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*(a-b)*
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*(a+b+b
*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4141, 1975, 478, 523, 217, 206, 377, 203}

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b\left(1+x^2\right)}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+b+(-a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx,\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,d\right.}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\text{t}}{\sqrt{a+b}}\right.}{f}$$

$$= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

Mathematica [C] time = 4.19, size = 526, normalized size = 4.46

$$\frac{e^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{i\sqrt{a}\sqrt{b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-i\sqrt{a}\sqrt{b}\log\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*
x)))^2 + (-2*Sqrt[a]*Sqrt[b]*f*x + I*Sqrt[a]*Sqrt[b]*Log[a + 2*b + a*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]] - I*Sqrt[a]*Sqrt[b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] - a*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((2*I)
*(e + f*x))))] + b*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*
E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((
2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]])
```

fricas [B] time = 1.15, size = 1471, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)
*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*(a - b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) + sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
```

```
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
e) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 +
 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2
)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/(b*f*cos(f*x + e)), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a
- b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a} \tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

maple [C] time = 1.58, size = 1331, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/2/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a+2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*b+2*2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*s
in(f*x+e)*cos(f*x+e)^2*a-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*b-4*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a
```

```
-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+c
os(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**2, x)
```

3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
```

```
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.81, size = 1227, normalized size = 15.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
```

```
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.76, size = 589, normalized size = 7.46

$$\sqrt{2}\left(-\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin\left(fx+e\right)}, \sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^{2}+6ab-b^{2}}{\left(a+b\right)^{2}}}\right)a-\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}\cdots}{a+b}}}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*(-EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a-EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b+2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b+2*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a)*cos(f*x+e)*sin(f*x+e)^2*((b+a*co
s(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)/(-1+
cos(f*x+e))/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{b}{\cos\left(e + f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4141, 1975, 475, 12, 377, 203}

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x,\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(}{\sqrt{a+b+b}}\right.}{f}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

Mathematica [A] time = 0.57, size = 130, normalized size = 1.88

$$-\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a}\sin(e+fx)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a+b}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\right)}{f\sqrt{a+b}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[a + b]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[2]*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/S
qrt[a + b]]*Sin[e + f*x]))/(Sqrt[a + b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]))
```

fricas [B] time = 0.69, size = 499, normalized size = 7.23

$$\left[\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8 - 256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4 + a^4 -\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))*sin(f*x + e) - 8*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e))/(f*sin(f*x + e)), 1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a
^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^
2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a}\, \cot \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

maple [C] time = 1.70, size = 1004, normalized size = 14.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(a*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)-2*2^(1/2)*((I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*co
s(f*x+e)*sin(f*x+e)*a+a*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin
(f*x+e)-2*a*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(b+a*cos(f*x
+e)^2)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \cot\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e+fx\right)}\, \cot^2\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f(a+b)}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/3*(3*a+2*
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.25, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4141, 1975, 475, 583, 12, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{b-3(a+b)-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, \right.}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+}}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+}}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+}}{3f}$$

$$= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e}}{3(a+b)f}$$

Mathematica [A] time = 0.74, size = 176, normalized size = 1.54

$$-\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{\csc^3(e+fx)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{a+b} - 3\csc(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sqrt{a}\,\text{s}}{}\right.\right.}{3f\sqrt{a\cos(2e+2fx)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[2]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((Csc[e + f*x]^3*(a +
 b - a*Sin[e + f*x]^2)^(3/2))/(a + b) - 3*Csc[e + f*x]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*(1 + (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
 f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))))/(f*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]])
```

fricas [B] time = 1.24, size = 629, normalized size = 5.52

$$\left[\frac{3\left((a+b)\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*log(128*a^4*cos(f*x + e)
^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*
a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(
a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((4*a + 3*b)*cos(
f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), -1/12*(3*((
a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*
a + 3*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\,\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

maple [C] time = 1.57, size = 3855, normalized size = 33.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^3*sin(
f*x+e)*a^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^3*s
in(f*x+e)*a*b-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2-6*2^(1/2)*((I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*cos(f*x+e)^3*sin(f*x+e)*a*b+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(
f*x+e)*a^2-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a*b-3*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-3*sin(f*x+e)*cos
(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+6*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*cos(f*x+e)*sin(f*x+e)*a^2+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
```

```
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b-
3*a^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-3*a*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*s
in(f*x+e)*a^2+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+4*cos(f*x+e)^4*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2+
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+3*cos(f*x+e)^2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b
+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)^3/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\, \cot\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec^{2}\left(e + fx\right)} \cot^{4}\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b}}{5}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-1/15*(15*a
^2+25*a*b+8*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/15*(-4*b
-5*a)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b
+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.33, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 475, 583, 12, 377, 203}

$$-\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)^2*f) - ((b - 5*(a + b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(15*(a + b)*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(
5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{b-5(a+b)-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx,\right.}{5f}$$

$$= -\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a}}{}$$

$$= -\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+}{}$$

$$= -\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+}{}$$

$$= -\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+}{}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+}}{15(a+b)^2f}$$

Mathematica [A] time = 1.65, size = 178, normalized size = 1.07

$$-\frac{\cot(e+fx)\left(-\left(11a^2+21ab+10b^2\right)\csc^2(e+fx)+23a^2+3(a+b)^2\csc^4(e+fx)+40ab+15b^2\right)\sqrt{a+b\sec}}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e
+ 2*f*x]])) - (Cot[e + f*x]*(23*a^2 + 40*a*b + 15*b^2 - (11*a^2 + 21*a*b +
10*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sqrt[a + b*Sec[e + f*x
]^2])/(15*(a + b)^2*f)
```

fricas [B] time = 3.95, size = 849, normalized size = 5.08

$$\left[\frac{15\left(\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(
f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 40*a*b + 15*b^2)
*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*
a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^
2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/60*(15*((a^2 + 2*a*b + b^2)*cos
(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e
)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 40*a*b + 15*b^2)*cos(f*x +
e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*a*b + 8*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b
 + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 +
2*a*b + b^2)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

maple [C] time = 1.90, size = 8605, normalized size = 51.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\, \cot\left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{6}\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\,\cot^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

3.389 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=135

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f-1/5*(a+2*b)*(a+b*sec(f*x+e)^2)^(5/2)/b^2/f+1/7*(a+b*sec(f*x+e)^2)^
(7/2)/b^2/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 88, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) - ((a + 2*b)*(a + b
*Sec[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Sec[e + f*x]^2)^(7/2)/(7*b^2*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}{x}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^{3/2}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$

$$=\frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^{3/2}}{b}+\frac{(a+bx)^{3/2}}{x}+\frac{(a+bx)^{5/2}}{b}\right)dx,x,\sec^2(e+fx)\right)}{2f}$$

$$=-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}+\frac{\text{Subst}\left(\cdots\right.}{}$$

$$=\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\,\text{se}\cdots\right.}{}$$

$$=\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\,\text{sec}\cdots\right.}{5b^2f}$$

$$=\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\,\text{sec}\cdots\right.}{5b^2f}$$

$$=-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2\cdots\right.}{3\cdots}$$

Mathematica [F] time = 2.96, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5, x]
```

fricas [B] time = 12.89, size = 527, normalized size = 3.90

$$\left[\frac{105\,a^{\frac{3}{2}}b^2\cos\left(fx+e\right)^6\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\,\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/840*(105*a^(3/2)*b^2*cos(f*x + e)^6*log(128*a^4*cos(f*x + e)^8 + 256*a^3
*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 +
b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)) - 8*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84
*a*b^2 + 35*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)
^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6), 1/
420*(105*sqrt(-a)*a*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^6 - 4*(2*(3*a^3
 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84*a*b^2 + 35*b^3)*cos(
f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)4*(1/105*(-105*a^2*(
-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2
*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+
b))^13*sign(cos(f*x+exp(1)))-sqrt(a+b)*(315*a^8+4992*b^8+53952*a*b^7-133053
*a^2*b^6+133050*a^3*b^5-81403*a^4*b^4+28364*a^5*b^3-4515*a^6*b^2+1050*a^7*b
)*sign(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*
(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*t
an(1/2*(f*x+exp(1)))^2+a+b))^3*(630*a^7-36288*b^7-300608*a*b^6-492786*a^2*b
^5+732270*a^3*b^4-112084*a^4*b^3-185220*a^5*b^2+99750*a^6*b)*sign(cos(f*x+e
xp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+
b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(
1)))^2+a+b))^5*(-51345*a^6+20608*b^6+387072*a*b^5+93471*a^2*b^4-690004*a^3*
b^3+332010*a^4*b^2+65100*a^5*b)*sign(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(1/2*(
f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a
```

```
*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^7*(-6300*a^5+200
96*b^5-183680*a*b^4+19796*a^2*b^3+320460*a^3*b^2-202020*a^4*b)*sign(cos(f*x
+exp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^
4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+ex
p(1)))^2+a+b))^9*(45675*a^4-9408*b^4+63616*a*b^3-15645*a^2*b^2-82530*a^3*b)
*sign(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(
f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*ta
n(1/2*(f*x+exp(1)))^2+a+b))^11*(9030*a^3-2240*b^3-6720*a*b^2+1470*a^2*b)*si
gn(cos(f*x+exp(1)))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x
+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1
/2*(f*x+exp(1)))^2+a+b))*(2205*a^8-25536*b^8-303744*a*b^7+407925*a^2*b^6-74
186*a^3*b^5-150493*a^4*b^4+129556*a^5*b^3-41685*a^6*b^2+10710*a^7*b)*sign(c
os(f*x+exp(1)))-1575*a^2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt
(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1
)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^12*sign(cos(f*x+exp(1)))-sqrt(a+b)*
(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/
2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a
+b))^2*(5250*a^7+48384*b^7+547008*a*b^6-602742*a^2*b^5+7658*a^3*b^4+269444*
a^4*b^3-159180*a^5*b^2+40530*a^6*b)*sign(cos(f*x+exp(1)))-sqrt(a+b)*(-sqrt(
a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+
exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4*
(-22575*a^6-1792*b^6-336000*a*b^5+952161*a^2*b^4-553196*a^3*b^3-69930*a^4*b
^2+143220*a^5*b)*sign(cos(f*x+exp(1)))-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+e
xp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(
1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^6*(-49140*a^5-21504*b
^5+90496*a*b^4-565796*a^2*b^3+494340*a^3*b^2-65100*a^4*b)*sign(cos(f*x+exp(
1)))-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(
1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f
*x+exp(1)))^2+a+b))^8*(37485*a^4-6272*b^4-21056*a*b^3+202965*a^2*b^2-198030
*a^3*b)*sign(cos(f*x+exp(1)))-sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2
+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+
exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^10*(27090*a^3+8960*b^3-6720*a*
b^2-29190*a^2*b)*sign(cos(f*x+exp(1))))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(
f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a
*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(
1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^
4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^7+
1/2*a^2*sign(cos(f*x+exp(1)))*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2-
sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(
1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-a))/sqrt(-a))/f
```

maple [B] time = 1.69, size = 2606, normalized size = 19.30

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/420/f*4^(1/2)*(-1+cos(f*x+e))^3*(-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(7/2)*b^4+84*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(7/2)*b^4-30*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(7/2)*b^4-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(7/2)*a*b^3-210*cos(f*x+e)^7*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b^
2-945*cos(f*x+e)^7*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b^3-1575*cos(f
*x+e)^7*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
```

```
/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^4-1155*cos(f*x+e)^7*ln(
-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^5-315*cos(f*x+e)^7*ln(-4*(-1+cos(f
*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin
(f*x+e)^2/(a+b)^(1/2))*a^2*b^6+210*cos(f*x+e)^7*ln(-2*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))*a^6*b^2+945*cos(f*x+e)^7*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a
^5*b^3+1575*cos(f*x+e)^7*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^4+1155
*cos(f*x+e)^7*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^5+315*cos(f*x+e)^
7*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*
cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^6+12*cos(f*x+e)^7*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^4+12*cos(f*x+e)^6*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^4-70*cos(f*x+e)^5*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^4-70*cos(f*x+e)^4*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^4+84*cos(f*x+e)^3*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^4+96*cos(f*x+e)^7*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3*b+210*cos(f*x+e)^7*a^
(5/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4
*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^
(7/2)*b^2+210*cos(f*x+e)^7*a^(3/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*b^3-196*cos(f*x+e)^7*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b^2-280*cos(f*x+e)^7*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^3+96*cos(f*x+e)^6*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3*b-196*cos(f*x+e)^6*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b^2-280*cos(f*x+e)^6*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^3-6*cos(f*x+e)^
5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3*b+162*cos(f*x
+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b^2+98*co
s(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^3-6*
cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3*b+
162*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^
2*b^2+98*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/
2)*a*b^3-48*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(7/2)*a^2*b^2+36*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(7/2)*a*b^3-48*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(7/2)*a^2*b^2+36*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(7/2)*a*b^3-30*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(7/2)*a*b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+
e)^6/cos(f*x+e)^4/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/b^2/(a+b)^(9/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tan \left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \tan^{5}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

3.390 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=104

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/3*(a+b*sec(f*x+e)^2)^
(3/2)/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b/f-a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (a*Sqrt[a + b*Sec
[e + f*x]^2])/f - (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) + (a + b*Sec[e + f*x]^
2)^(5/2)/(5*b*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+b}}{x}\right.}{}$$

$$= -\frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\sec^2(e+f\right.}{5bf}$$

$$= -\frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\sec^2(e+f\right.}{5bf}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\left(a+b\sec^2(e\right.}{3f}$$

Mathematica [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3, x]
```

fricas [B] time = 3.59, size = 443, normalized size = 4.26

$$\left[\frac{15\,a^{\frac{3}{2}}b\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/120*(15*a^(3/2)*b*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) + 8*((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2
+ 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4),
 -1/60*(15*sqrt(-a)*a*b*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 - 4*((3*a^2 -
 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2 + 3*b^2)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)4*(1/15*(15*a^2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1
)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*
(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^9*sign(cos(f*x+exp(1)))+s
qrt(a+b)*(-15*a^6+132*b^6+984*a*b^5-1543*a^2*b^4+980*a^3*b^3-390*a^4*b^2+60
*a^5*b)*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*ta
n(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2
+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^3*(-420*a^5-160*b^5-80*a*b^4-4500*a^2*b^
3+1380*a^3*b^2+420*a^4*b)*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f*x+ex
p(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1
/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^5*(210*a^4-32*b^4+720*
a*b^3+2130*a^2*b^2-1260*a^3*b)*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*
tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^7*(300*a^3+160*b^
3-240*a*b^2-420*a^2*b)*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1
)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*
(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(-105*a^6-480*b^6-4240*a*
b^5+2455*a^2*b^4+860*a^3*b^3-1110*a^4*b^2+300*a^5*b)*sign(cos(f*x+exp(1)))+
sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^
```

```
4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+ex
p(1)))^2+a+b))^2*(-300*a^5+560*b^5+4880*a*b^4-1740*a^2*b^3-1140*a^3*b^2+780
*a^4*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2
+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+
exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4*(-210*a^4-40*b^4-3360*a*b^3+
3030*a^2*b^2-420*a^3*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-sqrt(a+b)*tan(1/2
*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2
*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^6*(420*a^3-80*
b^3+1200*a*b^2-1260*a^2*b)*sign(cos(f*x+exp(1)))+sqrt(a+b)*(-sqrt(a+b)*tan(
1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^
4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^8*(105*a^2-
60*b^2-120*a*b)*sign(cos(f*x+exp(1))))/(-2*sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f
*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*
tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))-(-sqrt(a+b)*tan(1
/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4
-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2-a+3*b)^5-1
/2*a^2*sign(cos(f*x+exp(1)))*atan(1/2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2-s
qrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1
/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sqrt(-a))/sqrt(-a))/f
```

maple [B] time = 1.59, size = 2150, normalized size = 20.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x)
```

```
[Out] -1/60/f*4^(1/2)*(-1+cos(f*x+e))^3*(6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(7/2)*b^3-135*cos(f*x+e)^5*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2)
)*a^5*b^2-225*cos(f*x+e)^5*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^3-16
5*cos(f*x+e)^5*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^4-45*cos(f*x+e)^
5*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*
cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^5+30*cos(f*x+e)^5*ln(-2*(-1+c
os(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)
/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b+135*cos(f*x+e)^5*ln(-2*(-1+cos(f*x+e))*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/
(a+b)^(1/2))*a^5*b^2+225*cos(f*x+e)^5*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))
*a^4*b^3+165*cos(f*x+e)^5*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^4-10*
cos(f*x+e)^3*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-10
*cos(f*x+e)^2*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+6
*cos(f*x+e)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+6*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2+6*cos(f*x+e)^5
*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3+6*cos(f*x+e)^4
*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3+45*cos(f*x+e)^
5*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*
cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^5-30*cos(f*x+e)^5*ln(-4*(-1+c
```

```
os(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)
/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b+30*cos(f*x+e)^5*(a+b)^(7/2)*ln(4*cos(f*x+e
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(3/2)*b^2+30*cos(f*x+e)^
5*(a+b)^(7/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*a^(5/2)*b-34*cos(f*x+e)^5*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2*b-40*cos(f*x+e)^5*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b^2-34*cos(f*x+e)^4*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a^2*b-40*cos(f*x+e)^4*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a*b^2+12*cos(f*x+e)^3*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^2*b+2*cos(f*x+e)^3*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b^2+12*cos(f*x+e)^2*(a+b)^(7/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+2*cos(f*x+e)^2*(a+b)^(7/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+6*cos(f*x+e)*(a+b)^(7/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/
sin(f*x+e)^6/cos(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/b/(a+
b)^(9/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tan\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^{3}\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^{2}}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + fx \right)}\right)^{\frac{3}{2}} \tan^{3}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

3.391 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
```

```
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx,x,\sec(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.}{}$$
$$=\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+}\right.}{}$$
$$=-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec\right.}{}$$

Mathematica [C] time = 0.23, size = 84, normalized size = 1.08

$$\frac{2b\left(a+b\sec^2(e+fx)\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{a\cos^2(e+fx)}{b}\right)}{3f\sqrt{\frac{a\cos^2(e+fx)}{b}+1}\,(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cos[e + f*x]^2)/b)]*(a + b*Se
c[e + f*x]^2)^(3/2))/(3*f*Sqrt[1 + (a*Cos[e + f*x]^2)/b]*(a + 2*b + a*Cos[2
*(e + f*x)]))
```

fricas [B] time = 1.06, size = 373, normalized size = 4.78

$$\left[\frac{3\,a^{\frac{3}{2}}\cos\left(fx+e\right)^2\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/24*(3*a^(3/2)*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8
```

```
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
 + 8*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^
2 + 4*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)4*(1/3*(sqrt(a+b)*(-7*b^4-28*a*b^3+21*a^2*b^2-6*a^3*b)*sign(cos(f*x+exp(
1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*t
an(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1))
)^2+a+b))^3*(-2*b^3-30*a*b^2+12*a^2*b)*sign(cos(f*x+exp(1)))+(3*b^2+6*a*b)*
(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/
2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a
+b))^5*sign(cos(f*x+exp(1)))+(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan
(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+
2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*(15*b^4+84*a*b^3+3*a^2*b^2-18*a^3*b)*sign
(cos(f*x+exp(1)))+sqrt(a+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(
1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2
*b*tan(1/2*(f*x+exp(1)))^2+a+b))^2*(-6*b^3-42*a*b^2-12*a^2*b)*sign(cos(f*x+
exp(1)))+sqrt(a+b)*(-3*b^2+18*a*b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt
(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1
)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))^4*sign(cos(f*x+exp(1))))/(-2*sqrt(a
+b)*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*ta
n(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))
^2+a+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4
+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp
(1)))^2+a+b))^2-a+3*b)^3+1/2*a^2*sign(cos(f*x+exp(1)))*atan(1/2*(-sqrt(a+b)
*tan(1/2*(f*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2
*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+
b))/sqrt(-a))/sqrt(-a))/f
```

maple [A] time = 0.30, size = 81, normalized size = 1.04

$$\frac{\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{3f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f}+\frac{a\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x)
```

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e
)^2)^(1/2))/sec(f*x+e))+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

mupad [B] time = 7.16, size = 66, normalized size = 0.85

$$\frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}{3\,f}-\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sqrt{a}}\right)}{f}+\frac{a\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b/cos(e + f*x)^2)^(3/2)/(3*f) - (a^(3/2)*atanh((a + b/cos(e + f*x)^2)^
(1/2)/a^(1/2)))/f + (a*(a + b/cos(e + f*x)^2)^(1/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

3.392 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b)^(3/2)*arctanh((a+
b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f+b*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4139, 446, 84, 156, 63, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((a + b)^(3/2)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f + (b*Sqrt[a + b*Sec[e + f*
x]^2])/f
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)}\,dx,x,\sec(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{(-1+x)x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\text{Subst}\left(\int\frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2f}+$$
$$=\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+}\right.}{bf}$$
$$=\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}+$$

Mathematica [C] time = 5.25, size = 506, normalized size = 5.56

$$\frac{\sqrt{2}\,e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{a^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+a^{3/2}\log}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*((2*b)/(1 + E^((2*I)*(e + f*x))) + ((-2*I)*a^(
3/2)*f*x + 2*(a + b)^(3/2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] + a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((
2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]] - 2*a*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]] - 2*b*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^
((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2))
```

fricas [B] time = 1.05, size = 1075, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*(a + b)^(3/2)
*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x +
e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)) + 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/8*(4*(a +
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 16
0*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*
x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*b*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*
cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b
^2)) - (a + b)^(3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b
 + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x +
e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e
)^4 - 2*cos(f*x + e)^2 + 1)) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*(a + b)*sqrt(-a - b)*ar
ctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 4*b*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)4*((-b^2*sqr
t(a+b)*sign(cos(f*x+exp(1)))+b^2*(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a
*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1))
)^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))*sign(cos(f*x+exp(1))))/(-2*sqrt(a+b)*
```

```
(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/
2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a
+b))-(-sqrt(a+b)*tan(1/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*t
an(1/2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1))
)^2+a+b))^2-a+3*b)-1/2*a^2*sign(cos(f*x+exp(1)))*atan(1/2*(-sqrt(a+b)*tan(1
/2*(f*x+exp(1)))^2-sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+
exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))/sq
rt(-a))/sqrt(-a)-1/8*sqrt(a+b)*(-a-b)*sign(cos(f*x+exp(1)))*ln(abs(sqrt(a+b
)*tan(1/2*(f*x+exp(1)))^2+sqrt(a+b)-sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/
2*(f*x+exp(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a
+b)))+1/8*sqrt(a+b)*(-a-b)*sign(cos(f*x+exp(1)))*ln(abs(-sqrt(a+b)*tan(1/2*
(f*x+exp(1)))^2+sqrt(a+b)+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp
(1)))^4-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b)))+sqrt
(a+b)*(-a^2-b^2-2*a*b)*sign(cos(f*x+exp(1)))*ln(abs((a+b)*(-sqrt(a+b)*tan(1
/2*(f*x+exp(1)))^2+sqrt(a*tan(1/2*(f*x+exp(1)))^4+b*tan(1/2*(f*x+exp(1)))^4
-2*a*tan(1/2*(f*x+exp(1)))^2+2*b*tan(1/2*(f*x+exp(1)))^2+a+b))+sqrt(a+b)*(a
-b)))/(8*a+8*b))/f
```

maple [B] time = 1.84, size = 2424, normalized size = 26.64

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^2*(-1+cos(f*x+e))
^3*(2*a^(5/2)*cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2))*(a+b)^(7/2)+2*a^(3/2)*cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*b+2*cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b+2*cos(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a*b*(a+b)^(7/2)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^2*(a+b)^(7/2)+3*cos(f*x+e)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b+
9*cos(f*x+e)*ln(-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^2+9*cos(f*x+e)*ln(
-4*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^3+3*cos(f*x+e)*ln(-4*(-1+cos(f*x+e
))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x
+e)^2/(a+b)^(1/2))*a^2*b^4+cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))
*a^6+3*cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b+6*cos(f*x+e)*
ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*co
s(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^2+11*cos(f*x+e)*ln(-2*(-1+cos(f
*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin
(f*x+e)^2/(a+b)^(1/2))*a^3*b^3+12*cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)
^(1/2))*a^2*b^4+6*cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+
```

```
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^5+co
s(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^6-cos(f*x+e)*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*
a^6-6*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^5*b-15*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4*b^2-20*c
os(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+
b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/(-1+cos(f*x+e)))*a^3*b^3-15*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^4-6*cos(f*x
+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/
2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/
(-1+cos(f*x+e)))*a*b^5-cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^6)*4^(1/2)/sin(f*x+e)^6/((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/(a+b)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

3.393 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2*a-b)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-1/2*(a+b)*cot(f*x+e)^2*(a
+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4139, 446, 98, 156, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*f) - ((a + b)*
Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a^2+\frac{1}{2}(a-b)bx}{(-1+x)x\sqrt{a+bx}}\,d\right.}{2f}\\
&= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{2f}\\
&= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{b}\\
&= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+}}{\sqrt{a+b}}\right.}{2f}
\end{aligned}
$$

Mathematica [C] time = 5.44, size = 622, normalized size = 5.46

$$
\frac{\sqrt{2}\,e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{(a+b)\left(1+e^{2i(e+fx)}\right)}{\left(-1+e^{2i(e+fx)}\right)^2} - \frac{a^{3/2}\sqrt{a+b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+\right.}{}\right.}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((a + b)*(1 + E^((2*I)*(e + f*x))))/(-1 + E^(
(2*I)*(e + f*x)))^2 - ((-2*I)*a^(3/2)*Sqrt[a + b]*f*x + (2*a^2 + a*b - b^2)
*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + a^(3/2)*Sqrt[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
```

```
f*x)))^2]] - 2*a^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x
)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] - a*b*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt
[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + b^
2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]
*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec
[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 1.03, size = 1300, normalized size = 11.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ (a*cos(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos
(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 -
8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^
4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b
+ b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a +
b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x +
e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)^2 - 2*((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*
((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a*cos(f*x + e)^2
- a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^
2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)
^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(f*cos(f*x + e)^2
 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e)^2 + 2*(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a -
b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1
/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 +
(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a - b)*cos(f*x
 + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*s
qrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*
x + e)^2 + a*b + b^2)))/(f*cos(f*x + e)^2 - f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
```

```
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Evaluation time: 2.67Error: Bad
 Argument Type
```

maple [B] time = 1.86, size = 1609, normalized size = 14.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*(-1+cos(f*x+e))^2*cos
(f*x+e)^3*(4*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))
*cos(f*x+e)*(a+b)^(3/2)*a^(3/2)-4*a^(3/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*(a+b)^(3/2)*a-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)*(a+b)^(3/2)*b-2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*a^3-3*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e
)))*cos(f*x+e)*a^2*b+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*b^3+2*ln(-2*(-1+cos(f*x+e))*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2
/(a+b)^(1/2))*cos(f*x+e)*a^3+3*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*
x+e)*a^2*b-ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*b^3+2*a^3*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f
*x+e)))+3*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(-1+cos(f*x+e)))*b-b^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-2*ln(-2*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
```

```
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))*a^3-3*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b+ln(-2*(-1+cos(
f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/si
n(f*x+e)^2/(a+b)^(1/2))*b^3)/sin(f*x+e)^6/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(3/2)/(a+b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.394 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/8*(8*a^2+4*a*b-b^2)*a
rctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/8*(4*a-b)*cot(
f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/f-1/4*(a+b)*cot(f*x+e)^4*(a+b*sec(f*x+e)^
2)^(1/2)/f
```

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4139, 446, 98, 151, 156, 63, 208}

$$-\frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 4*a*b -
 b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*Sqrt[a + b]*f) +
((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - ((a + b)*Cot[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^3x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{2a^2+\frac{1}{2}(3a-b)\ldots}{(-1+x)^2x\sqrt{a+\ldots}}\ldots\right.}{4\ldots}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{\ldots}}{4f\ldots}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{\ldots}}{4f\ldots}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{\ldots}}{4f\ldots}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(\ldots}}{\sqrt{a+b}}\ldots\right.}{8\sqrt{a+b}\,f}$$

Mathematica [C] time = 5.59, size = 684, normalized size = 4.30

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{4a^{3/2}\sqrt{a+b}\,\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+4a^{3/2}\sqrt{a}\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(-(((1 + E^((2*I)*(e + f*x)))*(b*(1 + 6*E^((2*I)*(e +
f*x)) + E^((4*I)*(e + f*x))) + a*(6 - 4*E^((2*I)*(e + f*x)) + 6*E^((4*I)*(e
 + f*x)))))/(-1 + E^((2*I)*(e + f*x)))^4) + ((-8*I)*a^(3/2)*Sqrt[a + b]*f*x
 + (8*a^2 + 4*a*b - b^2)*Log[1 - E^((2*I)*(e + f*x))] + 4*a^(3/2)*Sqrt[a +
b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + 4*a^(3/2)*Sqrt[a + b]*Log[a + a*E^((
2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 8*a^2*Log[a + b + a*E^((2*I)*(e +
 f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) +
 a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*a*b*Log[a + b + a*E^((2*I)*(e + f*x))
+ b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
 E^((2*I)*(e + f*x)))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 3.16, size = 1801, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 +
 a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - ((8*a^2 + 4*a*b
 - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4
*a*b - b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 -
 (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f
), 1/16*(((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*co
s(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + a*b)*cos(f*x + e)^4 - 2
*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)) - 2*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*
b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/32*(8*((
```

```
a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) + ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a
^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e
)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/16*(4*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(-a)*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)
*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*c
os(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^2 + 7*a*b + b^2)*cos(f
*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2
+ (a + b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Evaluation time: 4.86Error: Bad Argument Type
```

maple [B] time = 1.66, size = 4955, normalized size = 31.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)

[Out] -1/32/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*(-1+cos(f*x+e))^2*c
os(f*x+e)^3*(8*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-20*ln(-2*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))*a^3*b-15*a^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^2-2*ln(-2*(
-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b
)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e
)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^3*a-8*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^
2/(a+b)^(1/2))*a^4-8*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-16*a^(3/2)*ln(4*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)-cos(f*x+e)^3*
ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+
cos(f*x+e)))*b^4+cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^4+8*c
os(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-cos(f*x+e)^2*ln(-2*(-
1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)
^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)
+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^4-8*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4+cos(f*x+e)*
ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+
cos(f*x+e)))*b^4+8*cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-cos
(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^4+20*a^3*ln(-4*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b+15*
a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/
2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/
(-1+cos(f*x+e)))*b^2+2*b^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a+15*cos(f*x+e)^3*ln(-4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*
b^2+2*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^3-20*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e
)^2/(a+b)^(1/2))*a^3*b-15*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2)
)*a^2*b^2-2*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(
```

```
f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^3-20*cos
(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+
b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/(-1+cos(f*x+e)))*a^3*b-15*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-2*cos(f*x
+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(
1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/(-1+cos(f*x+e)))*a*b^3+20*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/
2))*a^3*b+15*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2+2*c
os(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^3-20*cos(f*x+e)*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f
*x+e)))*a^3*b-15*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-2*cos(f*x+e)*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*
a*b^3+20*cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b+15*cos(f*x+
e)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2+2*cos(f*x+e)*ln(-2*(-1+cos
(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/s
in(f*x+e)^2/(a+b)^(1/2))*a*b^3+12*(a+b)^(5/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a+2*(a+b)^(5/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b-8*(a+b)^(5/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a+2*(a+b)^(5/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b-16*(a+b)^(5/2)*cos(f*x+e)^3*a^(3/2)*ln(4*cos(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))+16*(a+b)^(5/2)*cos(f*x+e)^2*a^(
3/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*
a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))+16*(a+b
)^(5/2)*cos(f*x+e)*a^(3/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2))+20*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+cos(f*x+e)^2*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))
*b^4-b^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b
)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+b)/(-1+cos(f*x+e)))-8*a^4*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))+b^4*ln(-2
*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))+8*a^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e))))/sin(f*x+e)^8/((b+a*cos
```

```
(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/(a+b)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{5}\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.395 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx$

Optimal. Leaf size=290

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{192bf}-\frac{\left(3a^3+17a^2b-55a\right.}{}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/128*(3*a
^4+20*a^3*b+90*a^2*b^2-60*a*b^3-5*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*ta
n(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+17*a^2*b-55*a*b^2-5*b^3)*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-50*a*b-5*b^2)*(a+b+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/48*(9*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan
(f*x+e)^5/f+1/8*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A] time = 0.57, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4141, 1975, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{192bf}-\frac{\left(17a^2b+3a^3-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan}}{128b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3 + 17
*a^2*b - 55*a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 50*a*b - 5*b^2)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(192*b*f) + ((9*a + b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^
2])/(48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/(8*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^{3/2}}{1+x^2}dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6((a+b)(8a+b)+\ldots}{(1+x^2)\sqrt{a+\ldots}}\right.}{}$$

$$= \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{\ldots}}{}$$

$$= \frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+\ldots}{}$$

$$= -\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx\ldots}}{128b^2f}$$

$$= -\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx\ldots}}{128b^2f}$$

$$= -\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx\ldots}}{128b^2f}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^4+20a^3b+90a^2b^2-60ab^3\ldots\right.}{128\ldots}$$

Mathematica [**A**] time = 6.19, size = 353, normalized size = 1.22

$$-\frac{\tan(e+fx)\sec^6(e+fx)\left(9a^3\cos(6(e+fx))+90a^3+57a^2b\cos(6(e+fx))+498a^2b+\left(135a^3+759a^2b-2\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -1/32*((128*a^(3/2)*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]] - ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(S
qrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]
^3*(a + b*Sec[e + f*x]^2)^(3/2))/(Sqrt[2]*b^2*f*(a + 2*b + a*Cos[2*e + 2*f*
x])^(3/2)) - ((90*a^3 + 498*a^2*b - 1594*a*b^2 - 626*b^3 + (135*a^3 + 759*a
^2*b - 2303*a*b^2 + 513*b^3)*Cos[2*(e + f*x)] + 2*(27*a^3 + 159*a^2*b - 523
*a*b^2 - 191*b^3)*Cos[4*(e + f*x)] + 9*a^3*Cos[6*(e + f*x)] + 57*a^2*b*Cos[
6*(e + f*x)] - 337*a*b^2*Cos[6*(e + f*x)] + 15*b^3*Cos[6*(e + f*x)])*Sec[e
 + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(12288*b^2*f)
```

fricas [**A**] time = 25.23, size = 1973, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/1536*(192*sqrt(-a)*a*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
- 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*
b^3 - 5*b^4)*sqrt(b)*cos(f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*
x + e)^6 - 2*(3*a^2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(
9*a*b^3 - 17*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7), 1/768*(96*sqrt(-a)*a*b^3*cos(f*x +
 e)^7*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*
(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 2*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^
2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^3*f*cos(f*x + e)^7), 1/1536*(384*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(b)*cos(f*x +
e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((9*a^3
*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 - 122*a
*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x + e)^
2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x
 + e)^7), 1/768*(192*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2
- a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 -
 (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^7 + 3*(3*a^4 +
 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^7 - 2*((
9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 -
122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^7)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

maple [C] time = 2.44, size = 3583, normalized size = 12.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x)
```

```
[Out] -1/384/f*sin(f*x+e)*(60*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*a^3*b-48*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)*b^4+380*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-38
0*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-57*cos(f*x+e)^
8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+337*cos(f*x+e)^8*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-15*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^3+301*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^2*b^2-455*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
^3+57*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-337*cos(f*
x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+15*cos(f*x+e)^9*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^3*b-301*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*b^2+455*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b^3+120*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^3+3*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a^3*b-78*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b^2+78*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^4*a^2*b^2-120*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^3*a*b^3-15*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*b^4-18*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4+30*
sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^4+9*sin(f*x+e)*cos(f*x+e)^8*2^(1/
2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4+9*cos(f*x+e)^9*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-9*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^4+15*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-
15*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-118*cos(f*x+e)^
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+118*cos(f*x+e)^4*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+136*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*b^4-136*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^4+48*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-114*sin(f*x+e)*cos(f*x+e
)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
```

```
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2-180*sin(f*x+e)*
cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3-120*sin(
f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b-540*sin(f*x+e)*cos(f*x+e)^8*2^(1/
2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*a^2*b^2+360*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3+768*si
n(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x
+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)^5/b^2/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tan\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^{6}\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.396 $\int \left(a+b\sec^2(e+fx)\right)^{3/2} \tan^4(e+fx)\,dx$

Optimal. Leaf size=214

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh}{16b^{3/2}f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/16*(a-b)*
(a^2+10*a*b+b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(
3/2)/f+1/16*(a^2-8*a*b-b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/24*
(7*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/6*b*(a+b+b*tan(f*x+e)^2
)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.48, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{a^{3/2}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a + b)*Tan[e + f*x]^3*Sqrt[a + b
 + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x
]^2])/(6*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4\left((a+b)(6a+b)+b(7a\right.}{\left(1+x^2\right)\sqrt{a+b+bx}}\right.}{6f}$$

$$= \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+}}{6}$$

$$= \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3}{}$$

$$= \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3}{}$$

$$= \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a+}}\right.}{16b^{3/2}f}$$

Mathematica [A] time = 4.10, size = 258, normalized size = 1.21

$$\frac{\tan(e+fx)\sec^4(e+fx)\left(4\left(3a^2-24ab-11b^2\right)\cos(2(e+fx))+\left(3a^2-38ab+3b^2\right)\cos(4(e+fx))+9a^2-58ab\right.}{384bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] ((16*a^(3/2)*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
] - ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b
 - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)
)/(4*Sqrt[2]*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((9*a^2 - 58*a*b +
 17*b^2 + 4*(3*a^2 - 24*a*b - 11*b^2)*Cos[2*(e + f*x)] + (3*a^2 - 38*a*b +
3*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e +
f*x])/(384*b*f)
```

fricas [A] time = 7.52, size = 1777, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(b)*
cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(
f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) +
 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^2*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*
a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 + 2*((3*a^2*b - 38*a*b^2 + 3*b^3)*co
s(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^5), -1/192*(48*a
^(3/2)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
+ (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f
*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3
)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) - 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b
^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^5), -1/96*(24*a^(3/2)*b^2*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f
*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2
*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*co
s(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^
2*f*cos(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)^{2}+a\right)^{\frac{3}{2}} \tan \left(f x+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

maple [C] time = 1.91, size = 2757, normalized size = 12.88

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x)

[Out] 1/48/f*sin(f*x+e)*(54*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+22*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2+17*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^2*b-17*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^4*a^2*b-22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^2*a*b^2-21*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^2*b-27*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a*b^2-54*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*
b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^3+96*2^(1/2)*((I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*sin(f*x+e)*cos(f*x+e)^6*a^2*b+52*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a*b^2-52*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b^2-38*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*c
os(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+38*cos(f*x+e)^6*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-3*cos(f*x+e)^6*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^7*a^3+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-14*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+14*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^3-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^3+3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a^3-6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+6*sin(f*x+e)*cos
(f*x+e)^6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*a^(1/2)
```

```
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e
)^2)^2/cos(f*x+e)^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tan \left(fx+e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan \left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos \left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \tan^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

3.397 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=166

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}f}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+}}{8f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/8*(3*a^2
-6*a*b-b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2
)+1/8*(5*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*b*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.36, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}f}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^2 - 6*a*b - b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
```

```
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2((a+b)(4a+b)+b(5a}{\left(1+x^2\right)\sqrt{a+b+bx}}\right.}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+}}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+}}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+}}{4f}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e}{\sqrt{a+b+b\tan}}\right.}{8\sqrt{b}f}$$

Mathematica [C] time = 6.18, size = 703, normalized size = 4.23

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{4ia^{3/2}\sqrt{b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-4ia^{3/2}\sqrt{b}\,\mathrm{l}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*(-1 + E^((2*I)*(e + f*x)))*(5*a*(1 + E^((2*I)*(
e + f*x)))^2 - b*(1 - 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(1 + E
^((2*I)*(e + f*x)))^4 + (-8*a^(3/2)*Sqrt[b]*f*x + (4*I)*a^(3/2)*Sqrt[b]*Log
[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]] - (4*I)*a^(3/2)*Sqrt[b]*Log[a + a*E^((2*I)*(
e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x))))] + 6*a*b*Log[(4*(Sqr
t[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x)))
)] + b^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 +
```

```
E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b +
 a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 2.14, size = 1627, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (3*a^2 - 6*a*b - b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b - b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*
a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*
b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^
4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f
*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + (3*a^2 - 6*a*b -
 b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)
^3), 1/32*(8*a^(3/2)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos
(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*
a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 - (3*a^2 - 6*a*b - b^2
)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + 4*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3), 1/16*(4*a^(3/2)*b*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e)))*cos(f*x + e)^3 + (3*a^2 - 6*a*b - b^2)*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x +
e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

maple [C] time = 1.53, size = 2002, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/8/f*sin(f*x+e)*(5*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^2+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*a*b+sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*b^2+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-12*s
in(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2
)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*b^2-16*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+5*cos(f*x+e)^
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)^5*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b-5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^2+cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+7*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b-cos(f*x+e)^3*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^2*a*b+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2
+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2-2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+c
os(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

3.398 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4128, 416, 523, 217, 206, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}} dx, x, \tan\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}} dx, x, \tan\right.}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2} dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+}}\right.}{f}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan}{}$$

Mathematica [C] time = 1.79, size = 527, normalized size = 4.47

$$\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+ia^{3/2}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
```

```
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 1.12, size = 1457, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.44, size = 1557, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*(6*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*a*b+2*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*sin(
f*x+e)*cos(f*x+e)^2*b^2-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*c
os(f*x+e)^2*sin(f*x+e)*a^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*sin(f*x+e)*cos(f*x+e)^2*b^2+4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a
^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b-((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*
cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

3.399 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(3/2)*ar
ctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-(a+b)*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.23, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4141, 1975, 474, 523, 217, 206, 377, 203}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/f - ((a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
```

```
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^{3/2}}{x^2\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^2\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a^2+b^2+b^2}{\left(1+x^2\right)\sqrt{a+b}}\right.}{}$$

$$= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a}}\right.}{}$$

$$= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2}dx\right.}{}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a}{}$$

Mathematica [C] time = 6.12, size = 410, normalized size = 3.69

$$\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-2\left(a^{3/2}fx+b^{3/2}\log\left(\frac{f\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\right)}{b^2\left(1+e^{2i(e+fx)}\right)}\right)\right)}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-2*I)*(a + b))/(-1 + E^((2*I)*(e + f*x))) +
 (I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - I*a^(3/2)*Log[a + a*E^((2*I
)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*(a^(3/2)*f*x + b^(3/2)*Log[((Sqrt[
b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2
))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

fricas [B] time = 1.21, size = 1446, normalized size = 13.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4)*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e))/(f*sin(f*x + e)), 1/8*(4*sqrt(-b)*b*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e)
+ sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e))/(f*sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + b^(
3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
```

```
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt(-b)*b*arct
an(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*si
n(f*x + e) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e))/(f*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

maple [C] time = 2.03, size = 1952, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2*2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
b^2*sin(f*x+e)*cos(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2+a^2*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-2^(1/2)*((I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
```

```
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)+2*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*
x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*cos(f*x+e)^2*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a
*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*b^2)/sin(f*x+e)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \cot^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

3.400 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{3f}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a-b)
*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b)*cot(f*x+e)^3*(a+b+b*tan(
f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.28, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 474, 583, 12, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f) - ((a + b)*C
ot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^{3/2}}{x^4\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^4\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-(3a-b)}{x^2(1+}\right.}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+f}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+f}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+f}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\,\text{ta}}}{3f}$$

Mathematica [C] time = 0.31, size = 100, normalized size = 0.89

$$-\frac{2(a+b)\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{a\sin^2(e+fx)}{a+b}\right)}{3f\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\,(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + b)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e + f
*x]^2)/(a + b)]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*(e +
f*x)])*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

fricas [B] time = 1.54, size = 597, normalized size = 5.33

$$\left[\frac{3\left(a\cos\left(fx+e\right)^{2}-a\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a*cos(f*x + e)^2 - a)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*(4*a*cos(f*x + e)^3 - (3*a
- b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x
 + e)^2 - f)*sin(f*x + e)), -1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*(4*a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^2 - f)*sin(f*x +
e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

maple [C] time = 1.87, size = 2014, normalized size = 17.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f*(-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+co
```

```
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2+3*2^(1/2)*((I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2-6*2^(1/2)*((I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*cos(f*x+e)^2*sin(f*x+e)*a^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*
a^2-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*sin(f*x+e)
*a^2+4*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+6*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*sin(f*x+e)*a^2-3*a^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*sin(f*x+e)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2+5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b-3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos
(f*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)^2/sin(
f*x+e)^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+fx\right)^4\left(a+\frac{b}{\cos\left(e+fx\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.401 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}-\frac{(a+b)\cot^5(e+fx)}{}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*a
^2+10*a*b-2*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f+1/15*(5*a-b)
*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/5*(a+b)*cot(f*x+e)^5*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.36, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 474, 583, 12, 377, 203}

$$-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^5(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 10*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*f) - ((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^{3/2}}{x^6\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^6\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}+\frac{\text{Subst}\left(\int \frac{-(5a-b)(a+b}{x^4\left(1+x^2\right)}\right.}{}$$

$$=\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f}-\frac{(a+b)\cot^5(e+fx)}{}$$

$$=-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b}{}$$

$$=-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b}{}$$

$$=-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+\frac{(5a-b}{}$$

$$=-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a}}{15(a+b)f}$$

Mathematica [C] time = 1.35, size = 139, normalized size = 0.84

$$\frac{2\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(\frac{5(a+b)^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{a\sin^2(e+fx)}{a+b}\right)}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}}-\frac{3}{4}\csc^2(e+fx)(a\cos(2(e+fx))+a+2b)\right.}{15f(a+b)(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-3*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Csc[e + f*x]^2)/4 + (5*(a + b)^2*Hypergeometric2F1[-3/2, -3/2, -1
/2, (a*Sin[e + f*x]^2)/(a + b)])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))
/(15*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

fricas [B] time = 5.50, size = 767, normalized size = 4.65

$$\left[\frac{15\left(\left(a^2+ab\right)\cos\left(fx+e\right)^4-2\left(a^2+ab\right)\cos\left(fx+e\right)^2+a^2+ab\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2
 + a*b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) - 8*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*
b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b
)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e)), 1/60*(15*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*arctan(1/4*(8*
a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(
f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
*sin(f*x + e) - 4*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*b - 5*
b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*co
s(f*x + e)^2 + (a + b)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

maple [C]   time = 2.02, size = 5850, normalized size = 35.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.402 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2)
)/f/a^(1/2)-(a+2*b)*(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4139, 446, 88, 63, 208}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) - ((a + 2*b)*Sqr
t[a + b*Sec[e + f*x]^2])/(b^2*f) + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b^2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}}+\frac{1}{x\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{s}\right.}{2f}$$
$$= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}$$

Mathematica [F] time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 1.15, size = 410, normalized size = 4.61

$$\left[\frac{3\sqrt{a}\,b^2\cos\left(fx+e\right)^2\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/24*(3*sqrt(a)*b^2*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) - 8*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt((a*cos(f*x + e)^2 + b)
```

```
/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*b^2*arctan(1/4
*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)
^2 + a*b^2))*cos(f*x + e)^2 - 4*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*32*(1/48*(-3*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x
+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*
x+exp(1))/2)^2+a+b))^5-21*sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(
a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2
*b*tan((f*x+exp(1))/2)^2+a+b))^4-(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*t
an((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*
tan((f*x+exp(1))/2)^2+a+b))*(-9*a^2+111*b^2+54*a*b)-(6*a-34*b)*(-tan((f*x+e
xp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-
2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^3-sqrt(a+b)*(-30*
a-54*b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*ta
n((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b
))^2-sqrt(a+b)*(9*a^2-47*b^2+10*a*b))/(-2*sqrt(a+b)*(-tan((f*x+exp(1))/2)^2
*sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*
x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-tan((f*x+exp(1))/2)^2*sqrt
(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp
(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2-a+3*b)^3/sign(tan((f*x+exp(1))/
2)^2-1)+1/32*atan(1/2*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*ta
n((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*t
an((f*x+exp(1))/2)^2+a+b))/sqrt(-a))/sqrt(-a)/sign(tan((f*x+exp(1))/2)^2-1)
)
```

maple [B] time = 1.78, size = 358, normalized size = 4.02

$$\frac{\left(\sin^2\left(fx+e\right)\right)\left(2\left(\cos^4\left(fx+e\right)\right)a^{\frac{5}{2}}+3\left(\cos^4\left(fx+e\right)\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\ \ln\left(4\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)^2*(2*cos(f*x+e)^4*a^(5/2)+3*cos(f*x+e)^4*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2))*b^2+6*cos(f*x+e)^4*a^(3/2)*b+3*cos(f*x+e)^3*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2))*b^2+cos(f*x+e)^2*a^(3/2)*b+6*cos(f*x+e)^2*a^(1/2)*b^2-a^(1/
2)*b^2)/cos(f*x+e)^4/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)^2-
1)/b^2/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{5}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^{5}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+(a+b*sec(f*x+e)^2)^(1/2
)/b/f
```

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4139, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
+ f*x]^2]/(b*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
```

```
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

Mathematica [F] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.69, size = 328, normalized size = 5.86

$$\left[\frac{\sqrt{a}\,b\log\left(128\,a^4\cos\left(fx+e\right)^8 + 256\,a^3b\cos\left(fx+e\right)^6 + 160\,a^2b^2\cos\left(fx+e\right)^4 + 32\,ab^3\cos\left(fx+e\right)^2 + b^4 + 8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*a*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f), -1/4*(sqrt(-a)*b*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*8*(1/4*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan(
(f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan
((f*x+exp(1))/2)^2+a+b))/(-2*sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sq
rt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-tan((f*x+exp(1))/2)^2*sqrt(a+b)+sqrt(a*
tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b
*tan((f*x+exp(1))/2)^2+a+b))^2-a+3*b)/sign(tan((f*x+exp(1))/2)^2-1)-1/8*ata
n(1/2*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2
)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/
2)^2+a+b))/sqrt(-a))/sqrt(-a)/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [B] time = 1.50, size = 303, normalized size = 5.41

$$-\frac{\left(\sin^2\left(fx+e\right)\right)\left(a^{\frac{3}{2}}\left(\cos^2\left(fx+e\right)\right)+\left(\cos^2\left(fx+e\right)\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\,\ln\left(4\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^2}}\,\sqrt{a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*sin(f*x+e)^2*(a^(3/2)*cos(f*x+e)^2+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2))*b+cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+a^(1/2)*b)/cos(f*x
+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)^2-1)/b/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan\left(e+f\,x\right)^3}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e + fx\right)}{\sqrt{a + b\sec^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.404 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4139, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.58, size = 261, normalized size = 7.91

$$\left[\frac{\log\left(128\,a^4\cos\left(fx+e\right)^8 + 256\,a^3b\cos\left(fx+e\right)^6 + 160\,a^2b^2\cos\left(fx+e\right)^4 + 32\,ab^3\cos\left(fx+e\right)^2 + b^4 - 8\left(16\,a^3\right.\right.}{8\sqrt{a}\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*co
s(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 2
4*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(sqrt(a)*f), 1/4*sqrt(-a)*
arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*co
s(f*x + e)^2 + a*b^2))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*atan(1/2*(-tan((f*x+exp(1))/2)^
```

```
2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-
2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a))/sqrt(-a
)/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 0.25, size = 42, normalized size = 1.27

$$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] -1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [B] time = 5.09, size = 27, normalized size = 0.82

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] -atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2/Sqrt[a]]/(Sqrt[a]*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [F] time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.77, size = 1015, normalized size = 14.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*((a + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(
a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*co
s(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a
 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1)))/((a^2 + a*b)*f), 1/8*(4*a*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a + b)*sqrt(a)*log(128*a
^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 +
 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*
x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^2 + a*b)*f), -1/4*(sqrt(-a)*(a + b)
*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*c
os(f*x + e)^2 + a*b^2)) - sqrt(a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*
```

```
x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^2 + a*b)*f), -1/4*(sqr
t(-a)*(a + b)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^
4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*a*sqrt(-a - b)*arctan(1/2*((2*a +
b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^2 + a*b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not
 checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by
 intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evalu
ation time: 0.64Error: Bad Argument Type
```

maple [B] time = 1.88, size = 376, normalized size = 5.37

$$-\frac{\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^{2}}}\,\left(\ln\left(-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^{2}}}\,\cos\left(fx+e\right)\sqrt{a+b}+\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\left(1+\cos\left(fx+e\right)\right)^{2}}}\,\sqrt{a+b}-a\cos\left(fx+e\right)+b\right)}{\sin\left(fx+e\right)^{2}\sqrt{a+b}}\right)\sqrt{a}\;+2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2*(-1+cos(f*x+e))*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^
2/(a+b)^(1/2))*a^(1/2)+2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2))*(a+b)^(1/2)-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^(1/2))*sin(f*x+e)^2/((b+a*cos(f*x+e
)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))/(a+b)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)}{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.406 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-a
rctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*cot(f*x+e)^2*(a+b*se
c(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 103, 156, 63, 208}

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ((2*a + 3*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)*f) - (Cot[
e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{(2a}{}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right.}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} - \frac{\cot^2(e+fx)\sqrt{a}}{2(a}$$

Mathematica [F] time = 4.76, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 1.02, size = 1550, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e
)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
2*a^2 - 3*a*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(
-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x +
e)^2 + a*b + b^2)) + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*
b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)
^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^3 + 2*a^2*b + a
*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a^2 + 2*a*b +
 b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*
x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
 ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2
 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4
*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^
3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/4*(2*
(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + ((
a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*
(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^
2 + a*b^2)) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(-a - b)
*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^3 +
 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
```

```
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Discontinuities at zeroes of cos(f
*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)W
arning, integration of abs or sign assumes constant sign by intervals (corr
ect if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.45
Error: Bad Argument Type
```

maple [B] time = 2.01, size = 4245, normalized size = 36.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*(2*(a+b)^(3/2)*a^(3/2)*cos(f*x+e)^3+2*a^(5/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))-2*a^(5/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-2*a^(5/2)*cos(f*x+e)^2*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^
(1/2))+2*a^(5/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+c
os(f*x+e)))+2*(a+b)^(3/2)*a^(1/2)*cos(f*x+e)*b+4*(a+b)^(3/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2))*a+4*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*
cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+5*a^(3/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+
b)^(1/2))*b-5*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e
)))*b+3*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f
*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
```

((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin
(f*x+e)^2/(a+b)^(1/2))*b^2-3*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(
1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/(-1+cos(f*x+e)))*b^2-2*a^(5/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))+2*a^(5/2)*cos(f*x+e)^3
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))+2*a^(5/2)*cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a
+b)^(1/2))-2*a^(5/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
ln(-4(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1
+cos(f*x+e)))+3*a^(1/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(
1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/(-1+cos(f*x+e)))*b^2-4*(a+b)^(3/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2))*a-4*(a+b)^(3/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*
a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b-5*a^(
3/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos
(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/s
in(f*x+e)^2/(a+b)^(1/2))*b+5*a^(3/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
x+e)(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b-3*a^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^2+3*a^(1/
2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^
2+4*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4
*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x
+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a+4*(a+b)^(3/2)*
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+5*a^(3/2)*cos(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b-
5*a^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)
))*b+3*a^(1/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2
*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^2-3*a^(1/2)*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^2-4*(a+b)^(3/2)*cos(f*x+e)^2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)

```
/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2))*a-4*(a+b)^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*b-5*a^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+b)^(1/2))*b+5*a^(3/2)*cos(f*x+e)^2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b-3*a^(1/2)*cos(f*x+e)
^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+b)/sin(f*x+e)^2/(a+
b)^(1/2))*b^2)*sin(f*x+e)^2/(-1+cos(f*x+e))^2/cos(f*x+e)/((b+a*cos(f*x+e)^2
)/cos(f*x+e)^2)^(1/2)/(1+cos(f*x+e))^2/(a+b)^(5/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^{3}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.407 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(8a^2+20ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}+\frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\,s}}{8f(a+b)^2}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a
+b)^(5/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/8*(4*a+7*
b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f-1/4*cot(f*x+e)^4*(a+b*se
c(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4139, 446, 103, 151, 156, 63, 208}

$$-\frac{\left(8a^2+20ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}+\frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\,s}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ((8*a^2 + 20*a*b
+ 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(5/2)
*f) + ((4*a + 7*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)^2*
f) - (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*(a + b)*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3 x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{3bx}{2}}{(-1+x)^2 x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2 f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} +$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2 f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} -$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2 f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} - \frac{\left(8a^2+20ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2} f} + \frac{(4a+}{}$$

Mathematica [F] time = 7.30, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 2.37, size = 2257, normalized size = 13.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*l
og(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
+ e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*
b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^3 + 20*a^2*b + 15*a*b^2)
*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*
b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^
4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 +
b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)
*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos
(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), 1/16*(((8*a^3 + 20*a^2*b + 15*a*b^2)*co
s(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2
)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 +
 a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
+ e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(3*(2*a^3 +
 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*co
s(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/32*(8*((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*co
s(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2
)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*
a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/1
6*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*
```

```
b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arct
an(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*
x + e)^2 + a*b^2)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3
+ 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqr
t(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2))
 + 2*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Warning, integration of abs or sign assumes constant sign by interv
als (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Disconti
nuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Warning, integration of abs or sign assumes constant sign by interv
```

```
als (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation t
ime: 2.06Error: Bad Argument Type
```

maple [B] time = 2.67, size = 10441, normalized size = 62.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=173

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\ \cdot}{\sqrt{a+b\,\mathrm{ta}}}\right)}{\sqrt{a}}$$

```
[Out] 1/8*(3*a^2+10*a*b+15*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(
1/2)-1/8*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f
```

Rubi [A] time = 0.32, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 479, 582, 523, 217, 206, 377, 203}

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\tan^3(e+fx)\cdot$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(4*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b(1+x^2)}}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b+bx^2}}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+7b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}dx, x, \tan(e\right.}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(}{4bf}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{(}{}$$

Mathematica [A] time = 4.35, size = 230, normalized size = 1.33

$$-\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{8b^2\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}} - \frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{b}}\right)}{8\sqrt{2}b^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/8*(((8*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
)/Sqrt[a] - ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[
a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec
[e + f*x])/(Sqrt[2]*b^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - ((a + 2*b + a*Cos[2
*(e + f*x)])*(3*a + 5*b + 3*(a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*x]^4*Tan[
e + f*x])/(32*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [B] time = 2.85, size = 1673, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
```

```
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x
 + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e
)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(2*a
*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), -1/16*(2*sqrt(-a)*b^3*c
os(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a
^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^
3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*co
s(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +
 e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(2*a*b^2
- 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/32*(8*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x + e)^
3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 +
4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(2*a*b^2 -
 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/16*(4*sqrt(a)*b^3*arctan(1/
4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2
)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3
*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x +
 e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e
)^3 + 2*(2*a*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)

maple [C] time = 2.34, size = 1993, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)

[Out] -1/8/f*sin(f*x+e)*(3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+

```
e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a^2+10*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a*b+7*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*b^2-6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-
20*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-30*sin(f*x+e)*cos(f*x+e)^4*2
^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*b^2+16*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+3*cos(f*
x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+9*cos(f*x+e)^5*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^2-9*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b+9*cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*cos(f*x+e)^2*a*b-9*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2+2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*c
os(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^
2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=120

$$-\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] -1/2*(a+3*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)
/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/2*(a+b
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A] time = 0.23, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4141, 1975, 479, 523, 217, 206, 377, 203}

$$-\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) -
 ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
```

```
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+3b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+}}{2}$$

Mathematica [A] time = 2.81, size = 196, normalized size = 1.63

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)}{4bf\sqrt{a+b\sec^2(e+fx)}} + \frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+}}\right)}{\sqrt{a}}\right.}{2\sqrt{2}bf\sqrt{a+b\sec^2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (((2*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
a] - ((a + 3*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^
2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(2*Sqrt[2]*b
*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^2*Tan[e + f*x])/(4*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [B] time = 1.13, size = 1507, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(sqrt(-a)*b
^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
+ e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*
a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e)) + 2*(a^2 + 3*a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*co
s(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*a*b*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(2*sqrt
(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e)))*cos(f*x + e) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/
4*(sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a^2 + 3*a*b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) - 2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
a*b^2*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.73, size = 1322, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a+6*2^(1/2)*((
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*sin(f*x+e)*cos(f*x+e)^2*b-4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*
b-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a
-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*b-
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/((b
+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)^3/b/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}\,f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 483, 217, 206, 377, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}\,f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [F] time = 2.71, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2], x]
```

fricas [B] time = 0.91, size = 1259, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
```

```
70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3
)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e)) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4))/(a*b*f), 1/8*(4*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*c
os(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8
 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos
(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^
3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 -
 (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
))) + a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*c
os(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 2*a*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a*b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.52, size = 404, normalized size = 5.05

$$\frac{2\sqrt{2}\,\sqrt{\frac{i\sqrt{a}\,\sqrt{b}\,\cos\left(fx+e\right)-i\sqrt{a}\,\sqrt{b}+a\cos\left(fx+e\right)+b}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\,\sqrt{-\frac{2\left(i\sqrt{a}\,\sqrt{b}\,\cos\left(fx+e\right)-i\sqrt{a}\,\sqrt{b}-a\cos\left(fx+e\right)-b\right)}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\,\left(\operatorname{EllipticPi}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{}}{\sin\left(fx+\right.}\right.\right.}{f\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\,\cos\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 2/f*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))-EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
```

```
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+
e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.411 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4128, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [B] time = 0.11, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

fricas [B] time = 0.73, size = 408, normalized size = 10.46

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.64, size = 380, normalized size = 9.74

$$-\frac{\sqrt{2}\sqrt{\frac{i\sqrt{a}\sqrt{b}\cos\left(fx+e\right)-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\sqrt{-\frac{2\left(i\sqrt{a}\sqrt{b}\cos\left(fx+e\right)-i\sqrt{a}\sqrt{b}-a\cos\left(fx+e\right)-b\right)}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\left(\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\ldots}{\sin(\ldots}\right.\right.}{f\sqrt{\frac{b+a\left(\cos^2\left(fx+e\right)\right)}{\cos\left(fx+e\right)^2}}\cos\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(
1/2)/cos(f*x+e)/(-1+cos(f*x+e))/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [B] time = 0.63, size = 992, normalized size = 25.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2
*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 +
 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f
*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*si
n(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a +
2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.19, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4141, 1975, 480, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-b}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f\right.}{(a+b)f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

Mathematica [A] time = 0.20, size = 127, normalized size = 1.72

$$-\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left((a+b)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)+\sqrt{a}\csc(e+fx)\sqrt{-a\sin^2(e+fx\right.}{\sqrt{2}\sqrt{a}f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] + Sqrt[a]*Csc[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*f*Sqrt[a + b*Sec[e + f
*x]^2]))
```

fricas [B] time = 0.72, size = 525, normalized size = 7.09

$$\left[-\frac{\sqrt{-a}\,(a+b)\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4 \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), 1/4*((a + b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*sin(f*x + e) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.86, size = 1865, normalized size = 25.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*(a*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+
e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)*cos(f*x+e)-
2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
```

```
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*a-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)*
cos(f*x+e)+a*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)+2^(
1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*a*2^(1/2)*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/cos(f
*x+e)/sin(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^{2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{3f(a+b)^2}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/3*(3*a+5*
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.25, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4141, 1975, 480, 583, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) +
 ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)\sqrt{a+b\left(1+x^2\right)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3a-5b-2bx^2}{x^2\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f\right.}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+f}}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+f}}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+f}}{3(a+b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \cot^3(e$$

Mathematica [A] time = 1.83, size = 168, normalized size = 1.41

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{\csc^3(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+}{6f(a+b)^2\sqrt{a+b\,s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2]) - ((a + 2*b + a*Cos[2*(e + f*x)])*(-a - 2*b + (2*a + 3*b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

fricas [B] time = 1.19, size = 723, normalized size = 6.08

$$\left[-\frac{3\left(\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2-a^2-2\,ab-b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)
*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
- 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
+ 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x +
 e) - 8*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f*cos
(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e)), -1/12*(3*((a^2 + 2*
a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f
*x + e) - 4*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f
*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 1.92, size = 5619, normalized size = 47.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.414 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=172

$$-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^5(e+fx)\sqrt{a+b\,\mathrm{ta}}}{5f(a+}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-1/15*(15*a
^2+40*a*b+33*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f+1/15*(5*a
+9*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a
+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.34, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4141, 1975, 480, 583, 12, 377, 203}

$$-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^5(e+fx)\sqrt{a+b\,\mathrm{ta}}}{5f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(15*(a + b)^3*f) + ((5*a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5a-9b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f$$

$$= \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+$$

$$= -\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+f}{15$$

$$= -\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+f}{15$$

$$= -\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+f}{15$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+}{15(a+b)^3f}$$

Mathematica [A] time = 4.17, size = 199, normalized size = 1.16

$$-\frac{\csc(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\left(11a^2+26ab+15b^2\right)\csc^2(e+fx)+23a^2+3(a+b)^2 c\right.}{30f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2
*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])) - ((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*(23*a^2 + 60*a*b
 + 45*b^2 - (11*a^2 + 26*a*b + 15*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e +
 f*x]^4)*Sec[e + f*x])/(30*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

fricas [B] time = 3.44, size = 987, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b
 + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-
a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x
 + e) + 8*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a^3 + 94*a^2
*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), 1/60*(15*(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*sin(f*x + e) - 4*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a
^3 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

maple [C] time = 2.27, size = 11267, normalized size = 65.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.415 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(a+b)^2/a/b^2/f/(a+b*s
ec(f*x+e)^2)^(1/2)+(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A] time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4139, 446, 87, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + (a + b)^2/(a*b
^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Sqrt[a + b*Sec[e + f*x]^2]/(b^2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
```

```
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{ax\sqrt{a+bx}}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e\right.}{2af}$$

$$= \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{abf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

Mathematica [F] time = 4.92, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [B] time = 1.42, size = 458, normalized size = 5.20

$$\left[\frac{\left(ab^2\cos\left(fx+e\right)^2+b^3\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
```

```
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + 8*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
, 1/4*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(a^
2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*16*(2*(-1/4096*(-64*a^2
*b^3*sign(tan((f*x+exp(1))/2)^2-1)-128*a^3*b^2*sign(tan((f*x+exp(1))/2)^2-1
)-64*a^4*b*sign(tan((f*x+exp(1))/2)^2-1))/a^3/b^3-1/4096*tan((f*x+exp(1))/2
)^2*(64*a^2*b^3*sign(tan((f*x+exp(1))/2)^2-1)+128*a^3*b^2*sign(tan((f*x+exp
(1))/2)^2-1)+64*a^4*b*sign(tan((f*x+exp(1))/2)^2-1))/a^3/b^3)/sqrt(a*tan((f
*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((
f*x+exp(1))/2)^2+a+b)+2*(1/16*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+s
qrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)
^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/b/(-2*sqrt(a+b)*(-tan((f*x+exp(1))/2)^2*
sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x
+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))-(-tan((f*x+exp(1))/2)^2*sqrt(
a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))^2-a+3*b)/sign(tan((f*x+exp(1))/2)^
2-1)+1/32*atan(1/2*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan((
f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan(
(f*x+exp(1))/2)^2+a+b))/sqrt(-a))/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)^2-1))
)
```

maple [B] time = 1.98, size = 6593, normalized size = 74.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^5}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.416 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(-a-b)/a/b/f/(a+b*sec(f
*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4139, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - (a + b)/(a*b*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$
$$= -\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] time = 3.91, size = 187, normalized size = 2.97

$$\frac{3(a+b)\tan^4(e+fx)F_1\left(2;\frac{1}{2},\frac{3}{2};3;\sin^2(e+fx),\frac{a\sin^2\ldots}{a\ldots}\right.}{2f\left(a+b\sec^2(e+fx)\right)^{3/2}\left(\sin^2(e+fx)\left(3aF_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)F_1\left(3;\frac{3}{2},\frac{3}{2};4;\sin^2(e+\ldots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[2, 1/2, 3/2, 3, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^4)/(2*f*(a + b*Sec[e + f*x]^2)^(3/2)*(6*(a + b)*AppellF1[
2, 1/2, 3/2, 3, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1
[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[3, 3/2, 3/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

fricas [B] time = 0.81, size = 417, normalized size = 6.62

$$\left[-\frac{8\left(a^2+ab\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)^2-\left(ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(\ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)^2 - (a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*
a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), -1/4*(4*(a^2 + a*b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 +
b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x +
e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*
f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*4*(2*
(-1/256*(16*a^3*sign(tan((f*x+exp(1))/2)^2-1)+16*a^2*b*sign(tan((f*x+exp(1)
)/2)^2-1))/a^3/b-1/256*tan((f*x+exp(1))/2)^2*(-16*a^3*sign(tan((f*x+exp(1))
/2)^2-1)-16*a^2*b*sign(tan((f*x+exp(1))/2)^2-1))/a^3/b)/sqrt(a*tan((f*x+exp
(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+ex
p(1))/2)^2+a+b)-1/4*atan(1/2*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sq
rt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a))/sqrt(-a)/a/sign(tan((f*x+exp(1)
)/2)^2-1))
```

maple [B] time = 1.70, size = 2368, normalized size = 37.59

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(1+cos(f*x+e))^2*(-3*cos(f*x+e)^5*a^(11/2)*b+2*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a
*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^5+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3-3*cos(f*x+e)*a^(7/2)*b^3-3*cos(f*x+e)
*a^(5/2)*b^4-cos(f*x+e)*a^(3/2)*b^5-3*cos(f*x+e)^5*a^(9/2)*b^2-cos(f*x+e)^5
*a^(7/2)*b^3-2*cos(f*x+e)^3*a^(11/2)*b-6*cos(f*x+e)^3*a^(9/2)*b^2-6*cos(f*x
+e)^3*a^(7/2)*b^3-2*cos(f*x+e)^3*a^(5/2)*b^4-cos(f*x+e)*a^(9/2)*b^2+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)*a*b^5+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*c
os(f*x+e)^5*a^5*b+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a
```

```
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^5*a^4*b^2+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^5*a^3*b^3+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2))*cos(f*x+e)^4*a^5*b+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f
*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^4*a
^4*b^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^4*a^3*b^3+2*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2))*cos(f*x+e)^3*a^4*b^2+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*c
os(f*x+e)^3*a^3*b^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(
f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4
*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^3*a^2*b^4+
2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^2*a^4*b^2+4*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*cos(f*x+e)^2*a^3*b^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4
*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*
x+e)^2*a^2*b^4+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)*a^3*b^3+2*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)*a^2*b^4-cos(f*x+e)^5*a^(13/2))*cos(f*x+
e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(a+b)/(b+a*cos(f*x+e)^2)^4/sin
(f*x+e)^4/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/b
```

maxima [B] time = 0.95, size = 2532, normalized size = 40.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e
), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 4*a*b*cos(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(1/2*arctan2(a*sin(4*f*x + 4*e
) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*
x + 2*e) + a))^2 - 4*(a^2 + a*b)*sin(2*f*x + 2*e)*sin(1/2*arctan2(a*sin(4*f
*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*
cos(2*f*x + 2*e) + a)) - 4*(a^2 + 2*a*b + (a^2 + a*b)*cos(2*f*x + 2*e))*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a^2*cos(4*f*x + 4*e)^2 + a^
2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
+ 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*
f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((b*cos(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b
)*cos(2*f*x + 2*e) + a))^2 + b*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
```

```
2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a))^2)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2 + 16
*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a*sin(2*f*x + 2*e)*s
in(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f
*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a*cos(2*f*x + 2*e) + a +
2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)))*sqrt(a) + 4*sqrt(a^2*
cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2
*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2
+ 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*(a*cos(1/2
*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a))^2)) + (b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*
b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2 + b*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*log(4*sqrt(a^2*c
os(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*
f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 +
a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^
2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(
2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
 + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x +
 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(
2*f*x + 2*e) + a))^2 + 16*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2
+ 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 +
2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*
b)*cos(2*f*x + 2*e))^(1/4)*(a + b)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*
e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f
*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2))*sqrt(a))/((a^2*b*cos(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a^2*b*sin(1/2*arctan2(a*sin(4*f*x +
4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2
*f*x + 2*e) + a))^2)*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(
a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^
2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*co
s(2*f*x + 2*e))^(1/4)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.417 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sec(f*x+e)^
2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sec[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
```

```
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}\\ &= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 6.36, size = 382, normalized size = 6.70

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)^{3/2}\left(-\frac{2}{b\sqrt{a\cos(2(e+fx))+a+2b}}+\frac{\sqrt{2}e^{i(e+fx)}\sec(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{\sqrt{a}\ldots}{b\left(a\left(1+e\ldots\right.\right.}\right.}{\ldots}\right)}{16f\left(a+b\sec^{2}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sec[e + f*x]^2*(-2/(b*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]]) + (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)
*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*((Sqrt[a]*(a + 4*b)*(1 + E^((2*I)*(e +
 f*x))))/(b*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)) + ((
4*I)*f*x - 2*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*Log[a + a*E^((2*I)*(e +
f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])*Sec[e + f*x])/a^(3/2)))/(16*f*(a + b*Sec[e + f*x]^2)^(3/
2))
```

fricas [B] time = 0.62, size = 392, normalized size = 6.88

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)^2+\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*c
os(f*x + e)^2 + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*
a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^
3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(a^
3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)^2 + (a*cos(f*x + e)^2 + b)*sqrt(-a)*arctan(1/4*(8*a^2
*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*
b^2)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(2*(1/8*b*tan((f*
x+exp(1))/2)^2/a/sign(tan((f*x+exp(1))/2)^2-1)/b-1/8/a/sign(tan((f*x+exp(1)
)/2)^2-1))/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*
x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)-1/2*atan(1/2*(-tan((f*x+exp(1
))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))
/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a))/s
qrt(-a)/a/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 0.21, size = 64, normalized size = 1.12

$$\frac{1}{af\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e
)^2)^(1/2))/sec(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [B] time = 5.56, size = 49, normalized size = 0.86

$$\frac{1}{a\,f\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] 1/(a*f*(a + b/cos(e + f*x)^2)^(1/2)) - atanh((a + b/cos(e + f*x)^2)^(1/2)/a
^(1/2))/(a^(3/2)*f)
```

sympy [A] time = 14.81, size = 53, normalized size = 0.93

$$\frac{1}{af\sqrt{a+b\sec^2\left(e+fx\right)}}+\frac{\mathrm{atan}\left(\frac{\sqrt{a+b\sec^2\left(e+fx\right)}}{\sqrt{-a}}\right)}{af\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] 1/(a*f*sqrt(a + b*sec(e + f*x)**2)) + atan(sqrt(a + b*sec(e + f*x)**2)/sqrt
(-a))/(a*f*sqrt(-a))
```

3.418 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4139, 446, 85, 156, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - b/(a*(a + b)*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af} + \frac{\text{Subst}}{}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf} +$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+f}}$$

Mathematica [F] time = 5.94, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [B] time = 1.22, size = 1569, normalized size = 15.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)^2 - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)
```

```
*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^
2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8
 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(a^3*cos(f*x + e)^
2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2
 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 4*(a^3*cos(f*x + e)^2 + a^2*
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*s
qrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*
cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^5 + 2*a^4*b + a^3*b
^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4*(4*(a^2*b + a
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a^2*b +
 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/
4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e
)^2 + a*b^2)) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4
*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt
(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2
*b*cos(f*x + e)^2 + a*b^2)) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-a - b)*a
rctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
```

```
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp
(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.1
2Error: Bad Argument Type
```

maple [B] time = 2.06, size = 9693, normalized size = 96.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.419 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b(a-2b)}{2af(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^2(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{2f(a+b}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/2*(2*a+5*b)*arctanh(
(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/2*(a-2*b)*b/a/(a+b)^2
/f/(a+b*sec(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/
2)
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4139, 446, 103, 152, 156, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b(a-2b)}{2af(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^2(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{2f(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((2*a + 5*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/2)*f) - ((a -
 2*b)*b)/(2*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - Cot[e + f*x]^2/(2*(
a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{3bx}{2}}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{(a\ldots}{2a(a+b)^2f\sqrt{\ldots}}$$

Mathematica [F] time = 9.97, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [B] time = 2.45, size = 2347, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b
^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a
)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a
^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^4 + 5*a^3*b)*cos(f*x
+ e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2
)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^
2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*s
qrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cos(f*x + e)
^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
- (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b
^2 + 3*a^3*b^3 + a^2*b^4)*f), -1/8*(2*((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2
*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a
- b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - ((
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a
*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a)*log(12
8*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^
4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos
(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b
^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f
*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 -
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/8*(2*((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 +
2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b
- 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*
b^2 + 2*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 +
 a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*
x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/4*(((a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4
```

```
- (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^
2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a
*b^2)) - ((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 +
 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*co
s(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2
*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^
2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
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check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
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to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
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 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or
 sign assumes constant sign by intervals (correct if the argument is real):
Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
```

```
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+ex
p(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
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(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, inte
gration of abs or sign assumes constant sign by intervals (correct if the a
rgument is real):Check [abs(t_nostep^2-1)]Evaluation time: 2.21Error: Bad A
rgument Type
```

maple [B] time = 2.25, size = 19968, normalized size = 130.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.420 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{b\left(4a^2+11ab-8b^2\right)}{8af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(8a^2+28ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{}{4f(}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/8*(8*a^2+28*a*b+35*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+1/8*b*(4*a^2
+11*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)+1/8*(4*a+9*b)*cot(f*x+e
)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*sec(f*
x+e)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4139, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(4a^2+11ab-8b^2\right)}{8af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(8a^2+28ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{}{4f(}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ((8*a^2 + 28*a*b
+ 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(7/2)
*f) + (b*(4*a^2 + 11*a*b - 8*b^2))/(8*a*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]
^2]) + ((4*a + 9*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cot[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^3\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{5bx}{2}}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right.}{4(a+b)f}$$

$$= \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2}{-}\right.}{}$$

$$= \frac{b\left(4a^2+11ab-8b^2\right)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{}{4(a+b)}$$

$$= \frac{b\left(4a^2+11ab-8b^2\right)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{}{4(a+b)}$$

$$= \frac{b\left(4a^2+11ab-8b^2\right)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{}{4(a+b)}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\left(8a^2+28ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} + \frac{}{8a(}$$

Mathematica [F] time = 13.43, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

fricas [B] time = 8.71, size = 3501, normalized size = 16.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/32*(4*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 +
a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^
2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2
- 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x
+ e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*c
os(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 +
10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)) + ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 +
```

8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), 1/16*(((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), -1/32*(8*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), -1/16*(4*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a

```
^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3
*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f
*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a
^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^5 + 19*a^4*b + 13*a^3*b^2
+ 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^
2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*
b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6
*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 +
 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
+ (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
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able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)W
arning, integration of abs or sign assumes constant sign by intervals (corr
ect if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
```

```
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Discontinuities a
t zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)W
arning, integration of abs or sign assumes constant sign by intervals (corr
ect if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 3.21
Error: Bad Argument Type
```

maple [B] time = 2.94, size = 32565, normalized size = 152.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}\left(e + fx\right)}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.421 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}+\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2ab^2f}-$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1/2*(3*a+5
*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/2*(3
*a+2*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/b^2/f-(a+b)*tan(f*x+e)^3/a/
b/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 470, 582, 523, 217, 206, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2ab^2f}-\frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - ((3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]
^2]])/(2*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(a*b*f*Sqrt[a + b + b*Tan[e
+ f*x]^2]) + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*b^2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(3a+2b)x^2\right)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} -$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} -$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{(a+b)\,\text{ta}}{abf\sqrt{a+b+}}$$

Mathematica [A] time = 9.69, size = 247, normalized size = 1.44

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\left(3a^2+4ab+2b^2\right)\cos(2(e+fx))+3a^2+6ab+2b^2\right)}{8ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\sec^3(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(((2*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
)/Sqrt[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Si
n[e + f*x]^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^
3)/(Sqrt[2]*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e
+ f*x)])*(3*a^2 + 6*a*b + 2*b^2 + (3*a^2 + 4*a*b + 2*b^2)*Cos[2*(e + f*x)])
*Sec[e + f*x]^4*Tan[e + f*x])/(8*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 3.10, size = 1895, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f
*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2
*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a
^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - ((3*a^4 + 5*a^3*b)*cos(f*
x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b +
b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2
+ 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), -1/8*(2*((3*
a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(-b
)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e
))) + (a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f*
x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*
b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^
4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(a^2*b^2 + (3*a^3*b + 4*
a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), 1/8*
(2*(a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + ((3*a
^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*
log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(a^2*b^2 + (
3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x
 + e)), 1/4*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(
8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*c
os(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*co
s(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)
)) - ((3*a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e)
)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*si
n(f*x + e))) + 2*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f
*x + e)^3 + a^2*b^4*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.79, size = 3860, normalized size = 22.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f*(4*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(
f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f
*x+e)*b^(5/2)*a+10*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*sin(f*x+e)*b^(3/2)*a^2-20*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^(3
/2)*a^2-8*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*((I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*cos(f*x+e)^4*2^(1/2)*b^(5/2)*sin(f*x+e)*a+4*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*b^(5/2)*a-4*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^(5/2)*a+8*cos(f*x+e)^5*b^(3/2)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+6*cos(f*x+e)^5*b^(1/2)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)^4*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^2+8*cos(f*x+e)^3*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a-6*cos(f*x+e)^4*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+8
*cos(f*x+e)^3*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-8*cos(f*x
+e)^2*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-8*cos(f*x+e)^2*b^(3
/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)*b^(5/2)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-2*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(7/2)
-4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(7/2)+6*cos(f*x+e
)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^(1/2)*a^3-
12*cos(f*x+e)^4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^(1/2)*a^3+10*cos(f*x+e)^2*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^(5/2)*a-20*cos(f*x
+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^(5/2)*a+6*cos(f*x+e)^2*2^(1/2)*((I*a^(1/
```

```
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*b^(3/2)*a^2-12*cos(f*x+e)^2*2^(1/
2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*sin(f*x+e)*b^(3/2)*a^2+7*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x
+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(
1/2)*4^(1/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-7*co
s(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(-1+cos(
f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2+7*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/
2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1
/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-7*cos(f*x+e)^
4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(-1+cos(f*x+e))*(
cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*b+4*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*sin(f*x+e)*b^(7/2)-8*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*((I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*2^(1/2)*b^(7/2)*sin(f*x+e)*cos(f*x+e)^2)*
sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(
3/2)/b^(5/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.422 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-(a+b)*tan(f*x+e)/a/b/
f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4141, 1975, 470, 523, 217, 206, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) +
 ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
 - ((a + b)*Tan[e + f*x])/(a*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+ax^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af} + \frac{\text{S}}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af} + \frac{\text{Subst}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+}}$$

Mathematica [A] time = 4.26, size = 201, normalized size = 1.73

$$\frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^{3/2}\left(\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{b}}\right)}{2\sqrt{2}abf\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a+b)\tan(e+}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (((b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[a]
 + (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a*b*f*(
a + b*Sec[e + f*x]^2)^(3/2)) - ((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[
e + f*x]^2*Tan[e + f*x])/(2*a*b*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 2.22, size = 1655, normalized size = 14.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 2*(a^3*cos(f*x + e)^2 +
 a^2*b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - 4*(a^3*cos(f*
x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e))) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a^3*b^2*f*cos(
f*x + e)^2 + a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*
sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e))) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(b)*log(((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/(a^3*b^2*f*cos(f*x + e)^2 +
a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*arcta
n(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
```

```
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2
*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f
*x + e))) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a^3*b^2*f*cos(f*x +
 e)^2 + a^2*b^3*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.72, size = 2662, normalized size = 22.95

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*(4*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(3/2)*sin(f*x+e)*
cos(f*x+e)^2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*cos(f*x+e)^2*
sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*b^(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a+4*2^(1
/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(5/2)*sin(f*x+e)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))-2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(5/2)*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)+4*cos(f*x+e)^2*sin(f*x+e)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*a^2-2*cos(f*x+e)^2*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*a^2+4*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
```

```
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^
(3/2)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a-2*sin(f*x+e)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2
)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a+cos(f*x+e)^2*sin(f*x+e)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos(f*
x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^2-cos(f*x+e)^2*sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*ar
ctanh(1/4*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f
*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*b^(3/2)*cos(f*x+e)^3*a-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(5/2)*c
os(f*x+e)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*cos(f*x+e)^2*a-
2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1/2)*a^2+sin(f*x+
e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(cos
(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a*b-sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*arctanh(1/4*
(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(5/2)-2
*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*a+2*cos(f*x+e)^
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1/2)*a^2+2*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*b^(3/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b
+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/b^(3/2)/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(e+f\,x\right)}^4}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}{\left(e+fx \right)}}{\left(a+b\sec^{2}{\left(e+fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.423 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\tan(e+fx)}{af\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+tan(f*x+e)
/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4141, 1975, 471, 377, 203}

$$\frac{\tan(e+fx)}{af\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) + Tan[e + f*x]/(a*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 2.44, size = 169, normalized size = 2.38

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}\sqrt{a+b}\sin(e+\right.}{4a^{3/2}f\sqrt{a+b}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(ArcSin[(Sqrt[a]*Sin[e
+ f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*Sqrt[
a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(a^(3/2)
*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)])
```

fricas [B] time = 1.09, size = 548, normalized size = 7.72

$$\left[\frac{8a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)-\left(a\cos(fx+e)^2+b\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256\left(a^4-a^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
+ e) - (a*cos(f*x + e)^2 + b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*co
s(f*x + e)^2 + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.64, size = 569, normalized size = 8.01

$$\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)\left(\sqrt{2}\,\sqrt{\frac{i\sqrt{a}\,\sqrt{b}\,\cos\left(fx+e\right)-i\sqrt{a}\,\sqrt{b}+a\cos\left(fx+e\right)+b}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\sqrt{-\frac{2\left(i\sqrt{a}\,\sqrt{b}\,\cos\left(fx+e\right)-i\sqrt{a}\,\sqrt{b}-a\cos\left(fx+e\right)-b\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\right.\ \mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin
(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*cos(f*x+e)-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)/(-1+cos(f
*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/a
```

maxima [B] time = 0.80, size = 2005, normalized size = 28.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] 1/2*(2*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 2*a*cos(1/2*ar
ctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e)
 + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e) + 2*(a*cos(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 - a*cos(2*f*x + 2*e))*sin(1/2*arctan2(
a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(
a + 2*b)*cos(2*f*x + 2*e) + a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x +
 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(
4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2
+ a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^
2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2
*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2
*e) + a))^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x +
2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*
a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4
*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(
a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*
cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
 a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*
e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a
^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e)
+ 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x
+ 2*e) + a)) + 2*a + 4*b) - (cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*
b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*
cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4
*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)
*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4
*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2
 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*
(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*s
in(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x +
4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^
2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*
b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*
cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a^2*cos(1/2*arc
tan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e)
+ 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a^2*sin(1/2*arctan2(a*sin(4*f*x +
4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2
*f*x + 2*e) + a))^2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(e + f x\right)}{\left(a + b \sec^{2}\left(e + f x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.424 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4128, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+ \right.}{af}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 1.36, size = 168, normalized size = 2.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}\,b\sin\right.}{4a^{3/2}f(a+b)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

fricas [B] time = 0.91, size = 601, normalized size = 7.81

$$\left[-\frac{8\,ab\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^2+ab\right)\cos\left(fx+e\right)^2+ab+b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
```

```
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

maple [C] time = 1.99, size = 1007, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(b+a*cos(f*x+e)^2)*(a*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
sin(f*x+e)+2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*a
*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*sin(f*x+e)-2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b)*sin(f*x+e)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/c
os(f*x+e)^3/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

maxima [B] time = 0.87, size = 2055, normalized size = 26.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

3.425 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{af(a+b)^2}-\frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(1/2)/a/(a+b)^2/f
```

Rubi [A] time = 0.27, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 472, 583, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{af(a+b)^2}-\frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((a - b
)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(a + b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{a(a+b)^2f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{a(a+b)^2f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{a(a+b)^2f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+}{}$$

Mathematica [A] time = 4.26, size = 182, normalized size = 1.53

$$-\frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{2\sqrt{2}a^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\csc(e+fx)\sec^3(e+fx)(a\cos(2(e+fx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*Sec
[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(a^2 + 2*a*b - b^2 +
(a^2 + b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]^3)/(4*a*(a + b)^2*f
*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 1.25, size = 741, normalized size = 6.23

$$\left[-\frac{\left(a^2b+2ab^2+b^3+\left(a^3+2a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqr
t(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(
f*x + e) + 8*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*cos(f*x + e))*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b + a^3*b^2)*f*
cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), 1/4*((a^2*
b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b +
 a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.96, size = 2841, normalized size = 23.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f/(b+a*cos(f*x+e)^2)^2*(2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2+4*co
s(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*b^2-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*sin(f
*x+e)*a^2-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*a*b-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)*c
os(f*x+e)+2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2+4*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+
e)+2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-a^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
```

```
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*sin(f*x+e)-2*a*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*b*sin(f*x+e)-2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+
e)-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2-cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)^3*((b+a*
cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)/(a+b)^2/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.426 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^2}-\frac{b\cot^3(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\text{(3}a$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+e
)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-b)*(a+3*b)*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/3*(a-3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x
+e)^2)^(1/2)/a/(a+b)^2/f
```

Rubi [A] time = 0.36, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 472, 583, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^2}-\frac{b\cot^3(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\text{(3}a$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Cot[e + f*x]^3)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a -
b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^3*f)
 - ((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^2
*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
```

```
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-3b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f\right.}{a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+}}{3a(a+b)^2f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan}}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan}}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan}}{3a(a+b)^3f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)}{}$$

Mathematica [A] time = 5.40, size = 224, normalized size = 1.29

$$\frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{2\sqrt{2}\,a^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^2\left(-\right.}{\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[e
+ f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^3*(((4*a
+ 9*b)*Csc[e + f*x])/(12*(a + b)^3*f) - Csc[e + f*x]^3/(12*(a + b)^2*f) - (
b^3*Sin[e + f*x])/(2*a*(a + b)^3*f*(a + 2*b + a*Cos[2*e + 2*f*x]))))/(a + b
*Sec[e + f*x]^2)^(3/2)
```

fricas [B] time = 3.82, size = 1061, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a
^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
- 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 +
8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*
a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin
(f*x + e) - 8*((4*a^4 + 9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4 + 4*a^3*
b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*
b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*
x + e)), -1/12*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3
*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x +
e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)
^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*co
s(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*a^4 + 9*a^3*b + 3*a*b^3)*
cos(f*x + e)^5 - (3*a^4 + 4*a^3*b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (
3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (
a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2
+ 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.12, size = 7541, normalized size = 43.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^4}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.427 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^3}-\frac{\left(15a^3+55a^2b+73ab\right.}{}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+55*a^2*b+73*a*b^2-15
*b^3)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^4/f+1/15*(5*a^2+14*a*b-
15*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/5*(a-5*b)*cot
(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

Rubi [A] time = 0.47, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 472, 583, 12, 377, 203}

$$\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^3}-\frac{\left(55a^2b+15a^3+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b}}{15af(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x]^5)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*
a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*a*(a + b)^4*f) + ((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(15*a*(a + b)^3*f) - ((a - 5*b)*Cot[e + f*x]^5*Sq
rt[a + b + b*Tan[e + f*x]^2])/(5*a*(a + b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
```

```
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-5b-6bx^2}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+\right.}{a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e}}{5a(a+b)^2f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+}}{15a(a+b)^3f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+}{15a(a+b)^4}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+}{15a(a+b)^4}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+}{15a(a+b)^4}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a}{}$$

Mathematica [A] time = 10.76, size = 237, normalized size = 0.98

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(-\left(23a^2+80ab+90b^2\right)\csc^2(e+fx)+\frac{30b^4}{a(a\cos(2(e+fx))+a+2}\right.}{60f(a+b)^4\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*Sec
[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^2*((30*b^4)/(a*(a + 2
*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 80*a*b + 90*b^2)*Csc[e + f*x]^2 + (a
+ b)*(11*a + 20*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x
]^2*Tan[e + f*x])/(60*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

fricas [B] time = 11.85, size = 1517, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6
 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3
*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b
^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^5
+ 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b +
80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 5
6*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2
 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e
)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos
(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^
5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)
*f)*sin(f*x + e)), 1/60*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 +
 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2
*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*a
rctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a
*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*s
in(f*x + e)))*sin(f*x + e) - 4*((23*a^5 + 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)
*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b + 80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 56*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*
b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2
+ 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^
5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^
5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 2.26, size = 14137, normalized size = 58.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6\left(e + fx\right)}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.428 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}+\frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(a+b)^2/a/b^2/f/(a
+b*sec(f*x+e)^2)^(3/2)+(1/a^2-1/b^2)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4139, 446, 87, 63, 208}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + (a + b)^2/(3*a
*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (a^(-2) - b^(-2))/(f*Sqrt[a + b*Sec[
e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}} + \frac{a^2-b^2}{a^2b(a+bx)^{3/2}} + \frac{1}{a^2x\sqrt{a+bx}}\right) dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{2a^2f}$$

$$= \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+}}$$

Mathematica [C] time = 7.82, size = 187, normalized size = 1.93

$$\frac{4(a+b)\tan^6(e+fx)F_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\,\mathrm{s}}{}\right.}{3f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(\sin^2(e+fx)\left(5aF_1\left(4;\frac{1}{2},\frac{7}{2};5;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)F_1\left(4;\frac{3}{2},\frac{5}{2};5;\sin^2(e\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*(a + b)*AppellF1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^6)/(3*f*(a + b*Sec[e + f*x]^2)^(5/2)*(8*(a + b)*AppellF1[
3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1
[4, 1/2, 7/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[4, 3/2, 5/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

fricas [B] time = 1.76, size = 564, normalized size = 5.81

$$\left[\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+2\,ab^3\cos\left(fx+e\right)^2+b^4\right)\sqrt{a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b^4)*sqrt(a)*lo
g(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
 e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b
*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(2*(a^4 - a^3*b - 2*a^2*b^2)*
cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2
 + a^3*b^4*f), 1/12*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b
^4)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4
 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 4*(2*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*
x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3
*b^4*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*16
*(2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-1/150994944*tan((f*x+ex
p(1))/2)^2*(-1572864*a^11*sign(tan((f*x+exp(1))/2)^2-1)+2359296*a^8*b^3*sig
n(tan((f*x+exp(1))/2)^2-1)+3145728*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1)-78
6432*a^10*b*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2-1/150994944*(4718592*a^
11*sign(tan((f*x+exp(1))/2)^2-1)+2359296*a^8*b^3*sign(tan((f*x+exp(1))/2)^2
-1)-9437184*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1)-7077888*a^10*b*sign(tan((
f*x+exp(1))/2)^2-1))/a^10/b^2)-1/150994944*(-4718592*a^11*sign(tan((f*x+exp
(1))/2)^2-1)-2359296*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1)+9437184*a^9*b^2*
sign(tan((f*x+exp(1))/2)^2-1)+7077888*a^10*b*sign(tan((f*x+exp(1))/2)^2-1))
/a^10/b^2)-1/150994944*(1572864*a^11*sign(tan((f*x+exp(1))/2)^2-1)-2359296*
a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1)-3145728*a^9*b^2*sign(tan((f*x+exp(1))
/2)^2-1)+786432*a^10*b*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2)/sqrt(a*tan(
(f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan
((f*x+exp(1))/2)^2+a+b)/(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*
a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)+1/16*atan(1/2*(-tan(
(f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f
*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b))/s
qrt(-a))/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [B] time = 3.58, size = 10947, normalized size = 112.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^5}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.429 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(-a-b)/a/b/f/(a+b*s
ec(f*x+e)^2)^(3/2)-1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4139, 446, 78, 51, 63, 208}

$$-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - (a + b)/(3*a*b*f*
(a + b*Sec[e + f*x]^2)^(3/2)) - 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= -\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,d\right.}{2a}$$

$$= -\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+}}$$

Mathematica [C] time = 10.17, size = 613, normalized size = 6.89

$$-\frac{e^{i(e+fx)}\sec^5(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-12\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-12\log\left(\sqrt{\right.}}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*
Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e +
f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(96*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^
```

```
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(
5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a
^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a
^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Se
c[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 1.50, size = 522, normalized size = 5.87

$$\left[\frac{3\left(a^2b\cos\left(fx+e\right)^4+2\,ab^2\cos\left(fx+e\right)^2+b^3\right)\sqrt{a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(3*a*b^2*cos(f*x + e)^2 + (a^3
+ 4*a^2*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^
5*b*f*cos(f*x + e)^4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f), -1/12*(3*(a
^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*
a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 +
 a*b^2)) + 4*(3*a*b^2*cos(f*x + e)^2 + (a^3 + 4*a^2*b)*cos(f*x + e)^4)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*b*f*cos(f*x + e)^4 + 2*a^4*b^
2*f*cos(f*x + e)^2 + a^3*b^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*(tan((f*x+exp(1))/2)^2*(ta
n((f*x+exp(1))/2)^2*(-1/2304*tan((f*x+exp(1))/2)^2*(576*a^8*b^3*sign(tan((f
*x+exp(1))/2)^2-1)+768*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1)+192*a^10*b*sig
n(tan((f*x+exp(1))/2)^2-1))/a^10/b^2-1/2304*(576*a^8*b^3*sign(tan((f*x+exp(
```

```
1))/2)^2-1)-2304*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1)-576*a^10*b*sign(tan(
(f*x+exp(1))/2)^2-1))/a^10/b^2)-1/2304*(-576*a^8*b^3*sign(tan((f*x+exp(1))/
2)^2-1)+2304*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1)+576*a^10*b*sign(tan((f*x
+exp(1))/2)^2-1))/a^10/b^2)-1/2304*(-576*a^8*b^3*sign(tan((f*x+exp(1))/2)^2
-1)-768*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1)-192*a^10*b*sign(tan((f*x+exp(
1))/2)^2-1))/a^10/b^2)/sqrt(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4
-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan((f*x+exp(1
))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(
1))/2)^2+a+b)+atan(1/2*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*t
an((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*
tan((f*x+exp(1))/2)^2+a+b))/sqrt(-a))/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)
^2-1))
```

maple [B] time = 3.20, size = 10839, normalized size = 121.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.430 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sec(f*x+e
)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 51, 63, 208}

$$\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sec[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
```

```
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, \right.}{2a^2f}$$

$$= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+}}$$

Mathematica [C] time = 7.43, size = 613, normalized size = 7.39

$$\frac{e^{i(e+fx)}\sec^5(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-12\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-12\log\left(\sqrt{a}\right.}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*
Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e +
f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(32*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(
5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a
^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a
^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Se
c[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 1.37, size = 494, normalized size = 5.95

$$\left[\frac{3\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{a}\,\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*(4*a^2*cos(f*x + e)^4 + 3*a*b*cos(f
*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), 1/12*(3*(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*
b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(4*a^2*cos(f
*x + e)^4 + 3*a*b*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)2/f*2*(2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-1/
36864*tan((f*x+exp(1))/2)^2*(-4608*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1)-61
44*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2-1/36864*(-4608*a^8*b^3*s
ign(tan((f*x+exp(1))/2)^2-1)+18432*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1))/a
^10/b^2)-1/36864*(4608*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1)-18432*a^9*b^2*
sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2)-1/36864*(4608*a^8*b^3*sign(tan((f*
x+exp(1))/2)^2-1)+6144*a^9*b^2*sign(tan((f*x+exp(1))/2)^2-1))/a^10/b^2)/sqr
t(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+2*b*tan((f*x+exp(1))/2)^2+a+b)/(a*tan((f*x+exp(1))/2)^4+b*tan((f*x+exp(1))
/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+a+b)-1/2*atan(1/2
*(-tan((f*x+exp(1))/2)^2*sqrt(a+b)-sqrt(a+b)+sqrt(a*tan((f*x+exp(1))/2)^4+b
*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+2*b*tan((f*x+exp(1))/2)^2+
a+b))/sqrt(-a))/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 0.23, size = 86, normalized size = 1.04

$$\frac{1}{3af\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{1}{a^{2}f\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/a/f/(a+b*sec(f*x+e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(5/
2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [B] time = 8.17, size = 68, normalized size = 0.82

$$\frac{\frac{a+\frac{b}{\cos(e+fx)^2}}{a^2}+\frac{1}{3a}}{f\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sqrt{a}}\right)}{a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b/cos(e + f*x)^2)/a^2 + 1/(3*a))/(f*(a + b/cos(e + f*x)^2)^(3/2)) - a
tanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(5/2)*f)
```

sympy [A] time = 24.14, size = 78, normalized size = 0.94

$$\frac{1}{3af\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec^2\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2\left(e+fx\right)}}{\sqrt{-a}}\right)}{a^2f\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] 1/(3*a*f*(a + b*sec(e + f*x)**2)**(3/2)) + 1/(a**2*f*sqrt(a + b*sec(e + f*x
)**2)) + atan(sqrt(a + b*sec(e + f*x)**2)/sqrt(-a))/(a**2*f*sqrt(-a))
```

3.431 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(2a+b)}{a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3*b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/
2)-b*(2*a+b)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4139, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a+b)}{a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - b/(3*a*(a + b)*f*(a + b*
Sec[e + f*x]^2)^(3/2)) - (b*(2*a + b))/(a^2*(a + b)^2*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+f\right.}{2a(a+b)f}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f\left(a+b\sec^2(e\right.}$$

Mathematica [F] time = 8.76, size = 0, normalized size = 0.00

$$\int \frac{\cot(e + fx)}{\left(a + b\sec^2(e + fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [B] time = 2.61, size = 2279, normalized size = 16.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f
*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 +
160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(
f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*
x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 6*(a^5*cos
(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 +
 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*(
(2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 8*((7
*a^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3
+ a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8
 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 +
 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 +
 a^3*b^5)*f), 1/24*(12*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)) + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*((7*a
^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 +
 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3
*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a
^3*b^5)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*(a^5*cos(f*x + e
)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b +
 b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b
)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((7*a^4*b +
 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7
*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b
^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5
)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
```

```
*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 6*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*
x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^
2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 4*((7*a^4*b + 11*a^3*b^2 + 4*a^2*b^
3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*
x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
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tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, in
tegration of abs or sign assumes constant sign by intervals (correct if the
 argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.93Error: Bad
 Argument Type
```

maple [B] time = 5.24, size = 68989, normalized size = 503.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.432 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b\left(a^2-6ab-2b^2\right)}{2a^2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{b(3a-2b)}{6af(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^2}{2f(a+b)\left(a+\right.}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*a+7*b)*arctanh(
(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f-1/6*(3*a-2*b)*b/a/(a+b)
^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^(
3/2)-1/2*b*(a^2-6*a*b-2*b^2)/a^2/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4139, 446, 103, 152, 156, 63, 208}

$$-\frac{b\left(a^2-6ab-2b^2\right)}{2a^2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(3a-2b)}{6af(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^2}{2f(a+b)\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ((2*a + 7*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(7/2)*f) - ((3*a
 - 2*b)*b)/(6*a*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/
(2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (b*(a^2 - 6*a*b - 2*b^2))/(2*a
^2*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{5bx}{2}}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}}{}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{2a^2(a}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{2a^2(a}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{2a^2(a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(3a}{6a(a+b)^2f\,(a+}$$

Mathematica [F] time = 19.51, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [B] time = 8.71, size = 3507, normalized size = 17.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
- a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*
b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^
2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
```

*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 -
 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4
*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2
+ 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*
((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((
3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 + 2*(
3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x + e)^4 + 3*(
a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*
b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4
- 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^
4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 + 4*a^4*b^5 + a^3*b^6)*f), -1/24*(6*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 - 2
*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4*a
^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*
a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 3*((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2
*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 -
 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2
*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x
 + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x
 + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a
^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3
 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b
^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/24*(6*((a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4
- 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*
b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^
5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*
cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 3*((2*a^6 + 7*a^
5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2
)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)
*cos(f*x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)
*cos(f*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b
^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8
*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 +
 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 +
 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/12*(3*((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*
a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^
4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4
- 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*((2*a^

```
6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14
*a^4*b^2)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^
2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + 2*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x
 + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x
 + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a
^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3
 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b
^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
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x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
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(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
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2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
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le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
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x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
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*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
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ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
```

```
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
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Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Discontinuities
at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^2-1)]Evaluation time: 4.45Error: Bad Argument Type
```

maple [B] time = 7.39, size = 105237, normalized size = 526.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.433 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{b\left(12a^2+39ab-8b^2\right)}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+36ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}+$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-1/8*(8*a^2+36*a*b+63*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(9/2)/f+1/24*b*(12*a
^2+39*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)+1/8*(4*a+11*b)*cot(f*
x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*sec
(f*x+e)^2)^(3/2)+1/8*b*(4*a^3+15*a^2*b-32*a*b^2-8*b^3)/a^2/(a+b)^4/f/(a+b*s
ec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4139, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(15a^2b+4a^3-32ab^2-8b^3\right)}{8a^2f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}+\frac{b\left(12a^2+39ab-8b^2\right)}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+36ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{8f(a+b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ((8*a^2 + 36*a*b
+ 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(9/2)
*f) + (b*(12*a^2 + 39*a*b - 8*b^2))/(24*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^(3/2)) + ((4*a + 11*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sec[e + f*x]
^2)^(3/2)) - Cot[e + f*x]^4/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (b
*(4*a^3 + 15*a^2*b - 32*a*b^2 - 8*b^3))/(8*a^2*(a + b)^4*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^3\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{7bx}{2}}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+f\right.}{4(a+b)f}$$

$$= \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}}{}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{4(}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{4(}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{4(}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{4(}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\left(8a^2+36ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} + \frac{}{24a}$$

Mathematica [F] time = 30.30, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

fricas [B] time = 33.46, size = 4751, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] [1/96*(12*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*
cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
```

b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), 1/48*(3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 6*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), -1/96*(24*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c

```
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3
*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 +
 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6
 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4
 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3
*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a
^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b +
96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^
4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*
a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*
a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^
10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^
3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*
a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 1
0*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), -1/48*(12*((a^7 + 5*a^6*b + 10*a^5*b^2
 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 +
 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5
*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 2
5*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b
 + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) - 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e
)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^
2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^
3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*
a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2
+ b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*
cos(f*x + e)^2 + a*b + b^2)) + 2*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4
*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a
^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6
- (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)
*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a
*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 +
 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
- 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*
x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
+ a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
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nostep/2)Warning, integration of abs or sign assumes constant sign by inter
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gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
 sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-
1)]Evaluation time: 6.57Error: Bad Argument Type
```

maple [B] time = 18.80, size = 145925, normalized size = 544.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.434 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{(a+b)\tan^3(e+fx}{3abf\left(a+b\tan^2(e+fx)\right.}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+(1/a^2-1/b^2)*tan(f*
x+e)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f
*x+e)^2)^(3/2)
```

Rubi [A] time = 0.34, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4141, 1975, 470, 578, 523, 217, 206, 377, 203}

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b\tan^2(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)
*f) - ((a + b)*Tan[e + f*x]^3)/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) +
 ((a^(-2) - b^(-2))*Tan[e + f*x])/(f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+3ax^2)}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+f\right.}{3abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\left(a^2-b^2\right)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\left(a^2-b^2\right)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\left(a^2-b^2\right)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a+b)\tan^3(e}{3abf\left(a+b+b\tan\right.}$$

Mathematica [B] time = 11.26, size = 316, normalized size = 2.01

$$\frac{\sec^5(e+fx)(a\cos(2e+2fx)+a+2b)^3\left(\frac{a^2(-\sin(e+fx))-2ab\sin(e+fx)-b^2\sin(e+fx)}{6a^2bf(a\cos(2e+2fx)+a+2b)^2} + \frac{-3a^2\sin(e+fx)+ab\sin(e+fx)+4b^2\sin(e+}{12a^2b^2f(a\cos(2e+2fx)+a+2b)}\right.}{\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(((b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/
Sqrt[a] - (a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2
]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*
a^2*b^2*f*(a + b*Sec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^5*((-(a^2*Sin[e + f*x]) - 2*a*b*Sin[e + f*x] - b^2*Sin[e + f*
x])/(6*a^2*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + (-3*a^2*Sin[e + f*x] + a
*b*Sin[e + f*x] + 4*b^2*Sin[e + f*x])/(12*a^2*b^2*f*(a + 2*b + a*Cos[2*e +
2*f*x]))))/(a + b*Sec[e + f*x]^2)^(5/2)
```

fricas [B] time = 3.88, size = 2035, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*
log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a
^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 6*(a^5*c
os(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((3*a^4*b - a^3*b^2 - 4*a^2*b
^3)*cos(f*x + e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4
 + 2*a^4*b^4*f*cos(f*x + e)^2 + a^3*b^5*f), 1/24*(12*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(a^2*b^3*cos(f*x + e)^4 +
2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x +
 e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*
f*cos(f*x + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*co
s(f*x + e)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 3*(a^5*cos(f*x + e)^4 + 2*a^4*
b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x + e)^3 + (
4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*f*cos(f*x
 + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e
)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e))) + 6*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x
 + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f
*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
 + e)^2 + b^2)*sin(f*x + e))) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x
+ e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4
*f*cos(f*x + e)^2 + a^3*b^5*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 1.94, size = 2256, normalized size = 14.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3-3*cos(f*x+e)^2*sin(f*x+e)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2+6*2^(1/2)*((I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)
*cos(f*x+e)^2*a*b^2-6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+3*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-3*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/
2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(
f*x+e)-6*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*cos(f*x+e)^3*a^3-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e
)^3*a^2*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^2-3*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^3+((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^2*a*b^2+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)*a^2*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2-3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-4*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^2*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*((2*I*
```

```
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*c
os(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(e+f\,x\right)}^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}{\left(e+fx \right)}}{\left(a+b\sec^{2}{\left(e+fx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.435 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(a-3*b)
*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)/a/b/f/(
a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 470, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 ((a + b)*Tan[e + f*x])/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a -
3*b)*Tan[e + f*x])/(3*a^2*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(a-2b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(\cdots}{3a^2bf\sqrt{a+b+b\,t\cdots}}$$

Mathematica [B] time = 6.02, size = 409, normalized size = 3.41

$$\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^{5/2}\left(\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{16\left(-a\sin^2(e+fx)+a+b\right)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{a^2(a+b)\sin^4(e+fx)}{\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\sqrt{a}\cdots}{\cdots}\right)}{a^3}\cdots\right)}{(\cdots}\cdots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((Sqrt[2]*Csc[e + f*x]
*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin
[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e
 + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/
(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Si
n[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(a
+ b - a*Sin[e + f*x]^2)^(3/2) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[e +
 f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (12*(b + (3*a + 2
*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)
])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 1.61, size = 661, normalized size = 5.51

$$\left[-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-a}\,\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6\cdots\right.}{\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(4*a^2*cos(f
*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a
^3*b^2*f), -1/12*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
```

```
6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 4*(4*a^2*cos(f*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e
)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 1.65, size = 1142, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+
e)^2*a-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x
+e)^2*a+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)-3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*b*sin(f*x+e)-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a-3*cos(f*x+e)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+3*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos
(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.436 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(2a+3b)\tan(e+fx)}{3a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(2*a+3
*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.26, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4141, 1975, 471, 527, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(2a+3b)\tan(e+fx)}{3a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + Tan[e + f*x]/(3*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((2*a + 3*b)*Ta
n[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Sub}}{}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Sub}}{}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Sub}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3}{3a^2(a+b)f\sqrt{a}}$$

Mathematica [B] time = 4.55, size = 410, normalized size = 3.45

$$\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^{5/2}\left(-\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{16\left(-a\sin^2(e+fx)+a+b\right)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{a^2(a+b)\sin^4(e+fx)}{\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\sqrt{a}\sqrt{a}\cdots}{\cdots}\right)}{a^3}\cdots\right)}{\left(-a\cdots\right.}\cdots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*(-((Sqrt[2]*Csc[e + f*
x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin
[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2
)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*
Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sq
rt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(
a + b - a*Sin[e + f*x]^2)^(3/2)) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[
e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (4*(b + (3*a +
 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 1.54, size = 773, normalized size = 6.50

$$\left[-\frac{3\left(\left(a^3+a^2b\right)\cos\left(fx+e\right)^4+ab^2+b^3+2\left(a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-\cdots\right.\right.}{\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(a^2*b + a*b^2)*c
os(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e)) - 8*((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b
^2 + a^3*b^3)*f), 1/12*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(
```

```
a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*((3*a^3 + 4*a^2*b
)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^
5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b^2 + a^3*b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 1.52, size = 2112, normalized size = 17.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*
sin(f*x+e)*a^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+3*a*2^(1/2)*(
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos
```

```
(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2
)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)
-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^3*a^2+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a
*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2-4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*cos(f*x+e)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2
)/cos(f*x+e)^2)^(5/2)/(a+b)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}{\left(e + f x \right)}}{\left(a + b \sec^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.437 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e-\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a}{3a^2(a+b)^2f}$$

Mathematica [C] time = 6.50, size = 1927, normalized size = 15.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
```

```
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

fricas [B] time = 1.36, size = 881, normalized size = 7.05

$$\left[-\frac{3\left(\left(a^4+2a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2ab^3+b^4+2\left(a^3b+2a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(12\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

maple [C] time = 2.20, size = 3024, normalized size = 24.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^3+6*cos(f*x+e)^2*sin(f*x+e)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)
*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*b+3*cos(f*x+e)^2*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^
(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a*b^2-6*2^(1/2)*((I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1
```

```
/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f
*x+e)*cos(f*x+e)^2*a^3-12*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b-6*
2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b^2+3*2^(1/2)*((I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^3*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-12*2^(1/2)*((I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+6*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^3*a*b^2-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+
e)^2*a^2*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*b^2+5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^2+3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^3-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/(a^2+2*a*b+b^2)/a^2/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

3.438 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}-\frac{b(7a+3b)\cot(e+f}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(7*a
+3*b)*cot(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(3*a+
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)/a/(
a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.38, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}-\frac{b(7a+3b)\cot(e+f}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*
(7*a + 3*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a^
2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
```

```
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-4bx^2}{x^2(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \text{ta}\right.}{3a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b}{3a^2(a+}$$

Mathematica [A] time = 7.21, size = 247, normalized size = 1.42

$$-\frac{\sec^5(e+fx)(a\cos(2e+2fx)+a+2b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{4\sqrt{2}\,a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{\csc(e+fx)\sec^5(e+fx)(a\cos(2(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*Sec
[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(3*a^4 + 8*a^3*b +
 5*a^2*b^2 - 12*a*b^3 - 4*b^4) + 4*(3*a^4 + 6*a^3*b + 8*a*b^3 + 3*b^4)*Cos[
2*(e + f*x)] + a*(3*a^3 + 9*a*b^2 + 4*b^3)*Cos[4*(e + f*x)])*Csc[e + f*x]*S
ec[e + f*x]^5)/(48*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 4.10, size = 1097, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(
f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x
 + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
 + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))*sin(f*x + e) + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5
 + (6*a^4*b - 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b
+ 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3
+ 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), 1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a
*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^
2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*
x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*s
in(f*x + e) - 4*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5 + (6*a^4*b
- 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2 - 8*a^2*b^3
- 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.27, size = 7586, normalized size = 43.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}{\left(e+fx \right)}}{\left(a+b\sec^{2}{\left(e+fx \right)}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.439 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=236

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}+\frac{(a-b)\left(3a^2+14ab+3b^2\right)\text{co}}{3a^2 j}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-b*(3*a+b)*c
ot(f*x+e)^3/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(a-b)*(3*a^2+14*a*
b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/3*(a^2-10*a*
b-3*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*
x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.48, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}+\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{3a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Cot[e + f*x]^3)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(
3*a + b)*Cot[e + f*x]^3)/(a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
 ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]
^2])/(3*a^2*(a + b)^4*f) - ((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(3*a^2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
```

```
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-b)-6bx^2}{x^4(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e\right.}{3a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} +$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a\ }{a^2(a+b)^2f\sqrt{}}$$

Mathematica [A] time = 13.95, size = 234, normalized size = 0.99

$$\frac{\sec^5(e+fx)(a\cos(2e+2fx)+a+2b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{4\sqrt{2}a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e
+ f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^5*(-4*(a
+ 3*b)*Csc[e + f*x] + (a + b)*Csc[e + f*x]^3 + (4*b^3*(6*a^2 + 13*a*b + 3*b
^2 + 2*a*(3*a + b)*Cos[2*(e + f*x)])*Sin[e + f*x])/(a^2*(a + 2*b + a*Cos[2*
(e + f*x)])^2)))/(24*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 13.45, size = 1579, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4
*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b
^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8
*(4*(a^6 + 3*a^5*b + 3*a^3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b -
 8*a^4*b^2 + 8*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4
*b^2 - 4*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*
b^3 - 11*a^2*b^4 - 3*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/(((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x +
 e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*c
os(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
a^3*b^6)*f)*sin(f*x + e)), -1/12*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^
6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x +
 e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*(4*(a^6 + 3*a^5*b + 3*a^
3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b - 8*a^4*b^2 + 8*a^3*b^3 -
a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4*b^2 - 4*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*b^3 - 11*a^2*b^4 - 3*a*b^
5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^9 + 4*a^
8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b -
2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b
+ 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2
 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.49, size = 15128, normalized size = 64.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.440 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=315

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5a^2f(a+b)^3}-\frac{b(11a+3b)\cot^5(e}{3a^2f(a+b)^2\sqrt{a+b\tan^2}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(11*
a+3*b)*cot(f*x+e)^5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^4+7
0*a^3*b+128*a^2*b^2-70*a*b^3-15*b^4)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/
a^2/(a+b)^5/f+1/15*(5*a^3+19*a^2*b-65*a*b^2-15*b^3)*cot(f*x+e)^3*(a+b+b*tan
(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/5*(a^2-20*a*b-5*b^2)*cot(f*x+e)^5*(a+b+b*t
an(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*
x+e)^2)^(3/2)
```

Rubi [A] time = 0.60, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5a^2f(a+b)^3}+\frac{\left(19a^2b+5a^3-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a+b\,t}}{15a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x]^5)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (
b*(11*a + 3*b)*Cot[e + f*x]^5)/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - ((15*a^4 + 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^2*(a + b)^5*f) + ((5*a^3 + 19*a^2*b
 - 65*a*b^2 - 15*b^3)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^
2*(a + b)^4*f) - ((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[
e + f*x]^2])/(5*a^2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-5b-8bx^2}{x^6(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \text{ta}\right.}{3a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(}{3a^2(a+}$$

Mathematica [A] time = 24.43, size = 272, normalized size = 0.86

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{20b^5(a+b)}{a^2(a\cos(2(e+fx))+a+2b)^2}+\frac{10b^4(15a+4b)}{a^2(a\cos(2(e+fx))+a+2b)}-\left(23a^2+1\right.\right.}{120f(a+b)^5\left(a+b\sec^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*Sec
```

```
[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^3*((-20*b^5*(a + b))/
(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (10*b^4*(15*a + 4*b))/(a^2*(a + 2*
b + a*Cos[2*(e + f*x)])) - (23*a^2 + 100*a*b + 150*b^2)*Csc[e + f*x]^2 + (a
 + b)*(11*a + 25*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*
x]^4*Tan[e + f*x])/(120*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

fricas [B] time = 43.49, size = 2059, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/120*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5
)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f
*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5
+ a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^
5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 25
6*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^7 + 100*a^6*b + 1
50*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*cos(f*x + e)^9 - (35*a^7 + 118*a^6*b
+ 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x +
 e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15
*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 105*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4
 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*
a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 +
a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a
^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3
 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b
+ 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2
 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
*sin(f*x + e)), 1/60*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b
^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 -
 a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b
^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
+ e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
 e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(
f*x + e) - 4*((23*a^7 + 100*a^6*b + 150*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*
cos(f*x + e)^9 - (35*a^7 + 118*a^6*b + 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b
^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^
4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 1
05*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e
)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 1
0*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 +
4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (
a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 +
a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 -
4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 +
 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

maple [C] time = 2.52, size = 22712, normalized size = 72.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.441 $\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=105

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{df(m+1)}$$

```
[Out] AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*(d*t
an(f*x+e))^(1+m)*(a+b+b*tan(f*x+e)^2)^p/d/f/(1+m)/((1+b*tan(f*x+e)^2/(a+b))
^p)
```

Rubi [A] time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4141, 1975, 511, 510}

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(1
+ m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+b+bx^2\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{(dx)^m\left(1\right.}{1+x}\right.}{f}$$

$$= \frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx)}{df(1+m)}$$

Mathematica [B] time = 3.49, size = 259, normalized size = 2.47

$$\frac{\sin(e+fx)\cos(e+fx)(d\tan(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{b\tan^2(}{a+}\right.}{f(m+1)\left(\frac{2\tan^2(e+fx)\left(bpF_1\left(\frac{m+3}{2};1-p,1;\frac{m+5}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)-(a+b)F_1\left(\frac{m+3}{2};-p,2;\frac{m+5}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\right)}{(m+3)(a+b)}+F_1\left(\frac{m+}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Tan[e + f
*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + (2*(b*p*AppellF1[(3 + m)/2, 1 - p, 1, (5 +
 m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[(
3 + m)/2, -p, 2, (5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
)*Tan[e + f*x]^2)/((a + b)*(3 + m))))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

maple [F] time = 2.89, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(d\tan\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \left(d\tan\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\tan\left(e+f\,x\right)\right)^m \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.442 $\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx$

Optimal. Leaf size=122

$$-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e}{a}\right.}{2af(p+1)}$$

```
[Out] -1/2*(a+2*b)*(a+b*sec(f*x+e)^2)^(1+p)/b^2/f/(1+p)-1/2*hypergeom([1, 1+p],[2
+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(1+p)/a/f/(1+p)+1/2*(a+b*sec(f*x
+e)^2)^(2+p)/b^2/f/(2+p)
```

Rubi [A] time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4139, 446, 88, 65}

$$-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e}{a}\right.}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

```
[Out] -((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*b^2*f*(1 + p)) - (Hypergeome
tric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(
1 + p))/(2*a*f*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(2*b^2*f*(2 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^p}{b} + \frac{(a+bx)^p}{x} + \frac{(a+bx)^{1+p}}{b}\right) dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} + \frac{\left(a+b\sec^2(e+fx)\right)^{2+p}}{2b^2f(2+p)} + \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} - \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right.}{2af(1+p}$$

Mathematica [A] time = 0.53, size = 94, normalized size = 0.77

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\left(b^2(p+2)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)+a\left(a-b(p+1)\sec^2(e+fx)+2b(p+2)\right.\right.}{2ab^2f(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

```
[Out] -1/2*((a + b*Sec[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a] + a*(a + 2*b*(2 + p) - b*(1 + p)*Sec[e
+ f*x]^2)))/(a*b^2*f*(1 + p)*(2 + p))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

maple [F] time = 1.48, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\tan^5\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^{5}\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^{2}}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.443 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^3(e+fx)\,dx$

Optimal. Leaf size=86

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

```
[Out] 1/2*(a+b*sec(f*x+e)^2)^(1+p)/b/f/(1+p)+1/2*hypergeom([1, 1+p],[2+p],1+b*sec
(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(1+p)/a/f/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4139, 446, 80, 65}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

```
[Out] (a + b*Sec[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sec^2(e+fx)\right)^{1+p}}{2bf(1+p)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sec^2(e+fx)\right)^{1+p}}{2bf(1+p)} + \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)(a+}{2af(1+p)}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 0.71

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\left(b\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)+a\right)}{2abf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
[Out] ((a + b*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a])*(a +
b*Sec[e + f*x]^2)^(1 + p))/(2*a*b*f*(1 + p))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

maple [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\tan^3\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^{3}\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.444 $\int \left(a+b\sec^2(e+fx)\right)^p \tan(e+fx)\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(1+p)/
a/f/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4139, 266, 65}

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e
 + f*x]^2)^(1 + p))/(2*a*f*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^p \tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*S
ec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

maple [F] time = 1.31, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\tan\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^p \tan\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x), x)
```

3.445 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=114

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+f}{a+b}\right.}{2f(p+1)(a+b)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^
(1+p)/(a+b)/f/(1+p)+1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*s
ec(f*x+e)^2)^(1+p)/a/f/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4139, 446, 86, 68, 65}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+f}{a+b}\right.}{2f(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a + b
*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f
*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x\left(-1+x^2\right)}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e\right.}{2f}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)} + \frac{{}_2F_1\left(1\right.}{}\end{aligned}$$

Mathematica [A] time = 2.13, size = 115, normalized size = 1.01

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(a+b\sec^2(e+fx)\right)^p\left((a+b)\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a+b}{a}\right)-a\,{}_2\right.}{4af(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Hypergeometric2F1[1, 1 + p, 2 + p,
 (a + b + b*Tan[e + f*x]^2)/a] - a*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*Tan[e + f*x]^2)/(a + b)])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*a*(
a + b)*f*(1 + p))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

maple [F] time = 2.03, size = 0, normalized size = 0.00

$$\int \cot\left(fx+e\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.446 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=157

$$\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)^2}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p\right.}{2af(p+1)}$$

```
[Out] -1/2*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1+p)/(a+b)/f+1/2*(-b*p+a+b)*hypergeom
([1, 1+p],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^(1+p)/(a+b)^2/
f/(1+p)-1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)
^(1+p)/a/f/(1+p)
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4139, 446, 103, 156, 68, 65}

$$\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)^2}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p\right.}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)*f) + ((a + b -
b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a
+ b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)^2*f*(1 + p)) - (Hypergeometric2F1[1
, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(
2*a*f*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{x\left(-1+x^2\right)^2}\,dx,x,\sec(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{(-1+x)^2x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=-\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f}-\frac{\text{Subst}\left(\int\frac{(a+bx)^p(a+b-bpx)}{(-1+x)x}\,dx,x\right.}{2(a+b)f}$$
$$=-\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f}+\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\sec^2(e\right.}{2f}$$
$$=-\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f}+\frac{(a+b-bp)\,{}_2F_1\left(1,1+p;2+\right.}{2(a}$$

Mathematica [A] time = 3.51, size = 139, normalized size = 0.89

$$-\frac{\tan^2(e+fx)\left((a+b)\cot^2(e+fx)+b\right)\left(a+b\sec^2(e+fx)\right)^p\left((a+b)^2\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a+b}{a}\right)-a(a\right.}{2af(p+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*((b + (a + b)*Cot[e + f*x]^2)*(a*(a + b)*(1 + p)*Cot[e + f*x]^2 + (a +
 b)^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] - a*
(a + b - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/(a
+ b)])*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2)/(a*(a + b)^2*f*(1 + p))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(fx+e\right)\right)\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{3}\left(a+\frac{b}{\cos\left(e+f\,x\right)^{2}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.447 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4141, 1975, 511, 510}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,d\right.}{f}$$
$$= \frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\right.}{5f}$$

Mathematica [B] time = 18.15, size = 2777, normalized size = 31.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^5*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/
2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)
*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])
*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)
/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f*(((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*((9*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p,
 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b
*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))
^p))/3 - (2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*
Sin[2*(e + f*x)]*Tan[e + f*x]*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan
[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^
2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + (2*p*
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]^2*((9*(a +
 b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometri
```

```
c2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*T
an[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^p*Tan[e + f*x]*((-18*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] +
2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Cos[e + f*x]^2*((2*b*p*Appe
llF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (2*b*p*Sec[e + f*x]^2*Tan
[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p)*(-3*Hypergeometric2F1[1
/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/
2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2))/(a + b) - (9*(a + b)*App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[
e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f
*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*
b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*App
ellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2 + (2*Hypergeometric2F
1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
- 3*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p) + 3*Sec[e + f*x]^
2*Tan[e + f*x]*(-Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a +
b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b)
)^p))/3))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

maple [F] time = 1.60, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(\tan^4\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^4 \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.448 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4141, 1975, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,d\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\right.}{3f}$$

Mathematica [B] time = 16.54, size = 2465, normalized size = 28.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^3*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a
 + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x
]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*((a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^(1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e +
f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
 + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 2*a*p*(a + 2*b +
a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*Tan[e + f*
x]*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*
Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*A
ppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
 - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2])*Tan[e + f*x]^2)) + 2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^p*Tan[e + f*x]^2*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x
]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^
2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a*Cos[2*(e
+ f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((-2*b*p*Hypergeometric2F1[1/2,
-p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b
*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) + (6*(a + b)*AppellF1[1/2, -p,
```

```
1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e
+ f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
 + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Cos[e +
 f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]
^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[
e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2
)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/
(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p,
 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Ta
n[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
 (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/
2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*Ap
pellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2)^2)))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\tan^2\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)^2 + a\right)^p \tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**2,x)
[Out] Timed out
```

3.449 $\int \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}dx, x, \tan(e+f\right.}{f}$$
$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{f}$$

Mathematica [B] time = 6.26, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
```

```
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

3.450 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=84

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4141, 1975, 511, 510}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
 + b))^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^p}{x^2\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{x^2\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2\left(1+x^2\right)}\,dx\right.}{f}$$
$$= -\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\left(a+b+\right.}{f}$$

Mathematica [B] time = 16.90, size = 2469, normalized size = 29.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/
(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*p*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Si
n[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b
*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - (a + 2*b +
a*Cos[2*(e + f*x)])^p*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F
1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) - 2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*Cot[e + f*x]*(Sec
[e + f*x]^2)^p*Sin[2*(e + f*x)]*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Ta
n[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin
[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a
*Cos[2*(e + f*x)])^p*Cot[e + f*x]*(Sec[e + f*x]^2)^p*((2*b*p*Hypergeometric
2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (6*(a + b)*AppellF1
```

```
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f
*x]*Sin[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a +
b)*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
 -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^
2) - (Csc[e + f*x]*Sec[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[
e + f*x]^2)/(a + b))] - (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e
+ f*x]^2)/(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e +
 f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*
(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*A
ppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)
) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)^2)))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int\left(\cot^2\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.451 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e
)^3*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4141, 1975, 511, 510}

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^p}{x^4\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{x^4\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^4\left(1+x^2\right)}\,dx\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\left(a+b\right.}{3f}$$

Mathematica [B] time = 18.35, size = 3033, normalized size = 34.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^7*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*Appe
llF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*T
an[e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f
*((2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*(Sec[e + f*x]^2)^p*(
(9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a +
 b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*T
an[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 - (a + 2*b + a*Cos[2*
(e + f*x)])^p*Cot[e + f*x]^2*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*((9*(a + b)*
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
in[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2
) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]
^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (2*a*p*(a + 2*b + a*Cos[2*(e + f*
x)])^(-1 + p)*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((9*(a + b
)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
```

```
^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*
x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x
)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*((18*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e +
f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (18*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Tan[e + f
*x]^3)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Sin[e +
f*x]^2*Tan[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
(2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p,
1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[
3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f
*x]^2) + (2*b*p*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b)
)^(-1 - p)*(Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))
] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e
 + f*x]^2))/(a + b) - (9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2*(4*(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (
a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]
^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2,
2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]
^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2
, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2)^2 - (-6*Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x] - 3*Sec[e + f*x]^2*Tan
[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]
- (1 + (b*Tan[e + f*x]^2)/(a + b))^p) - 3*Csc[e + f*x]*Sec[e + f*x]*(-Hyper
geometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + (1 + (b*Tan[e
 + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

3.452 $\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx$

Optimal. Leaf size=92

$$\frac{a\sec^4(e+fx)}{4f}-\frac{a\sec^2(e+fx)}{f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

```
[Out] -a*ln(cos(f*x+e))/f-a*sec(f*x+e)^2/f+1/3*b*sec(f*x+e)^3/f+1/4*a*sec(f*x+e)^
4/f-2/5*b*sec(f*x+e)^5/f+1/7*b*sec(f*x+e)^7/f
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 1802}

$$\frac{a\sec^4(e+fx)}{4f}-\frac{a\sec^2(e+fx)}{f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (b*Sec[e + f*x]^3)/(3*f
) + (a*Sec[e + f*x]^4)/(4*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]
^7)/(7*f)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^3\right)}{x^8}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^8}-\frac{2b}{x^6}+\frac{a}{x^5}+\frac{b}{x^4}-\frac{2a}{x^3}+\frac{a}{x}\right)dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 87, normalized size = 0.95

$$-\frac{a\left(-\tan^4(e+fx)+2\tan^2(e+fx)+4\log(\cos(e+fx))\right)}{4f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] (b*Sec[e + f*x]^3)/(3*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]^7)/
(7*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

fricas [A] time = 0.73, size = 81, normalized size = 0.88

$$-\frac{420\,a\cos\left(fx+e\right)^{7}\log\left(-\cos\left(fx+e\right)\right)+420\,a\cos\left(fx+e\right)^{5}-140\,b\cos\left(fx+e\right)^{4}-105\,a\cos\left(fx+e\right)^{3}+}{420\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a*cos(f*x + e)^7*log(-cos(f*x + e)) + 420*a*cos(f*x + e)^5 - 14
0*b*cos(f*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/(f
*cos(f*x + e)^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(a/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-a/2*ln(
abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1))+(1089*((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1))))^7*a-8463*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*a
+28749*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a-4480*((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1))))^4*b-51555*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^4*a-2240*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b+51555*((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^3*a-1344*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*b-28749*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+448*(1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1)))*b+8463*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
))*a-64*b-1089*a)*1/840/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^7)
```

maple [B] time = 1.10, size = 183, normalized size = 1.99

$$\frac{\left(\tan^{4}\left(fx+e\right)\right)a}{4f}-\frac{a\left(\tan^{2}\left(fx+e\right)\right)}{2f}-\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin^{6}\left(fx+e\right)\right)}{7f\cos\left(fx+e\right)^{7}}+\frac{b\left(\sin^{6}\left(fx+e\right)\right)}{35f\cos\left(fx+e\right)^{5}}-\frac{b\left(\sin^{6}\left(fx\right.\right.}{105f\cos\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x)
```

```
[Out] 1/4/f*tan(f*x+e)^4*a-1/2/f*a*tan(f*x+e)^2-a*ln(cos(f*x+e))/f+1/7/f*b*sin(f*
x+e)^6/cos(f*x+e)^7+1/35/f*b*sin(f*x+e)^6/cos(f*x+e)^5-1/105/f*b*sin(f*x+e)
^6/cos(f*x+e)^3+1/35/f*b*sin(f*x+e)^6/cos(f*x+e)+8/105/f*b*cos(f*x+e)+1/35/
f*b*cos(f*x+e)*sin(f*x+e)^4+4/105/f*b*cos(f*x+e)*sin(f*x+e)^2
```

maxima [A] time = 0.34, size = 73, normalized size = 0.79

$$-\frac{420\,a\log\left(\cos\left(fx+e\right)\right)+\frac{420\,a\cos\left(fx+e\right)^{5}-140\,b\cos\left(fx+e\right)^{4}-105\,a\cos\left(fx+e\right)^{3}+168\,b\cos\left(fx+e\right)^{2}-60\,b}{\cos\left(fx+e\right)^{7}}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(cos(f*x + e)) + (420*a*cos(f*x + e)^5 - 140*b*cos(f*x + e
)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x + e)^7)/f
```

mupad [B] time = 8.79, size = 227, normalized size = 2.47

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}-\frac{2\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-14\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+\left(32\,a+\frac{32\,b}{3}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+\left(\frac{16\,b}{3}-32\,a\right.}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((16*b)/105 - tan(e/2 + (f*x)/2)^2*(2
*a + (16*b)/15) + tan(e/2 + (f*x)/2)^4*(14*a + (16*b)/5) - tan(e/2 + (f*x)/
2)^6*(32*a - (16*b)/3) + tan(e/2 + (f*x)/2)^8*(32*a + (32*b)/3) - 14*a*tan(
e/2 + (f*x)/2)^10 + 2*a*tan(e/2 + (f*x)/2)^12)/(f*(7*tan(e/2 + (f*x)/2)^2 -
 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^
8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)
^14 - 1))
```

sympy [A] time = 8.41, size = 119, normalized size = 1.29

$$\begin{cases}\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^4\left(e+fx\right)}{4f}-\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^4\left(e+fx\right)\sec^3\left(e+fx\right)}{7f}-\frac{4b\tan^2\left(e+fx\right)\sec^3\left(e+fx\right)}{35f}+\frac{8b\sec^3\left(e+fx\right)}{105f} & \text{for } f\\ x\left(a+b\sec^3\left(e\right)\right)\tan^5\left(e\right) & \text{othe}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**3/(7*f) - 4*b*tan(e
+ f*x)**2*sec(e + f*x)**3/(35*f) + 8*b*sec(e + f*x)**3/(105*f), Ne(f, 0)),
(x*(a + b*sec(e)**3)*tan(e)**5, True))
```

3.453 $\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx$

Optimal. Leaf size=61

$$\frac{a\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^5(e+fx)}{5f}-\frac{b\sec^3(e+fx)}{3f}$$

```
[Out] a*ln(cos(f*x+e))/f+1/2*a*sec(f*x+e)^2/f-1/3*b*sec(f*x+e)^3/f+1/5*b*sec(f*x+
e)^5/f
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 1802}

$$\frac{a\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^5(e+fx)}{5f}-\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + (a*Sec[e + f*x]^2)/(2*f) - (b*Sec[e + f*x]^3)/(3*
f) + (b*Sec[e + f*x]^5)/(5*f)
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^3)}{x^6}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^6}-\frac{b}{x^4}+\frac{a}{x^3}-\frac{a}{x}\right)dx,x,\cos(e+fx)\right)}{f}\\ &= \frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.12, size = 59, normalized size = 0.97

$$\frac{a\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}+\frac{b\sec^5(e+fx)}{5f}-\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] -1/3*(b*Sec[e + f*x]^3)/f + (b*Sec[e + f*x]^5)/(5*f) + (a*(2*Log[Cos[e + f*
x]] + Tan[e + f*x]^2))/(2*f)
```

fricas [A] time = 0.60, size = 59, normalized size = 0.97

$$\frac{30\,a\cos\left(fx+e\right)^{5}\log\left(-\cos\left(fx+e\right)\right)+15\,a\cos\left(fx+e\right)^{3}-10\,b\cos\left(fx+e\right)^{2}+6\,b}{30\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a*cos(f*x + e)^5*log(-cos(f*x + e)) + 15*a*cos(f*x + e)^3 - 10*b*c
os(f*x + e)^2 + 6*b)/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-a/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+a/2*ln
(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1))+(-137*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^5*a+805*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a
-240*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*b-1730*((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^3*a-80*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b
+1730*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-80*(1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1)))*b-805*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+16*b+13
7*a)*1/120/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^5)
```

maple [B] time = 0.98, size = 126, normalized size = 2.07

$$\frac{a\left(\tan^{2}\left(fx+e\right)\right)}{2f}+\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin^{4}\left(fx+e\right)\right)}{5f\cos\left(fx+e\right)^{5}}+\frac{b\left(\sin^{4}\left(fx+e\right)\right)}{15f\cos\left(fx+e\right)^{3}}-\frac{b\left(\sin^{4}\left(fx+e\right)\right)}{15f\cos\left(fx+e\right)}-\frac{b\cos\left(fx+e\right)\left(\mathrm{s}\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*a*tan(f*x+e)^2+a*ln(cos(f*x+e))/f+1/5/f*b*sin(f*x+e)^4/cos(f*x+e)^5+1
/15/f*b*sin(f*x+e)^4/cos(f*x+e)^3-1/15/f*b*sin(f*x+e)^4/cos(f*x+e)-1/15/f*b
*cos(f*x+e)*sin(f*x+e)^2-2/15/f*b*cos(f*x+e)
```

maxima [A] time = 0.35, size = 51, normalized size = 0.84

$$\frac{30\,a\log\left(\cos\left(fx+e\right)\right)+\frac{15\,a\cos\left(fx+e\right)^{3}-10\,b\cos\left(fx+e\right)^{2}+6\,b}{\cos\left(fx+e\right)^{5}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/30*(30*a*log(cos(f*x + e)) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x + e)^2 +
 6*b)/cos(f*x + e)^5)/f
```

mupad [B] time = 8.39, size = 167, normalized size = 2.74

$$\frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\left(-6\,a-4\,b\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\left(6\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(-2\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{4\,b}{15}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

```
[Out] ((4*b)/15 - tan(e/2 + (f*x)/2)^2*(2*a + (4*b)/3) - tan(e/2 + (f*x)/2)^6*(6*
a + 4*b) + tan(e/2 + (f*x)/2)^4*(6*a - (4*b)/3) + 2*a*tan(e/2 + (f*x)/2)^8)
/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/
2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (2*a*atanh(ta
n(e/2 + (f*x)/2)^2))/f
```

sympy [A] time = 2.98, size = 82, normalized size = 1.34

$$\begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^3\left(e+fx\right)}{5f}-\frac{2b\sec^3\left(e+fx\right)}{15f} & \text{for } f\neq 0 \\ x\left(a+b\sec^3\left(e\right)\right)\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**3/(5*f) - 2*b*sec(e + f*x)**3/(15*f), Ne(f, 0
)), (x*(a + b*sec(e)**3)*tan(e)**3, True))
```

3.454 $\int \left(a+b \sec^3(e+fx)\right) \tan(e+fx)\,dx$

Optimal. Leaf size=30

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

```
[Out] -a*ln(cos(f*x+e))/f+1/3*b*sec(f*x+e)^3/f
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 14}

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^4}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a}{x}\right)dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

fricas [A] time = 0.52, size = 37, normalized size = 1.23

$$-\frac{3\,a\cos\left(fx+e\right)^{3}\log\left(-\cos\left(fx+e\right)\right)-b}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="fricas")
[Out] -1/3*(3*a*cos(f*x + e)^3*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(a/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-a/2*ln(
abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1))+(11*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^3*a-33*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-12*
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+33*(1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))*a-11*a-4*b)*1/12/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^
3)
```

maple [A] time = 0.36, size = 28, normalized size = 0.93

$$\frac{b\left(\sec^{3}\left(fx+e\right)\right)}{3f}+\frac{a\ln\left(\sec\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e),x)
[Out] 1/3*b*sec(f*x+e)^3/f+1/f*a*ln(sec(f*x+e))
```

maxima [A] time = 0.34, size = 28, normalized size = 0.93

$$-\frac{a\log\left(\cos\left(fx+e\right)^{3}\right)-\frac{b}{\cos\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="maxima")
[Out] -1/3*(a*log(cos(f*x + e)^3) - b/cos(f*x + e)^3)/f
```

mupad [B] time = 5.26, size = 83, normalized size = 2.77

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}-\frac{2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,b}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((2*b)/3 + 2*b*tan(e/2 + (f*x)/2)^4)/
(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6
- 1))
```

sympy [A] time = 0.88, size = 42, normalized size = 1.40

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b\sec^3\left(e+fx\right)}{3f} & \text{for } f \neq 0 \\ x\left(a+b\sec^3\left(e\right)\right)\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**3/(3*f), Ne(f
, 0)), (x*(a + b*sec(e)**3)*tan(e), True))
```

3.455 $\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(\cos(e+fx)+1)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] 1/2*(a+b)*ln(1-cos(f*x+e))/f+1/2*(a-b)*ln(1+cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 1802}

$$\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(\cos(e+fx)+1)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((a + b)*Log[1 - Cos[e + f*x]])/(2*f) + ((a - b)*Log[1 + Cos[e + f*x]])/(2*
f) + (b*Sec[e + f*x])/f
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^2\left(1-x^2\right)}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{2(-1+x)}+\frac{b}{x^2}+\frac{-a+b}{2(1+x)}\right)dx,x,\cos(e+fx)\right)}{f}\\ &= \frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.06, size = 65, normalized size = 1.20

$$\frac{a(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}+\frac{b\sec(e+fx)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b*Log[Cos[(e + f*x)/2]])/f) + (b*Log[Sin[(e + f*x)/2]])/f + (a*(Log[Cos[
e + f*x]] + Log[Tan[e + f*x]]))/f + (b*Sec[e + f*x])/f
```

fricas [A] time = 0.46, size = 61, normalized size = 1.13

$$\frac{(a-b)\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+(a+b)\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+2b}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) + (a + b)*cos(f*x + e
)*log(-1/2*cos(f*x + e) + 1/2) + 2*b)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((a+b)/4*ln(abs(1-c
os(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))-a/2*ln(abs((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1)))+1))-b/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1))
```

maple [A] time = 0.69, size = 48, normalized size = 0.89

$$\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x)
```

```
[Out] a*ln(sin(f*x+e))/f+1/f*b/cos(f*x+e)+1/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.35, size = 45, normalized size = 0.83

$$\frac{(a-b)\log\left(\cos\left(fx+e\right)+1\right)+(a+b)\log\left(\cos\left(fx+e\right)-1\right)+\frac{2b}{\cos\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(cos(f*x + e) + 1) + (a + b)*log(cos(f*x + e) - 1) + 2*b/co
s(f*x + e))/f
```

mupad [B] time = 4.64, size = 72, normalized size = 1.33

$$\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\ln\left({\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+1\right)}{f}-\frac{2b}{f\left({\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2-1\right)}+\frac{b\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (a*log(tan(e/2 + (f*x)/2)))/f - (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (2*b)
/(f*(tan(e/2 + (f*x)/2)^2 - 1)) + (b*log(tan(e/2 + (f*x)/2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^3\left(e + fx\right)\right)\cot\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x), x)
```

3.456 $\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal. Leaf size=72

$$-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(\cos(e+fx)+1)}{4f}-\frac{\csc^2(e+fx)(a+b\cos(e+fx))}{2f}$$

```
[Out] -1/2*(a+b*cos(f*x+e))*csc(f*x+e)^2/f-1/4*(2*a-b)*ln(1-cos(f*x+e))/f-1/4*(2*
a+b)*ln(1+cos(f*x+e))/f
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4138, 1814, 633, 31}

$$-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(\cos(e+fx)+1)}{4f}-\frac{\csc^2(e+fx)(a+b\cos(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((a + b*Cos[e + f*x])*Csc[e + f*x]^2)/(2*f) - ((2*a - b)*Log[1 - Cos[e + f
*x]])/(4*f) - ((2*a + b)*Log[1 + Cos[e + f*x]])/(4*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{b+ax^3}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f} + \frac{\text{Subst}\left(\int \frac{-b+2ax}{1-x^2}\,dx, x, \cos(e+f\right.}{2f}$$
$$= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos\right.}{4f}$$
$$= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f} - \frac{(2a-b)\log(1-\cos(e+fx))}{4f} -$$

Mathematica [A] time = 1.01, size = 114, normalized size = 1.58

$$-\frac{a\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f} - \frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\log\left(\text{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/8*(b*Csc[(e + f*x)/2]^2)/f - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Si
n[(e + f*x)/2]])/(2*f) - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[T
an[e + f*x]]))/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

fricas [A] time = 0.54, size = 99, normalized size = 1.38

$$\frac{2\,b\cos\left(fx+e\right) - \left((2\,a+b)\cos\left(fx+e\right)^2 - 2\,a-b\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right) - \left((2\,a-b)\cos\left(fx+e\right)^2 - 2\,a\right.}{4\left(f\cos\left(fx+e\right)^2 - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*cos(f*x + e) - ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*log(1/2*cos(f*
x + e) + 1/2) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*log(-1/2*cos(f*x + e)
+ 1/2) + 2*a)/(f*cos(f*x + e)^2 - f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*b-(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)/16+(-2*
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1)))*a-b-a)*1/16/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+a/2*ln(abs((1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(b-2*a)/8*ln(abs(1-cos(f*x+exp(1))
)/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 1.10, size = 69, normalized size = 0.96

$$-\frac{a\left(\cot^{2}\left(fx+e\right)\right)}{2f}-\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{b\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/2*a*cot(f*x+e)^2/f-a*ln(sin(f*x+e))/f-1/2/f*b*csc(f*x+e)*cot(f*x+e)+1/2/
f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.34, size = 62, normalized size = 0.86

$$-\frac{(2a+b)\log\left(\cos\left(fx+e\right)+1\right)+(2a-b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(b\cos\left(fx+e\right)+a\right)}{\cos\left(fx+e\right)^{2}-1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/4*((2*a + b)*log(cos(f*x + e) + 1) + (2*a - b)*log(cos(f*x + e) - 1) - 2
*(b*cos(f*x + e) + a)/(cos(f*x + e)^2 - 1))/f
```

mupad [B] time = 4.65, size = 86, normalized size = 1.19

$$\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{a}{8}-\frac{b}{8}\right)}{f}-\frac{\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{a}{8}+\frac{b}{8}\right)}{f}-\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (tan(e/2 + (f*x)/2)^2*(a/8 - b/8))/f
- (cot(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (log(tan(e/2 + (f*x)/2))*(a - b/2)
)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{3}\left(e+fx\right)\right)\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x)**3, x)
```

3.457 $\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=219

$$\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{4/3}f}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{4/3}f}-$$

```
[Out] -1/3*(a^(2/3)-2*b^(2/3))*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(4/3)/f+1
/6*(a^(2/3)-2*b^(2/3))*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*
x+e)^2)/a^(1/3)/b^(4/3)/f-1/3*ln(b+a*cos(f*x+e)^3)/a/f+sec(f*x+e)/b/f-1/3*(
a^(2/3)+2*b^(2/3))*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2
))/a^(1/3)/b^(4/3)/f*3^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4138, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{4/3}f}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{4/3}f}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -(((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]
*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)*f)) - ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)*f) + ((a^(2/3) - 2*b^(2/3))*
Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)*f) - Log[b + a*Cos[e + f*x]^3]/(3*a*f) + Sec[e + f*x]/(b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-2b-ax+bx^2}{b(b+ax^3)}\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{-2b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{bf}$$

$$= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{-2b-ax}{b+ax^3}\,dx, x, \cos(e+\right.}{bf}$$

$$= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\left(-a\sqrt[3]{b}-4\sqrt[3]{a}\,b\right)+\sqrt[3]{a}\left(-a\sqrt[3]{b}+2\sqrt[3]{a}\right.}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+a^{2/3}x^2}\right.}{3\sqrt[3]{a}\,b^{5/3}f}$$

$$= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+f}{bf}$$

$$= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{4/3}f} + \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(\right.}{6\sqrt[3]{a}\,b^{4/3}}$$

$$= -\frac{\left(a^{2/3}+2b^{2/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{4/3}f} - \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{4/3}f} +$$

Mathematica [C] time = 0.36, size = 251, normalized size = 1.15

$$-\text{RootSum}\left[\#1^3a-\#1^3b-6\#1^2a+12\#1a-8a\&, \frac{\#1^2ab\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)-\#1^2b^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)-4a^2\text{lo}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (3*b*Log[Sec[(e + f*x)/2]^2] - RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 -
 b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + T
an[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 8*a*b*Log[
1 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2
 - b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1
^2) & ] + 3*a*Sec[e + f*x])/(3*a*b*f)
```

fricas [C] time = 1.67, size = 4427, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/
27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2
```

```
*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) +
1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(
a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))
*a*b*f*cos(f*x + e)*log(1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)
/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2
*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 - ((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^
2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f
^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4
*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^
3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f + 4*a^2*b + 5*b^3 + (a^3 + 8*a*b^2)*co
s(f*x + e)) - (((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2)
)/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a
^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 +
b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/
(a*f))*a*b*f*cos(f*x + e) + 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(
a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^
2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
 b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^
2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3
) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*
(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 -
2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3)
+ 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*
(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 +
 36*b^2)/(a^2*b^2*f^2))*cos(f*x + e) - 18*b*cos(f*x + e))*log(1/36*((-I*sqr
t(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/
54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4
 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^
3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/
54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 -
((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f
^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1
/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27
/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f
^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f
+ 4*a^2*b + 5*b^3 - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2
 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*
f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3
*b^4*f^3))^(1/3) + 6/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) +
 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^
2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a
^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1
/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^
4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-
I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3)
 + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54
*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a
^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3)
 - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 2
88*a^2 + 36*b^2)/(a^2*b^2*f^2)) - 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - (((-I*s
qrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) +
```

```
1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a
^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*
f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) -
1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b*f*cos(f*x
+ e) - 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b
^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18
*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^
(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3)
 + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4
*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a^2*f^2) -
 (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*
f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3
*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/
(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^
4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^2*b^2*f^2
))*cos(f*x + e) - 18*b*cos(f*x + e))*log(-1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^
2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*
b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/
(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b
^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2
+ b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 + ((-I*sqrt(3) + 1)*(1
/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*
b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2
 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a
^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*
a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f - 4*a^2*b - 5*b^3 -
1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2
))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(
a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 +
 b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6
/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (
2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^
3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b
^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a
*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)
/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a
^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2
)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
+ 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2
*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^
2*b^2*f^2)) + 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - 36*a)/(a*b*f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^5}{b\sec\left(fx+e\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^3 + a), x)
```

maple [A] time = 0.66, size = 274, normalized size = 1.25

$$\frac{2\ln\left(\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3fa\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos^{2}\left(fx+e\right)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{3fa\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos\left(fx+e\right)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3fa\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\ln(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x)
```

```
[Out] 2/3/f/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))-1/3/f/a/(1/a*b)^(2/3)*ln
(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+2/3/f/a/(1/a*b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))-1/3/f/b/(1/a*b)
^(1/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/f/b/(1/a*b)^(1/3)*ln(cos(f*x+e)^2-(
1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3/f/b*3^(1/2)/(1/a*b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))-1/3*ln(b+a*cos(f*x+e)^3)/a/f+1
/f/b/cos(f*x+e)
```

maxima [A] time = 0.44, size = 218, normalized size = 1.00

$$\frac{\frac{2\sqrt{3}\left(2ab\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{b}{a}\right)+3a^{2}\left(\frac{b}{a}\right)^{\frac{2}{3}}+2b^{2}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos\left(fx+e\right)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab^{2}}-\frac{3\left(2b\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)-a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos\left(fx+e\right)^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{6\cdots}{\cdots}}{18f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(2*a*b*(3*(b/a)^(1/3) - b/a) + 3*a^2*(b/a)^(2/3) + 2*b^2)*a
rctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b^2) - 3*
(2*b*((b/a)^(2/3) + 1) - a*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*co
s(f*x + e) + (b/a)^(2/3))/(a*b*(b/a)^(2/3)) - 6*(b*((b/a)^(2/3) - 2) + a*(b
/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a*b*(b/a)^(2/3)) + 18/(b*cos(f*
x + e)))/f
```

mupad [B] time = 7.26, size = 7402, normalized size = 33.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^3),x)
```

```
[Out] symsum(log(-(262144*(148*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z +
 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^17 - 1920*a*b^15 - 156*b^16*
cos(e + f*x) + 300*b^16 + 16*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4
*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*b^18 + 5232*a^2*b^14 - 7
872*a^3*b^13 + 7080*a^4*b^12 - 3840*a^5*b^11 + 1200*a^6*b^10 - 192*a^7*b^9
+ 12*a^8*b^8 - 5916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15 + 4820*root(27*a^3*b^4*z^3
 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)
```

*a^3*b^14 + 5933*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13 - 12882*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^5*b^12 + 8891*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^6*b^11 - 2872*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^7*b^10 + 447*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9 - 26*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^9*b^8 + root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^10*b^7 + 1396*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a*b^17 + 192*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a*b^18 - 3936*a^2*b^14*cos(e + f*x) + 7152*a^3*b^13*cos(e + f*x) - 7800*a^4*b^12*cos(e + f*x) + 5136*a^5*b^11*cos(e + f*x) - 1920*a^6*b^10*cos(e + f*x) + 336*a^7*b^9*cos(e + f*x) - 12*a^8*b^8*cos(e + f*x) - 768*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^2*b^16 + 4772*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^3*b^15 - 13924*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^4*b^14 + 6927*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^5*b^13 + 5747*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^6*b^12 - 5944*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^7*b^11 + 2004*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^8*b^10 - 239*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^9*b^9 + 13*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^10*b^8 + 4296*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^2*b^17 - 11856*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^3*b^16 + 16956*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^4*b^15 - 17916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^5*b^14 + 11175*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^6*b^13 - 4608*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^7*b^12 + 2118*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^8*b^11 - 372*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^9*b^10 + 15*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^10*b^9 + 864*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^2*b^18 + 3240*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^3*b^17 - 12996*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^4*b^16 + 4140*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^5*b^15 + 16668*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^6*b^14 - 16011*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^7*b^13 + 4959*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^8*b^12 - 873*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^9*b^11 + 9*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^10*b^10 + 1728*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^3*b^18 - 5724*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2 *z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^4*b^17 + 6912*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^5*b^16 - 3024*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^6*b^15 - 1080*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^7*b^14 + 1836*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^8*b^13 - 648*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^9*b^12 + 1296*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^4*b^18 - 7452*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^5*b^17 + 14904*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^6*b^16 - 12960*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^7*b^15 + 4536*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^8*b^14 - 324*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^9*b^13 + 1456*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a*b^16 - 52*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^17*cos(e + f*x) + 1200*a*b^15*cos(e + f*x) - 880*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a*b^16*cos(e + f*x) + 4764*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15*cos(e + f*x) - 6932*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^3*b^14*cos(e + f*x) - 1109*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13*cos(e + f*x) + 12234*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^5*b^12*cos(e + f*x) - 12299*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^6*b^11*cos(e + f*x) + 5032*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^7*b^10*cos(e + f*x) - 807*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9*cos(e + f*x) + 50*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^9*b^8*cos(e + f*x) - root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^10*b^7*cos(e + f*x) - 548*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a*b^17*cos(e + f*x) + 160*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^2*b^16*cos(e + f*x) - 1380*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^3*b^15*cos(e + f*x) + 12140*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^4*b^14*cos(e + f*x) - 14767*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^5*b^13*cos(e + f*x) - 1659*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^6*b^12*cos(e + f*x) + 9272*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^7*b^11*cos(e + f*x) - 3691*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^8*b^10*cos(e + f*x) + 510*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^9*b^9*cos(e + f*x) - 38*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^10*b^8*cos(e + f*x) + root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^11*b^7*cos(e + f*x) - 1992*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^2*b^17*cos(e + f*x) + 8112*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^3*b^16*cos(e + f*x) - 18300*root(27*a^3*b^4*z^3

```
 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)
^3*a^4*b^15*cos(e + f*x) + 19788*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a
*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^5*b^14*cos(e + f*x
) - 10095*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z -
 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^6*b^13*cos(e + f*x) + 6000*root(27*a^3*b^
4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)^3*a^7*b^12*cos(e + f*x) - 4134*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 +
 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^8*b^11*cos(e +
 f*x) + 660*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
 - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^9*b^10*cos(e + f*x) - 39*root(27*a^3*b^
4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)^3*a^10*b^9*cos(e + f*x) - 2376*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 +
 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^3*b^17*cos(e +
 f*x) + 11124*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2
*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^4*b^16*cos(e + f*x) - 10044*root(27*a
^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 +
a^4, z, k)^4*a^5*b^15*cos(e + f*x) - 10260*root(27*a^3*b^4*z^3 + 27*a^2*b^4
*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^6*b^14*c
os(e + f*x) + 19899*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^7*b^13*cos(e + f*x) - 10287*roo
t(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 +
b^4 + a^4, z, k)^4*a^8*b^12*cos(e + f*x) + 2025*root(27*a^3*b^4*z^3 + 27*a^
2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^9*b
^11*cos(e + f*x) - 81*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18
*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^10*b^10*cos(e + f*x) + 1404*r
oot(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2
+ b^4 + a^4, z, k)^5*a^4*b^17*cos(e + f*x) - 6048*root(27*a^3*b^4*z^3 + 27*
a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^5
*b^16*cos(e + f*x) + 8208*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^6*b^15*cos(e + f*x) - 237
6*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b
^2 + b^4 + a^4, z, k)^5*a^7*b^14*cos(e + f*x) - 2700*root(27*a^3*b^4*z^3 +
27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*
a^8*b^13*cos(e + f*x) + 1512*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4
*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^9*b^12*cos(e + f*x) +
3564*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^
2*b^2 + b^4 + a^4, z, k)^6*a^5*b^17*cos(e + f*x) - 12312*root(27*a^3*b^4*z^
3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k
)^6*a^6*b^16*cos(e + f*x) + 15552*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*
a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^7*b^15*cos(e + f*
x) - 8424*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z -
 2*a^2*b^2 + b^4 + a^4, z, k)^6*a^8*b^14*cos(e + f*x) + 1620*root(27*a^3*b^
4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)^6*a^9*b^13*cos(e + f*x)))/cos(e/2 + (f*x)/2)^2)*root(27*a^3*b^4*z^3 +
 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k),
k, 1, 3)/f + log(1/cos(e/2 + (f*x)/2)^2)/(a*f) + 1/(b*f*cos(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{a+b\sec^3\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**3), x)
```

3.458 $\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=166

$$\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}f}-\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{2/3}f}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}f}+\frac{\log}{}$$

```
[Out] -1/3*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(2/3)/f+1/6*ln(b^(2/3)-a^(1/3
)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/b^(2/3)/f+1/3*ln(b+a*cos
(f*x+e)^3)/a/f+1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2
))/a^(1/3)/b^(2/3)/f*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4138, 1871, 200, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}f}-\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{2/3}f}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}f}+\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/
3)*b^(2/3)*f) - Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)*f) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(6*a^
(1/3)*b^(2/3)*f) + Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, \cos(e+fx)\right)}{3b^{2/3}f} - \frac{\text{Subst}\left(\int \frac{2}{b^{2/3}-\sqrt[3]{\cdots}}\right.}{}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f} + \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2\cdots}}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+a\cdots}\right.}{6\sqrt[3]{a}\cdots}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\,b^{2/3}f}\cdots$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+\cdots\right.}{6\sqrt[3]{a}\,b^{2\cdots}}$$

Mathematica [C] time = 0.25, size = 242, normalized size = 1.46

$$\text{RootSum}\left[\#1^3a-\#1^3b-3\#1^2a-3\#1^2b+3\#1a-3\#1b-a-b\&,\frac{\#1^2a\log\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\#1\right)-\#1^2b\log\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\#1\right)-}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (-3*Log[Sec[(e + f*x)/2]^2] + RootSum[-a - b + 3*a*#1 - 3*b*#1 - 3*a*#1^2 -
 3*b*#1^2 + a*#1^3 - b*#1^3 & , (-(a*Log[-#1 + Tan[(e + f*x)/2]^2]) - b*Log
[-#1 + Tan[(e + f*x)/2]^2] - 4*a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 - 2*b*Log
[-#1 + Tan[(e + f*x)/2]^2]*#1 + a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2 - b*Lo
g[-#1 + Tan[(e + f*x)/2]^2]*#1^2)/(a - b - 2*a*#1 - 2*b*#1 + a*#1^2 - b*#1^
2) & ])/(3*a*f)
```

fricas [C] time = 36.12, size = 2278, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*sqrt(1/3)*a*f*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*
b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(
3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a
^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2))*arctan(-1/8*(2*sqrt(1/3)*
sqrt((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b
^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)^2 -
4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3) + 1
)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/
3) - 2/(a*f)) + 4*b^2)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f
^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*b*f^2 + 2*b*f)*sqr
t(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(
a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*
f))*a*f + 4)/(a^2*f^2)) + sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) +
1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b
^2*f^3 - 8*a*b*f*cos(f*x + e) + 4*b^2*f - 4*(a^2*b*f^2*cos(f*x + e) - a*b^2
*f^2)*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 -
b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f)))*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*
f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^
2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54
*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)))/a) - 6*sq
rt(1/3)*a*f*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) -
1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3
) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3)
)^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*
sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^
2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)^2 - 4*a*b*cos(f
*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3) + 1)*(-1/54/(a
^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f
)) + 4*b^2)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*
(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*b*f^2 + 2*b*f)*sqrt(((3*(I*sq
rt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*
f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) +
1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4
)/(a^2*f^2)) - sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2
```

```
*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^3 - 8*
a*b*f*cos(f*x + e) + 4*b^2*f - 4*(a^2*b*f^2*cos(f*x + e) - a*b^2*f^2)*(3*(I
*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b
^2*f^3))^(1/3) - 2/(a*f)))*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54
/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 +
 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)))/a) + (3*(I*sqrt(3) +
 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(
1/3) - 2/(a*f))*a*f*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b
^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 +
a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + (a^2*b*f*cos(f*x + e) + a*b^2*f)*
(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(
a^3*b^2*f^3))^(1/3) - 2/(a*f)) + b^2) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3
) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f
 + 6)*log((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^
2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)
^2 - 4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3
) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3)
)^(1/3) - 2/(a*f)) + 4*b^2))/(a*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{3}}{b\sec\left(fx+e\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/(b*sec(f*x + e)^3 + a), x)
```

maple [A] time = 0.77, size = 141, normalized size = 0.85

$$-\frac{\ln\left(\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3fa\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos^{2}\left(fx+e\right)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6fa\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos\left(fx+e\right)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3fa\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/f/a/(1/a*b)^(2/3)*l
n(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))-1/3/f/a/(1/a*b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))+1/3*ln(b+a*cos
(f*x+e)^3)/a/f
```

maxima [A] time = 0.44, size = 159, normalized size = 0.96

$$-\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{2b}{a}\right)+2b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos\left(fx+e\right)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)\log\left(\cos\left(fx+e\right)^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{6\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{b}{a}\right.\right.}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*(a*(3*(b/a)^(1/3) - 2*b/a) + 2*b)*arctan(-1/3*sqrt(3)*((b/
a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b) - 3*(2*(b/a)^(2/3) + 1)*log(c
os(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a*(b/a)^(2/3)) - 6
*((b/a)^(2/3) - 1)*log((b/a)^(1/3) + cos(f*x + e))/(a*(b/a)^(2/3)))/f
```

mupad [B] time = 8.54, size = 1620, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

```
[Out] symsum(log(262144*(a - b)^2*(8*a - 8*b + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2 + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^
2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^2 - 3*root(27*a^3*b^2*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3 - 24*root(27*a^3*b^2*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^2 - 36*root(27*a^3*b^2*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^3*b + 28*root(27*a^3*b^
2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b + 36*root(27*a^3*
b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^2*b^2)*(16*a^2*
tan(e/2 + (f*x)/2)^2 + 32*b^2*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3*b^2*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^3 - 4*root(27*a^3*b^2*z^3
 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^3 - 8*a^2 + 8*b^2 + 3*ro
ot(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^4 - 3
*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^4*
tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
+ a^2 - b^2, z, k)^2*a*b^3 + 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b
^2*z + a^2 - b^2, z, k)^2*a^3*b + 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +
 9*a*b^2*z + a^2 - b^2, z, k)^3*a^4*b - 48*a*b*tan(e/2 + (f*x)/2)^2 + 24*ro
ot(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^2*b^2
 - 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3
*a^2*b^3 + 14*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2,
 z, k)*a^3*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z + a^2 - b^2, z, k)*b^3*tan(e/2 + (f*x)/2)^2 - 32*root(27*a^3*b^2*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b^2 - 32*root(27*a^3*
b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2*b - 146*root(27
*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b^2*tan(e/2
+ (f*x)/2)^2 + 64*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 -
b^2, z, k)*a^2*b*tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^3*tan(e/2 + (f*x)/2)^2 + 57*root(
27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3*b*tan(
e/2 + (f*x)/2)^2 - 54*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^
2 - b^2, z, k)^3*a^4*b*tan(e/2 + (f*x)/2)^2 + 84*root(27*a^3*b^2*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^2*b^2*tan(e/2 + (f*x)/2)^2 -
36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^
2*b^3*tan(e/2 + (f*x)/2)^2 + 198*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z + a^2 - b^2, z, k)^3*a^3*b^2*tan(e/2 + (f*x)/2)^2))*root(27*a^3*b^2*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k), k, 1, 3)/f - log(tan(e
/2 + (f*x)/2)^2 + 1)/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e+fx\right)}{a+b\sec^3\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

3.459 $\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

```
[Out] -1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 260}

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

fricas [A] time = 0.50, size = 21, normalized size = 0.91

$$-\frac{\log\left(a\cos\left(fx+e\right)^3+b\right)}{3\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/2/a*ln(abs((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-1/6/a*ln(abs(((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^3*b-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a+3*((1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+3*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^2*a+3*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-3*(1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1)))*a+b+a)))
```

maple [A] time = 0.17, size = 37, normalized size = 1.61

$$-\frac{\ln\left(a+b\left(\sec^{3}\left(fx+e\right)\right)\right)}{3fa}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f/a*ln(a+b*sec(f*x+e)^3)+1/f/a*ln(sec(f*x+e))
```

maxima [A] time = 0.34, size = 21, normalized size = 0.91

$$-\frac{\log\left(a\cos\left(fx+e\right)^{3}+b\right)}{3\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

mupad [B] time = 5.19, size = 114, normalized size = 4.96

$$\frac{3\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)-\ln\left(a+b-3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+3\,b\tan\left(\frac{e}{2}+\right.\right.}{3\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

```
[Out] (3*log(tan(e/2 + (f*x)/2)^2 + 1) - log(a + b - 3*a*tan(e/2 + (f*x)/2)^2 + 3
*a*tan(e/2 + (f*x)/2)^4 - a*tan(e/2 + (f*x)/2)^6 + 3*b*tan(e/2 + (f*x)/2)^2
 + 3*b*tan(e/2 + (f*x)/2)^4 + b*tan(e/2 + (f*x)/2)^6))/(3*a*f)
```

sympy [A] time = 43.39, size = 170, normalized size = 7.39

$$\begin{cases} \frac{\tilde{\infty} x \tan(e)}{\sec^3(e)} & \text{for } a = 0 \wedge \\ \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{for } b = 0 \\ \frac{x\tan(e)}{a+b\sec^3(e)} & \text{for } f = 0 \\ -\frac{1}{3bf\sec^3(e+fx)} & \text{for } a = 0 \\ -\frac{\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sec(e+fx)\right)}{3af} + \frac{\log(\tan^2(e+fx)+1)}{2af} - \frac{\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\sec(e+fx)+4\sec^2(e+fx)\right)}{3af} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(ta
n(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0)), (x*tan(e)/(a + b*sec(e)**3), Eq(f, 0
)), (-1/(3*b*f*sec(e + f*x)**3), Eq(a, 0)), (-log(-(-1)**(1/3)*a**(1/3)*(1/
b)**(1/3) + sec(e + f*x))/(3*a*f) + log(tan(e + f*x)**2 + 1)/(2*a*f) - log(
4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)*s
ec(e + f*x) + 4*sec(e + f*x)**2)/(3*a*f), True))
```

3.460 $\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=295

$$-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}f\left(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3}\right)}-\frac{b^2\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)}+\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{\ }\right.}{6\sqrt[3]{a}f\left(a^2-b^2\right)}$$

```
[Out] 1/2*ln(1-cos(f*x+e))/(a+b)/f+1/2*ln(1+cos(f*x+e))/(a-b)/f-1/3*(a^(2/3)+b^(2
/3))*b^(2/3)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^2)/f+1/6*(a^(2/3
)+b^(2/3))*b^(2/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)
^2)/a^(1/3)/(a^2-b^2)/f-1/3*b^2*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)/f-1/3*b^(2
/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)/(a^(
4/3)+a^(2/3)*b^(2/3)+b^(4/3))/f*3^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4138, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^2\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)}+\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)}-\frac{b^{2/3}\left(a^{2/3}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(S
qrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))*f)) + Log[1 - Cos[e +
f*x]]/(2*(a + b)*f) + Log[1 + Cos[e + f*x]]/(2*(a - b)*f) - ((a^(2/3) + b^(
2/3))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)*f
) + ((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x]
 + a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)*f) - (b^2*Log[b + a*Cos[
e + f*x]^3])/(3*a*(a^2 - b^2)*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} - \frac{1}{2(a-b)(1+x)} - \frac{b(b-ax+bx^2)}{(-a^2+b^2)(b+ax^3)}\right) dx, x, \cos(e+fx)\right)}{f}$$
$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+f\right.}{(a^2-b^2)f}$$
$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{b-ax}{b+ax^3}\,dx, x, \cos(e+fx)\right.}{(a^2-b^2)f}$$
$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a(a^2-b^2)f} - \frac{\sqrt[3]{b}\,\text{S}}{}$$
$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos\right.}{3\sqrt[3]{a}\,(a^2-b^2)f}$$
$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos\right.}{3\sqrt[3]{a}\,(a^2-b^2)f}$$
$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f} + \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} -$$

Mathematica [C] time = 0.40, size = 290, normalized size = 0.98

$$\frac{3\left(a(a-b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right) + a(a+b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + b^2\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\right) - b\text{RootSum}\left[\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (3*(a*(a + b)*Log[Cos[(e + f*x)/2]] + b^2*Log[Sec[(e + f*x)/2]^2] + a*(a -
b)*Log[Sin[(e + f*x)/2]]) - b*RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 -
b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + Ta
n[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 2*a*b*Log[1
 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2
- b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^
2) & ])/(3*a*(a - b)*(a + b)*f)
```

fricas [C] time = 1.66, size = 6482, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*(a^3 - a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f
^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 -
 a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*
b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^
3*f - a*b^2*f))*f*log(1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^
2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^
2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/
54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/
18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^
3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/
(a^3*f - a*b^2*f))*a*b^2*f - 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2
 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*
f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5
*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/
(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f))
 - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*
(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - a*b*cos(f*x + e) + 2*b^2
) - ((a^3 - a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))
*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/
((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f
^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f
 - a*b^2*f))*f + 3*sqrt(1/3)*(a^3 - a*b^2)*f*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b
^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) +
1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*
f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*
a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f
^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b^4/(a^3*f - a*b^2*f)^2
+ b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f
)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*
f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(
a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) -
 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(
I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f
^2)) - 18*b^2)*log(1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f
^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 -
 a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*
b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^
3*f - a*b^2*f))*a*b^2*f - 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2 +
b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^
3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^
3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^
3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1
/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*
sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 + 1/12*sqrt(1/3)*(a^4 - a^2*b
^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) +
1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*
f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*
a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f^2
*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^
```

```
2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^
4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2
*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2
*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^
5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1
) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a
*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3)
+ 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^
3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^
2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a
^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^
2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f
)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f^2)) + 2*a*b*cos(f*x + e) + 2*b^2) - ((a^3
- a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(
3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*
(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^
2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2
- a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54
*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f)
)*f - 3*sqrt(1/3)*(a^3 - a*b^2)*f*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/
(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*
b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*
f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1
/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*
(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^
2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - 144*a
^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4
*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18
*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*
b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*
b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/
(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3)
+ 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f^2)) - 18*
b^2)*log(-1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*
sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*
f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2
 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4
*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) +
 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b
^2*f))*a*b^2*f + 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*
f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b
^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(
a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) +
 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 + 1/12*sqrt(1/3)*(a^4 - a^2*b^2)*((b^4
/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27
*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2
*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(
1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)
*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b
^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f^2*sqrt(-((
a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b
^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f
^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1
/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1
/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a
^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2
/(a^3*f - a*b^2*f))^2*f^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b
```

```
^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/
27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b
^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))
^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^
2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 -
 b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 -
 2*a^4*b^2 + a^2*b^4)*f^2)) - 2*a*b*cos(f*x + e) - 2*b^2) - 18*(a^2 + a*b)*
log(1/2*cos(f*x + e) + 1/2) - 18*(a^2 - a*b)*log(-1/2*cos(f*x + e) + 1/2))/
((a^3 - a*b^2)*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(fx+e)}{b\sec(fx+e)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^3 + a), x)
```

maple [A] time = 1.04, size = 393, normalized size = 1.33

$$-\frac{b^2\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3f(a-b)(a+b)a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{b^2\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6f(a-b)(a+b)a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{b^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3f(a-b)(a+b)a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f*b^2/(a-b)/(a+b)/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/f*b
^2/(a-b)/(a+b)/a/(1/a*b)^(2/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/
a*b)^(2/3))-1/3/f*b^2/(a-b)/(a+b)/a/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))-1/3/f*b/(a-b)/(a+b)/(1/a*b)^(1/3)*ln(cos(
f*x+e)+(1/a*b)^(1/3))+1/6/f*b/(a-b)/(a+b)/(1/a*b)^(1/3)*ln(cos(f*x+e)^2-(1/
a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3/f*b/(a-b)/(a+b)*3^(1/2)/(1/a*b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))-1/3/f*b^2/(a-b)/(a+b
)/a*ln(b+a*cos(f*x+e)^3)+1/f/(2*a+2*b)*ln(-1+cos(f*x+e))+1/f/(2*a-2*b)*ln(1
+cos(f*x+e))
```

maxima [A] time = 0.44, size = 306, normalized size = 1.04

$$-\frac{\frac{2\sqrt{3}\left(ab^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{2b}{a}\right)-3a^2b\left(\frac{b}{a}\right)^{\frac{2}{3}}-2b^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}-a^2b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{3\left(b^2\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-ab\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}-ab^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*(a*b^2*(3*(b/a)^(1/3) + 2*b/a) - 3*a^2*b*(b/a)^(2/3) - 2*b
^3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^4*(
b/a)^(2/3) - a^2*b^2*(b/a)^(2/3))*(b/a)^(1/3)) + 3*(b^2*(2*(b/a)^(2/3) - 1)
 - a*b*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(
2/3))/(a^3*(b/a)^(2/3) - a*b^2*(b/a)^(2/3)) + 6*(b^2*((b/a)^(2/3) + 1) + a*
b*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^3*(b/a)^(2/3) - a*b^2*(b/
a)^(2/3)) - 9*log(cos(f*x + e) + 1)/(a - b) - 9*log(cos(f*x + e) - 1)/(a +
b))/f
```

mupad [B] time = 7.90, size = 11182, normalized size = 37.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

```
[Out] log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(f*(a + b)) - log(1/cos(e/2 + (f
*x)/2)^2)/(f*(a + b)) + (a*symsum(log((262144*(832*root(27*a^3*b^2*z^3 - 27
*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*b^5 - 840*b^6
*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5 - 21*root(27*
a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4
- 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9
*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b^2*z^3 - 27*a
^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e + f*x) - a^2*b
^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
 9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^3 - 27*a^5*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*root(27*a^3*b^2*
z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4 - 9*ro
ot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*
a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6 + 216667*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5 - 44745*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a
^5*b^4 + 1584*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
 - b^2, z, k)^3*a^6*b^3 - 12720*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8 + 14028*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7 + 156387*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*
b^6 - 457125*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^4*a^5*b^5 + 228117*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4 - 24723*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3 + 486*root(27*a
^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2
 + 864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^5*a^2*b^9 + 18792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
+ 9*a*b^2*z - b^2, z, k)^5*a^3*b^8 - 151488*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7 + 577008*root(27*a^3*
b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^5*b^6 -
414504*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^5*a^6*b^5 - 144432*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4 + 63702*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3 + 486*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2 - 172
8*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k
```

)^6*a^3*b^9 + 3672*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^4*b^8 + 69444*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7 - 637794*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*b^6 + 1468908*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^7*b^5 - 1112400*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^4 + 210384*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3 - 486*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2 + 1296*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^4*b^9 - 23004*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^5*b^8 + 195534*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7 - 778734*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b^6 + 1175796*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^8*b^5 - 690768*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4 + 120366*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3 - 486*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2 - 8702*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*b^6 - 272*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7*cos(e + f*x) + 62*a*b^5*cos(e + f*x) + 13774*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*b^6*cos(e + f*x) - 4098*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5*cos(e + f*x) + 122*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4*cos(e + f*x) + 2088*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a*b^7*cos(e + f*x) - 980*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8*cos(e + f*x) - 85013*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6*cos(e + f*x) + 55956*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5*cos(e + f*x) - 8075*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4*cos(e + f*x) + 117*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^5*b^3*cos(e + f*x) + 818*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7*cos(e + f*x) + 217434*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6*cos(e + f*x) - 285091*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5*cos(e + f*x) + 82633*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^5*b^4*cos(e + f*x) - 6984*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^6*b^3*cos(e + f*x) + 3792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8*cos(e + f*x) - 42132*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7*cos(e + f*x) - 54423*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*b^6*cos(e + f*x) + 435417*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^5*b^5*cos(e + f*x) - 280113*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4*cos(e + f*x) + 49239*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2*cos(e + f*x) - 4968*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^3*b^8*cos(e + f*x) + 99864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7*cos(e + f*x) - 643536*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^5*b^6*cos(e + f*x) + 636552*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^6*b^5*cos(e + f*x) - 936*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4*cos(e + f*x) - 28170*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^

2*z - b^2, z, k)^5*a^8*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2*z^3 - 27*a^5
*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2*cos(e + f*x) - 237
6*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k
)^6*a^4*b^8*cos(e + f*x) + 972*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^
2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7*cos(e + f*x) + 457758*root(27*a^3*
b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*b^6*co
s(e + f*x) - 1352916*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*
a*b^2*z - b^2, z, k)^6*a^7*b^5*cos(e + f*x) + 1122336*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^4*cos(e + f*x
) - 229176*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
b^2, z, k)^6*a^9*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2*cos(e + f*x) + 7452*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a
^5*b^8*cos(e + f*x) - 139482*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*
z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7*cos(e + f*x) + 729810*root(27*a^3*b^
2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b^6*cos(
e + f*x) - 1208844*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*
b^2*z - b^2, z, k)^7*a^8*b^5*cos(e + f*x) + 752328*root(27*a^3*b^2*z^3 - 27
*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4*cos(e + f*x) -
 144666*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2
, z, k)^7*a^10*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 2
7*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2*cos(e + f*x)))/cos(e/2 +
(f*x)/2)^2)*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
 b^2, z, k), k, 1, 3))/(f*(a + b)) + (b*symsum(log((262144*(832*root(27*a^3
*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*
b^5 - 840*b^6*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3
 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*
a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5
 - 21*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z
, k)*a^3*b^4 - 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z - b^2, z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a
^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b
^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e +
 f*x) - a^2*b^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*
a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^3
 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a
^4*b^4 - 9*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
b^2, z, k)^2*a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*
z^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z^3 - 27*a^
5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6 + 216667*root(27*
a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^
5 - 44745*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b
^2, z, k)^3*a^5*b^4 + 1584*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^
2 + 9*a*b^2*z - b^2, z, k)^3*a^6*b^3 - 12720*root(27*a^3*b^2*z^3 - 27*a^5*z
^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8 + 14028*root(27*a^3*
b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7 +
156387*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
 z, k)^4*a^4*b^6 - 457125*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
 + 9*a*b^2*z - b^2, z, k)^4*a^5*b^5 + 228117*root(27*a^3*b^2*z^3 - 27*a^5*z
^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4 - 24723*root(27*a^3*
b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3 +
486*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z,
 k)^4*a^8*b^2 + 864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z - b^2, z, k)^5*a^2*b^9 + 18792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27
*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^3*b^8 - 151488*root(27*a^3*b^2*z^
3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7 + 577008

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*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^5*a^5*b^6 - 414504*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z - b^2, z, k)^5*a^6*b^5 - 144432*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 2
7*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^7*b^4 + 63702*root(27*a^3*b^2*z^
3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3 + 486*ro
ot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*
a^9*b^2 - 1728*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*
z - b^2, z, k)^6*a^3*b^9 + 3672*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^4*b^8 + 69444*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7 - 637794*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*
b^6 + 1468908*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
 - b^2, z, k)^6*a^7*b^5 - 1112400*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2
*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^4 + 210384*root(27*a^3*b^2*z^3 -
27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3 - 486*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10
*b^2 + 1296*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
 b^2, z, k)^7*a^4*b^9 - 23004*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^5*b^8 + 195534*root(27*a^3*b^2*z^3 - 27*a
^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7 - 778734*root(27
*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b
^6 + 1175796*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^7*a^8*b^5 - 690768*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4 + 120366*root(27*a^3*b^2*z^3 - 27
*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3 - 486*root(27
*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*
b^2 - 8702*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -
b^2, z, k)*a*b^6 - 272*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z - b^2, z, k)*b^7*cos(e + f*x) + 62*a*b^5*cos(e + f*x) + 13774*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*b
^6*cos(e + f*x) - 4098*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z - b^2, z, k)*a^2*b^5*cos(e + f*x) + 122*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4*cos(e + f*x) + 20
88*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z,
k)^2*a*b^7*cos(e + f*x) - 980*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8*cos(e + f*x) - 85013*root(27*a^3*b^2*
z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6*cos(e
+ f*x) + 55956*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*
z - b^2, z, k)^2*a^3*b^5*cos(e + f*x) - 8075*root(27*a^3*b^2*z^3 - 27*a^5*z
^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4*cos(e + f*x) + 117*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2
*a^5*b^3*cos(e + f*x) + 818*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z
^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7*cos(e + f*x) + 217434*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6*cos(e
 + f*x) - 285091*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^
2*z - b^2, z, k)^3*a^4*b^5*cos(e + f*x) + 82633*root(27*a^3*b^2*z^3 - 27*a^
5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^5*b^4*cos(e + f*x) - 69
84*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z,
k)^3*a^6*b^3*cos(e + f*x) + 3792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*
b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8*cos(e + f*x) - 42132*root(27*a^3
*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7*c
os(e + f*x) - 54423*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z - b^2, z, k)^4*a^4*b^6*cos(e + f*x) + 435417*root(27*a^3*b^2*z^3 - 2
7*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^5*b^5*cos(e + f*x)
- 280113*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^
2, z, k)^4*a^6*b^4*cos(e + f*x) + 49239*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3*cos(e + f*x) - 3402*root(
27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8
*b^2*cos(e + f*x) - 4968*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
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+ 9*a*b^2*z - b^2, z, k)^5*a^3*b^8*cos(e + f*x) + 99864*root(27*a^3*b^2*z^3
 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7*cos(e + f
*x) - 643536*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^5*a^5*b^6*cos(e + f*x) + 636552*root(27*a^3*b^2*z^3 - 27*a^5*z
^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^6*b^5*cos(e + f*x) - 936*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5
*a^7*b^4*cos(e + f*x) - 28170*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3*cos(e + f*x) - 3402*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2*cos(e
 + f*x) - 2376*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*
z - b^2, z, k)^6*a^4*b^8*cos(e + f*x) + 972*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7*cos(e + f*x) + 457758
*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^6*a^6*b^6*cos(e + f*x) - 1352916*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2
*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^7*b^5*cos(e + f*x) + 1122336*root(27*
a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^8*b^
4*cos(e + f*x) - 229176*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
 9*a*b^2*z - b^2, z, k)^6*a^9*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2*cos(e + f*
x) + 7452*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b
^2, z, k)^7*a^5*b^8*cos(e + f*x) - 139482*root(27*a^3*b^2*z^3 - 27*a^5*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7*cos(e + f*x) + 729810*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7
*a^7*b^6*cos(e + f*x) - 1208844*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^8*b^5*cos(e + f*x) + 752328*root(27*a^3
*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4*c
os(e + f*x) - 144666*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*
a*b^2*z - b^2, z, k)^7*a^10*b^3*cos(e + f*x) + 3402*root(27*a^3*b^2*z^3 - 2
7*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2*cos(e + f*x)
))/cos(e/2 + (f*x)/2)^2)*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
+ 9*a*b^2*z - b^2, z, k), k, 1, 3))/(f*(a + b)) - (b*log(1/cos(e/2 + (f*x)/
2)^2))/(a*f*(a + b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{a+b\sec^3\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**3), x)
```

3.461 $\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=393

$$-\frac{b^2\left(2a^2+b^2\right)\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)^2}+\frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+\right.}{6\sqrt[3]{a}f\left(a^2-b^2\right)^2}$$

```
[Out] -1/4/(a+b)/f/(1-cos(f*x+e))-1/4/(a-b)/f/(1+cos(f*x+e))-1/4*(2*a+5*b)*ln(1-c
os(f*x+e))/(a+b)^2/f-1/4*(2*a-5*b)*ln(1+cos(f*x+e))/(a-b)^2/f-1/3*b^(4/3)*(
a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^
2)^2/f+1/6*b^(4/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(2/3)-a^(1/3)*b^(1/3)
*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/(a^2-b^2)^2/f-1/3*b^2*(2*a^2+b^2)
*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)^2/f+1/3*b^(4/3)*(a^2-3*a^(2/3)*b^(4/3)+2*
b^2)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)/(a^
2-b^2)^2/f*3^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4138, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^2\left(2a^2+b^2\right)\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)^2}+\frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+\right.}{6\sqrt[3]{a}f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[
e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*f) - 1/(4*(a +
 b)*f*(1 - Cos[e + f*x])) - 1/(4*(a - b)*f*(1 + Cos[e + f*x])) - ((2*a + 5*
b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2*f) - ((2*a - 5*b)*Log[1 + Cos[e + f*
x]])/(4*(a - b)^2*f) - (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2*f) + (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2
/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*f) - (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{2a+5b}{4(a+b)^2(-1+x)} - \frac{1}{4(a-b)(1+x)^2} + \frac{2a-5b}{4(a-b)^2(1+x)} + \frac{b^2\left(a^2+2b^2-3abx+\left(2a^2\cdots\right.\right.}{\left(a^2-b^2\right)^2\left(b+ax^3\cdots\right.}\right.\right.}{f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+\cdots}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+\cdots}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+\cdots}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+\cdots}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+\cdots}{4(a+b)^2f}$$

$$= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f} - \frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{\cdots}{4(a-\cdots}$$

Mathematica [C] time = 1.97, size = 336, normalized size = 0.85

$$\frac{8b^2\left((b-a)\text{RootSum}\left[\#1^3a-\#1^3b-6\#1^2a+12\#1a-8a\&, \frac{2\#1^2a^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)+\#1^2b^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)+8a^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)-6\#1a^2\cdots}{\#1^2a-\#1^2b-4\#1a+4a}\right.\right.}{a\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((-3*Csc[(e + f*x)/2]^2)/(a + b) + (12*(-2*a + 5*b)*Log[Cos[(e + f*x)/2]])/
(a - b)^2 - (12*(2*a + 5*b)*Log[Sin[(e + f*x)/2]])/(a + b)^2 + (8*b^2*(3*(2
*a^2 + b^2)*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a
*#1^2 + a*#1^3 - b*#1^3 & , (8*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 4*a*b
*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 6*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*
#1 + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 + b^2*Log[1 - #1 + Tan[(e
+ f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(a*(a^2 - b^2)^2)
 - (3*Sec[(e + f*x)/2]^2)/(a - b))/(24*f)
```

fricas [C] time = 3.84, size = 10746, normalized size = 27.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*a^4 - 18*a^2*b^2 + 2*((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 -
 (a^5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)
 - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/
(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)
*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*
f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^
2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5
*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*
f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4
)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)
^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f
 + a*b^4*f))*log(1/12*(a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^
2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/
18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f -
2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a
*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*
b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((
a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1
/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b
^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/
(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 + 2*a^2*b^2 + 7*b^4 + 1/6*(a^5 + 16*
a^3*b^2 + 10*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*
b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4
/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6
*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27
*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)
*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 +
 a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*
b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))
^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f
))*f - (a^3*b + 8*a*b^3)*cos(f*x + e)) - 18*(a^3*b - a*b^3)*cos(f*x + e) +
(36*a^2*b^2 + 18*b^4 - 18*(2*a^2*b^2 + b^4)*cos(f*x + e)^2 - ((a^5 - 2*a^3*
b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2
 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^
4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2
)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^
3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)
- 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 +
b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*
b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54
*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*
b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f)) + 3*sqrt(1/3)*((a^5 - 2*a^3*b^2
 + a*b^4)*f*cos(f*x + e)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4
 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b
^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^
5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^
5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2
*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^
4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2
)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f +
 a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3
```

```
) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2
*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)
 - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/
(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)
*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*
f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^
2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5
*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*
f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4
)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)
^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f
 + a*b^4*f))*f)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2))
)*log(1/12*(a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2
*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt
(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b
^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f
 + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 +
 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^
3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2
*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*
b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a
^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*
a^3*b^2*f + a*b^4*f))^2*f^2 + 2*a^2*b^2 + 7*b^4 + 1/6*(a^5 + 16*a^3*b^2 + 1
0*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^
2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 -
 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^
4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2
 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2
- b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^
3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a
^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^
2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*s
qrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f + 1/4*
sqrt(1/3)*((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2
*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt
(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b
^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f
 + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 +
 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^
3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2
*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*
b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a
^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*
a^3*b^2*f + a*b^4*f))*f^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^
4 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*
b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a
^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a
^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b
^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^
2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(
3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(
2*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2
) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)
/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4
)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4
```

```
*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a
^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^
5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2
*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^
4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2
)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*
f + a*b^4*f))*f)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2)
) + 2*(a^3*b + 8*a*b^3)*cos(f*x + e)) + (36*a^2*b^2 + 18*b^4 - 18*(2*a^2*b^
2 + b^4)*cos(f*x + e)^2 - ((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^
5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (
2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/
54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4
/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f))
 - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 +
8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2
*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2
+ a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/
(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a
*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a
*b^4*f)) - 3*sqrt(1/3)*((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^5 -
 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4
*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2
+ a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I
*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*
a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*
b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f
)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a
^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^
2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2
*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^
4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f
 - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4
/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a
^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3
 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^
2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5
*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3
))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a
^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b
^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)
^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) -
6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f)/((a^10 - 4*a^8*b^2
+ 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2)))*log(-1/12*(a^6 - 2*a^4*b^2 + a^2*
b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a
^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a
^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b
^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^
2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(
3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 2*a^
2*b^2 - 7*b^4 - 1/6*(a^5 + 16*a^3*b^2 + 10*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^
2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/
18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f -
2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a
*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*
```

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b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((
a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1
/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b
^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/
(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f + 1/4*sqrt(1/3)*((a^6 - 2*a^4*b^2 + a^2*
b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a
^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a
^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b
^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^
2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(
3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f^2 - 2*(a^5
 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4 + 720*a^2*b^6 - 36*b^8 - (a^10 -
 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^
2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(
-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(
2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^
3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4
*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/
(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*
f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*
(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*
b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5
*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b
^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2
*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f
^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 +
a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a
^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f
^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2
*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3
*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*
f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1)
- 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f)/((a^10 - 4*a^8*b^
2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2)) - 2*(a^3*b + 8*a*b^3)*cos(f*x +
e)) + 9*(2*a^4 - a^3*b - 8*a^2*b^2 - 5*a*b^3 - (2*a^4 - a^3*b - 8*a^2*b^2 -
 5*a*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 9*(2*a^4 + a^3*b -
8*a^2*b^2 + 5*a*b^3 - (2*a^4 + a^3*b - 8*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)
*log(-1/2*cos(f*x + e) + 1/2))/((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2
- (a^5 - 2*a^3*b^2 + a*b^4)*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^3}{b\sec\left(fx+e\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^3/(b*sec(f*x + e)^3 + a), x)
```

maple [B] time = 1.33, size = 676, normalized size = 1.72

$$-\frac{b^{2} a \ln\left(\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3f\left(a-b\right)^{2}\left(a+b\right)^{2}\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{2b^{4}\ln\left(\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3f\left(a-b\right)^{2}\left(a+b\right)^{2}a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{b^{2} a \ln\left(\cos^{2}\left(fx+e\right)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right.\right.}{6f\left(a-b\right)^{2}\left(a+b\right)^{2}\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f*b^2/(a-b)^2/(a+b)^2*a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))-2/3
/f*b^4/(a-b)^2/(a+b)^2/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/f*b
^2/(a-b)^2/(a+b)^2*a/(1/a*b)^(2/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)
+(1/a*b)^(2/3))+1/3/f*b^4/(a-b)^2/(a+b)^2/a/(1/a*b)^(2/3)*ln(cos(f*x+e)^2-(
1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))-1/3/f*b^2/(a-b)^2/(a+b)^2*a/(1/a*b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))-2/3/f*b^4/
(a-b)^2/(a+b)^2/a/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)
*cos(f*x+e)-1))-1/f*b^3/(a-b)^2/(a+b)^2/(1/a*b)^(1/3)*ln(cos(f*x+e)+(1/a*b)
^(1/3))+1/2/f*b^3/(a-b)^2/(a+b)^2/(1/a*b)^(1/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/
3)*cos(f*x+e)+(1/a*b)^(2/3))+1/f*b^3/(a-b)^2/(a+b)^2*3^(1/2)/(1/a*b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))-2/3/f*b^2/(a-b)^2/(a+b)^
2*a*ln(b+a*cos(f*x+e)^3)-1/3/f*b^4/(a-b)^2/(a+b)^2/a*ln(b+a*cos(f*x+e)^3)+1
/f/(4*a+4*b)/(-1+cos(f*x+e))-1/2/f/(a+b)^2*ln(-1+cos(f*x+e))*a-5/4/f/(a+b)^
2*ln(-1+cos(f*x+e))*b-1/f/(4*a-4*b)/(1+cos(f*x+e))-1/2/f/(a-b)^2*ln(1+cos(f
*x+e))*a+5/4/f/(a-b)^2*ln(1+cos(f*x+e))*b
```

maxima [A] time = 0.43, size = 488, normalized size = 1.24

$$\frac{\frac{4\sqrt{3}\left(a^{2}b^{3}\left(9\left(\frac{b}{a}\right)^{\frac{2}{3}}+4\right)-a^{3}b^{2}\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{4b}{a}\right)-2ab^{4}\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{b}{a}\right)+2b^{5}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos\left(fx+e\right)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^{6}\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a^{4}b^{2}\left(\frac{b}{a}\right)^{\frac{2}{3}}+a^{2}b^{4}\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{6\left(a^{2}b^{2}\left(4\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)+2b^{4}\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-3ab^{3}\right.}{a^{5}\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a\ldots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b^3*(9*(b/a)^(2/3) + 4) - a^3*b^2*(3*(b/a)^(1/3) + 4*b
/a) - 2*a*b^4*(3*(b/a)^(1/3) + b/a) + 2*b^5)*arctan(-1/3*sqrt(3)*((b/a)^(1/
3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^6*(b/a)^(2/3) - 2*a^4*b^2*(b/a)^(2/3)
 + a^2*b^4*(b/a)^(2/3))*(b/a)^(1/3)) - 6*(a^2*b^2*(4*(b/a)^(2/3) - 1) + 2*b
^4*((b/a)^(2/3) - 1) - 3*a*b^3*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3
)*cos(f*x + e) + (b/a)^(2/3))/(a^5*(b/a)^(2/3) - 2*a^3*b^2*(b/a)^(2/3) + a*
b^4*(b/a)^(2/3)) - 12*(a^2*b^2*(2*(b/a)^(2/3) + 1) + b^4*((b/a)^(2/3) + 2)
+ 3*a*b^3*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^5*(b/a)^(2/3) - 2
*a^3*b^2*(b/a)^(2/3) + a*b^4*(b/a)^(2/3)) - 9*(2*a - 5*b)*log(cos(f*x + e)
+ 1)/(a^2 - 2*a*b + b^2) - 9*(2*a + 5*b)*log(cos(f*x + e) - 1)/(a^2 + 2*a*b
 + b^2) - 18*(b*cos(f*x + e) - a)/((a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2))
/f
```

mupad [B] time = 19.52, size = 58699, normalized size = 149.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

```
[Out] -(a^3*cos(e/2 + (f*x)/2)^4 + a^3*sin(e/2 + (f*x)/2)^4 - a*b^2*cos(e/2 + (f*
x)/2)^4 + a*b^2*sin(e/2 + (f*x)/2)^4 + 2*a^2*b*sin(e/2 + (f*x)/2)^4 - 8*a^3
*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*co
s(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 + 8*b^3*log((cos(e/2 + (f*x)/2)^2 +
 sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 +
 (f*x)/2)^2 + 8*a^3*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (
f*x)/2))*sin(e/2 + (f*x)/2)^2 - 8*a^4*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/
2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 +
 (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*
x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2
 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*
a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/
2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x
)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b
^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*si
n(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e
/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 +
 (f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x
)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2
*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2
 - 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 142
40*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 -
 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^
3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
 z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3
*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^
14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
```

a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54 *a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*cos(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*cos(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*cos(e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,

k)^7*a^7*b^11*cos(e/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^8*b^10*cos(e/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^10*b^8*cos(e/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^11*b^7*cos(e/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^12*b^6*cos(e/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b
^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^13*b^5*cos(e/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^14*b^4*cos(e/2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4
*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k
)^7*a^15*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^16*b^2*cos(e/2 + (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2
*a^2*b^11*sin(e/2 + (f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2
*a^3*b^10*sin(e/2 + (f*x)/2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2
*a^4*b^9*sin(e/2 + (f*x)/2)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*
a^5*b^8*sin(e/2 + (f*x)/2)^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^
6*b^7*sin(e/2 + (f*x)/2)^2 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*
b^6*sin(e/2 + (f*x)/2)^2 + 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5
*sin(e/2 + (f*x)/2)^2 - 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(
e/2 + (f*x)/2)^2 + 95632*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e
/2 + (f*x)/2)^2 - 439696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e
/2 + (f*x)/2)^2 - 1471448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(
e/2 + (f*x)/2)^2 - 606964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e
/2 + (f*x)/2)^2 + 521558*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/
2 + (f*x)/2)^2 - 415182*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2
+ (f*x)/2)^2 - 598284*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2
+ (f*x)/2)^2 + 24606*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 +
(f*x)/2)^2 + 20592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (
f*x)/2)^2 - 756*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x
)/2)^2 + 33984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)
/2)^2 + 32784*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/
2)^2 - 1159584*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)
/2)^2 + 567024*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b

^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)
/2)^2 + 6779964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)
/2)^2 + 4475790*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)
/2)^2 - 2069340*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*
b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)
/2)^2 - 421956*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/
2)^2 + 382248*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/
2)^2 - 554778*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/
2)^2 + 146880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/
2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)
^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2
 + 54432*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2
- 422856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2
+ 625176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2
+ 6126696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2
 - 2480004*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2
 - 15505344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^
2 - 346572*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2
 + 9474120*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^
2 + 24660*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2
 - 1571688*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^
2 + 232740*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^
2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2
+ 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 -
1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 31
9896*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 32
46912*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5
322240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 -
9560160*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 +
16055280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 +
 8485344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2

- 14873760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2*(a + b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3) + 8*a^2*b^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 + (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*sin(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -

54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -

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 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 2
6928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 304
2*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a
^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*roo
t(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^
4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^5*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 -
 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^6*a^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 2
```

7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^4*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^5*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^6*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^7*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^6*a^8*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27
*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b
^4, z, k)^6*a^9*b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*
a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^
4, z, k)^6*a^10*b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*
a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^
4, z, k)^6*a^11*b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a
^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4
, z, k)^6*a^12*b^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a
^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4
, z, k)^6*a^13*b^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^
3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
 z, k)^6*a^14*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^6*a^15*b^2*cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)^7*a^4*b^14*cos(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4
*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k
)^7*a^5*b^13*cos(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*
z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)
^7*a^6*b^12*cos(e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^7*b^11*cos(e/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^8*b^10*cos(e/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^10*b^8*cos(e/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^11*b^7*cos(e/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^12*b^6*cos(e/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z
^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^
7*a^13*b^5*cos(e/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7
*a^14*b^4*cos(e/2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*
a^15*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^
16*b^2*cos(e/2 + (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2
*b^11*sin(e/2 + (f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3
*b^10*sin(e/2 + (f*x)/2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4
*b^9*sin(e/2 + (f*x)/2)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 2
7*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*
b^8*sin(e/2 + (f*x)/2)^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^
7*sin(e/2 + (f*x)/2)^2 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^
7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*

sin(e/2 + (f*x)/2)^2 + 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*sin
(e/2 + (f*x)/2)^2 - 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(e/2
+ (f*x)/2)^2 + 95632*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e/2 +
(f*x)/2)^2 - 439696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e/2 +
(f*x)/2)^2 - 1471448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(e/2
+ (f*x)/2)^2 - 606964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e/2 +
(f*x)/2)^2 + 521558*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/2 +
(f*x)/2)^2 - 415182*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2 + (
f*x)/2)^2 - 598284*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2 + (f
*x)/2)^2 + 24606*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4
*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 + (f*x
)/2)^2 + 20592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (f*x)
/2)^2 - 756*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x)/2)
^2 + 33984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)/2)^
2 + 32784*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/2)^2
- 1159584*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)/2)^
2 + 567024*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)/2)^
2 + 6779964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)/2)^
2 + 4475790*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)/2)^
2 - 2069340*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)/2)^
2 - 421956*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/2)^2
+ 382248*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/2)^2
- 554778*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/2)^2
+ 146880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/2)^2
- 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
- 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)^2 +
1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2 + 5
4432*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2 - 42
2856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2 + 62
5176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2 + 61
26696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2 - 2
480004*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -

27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2 - 15505344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^2 - 346572*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2 + 9474120*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^2 + 24660*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2 - 1571688*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^2 + 232740*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 - 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 319896*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 3246912*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5322240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560160*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 16055280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 8485344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14873760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 +

838512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2*(a + b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3) + 8*a*b^2*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 - 8*a^2*b*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 - 20*a*b^2*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^2 + 12*a^2*b*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^2 + 8*a*b^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 + (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*sin(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7 *sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z

- b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7

*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*
cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*co
s(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*cos
(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*cos(
e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*cos(e
/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*cos(e
/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*cos(e
/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*cos(e
/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*cos(e
/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3
- 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*cos(e
/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*cos(e/
2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*cos(e/2
 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54
*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*cos(e/2 +
 (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*sin(e/2 +
(f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*sin(e/2 +
(f*x)/2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*sin(e/2 + (
f*x)/2)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a
^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*sin(e/2 + (f
*x)/2)^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4
*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*sin(e/2 + (f*x
)/2)^2 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b
^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*sin(e/2 + (f*x)/
2)^2 + 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*sin(e/2 + (f*x)/2)^
2 - 18*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(e/2 + (f*x)/2)^2 + 9
5632*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e/2 + (f*x)/2)^2 - 43
9696*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e/2 + (f*x)/2)^2 - 14
71448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(e/2 + (f*x)/2)^2 - 6
06964*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e/2 + (f*x)/2)^2 + 52
1558*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 2
7*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/2 + (f*x)/2)^2 - 415
182*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27
*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2 + (f*x)/2)^2 - 5982
84*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*
a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2 + (f*x)/2)^2 + 24606
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 + (f*x)/2)^2 + 20592*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (f*x)/2)^2 - 756*root

(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x)/2)^2 + 33984*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)/2)^2 + 32784*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/2)^2 - 1159584*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)/2)^2 + 567024*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)/2)^2 + 6779964*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)/2)^2 + 4475790*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)/2)^2 - 2069340*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)/2)^2 - 421956*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/2)^2 + 382248*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/2)^2 - 554778*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/2)^2 + 146880*root(5
4*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z
^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)^2 + 1728*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2 + 54432*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2 - 422856*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2 + 625176*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2 + 6126696*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2 - 2480004*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2 - 15505344*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^2 - 346572*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2 + 9474120*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^2 + 24660*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2 - 1571688*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^2 + 232740*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
- 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 - 1728*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 319896*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 3246912*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a

```
*b^4*z - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5322240*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560160*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 16055280*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 8485344*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14873760*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*
a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2
*(a + b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3
) - 8*a^3*b*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*symsum(log((131072*(9
80*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 + (f*x)/2)^2 + 1764*a^2*b^9
*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*x)/2)^2 + 640*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
 - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)
^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^
4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z,
k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root
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(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2
 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*
a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*cos(e/2 + (f*x
)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*
z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*cos(e/2 + (f*x)/2)^2 +
 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*sin(e/2 + (f*x)/2)^2 + 39032
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 + (f*x)/2)^2 + 30296*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*x)/2)^2 + 7420*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)^2 - 252*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 168*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*root(54*a^5*b^2*z^3 - 2
7*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -
b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(54*a^5*b^2*z^3 - 27*a^
3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4,
 z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4
*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k
)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^
3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a
^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 -
 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*
b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*
a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*
sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z
^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*sin(e
/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (
f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^
4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x
)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)
^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 +
107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 +
156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 +
55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21
772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27
*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 3536
4*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a
^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*cos(e/2 + (f*x)/2)^2 - 3051*root
(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4
*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*cos(e/2 + (f*x)/2)^2 + 18*root(54*a
^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2
 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*cos(e/2 + (f*x)/2)^2 + 73528*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^3*a^2*b^12*cos(e/2 + (f*x)/2)^2 + 222176*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^3*a^3*b^11*cos(e/2 + (f*x)/2)^2 - 101192*root(54*a^5

*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*cos(e/2 + (f*x)/2)^2 - 567064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*cos(e/2 + (f*x)/2)^2 - 125428*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*cos(e/2 + (f*x)/2)^2 + 278436*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*cos(e/2 + (f*x)/2)^2 + 66894*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*cos(e/2 + (f*x)/2)^2 - 26928*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*cos(e/2 + (f*x)/2)^2 - 3042*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*cos(e/2 + (f*x)/2)^2 + 648*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*cos(e/2 + (f*x)/2)^2 + 19104*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^2*b^13*cos(e/2 + (f*x)/2)^2 + 128832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^3*b^12*cos(e/2 + (f*x)/2)^2 - 191988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^4*b^11*cos(e/2 + (f*x)/2)^2 - 899856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^5*b^10*cos(e/2 + (f*x)/2)^2 + 183204*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*cos(e/2 + (f*x)/2)^2 + 1173036*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*cos(e/2 + (f*x)/2)^2 - 519612*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*cos(e/2 + (f*x)/2)^2 - 666384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^9*b^6*cos(e/2 + (f*x)/2)^2 + 368028*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^10*b^5*cos(e/2 + (f*x)/2)^2 + 46260*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^11*b^4*cos(e/2 + (f*x)/2)^2 - 46332*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^12*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a^13*b^2*cos(e/2 + (f*x)/2)^2 + 1728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^2*b^14*cos(e/2 + (f*x)/2)^2 + 34560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^3*b^13*cos(e/2 + (f*x)/2)^2 - 51480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^4*b^12*cos(e/2 + (f*x)/2)^2 - 677880*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^5*b^11*cos(e/2 + (f*x)/2)^2 + 773640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^6*b^10*cos(e/2 + (f*x)/2)^2 + 1207440*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^7*b^9*cos(e/2 + (f*x)/2)^2 - 1363176*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^8*b^8*cos(e/2 + (f*x)/2)^2 - 82728*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^9*b^7*cos(e/2 + (f*x)/2)^2 + 738792*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^10*b^6*cos(e/2 + (f*x)/2)^2 - 412704*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a

^11*b^5*cos(e/2 + (f*x)/2)^2 + 11304*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^12*b^4*cos(e/2 + (f*x)/2)^2 + 36288*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^13*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^5*a^14*b^2*cos(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^3*b^14*cos(e/2 + (f*x)/2)^2 + 7344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^4*b^13*cos(e/2 + (f*x)/2)^2 - 225504*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^5*b^12*cos(e/2 + (f*x)/2)^2 + 450468*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^6*b^11*cos(e/2 + (f*x)/2)^2 + 360072*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^7*b^10*cos(e/2 + (f*x)/2)^2 - 879984*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^8*b^9*cos(e/2 + (f*x)/2)^2 - 183600*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^9*b^8*cos(e/2 + (f*x)/2)^2 + 431352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^10*b^7*cos(e/2 + (f*x)/2)^2 + 165888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^11*b^6*cos(e/2 + (f*x)/2)^2 - 61344*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^12*b^5*cos(e/2 + (f*x)/2)^2 - 114480*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^13*b^4*cos(e/2 + (f*x)/2)^2 + 52164*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^14*b^3*cos(e/2 + (f*x)/2)^2 - 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^6*a^15*b^2*cos(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^4*b^14*cos(e/2 + (f*x)/2)^2 - 28512*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^5*b^13*cos(e/2 + (f*x)/2)^2 + 75816*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^6*b^12*cos(e/2 + (f*x)/2)^2 + 71280*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^7*b^11*cos(e/2 + (f*x)/2)^2 - 323352*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^8*b^10*cos(e/2 + (f*x)/2)^2 + 483408*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^10*b^8*cos(e/2 + (f*x)/2)^2 - 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^11*b^7*cos(e/2 + (f*x)/2)^2 - 307152*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^12*b^6*cos(e/2 + (f*x)/2)^2 + 142560*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^13*b^5*cos(e/2 + (f*x)/2)^2 + 62856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^14*b^4*cos(e/2 + (f*x)/2)^2 - 42768*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^15*b^3*cos(e/2 + (f*x)/2)^2 + 5832*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^7*a^16*b^2*cos(e/2 + (f*x)/2)^2 - 43760*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*sin(e/2 + (f*x)/2)^2 - 401720*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^

2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*sin(e/2 + (f*x)/
2)^2 - 563860*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*sin(e/2 + (f*x)/2
)^2 - 249110*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2
*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*sin(e/2 + (f*x)/2)
^2 - 59988*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z
^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*sin(e/2 + (f*x)/2)^2
 - 35586*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2
 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^7*b^6*sin(e/2 + (f*x)/2)^2 +
 5751*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^8*b^5*sin(e/2 + (f*x)/2)^2 - 18
*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^
2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^9*b^4*sin(e/2 + (f*x)/2)^2 + 95632*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^2*b^12*sin(e/2 + (f*x)/2)^2 - 439696*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^3*b^11*sin(e/2 + (f*x)/2)^2 - 1471448*
root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2
*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^4*b^10*sin(e/2 + (f*x)/2)^2 - 606964*
root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2
*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^5*b^9*sin(e/2 + (f*x)/2)^2 + 521558*r
oot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*
b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^6*b^8*sin(e/2 + (f*x)/2)^2 - 415182*ro
ot(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b
^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^7*b^7*sin(e/2 + (f*x)/2)^2 - 598284*roo
t(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^
4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a^8*b^6*sin(e/2 + (f*x)/2)^2 + 24606*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)^3*a^9*b^5*sin(e/2 + (f*x)/2)^2 + 20592*root(54
*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^
2 - 9*a*b^4*z - b^4, z, k)^3*a^10*b^4*sin(e/2 + (f*x)/2)^2 - 756*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^3*a^11*b^3*sin(e/2 + (f*x)/2)^2 + 33984*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^2*b^13*sin(e/2 + (f*x)/2)^2 + 32784*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^4*a^3*b^12*sin(e/2 + (f*x)/2)^2 - 1159584*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^4*b^11*sin(e/2 + (f*x)/2)^2 + 567024*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^5*b^10*sin(e/2 + (f*x)/2)^2 + 6779964*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^4*a^6*b^9*sin(e/2 + (f*x)/2)^2 + 4475790*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^4*a^7*b^8*sin(e/2 + (f*x)/2)^2 - 2069340*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
 9*a*b^4*z - b^4, z, k)^4*a^8*b^7*sin(e/2 + (f*x)/2)^2 - 421956*root(54*a^5
*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 -
9*a*b^4*z - b^4, z, k)^4*a^9*b^6*sin(e/2 + (f*x)/2)^2 + 382248*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^4*a^10*b^5*sin(e/2 + (f*x)/2)^2 - 554778*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^4*a^11*b^4*sin(e/2 + (f*x)/2)^2 + 146880*root(54*a^5*
b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9
*a*b^4*z - b^4, z, k)^4*a^12*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^4*a^13*b^2*sin(e/2 + (f*x)/2)^2 + 1728*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^5*a^2*b^14*sin(e/2 + (f*x)/2)^2 + 54432*root(54*a^5*b^2*z

^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^5*a^3*b^13*sin(e/2 + (f*x)/2)^2 - 422856*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^5*a^4*b^12*sin(e/2 + (f*x)/2)^2 + 625176*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^5*a^5*b^11*sin(e/2 + (f*x)/2)^2 + 6126696*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^5*a^6*b^10*sin(e/2 + (f*x)/2)^2 - 2480004*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^7*b^9*sin(e/2 + (f*x)/2)^2 - 15505344*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^5*a^8*b^8*sin(e/2 + (f*x)/2)^2 - 346572*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^9*b^7*sin(e/2 + (f*x)/2)^2 + 9474120*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^10*b^6*sin(e/2 + (f*x)/2)^2 + 24660*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^5*a^11*b^5*sin(e/2 + (f*x)/2)^2 - 1571688*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^12*b^4*sin(e/2 + (f*x)/2)^2 + 232740*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^5*a^13*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^5*a^14*b^2*sin(e/2 + (f*x)/2)^2 + 3456*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^3*b^14*sin(e/2 + (f*x)/2)^2 - 1728*root(54*a^5*b^2*z^3 -
 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^6*a^4*b^13*sin(e/2 + (f*x)/2)^2 - 319896*root(54*a^5*b^2*z^3 -
 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
- b^4, z, k)^6*a^5*b^12*sin(e/2 + (f*x)/2)^2 + 3246912*root(54*a^5*b^2*z^3
- 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z
 - b^4, z, k)^6*a^6*b^11*sin(e/2 + (f*x)/2)^2 - 5322240*root(54*a^5*b^2*z^3
 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*
z - b^4, z, k)^6*a^7*b^10*sin(e/2 + (f*x)/2)^2 - 9560160*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^6*a^8*b^9*sin(e/2 + (f*x)/2)^2 + 16055280*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^6*a^9*b^8*sin(e/2 + (f*x)/2)^2 + 8485344*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^6*a^10*b^7*sin(e/2 + (f*x)/2)^2 - 14873760*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^11*b^6*sin(e/2 + (f*x)/2)^2 - 1269216*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^6*a^12*b^5*sin(e/2 + (f*x)/2)^2 + 4449384*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^13*b^4*sin(e/2 + (f*x)/2)^2 - 901152*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^6*a^14*b^3*sin(e/2 + (f*x)/2)^2 + 7776*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^6*a^15*b^2*sin(e/2 + (f*x)/2)^2 + 2592*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^7*a^4*b^14*sin(e/2 + (f*x)/2)^2 - 58320*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^7*a^5*b^13*sin(e/2 + (f*x)/2)^2 + 603936*root(54*a^5*b^2*z^
3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4
*z - b^4, z, k)^7*a^6*b^12*sin(e/2 + (f*x)/2)^2 - 2230416*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^
4*z - b^4, z, k)^7*a^7*b^11*sin(e/2 + (f*x)/2)^2 + 536544*root(54*a^5*b^2*z
^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^

```
4*z - b^4, z, k)^7*a^8*b^10*sin(e/2 + (f*x)/2)^2 + 6518880*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^7*a^9*b^9*sin(e/2 + (f*x)/2)^2 - 5251392*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)^7*a^10*b^8*sin(e/2 + (f*x)/2)^2 - 5590944*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^7*a^11*b^7*sin(e/2 + (f*x)/2)^2 + 6456672*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^7*a^12*b^6*sin(e/2 + (f*x)/2)^2 + 838512*root(54*a^5*b^
2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a
*b^4*z - b^4, z, k)^7*a^13*b^5*sin(e/2 + (f*x)/2)^2 - 2340576*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^7*a^14*b^4*sin(e/2 + (f*x)/2)^2 + 522288*root(54*a^5*b
^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*
a*b^4*z - b^4, z, k)^7*a^15*b^3*sin(e/2 + (f*x)/2)^2 - 7776*root(54*a^5*b^2
*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*
b^4*z - b^4, z, k)^7*a^16*b^2*sin(e/2 + (f*x)/2)^2 + 17192*root(54*a^5*b^2*
z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b
^4*z - b^4, z, k)*a*b^11*cos(e/2 + (f*x)/2)^2))/(cos(e/2 + (f*x)/2)^2*(a +
b)^3*(a^2 - 2*a*b + b^2)))*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^
3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k), k, 1, 3))/(8*
a*f*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2*(a + b)^2*(a - b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

3.462 $\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=30

$$\text{Int}\left((d\tan(e+fx))^m \left(a+b(c\sec(e+fx))^n\right)^p, x\right)$$

[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]

[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx = \int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$$

Mathematica [A] time = 3.19, size = 0, normalized size = 0.00

$$\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]

[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")

[Out] integral(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="giac")

[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)

maple [A] time = 6.12, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(d\tan\left(e+f\,x\right)\right)^{m}\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*(d*tan(f*x+e))**m,x)
[Out] Timed out
```

3.463 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^5(e+fx)\,dx$

Optimal. Leaf size=226

$$\frac{\sec^4(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{n},-p;\frac{n+4}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{4f}-\frac{\sec^2(e+fx)\left(a+b\right.}{}$$

```
[Out] -hypergeom([1, 1+p],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(1
+p)/a/f/n/(1+p)-hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*
x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)+1/4*hypergeo
m([-p, 4/n],[(4+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*x+e)^4*(a+b*(c*sec(f*x+e
))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

Rubi [A] time = 0.52, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4139, 6742, 367, 12, 266, 65, 365, 364}

$$\frac{\sec^4(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{n},-p;\frac{n+4}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{4f}-\frac{\sec^2(e+fx)\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p))) - (Hypergeometric2F1[2/n, -p,
 (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f
*x])^n)^p)/(f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p) + (Hypergeometric2F1[4/n, -
p, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b(cx)^n)^p}{x} - 2x\left(a+b(cx)^n\right)^p + x^3\left(a+b(cx)^n\right)^p\right)dx, x, \text{se}\right.}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^3\left(a+b(cx)^n\right)^p\right.}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x^3(a+bx^n)^p}{c^3}\,dx, x, c\right.}{cf}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^3\left(a+bx^n\right)^p\,dx,\right.}{c^4 f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} + \frac{\left(\left(a+b(c\sec(e+fx))^n\right)^p\right.}{}$$

$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)\left(a+b(c\sec(e+fx))^n\right)^{1+p}}{afn(1+p)} -$$

Mathematica [A] time = 10.12, size = 245, normalized size = 1.08

$$\frac{\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}+1\right)^{-p}\left(-4\left(a+b\left(c\sqrt{\sec^2(e+fx)}\right)^n\right)\left(\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}+1\right)^p {}_2F_1\left(1,p+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*(-4*a*n*(1 + p)*Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*Sec[e + f*x]^2 + a*n*(1 + p)*
Hypergeometric2F1[4/n, -p, (4 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*
Sec[e + f*x]^4 - 4*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sqrt[Sec[e
+ f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n)*(1 + (b*(c*Sqrt[Sec[e +
 f*x]^2])^n)/a)^p))/(4*a*f*n*(1 + p)*(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)
^p)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

maple [F] time = 1.92, size = 0, normalized size = 0.00

$$\int\left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(\tan^{5}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\tan\left(e+f\,x\right)^{5}\,{\left(a+b\,{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^{n}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p,x)
```

```
[Out] int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.464 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^3(e+fx)\,dx$

Optimal. Leaf size=143

$$\frac{\sec^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{2f}+\frac{\left(a+b(c\sec(e+fx\right.}{}$$

```
[Out] hypergeom([1, 1+p],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(1+
p)/a/f/n/(1+p)+1/2*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec
(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

Rubi [A] time = 0.29, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4139, 6742, 367, 12, 266, 65, 365, 364}

$$\frac{\sec^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{2f}+\frac{\left(a+b(c\sec(e+fx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*(c
*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*x]
)^n)^p)/(2*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b(cx)^n)^p}{x} + x\,(a+b(cx)^n)^p\right)dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x\,(a+b(cx)^n)^p\right.}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x(a+bx^n)^p}{c}\,dx, x,\right.}{cf}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x\,(a+bx^n)^p\,dx\right.}{c^2 f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} + \frac{\left(\left(a+b(c\sec(e+fx))^n\right.\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)\left(a+b(c\sec(e+fx))^n\right)^{1+p}}{afn(1+p)} + \cdots$$

Mathematica [A] time = 4.59, size = 162, normalized size = 1.13

$$\frac{\left(a+b(c\sec(e+fx))^n\right)^p\left(\sec^2(e+fx)\left(\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)+\frac{2\left(a+b\left(c\sqrt{\sec^2(e+f\cdots\right.\right.}{\cdots}\right.}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*(c*Sqrt[Sec[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n))/(a*n*(1
 + p)) + (Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2
])^n)/a)]*Sec[e + f*x]^2)/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)^p))/(2*f)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int\left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(\tan^{3}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(e+f\,x\right)^{3}\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
```

```
[Out] int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p}\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**3,x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

3.465 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan(e+fx)\,dx$

Optimal. Leaf size=59

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

```
[Out] -hypergeom([1, 1+p],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(1
+p)/a/f/n/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4139, 367, 12, 266, 65}

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)\left(a+b(c\sec(e+fx))^n\right)^{1+p}}{afn(1+p)}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.00

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \tan\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^n\right)^p \tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)\,{\left(a+b\,{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^{n}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p}\tan\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e),x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x), x)
```

3.466 $\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 3.77, size = 0, normalized size = 0.00

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

maple [A] time = 1.82, size = 0, normalized size = 0.00

$$\int \cot\left(fx+e\right)\left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot\left(e+f\,x\right)\,{\left(a+b\,{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p}\cot\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x), x)
```

3.467 $\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 36.20, size = 0, normalized size = 0.00

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

maple [A] time = 2.13, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(fx+e\right)\right)\left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot\left(e+f\,x\right)^{3}\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.468 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\tan^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx=\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$$

Mathematica [A] time = 2.44, size = 0, normalized size = 0.00

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

maple [A] time = 1.64, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(\tan^{2}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(e+f\,x\right)^{2}\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**2,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

3.469 $\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 1.23, size = 0, normalized size = 0.00

$$\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

maple [A] time = 1.70, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n} b+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/cos(e + f*x))^n)^p,x)
```

```
[Out] int((a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p, x)
```

3.470 $\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 1.85, size = 0, normalized size = 0.00

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

maple [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(fx+e\right)\right)\left(a+b\left(c\sec\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\sec\left(fx+e\right)\right)^n b+a\right)^p \cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot\left(e+f\,x\right)^2\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^n\right)^p \cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```